Real-time Assessment of Allan Deviation and Time Deviation

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***Abstract*— In this paper the methods enabling real-time Allan deviation (ADEV) and time deviation (TDEV) are presented. The idea of real-time quasi-parallel computation of ADEV and TDEV is described first. The results of experimental tests of the methods proposed for different conditions (numbers and lengths of observation intervals simultaneously analyzed) are presented and discussed.**

I. INTRODUCTION

Allan deviation ADEV and time deviation TDEV are commonly used for describing the quality of synchronization signal in the telecommunication network [1, 2, 3]. The parameters allow the variations of time interval provided by the synchronization signal to be assessed and the type of phase noise affecting the signal to be recognized. The estimates of the parameters are computed for a series of observation intervals using the sequence of time error samples previously measured at some network interface. The evaluation of the synchronization signal is commonly a two-stage process. First, the sequence of time error samples between the analyzed signal and some reference has to be measured. Then, when the measurement is completed, the calculation of the parameter's estimate is performed. Such way causes an obvious delay of the evaluation process.

In this paper the real-time methods of ADEV and TDEV computation, which enable to reduce the evaluation time, are proposed. These methods allow to compute the estimates of ADEV and TDEV (which characterizes of more complex estimator's formula) in the real time, during the measurement process, simultaneously for a set of observation intervals. As result, the values of the parameter considered are known immediately after the measurement is finished. Additionally, the current value of the parameter considered can be observed during the measurement.

In order to calculate the ADEV or TDEV estimate simultaneously for several observation intervals in the real time, all necessary operations should be performed in the time period between two sampling instants, i.e. during the sampling interval τ_0. The ability of the real-time assessment depends on the several conditions: computation ability of the measurement equipment, sampling interval and number of the observation intervals considered.

The methods described in the paper are developed for a measurement system, where the time error counter and the computer controlling the measurement are two separate units. Therefore the computer may be changed depending on the computing requirements. In the paper the results of experimental tests of the methods proposed for different conditions are presented. The calculations were performed for the time error sequences taken with sampling interval τ_0=1/30 s, which is often used in the telecommunication applications. Different numbers and lengths of observation intervals simultaneously analyzed were considered. The results of calculation using personal computers with different processors and clock's frequency are presented and compared.

II. ALLAN DEVIATION AND TIME DEVIATION

Allan deviation and time deviation are computed basing on averaging of second differences of the phase process $x(t)$ of the analyzed timing signal. We can assume for the telecommunication applications, in the case of negligible influence of frequency drift, that ADEV and TDEV are estimated basing on the time error function measured between the analyzed timing signal and the reference one.

The formulae for the estimators of the Allan deviation ADEV and the time deviation TDEV take the form:

$$A\hat{D}EV(\tau) = \sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}\left(x_{i+2n} - 2x_{i+n} + x_i\right)^2} \quad (1)$$

$$T\hat{D}EV(\tau) = \sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{j+n-1}\left(x_{i+2n} - 2x_{i+n} + x_i\right)\right]^2} \quad (2)$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with interval τ_0; $\tau=n\tau_0$ is an observation interval. For TDEV computation the estimator formula (2) is changed in order to simplify the summing [4, 5] and takes the form:

$$T\hat{D}EV(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{N-3n+1}\cdot\frac{1}{n^2}\sum_{j=1}^{N-3n+1}S_j^2(n)} \quad (3)$$

where

$$S_j(n)=S_{j-1}(n)-x_{j-1}+3x_{j+n-1}-3x_{j+2n-1}+x_{j+3n-1} \quad (4)$$

$$S_1(n)=\sum_{i=1}^{n}(x_{i+2n}-2x_{i+n}+x_i) \quad (5)$$

III. Real-time computation scheme

The formulae of ADEV and TDEV estimators allow to perform the calculation of the parameter in the real-time, during the time error measurement. A general procedure of the real-time quasi-parallel ADEV or TDEV calculation for a series of observation intervals is as follows:

1. Measure a new time error sample and store it in data file.
2. Compute the relevant difference for a given n (observation interval $\tau=n\tau_0$) using current sample, and the samples measured n, $2n$ or $3n$ sampling intervals earlier.
3. Update the sum of squares,
4. Compute current average and its square root.
5. Repeat the steps 2-4 for next observation intervals (greater n).
6. Execute step 1 (measure a new sample).
7. When the measurement is finished, the values of the parameter's estimate for the observation intervals considered are known.

The steps 2-5 can be performed, when sufficient number of samples were measured, i.e. $2n+1$ samples for a given n.

The indexes in formulae for ADEV and TDEV estimators must be changed in the case of the real-time calculation. We haven't access to the time error samples indexed by $i+n$ or $i+2n$ for a time instant described by index i (currently measured sample). Therefore the formula for the estimator of Allan deviation, for a current index i of measured sample and observation interval $\tau=n\tau_0$, takes the form

$$A\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{2n^2\tau_0^2(i-2n)}\sum_{j=2n+1}^{i}(x_j-2x_{j-n}+x_{j-2n})^2} \quad (6)$$

Presenting the sum of squares of second differences in the form

$$S_i(n)=\sum_{j=2n+1}^{i}(x_j-2x_{j-n}+x_{j-2n})^2 \quad (7)$$

we can write the equation (6) in the form

$$A\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{2n^2\tau_0^2(i-2n)}S_i(n)} \quad (8)$$

Taking into consideration the sum of squares computed for a preceding sample, the operations of ADEV computation for i-th sampling interval are performed using the formula:

$$A\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{2n^2\tau_0^2(i-2n)}\left(S_{i-1}(n)+(x_i-2x_{i-n}+x_{i-2n})^2\right)} \quad (9)$$

The example of real-time ADEV computation for the three observation intervals – $3\tau_0$, $5\tau_0$, $7\tau_0$ – is presented in Fig. 1 and 2. The early stage of the process is presented in Fig. 1. Seven time error samples were measured until now and the window related to observation interval $3\tau_0$ is active. The first item of the sum $S(3)$ is computed using samples number 7, 4, and 1. Other windows – related with observation intervals $5\tau_0$ and $7\tau_0$ are not active yet, because of unsufficient number of samples currently measured. The middle stage of the measurement and computation process is presented in Fig. 2. Fifteen samples were measured until now. All the widows considered are active. The sums $S(3)$ and $S(5)$ are updated and the first item of the sum $S(7)$ is computed.

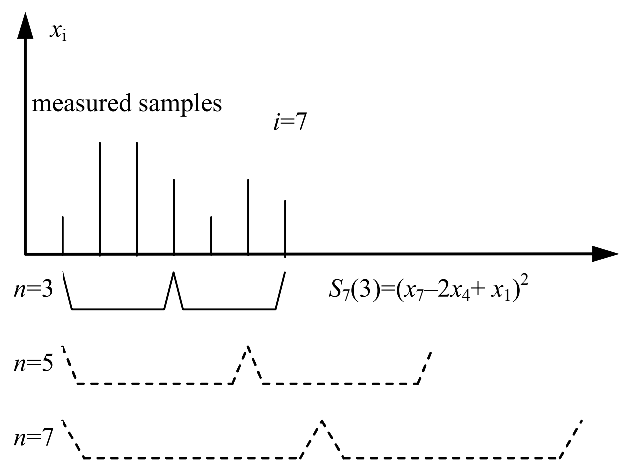

Figure 1. Real-time of ADEV calculation for observation intervals $3\tau_0$, $5\tau_0$, $7\tau_0$, sample number 7 is measured, computations for intervals $5\tau_0$ and $7\tau_0$ are not active yet.

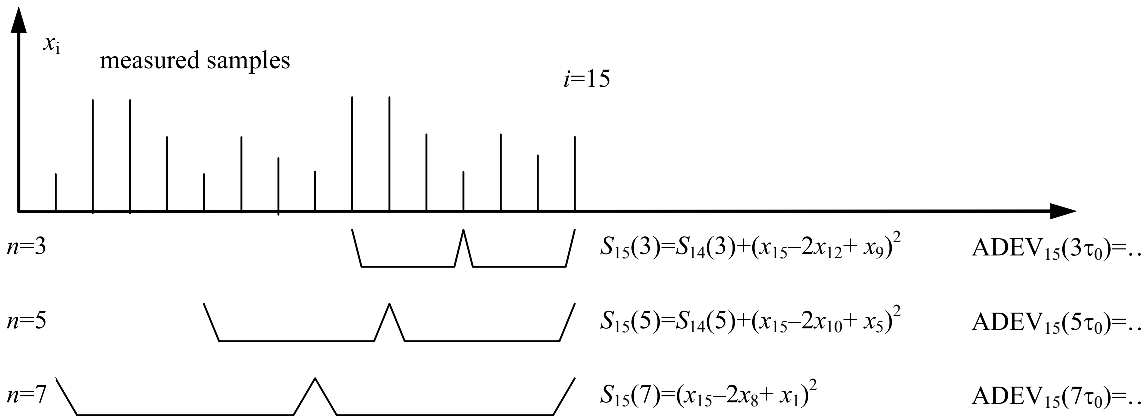

Figure 2. Real-time of ADEV calculation for observation intervals $3\tau_0$, $5\tau_0$, $7\tau_0$, sample number 15 is measured, computations for all intervals are active now.

The computation of time deviation is little more complex than Allan deviation. For the real-time computation the simplified formula (3-5) of the estimator (2) is applied. After change of the indexes, as for ADEV estimator, we have obtained:

$$T\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}S_{ov,i}(n)} \tag{10}$$

where $S_{ov,i}(n)$ is the overall sum updated for each sample i given in the form:

$$S_{ov,i}(n)=S_{ov,i-1}(n)+S_i^2(n) \tag{11}$$

where

$$S_i(n)=S_{i-1}(n)-x_{i-3n}+3x_{i+2n}-3x_{i+n}+x_i, i>3n \tag{12}$$

and

$$S_{3n}(n)=\sum_{j=2n+1}^{3n}\left(x_j-2x_{j-n}+x_{j-2n}\right) \tag{13}$$

Finally, the operations of TDEV computation for i-th sampling interval are performed using the formula (14).

$$T\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}\left[S_{ov,i-1}(n)+\left(S_{i-1}(n)-x_{i-3n}+3x_{i-2n}-3x_{i-n}+x_i\right)^2\right]} \tag{14}$$

The computation of TDEV for the first observation interval $\tau=n\tau_0$ begins, when the first $2n+1$ samples are measured – for this instant the first item of internal sum $S_{3n}(n)$ can be computed. The sum $S_{3n}(n)$ is updated until the sample number $3n$ is measured. Starting from this instant, the sum $S_i(n)$ is updated using the samples number i-$3n$, i-$2n$, i-n and i, according to (12), and the overall sum $S_{ov,i}(n)$ is updated according to (11). When the updating for a given n is finished, the conditions for the next (greater) observation intervals are checked, and necessary operations for the intervals are performed.

The example of real-time TDEV computation for the three observation intervals – $3\tau_0$, $5\tau_0$, $7\tau_0$ – is presented in Fig. 3-5. The early stage of the process is presented in Fig. 3. Seven time error samples were measured until now and the window related to observation interval $3\tau_0$ is active. The first item of the internal sum $S_{3n}(3)$ is computed using samples number 7, 4, and 1. Other windows – window related with overal sum of observation interval $3\tau_0$ and windows related with observation intervals $5\tau_0$ and $7\tau_0$ are not active yet, because of unsufficient total number of samples currently measured. The middle stage of the measurement and computation process is presented in Fig. 4. Ten samples were measured until now. The internal sum $S_{3n}(3)$ is computed (window is not active now) and the overal sum $S_{ov,i}(3)$ is updated using the samples number 1, 4, 7, and 10. The computations for greater observation intervals are still not active. The stage of the process after measurement of the sample number 16 is presented in Fig. 5. The overall sum $S_{ov,i}(3)$ is updated using the samples number 10, 13, and16. The internal sum $S_1(5)$ is computed and the overal sum $S_{ov,i}(5)$ is updated using the samples number 1, 6, 11, and 16. The internal sum for observation interval $7\tau_0$ is updated.

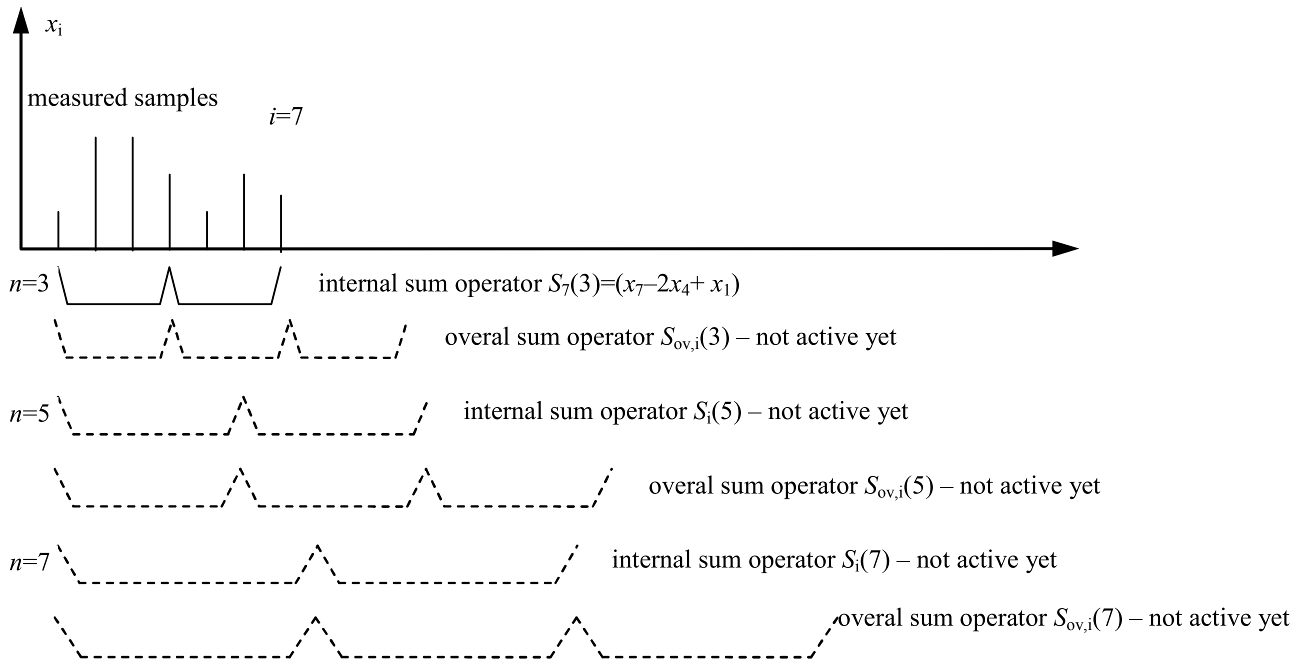

Figure 3. Real-time of TDEV calculation for observation intervals $3\tau_0$, $5\tau_0$, $7\tau_0$, sample number 7 is measured, first item of internal sum $S(3)$ is computed, computations for intervals $5\tau_0$ and $7\tau_0$ are not active yet.

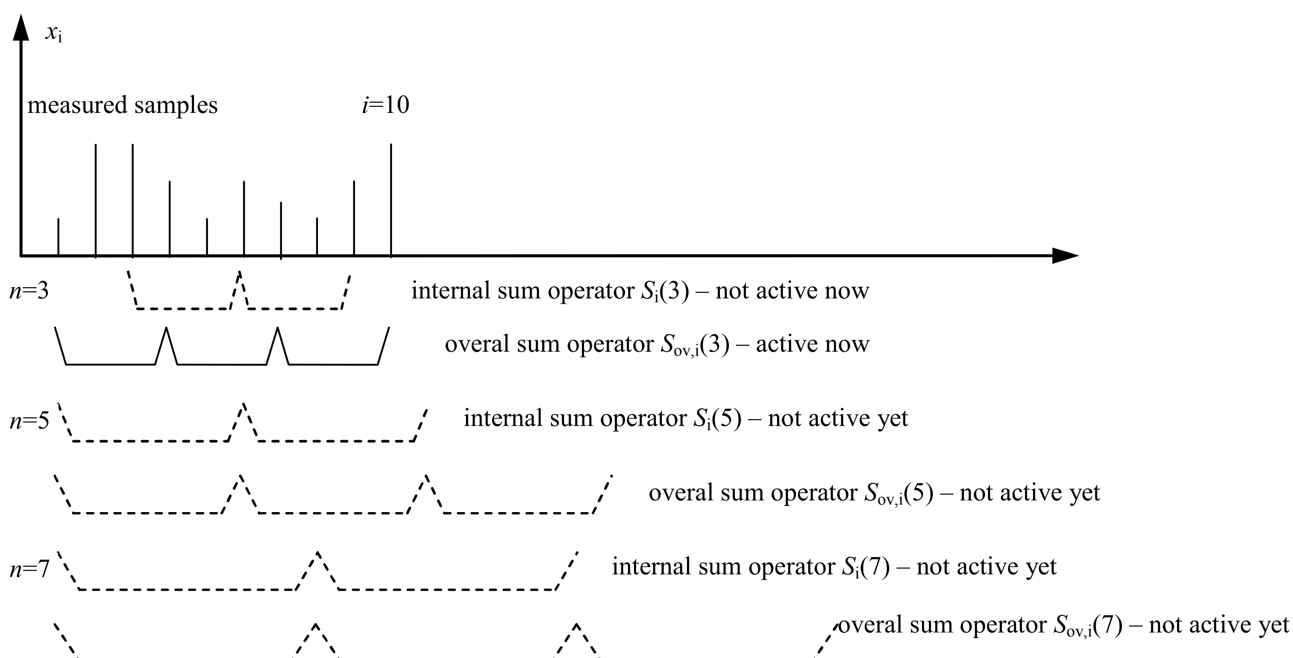

Figure 4. Real-time of TDEV calculation for observation intervals $3\tau_0$, $5\tau_0$, $7\tau_0$, sample number 10 is measured, the internal sum $S(3)$ is computed, the overal sum $S_{ov,i}(3)$ is updated, computations for intervals $5\tau_0$ and $7\tau_0$ are not active yet.

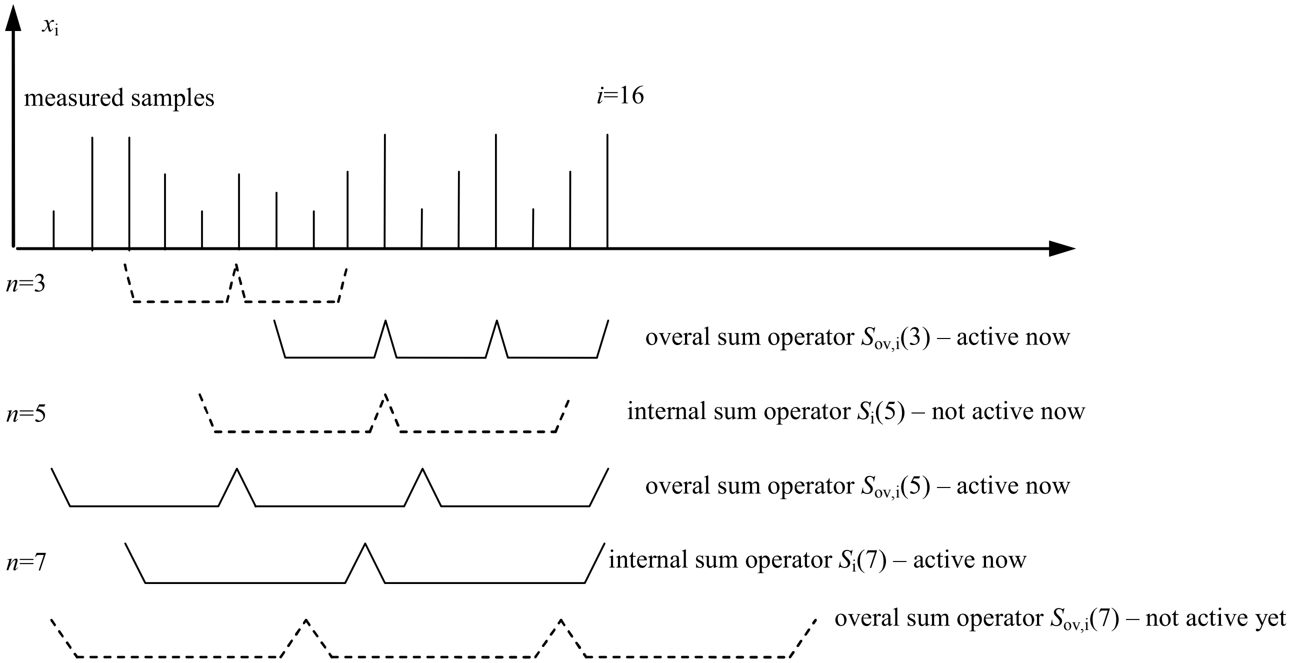

Figure 5. Real-time of TDEV calculation for observation intervals $3\tau_0$, $5\tau_0$, $7\tau_0$, sample number 16 is measured, the overal sum $S_{ov,i}(3)$ is updated, the internal sum $S(5)$ is computed, the overal sum $S_{ov,i}(5)$ is updated, the internal sum $S(7)$ is updated, the overal sum $S_{ov,i}(7)$ is not active yet.

On-line computation of TDEV is more complex than ADEV computation especially for the early stages of the measurement and computation process, when the internal and overall sums are computed and the computations for some greater observation intervals are not active yet (the conditions of beginning the computations must be checked for each step). In general, for ADEV real-time computation, three samples are engaged for a given observation interval (given n): one sample currently measured, and two samples from the past – measured n and $2n$ sampling intervals earlier. For TDEV real-time computation, four samples are engaged, except of the early stages mentioned above, when the internal sum is updated. The computation complexity does not depend on the observation interval value (defined by n) for both cases. It depends on the number of observation intervals simultaneously considered.

IV. Computation experiment

The methods of real-time ADEV and TDEV computation described above were tested in the calculation experiment, arranged similarly as the real-time MTIE tests [6]. The calculations were performed off-line but the on-line work was imitated. The data sequence contains time error samples taken with the sampling interval τ_0=1/30 s, representing white phase noise.

The calculations were performed for changing numbers of observation intervals arranged in the logarithmic scale in a range between 0.1 s and 1000 s. The starting (smallest) observation interval was τ_{min}=0.1 s (n=3). The longest observation interval was changed from 1 s till 1000 s. The calculations were performed for 5, 10, and 20 observation intervals per decade for each range.

Two personal computers with Pentium IV 1.4 GHz and Pentium IV 3.0 GHz were used in the experimental tests. The observed quantity was the maximum time spent for calculation within one sampling interval. We have assumed that this time cannot exceed the length of sampling interval τ_0=1/30 s = 0.0333… s.

The time of ADEV computation is presented in Table I (for Pentium IV 1.4 GHz) and Table II (for Pentium IV 3.0 GHz).

TABLE I. Time of ADEV calculation for computer with Pentium IV 1.4 GHz

Range of intervals [s]	Number of intervals per decade		
	5	10	20
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.0002	0.0003	0.0006
0.1-10	0.00035	0.0006	0.0012
0.1-100	0.0005	0.0009	0.0018
0.1-1000	0.0007	0.0012	0.0024

TABLE II. Time of ADEV calculation for computer with Pentium IV 3.0 GHz

Range of intervals [s]	Number of intervals per decade		
	5	10	20
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.00012	0.00025	0.0005
0.1-10	0.00024	0.00050	0.0010
0.1-100	0.00034	0.00078	0.0015
0.1-1000	0.00055	0.00110	0.0020

The time of TDEV computation is presented in Table III (for Pentium IV 1.4 GHz) and Table IV (for Pentium IV 3.0 GHz).

TABLE III. TIME OF TDEV CALCULATION FOR COMPUTER WITH PENTIUM IV 1.4 GHZ

Range of intervals [s]	**Number of intervals per decade**		
	5	**10**	**20**
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.00022	0.0004	0.0008
0.1-10	0.0004	0.0008	0.0016
0.1-100	0.0006	0.0012	0.0024
0.1-1000	0.0008	0.0016	0.0031

TABLE IV. TIME OF TDEV CALCULATION FOR COMPUTER WITH PENTIUM IV 3.0 GHZ

Range of intervals [s]	**Number of intervals per decade**		
	5	**10**	**20**
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.00018	0.0003	0.0006
0.1-10	0.0003	0.0006	0.0012
0.1-100	0.0005	0.0009	0.0018
0.1-1000	0.0007	0.0013	0.0026

The results obtained for both parameters were satisfactory for all cases considered. Even the most time consuming case – TDEV computation for the range of observation intervals from 0.1 s till 1000 s and 20 observation intervals for decade, using the computer with 1.4 GHz clock, brought good result. The maximum time of operations performed for one sampling interval does not exceed the sampling interval 1/30 s. The computation complexity does not depend on the length of observation interval, only the number of observation intervals considered is the limiting factor. Therefore, having limited computational abilities, we can exchange between the wide range of observation intervals and their number for one decade (resolution of the computation results on the scale of observation intervals).

V. CONCLUSIONS

The results of the experimental tests have proved the ability of the real-time computation of Allan deviation as well as time deviation. The computation can be performed simultaneously for numerous series of observation intervals (up to 80 simultaneously analyzed observation intervals were tested). The rearrangement of TDEV computation and rather small maximum time spent for computation within one sampling interval allow to consider joint computation of ADEV and TDEV performed in the real time.

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