Real-Time Assessment of Dynamic Allan Deviation and Dynamic Time Deviation

Andrzej Dobrogowski, Michal Kasznia
Chair of Telecommunication Systems and Optoelectronics
Poznan University of Technology
Poznan, Poland
dobrog@et.put.poznan.pl, mkasznia@et.put.poznan.pl

Abstract— **In this paper the methods enabling real-time assessment of dynamic Allan deviation and dynamic time deviation are presented. The idea of real-time analysis of timing signals using dynamic approach is described first. Then the algorithms of real-time computation of dynamic parameters are presented. The results of experimental tests of the methods proposed for different conditions are presented and discussed.**

I. Introduction

Allan deviation ADEV and time deviation TDEV are commonly used for describing the quality of synchronization signals in the telecommunication network [1, 2, 3]. The parameters allow the variations of time interval provided by the synchronization signal to be assessed and the type of phase noise affecting the signal to be recognized. The characterization of the timing signal using dynamic parameter (dynamic Allan deviation, DADEV) [4, 5] allows to recognize the variations of the phase noise affecting the analyzed signal. This approach was extended by the authors of this paper. The dynamic time deviation (DTDEV) was introduced and the time effective methods of calculation of the dynamic parameters were proposed [6, 7].

The estimates of the parameters are computed for a series of observation intervals using the sequence of time error samples previously measured at some network interface. The evaluation of the synchronization signal is commonly a two-stage process. The measurement of the sequence of time error samples is followed then by the calculation of the parameter's estimate. Application of the methods of real-time computation enables to simplify the evaluation process [10, 11]. Real-time tracking of the parameter value of the signal delivers timely information about behavior of the signal source and may result in a suitable activity, for example some reaction of the network maintenance team (as network reconfiguration could be the cause of the parameter variations).

In the paper the methods of the real-time computation of the dynamic parameters are proposed. These methods allow to compute the estimates of dynamic Allan deviation and dynamic time deviation (which characterizes of more complex estimator's formula) in the real time, during the time error measurement process, simultaneously for a set of observation intervals.

The computation of the dynamic parameters requires a specific arrangement of data. We can consider overlapping and non-overlapping segments of data used for calculation in order to obtain a set of curves presented in the form of 3D plot. The arrangement of the segments depends on the number of segments, their length and the length of the whole data sequence. This arrangement (overlapping or non-overlapping segments) influences the method of data organization applied for the real-time computation.

In the paper the results of experimental tests of the methods proposed for different conditions are presented. Different arrangements of the data segments simultaneously analyzed were considered.

II. Idea of Dynamic Parameters

The idea of dynamic parameters is quite simple. Instead of one curve representing the values of ADEV or TDEV as a function of observation interval τ, the set of the curves plotted in the form of three-dimensional graph as a function of observation interval τ and time t is considered. As a result we can recognize the changes of the type of phase noise affecting analyzed timing signal. The changes of the slope of the parameter's curve indicate the changes of the noise type [4, 5].

In order to obtain such form of graph, the following computation procedure must be performed. First, the data sequence (time series of time error measured at some network interface) is divided into equal segments (slices) with the length of T_s. We can consider overlapping data segments (Fig. 1) and non-overlapping data segments (Fig. 2). Then the calculation of the parameter's value for a required range of observation intervals for each data segment is done. The results of calculation are plotted in a form of three-dimensional graph (Fig. 3).

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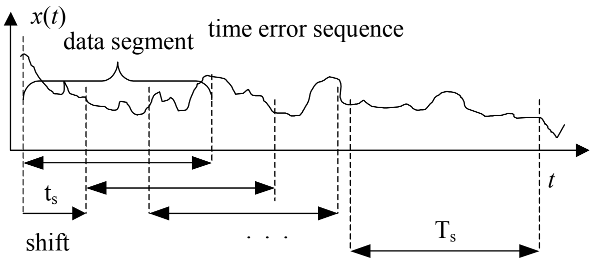

Figure 1. Overlapping segments of time error sequence

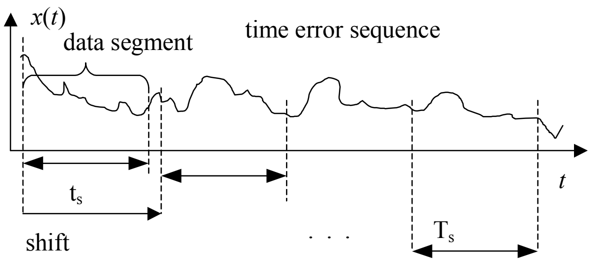

Figure 2. Non-overlapping segments of time error sequence

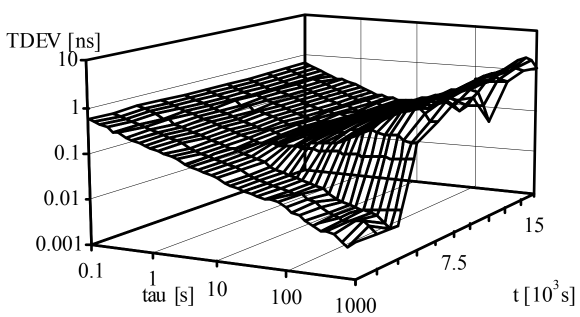

Figure 3. Example plot of dynamic time deviation

The procedure of dynamic ADEV and dynamic TDEV calculation depends on the set of quantities. These are: length of the whole data series T, length of the data segments created within the whole data series T_s, time shift between the initial points of the data segments t_s, and range of observation intervals ($\tau_{min} - \tau_{max}$).

The relation between the whole data series length, segments' lengths and the time shift t_s determines the number of segments and the arrangement of the segments (overlapping or non-overlapping). We must also take into consideration the relation between T_s and the maximum observation interval τ_{max}. In practice, the value of the parameter's estimate can be calculated, when the length of data series (in this situation: segment's length) is two times longer for ADEV and three times longer for TDEV. According to the telecommunication standards and recommendations, the length of data sequence used for ADEV or TDEV calculation must be 12 to 15 times longer than the maximum observation interval τ_{max} [1, 2, 3]. The minimum observation interval τ_{min} is determined by the sampling interval τ_0: τ_{min} must be three times longer than τ_0.

III. Real-Time Computation of ADEV and TDEV

Allan deviation and time deviation are computed based on the averaging of second differences of the phase process $x(t)$ of the analyzed timing signal. We can assume for the telecommunication applications, in the case of negligible influence of frequency drift, that ADEV and TDEV are estimated based on the time error function measured between the analyzed timing signal and the reference one [8].

The formulae for the estimators of Allan deviation ADEV and the time deviation TDEV take the form:

$$A\hat{D}EV(\tau) = \sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}\left(x_{i+2n} - 2x_{i+n} + x_i\right)^2} \quad (1)$$

$$T\hat{D}EV(\tau) = \sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{j+n-1}\left(x_{i+2n} - 2x_{i+n} + x_i\right)\right]^2} \quad (2)$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with interval τ_0; $\tau = n\tau_0$ is an observation interval. For TDEV computation the estimator formula (2) is changed in order to simplify the summing [8, 9] and takes the form:

$$T\hat{D}EV(n\tau_0) = \sqrt{\frac{1}{6}\cdot\frac{1}{N-3n+1}\cdot\frac{1}{n^2}\sum_{j=1}^{N-3n+1}S_j^2(n)} \quad (3)$$

where

$$S_j(n) = S_{j-1}(n) - x_{j-1} + 3x_{j+n-1} - 3x_{j+2n-1} + x_{j+3n-1} \quad (4)$$

$$S_1(n) = \sum_{i=1}^{n}\left(x_{i+2n} - 2x_{i+n} + x_i\right) \quad (5)$$

When computing in the real time, we do not have access to the time error samples indexed by $i+n$ or $i+2n$ for the current time instant described by index i, because these samples have not been measured yet. We have access to the sample currently measured (for the current sampling instant i) and the samples measured earlier (with indexes smaller than i) and saved in the memory of the measurement equipment. Therefore, the indexes in formulae for ADEV and TDEV estimators should be changed in the case of real-time calculation.

The rearrangement of indexes for both estimators was performed in [10]. As a result we have obtained the ADEV estimator's formula for a current instant i in the form depending on the sum of squares of second differences computed for the instant $i-1$

$$A\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{2n^2\tau_0^2(i-2n)}\left(A_{i-1}(n) + \left(x_i - 2x_{i-n} + x_{i-2n}\right)^2\right)} \quad (6)$$

were $A_i(n)$ is the sum of squares of second differences of time error samples

$$A_i(n) = \sum_{j=2n+1}^{i}\left(x_j - 2x_{j-n} + x_{j-2n}\right)^2, i > 2n \quad (7)$$

The rearrangement of time deviation estimator is a little more complex than for Allan deviation [10]. After changing the indexes with the use of the simplified formula (3-5), we have obtained:

$$T\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}S_{ov,i}(n)} \quad (8)$$

where $S_{ov,i}(n)$ is the overall sum updated for each sample i, given in the form:

$$S_{ov,i}(n) = S_{ov,i-1}(n) + S_i^2(n) \quad (9)$$

where

$$S_i(n) = S_{i-1}(n) - x_{i-3n} + 3x_{i+2n} - 3x_{i+n} + x_i, i > 3n \quad (10)$$

$$S_{3n}(n) = \sum_{j=2n+1}^{3n}\left(x_j - 2x_{j-n} + x_{j-2n}\right), j > 2n \quad (11)$$

Finally, the operations of TDEV computation for i-th sampling interval are performed using the formula (12) [10]. As a result of the rearrangement of the parameters' formulae, in order to compute ADEV and TDEV for a current sampling instant i and given observation interval $\tau = n\tau_0$, we need the values of appropriate sum $A_{i-1}(n)$, $S_{ov,i-1}(n)$, and $S_{i-1}(n)$, currently measured sample x_i and the samples x_{i-n}, x_{i-2n}, and x_{i-3n} previously measured and stored in the memory.

The formulae of the parameters given in the forms presented above allow us to perform the computation in the real time, during the time error measurement process. The calculation can be performed jointly for both parameters considered, as well as for single parameter [11]. The course of operations is as follows:

1. Measure a new time error sample and store it in a data file.
2. Compute the appropriated differences for a given n (observation interval $\tau = n\tau_0$) using the current sample, and the samples measured n, $2n$, or $3n$ sampling intervals earlier.
3. Update the appropriated sums.
4. Compute current averages and their square roots.
5. Execute Steps 2-4 for successive larger observation intervals (larger n).
6. Return to Step 1 (measure a new sample).
7. When the measurement is finished, the values of the parameter's estimate for the observation intervals considered are known.

The computations of ADEV and TDEV start when the sample no. $2n+1$ has been measured. The first value of ADEV estimate can be computed at this instant. However, for the TDEV the computation of the internal sum $S_i(n)$ only just starts. The first value of TDEV can be computed after the sample no. $3n+1$ has been measured. The example of the joint real-time computation process is presented in Fig. 4 [11].

$$T\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}\left[S_{ov,i-1}(n) + \left(S_{i-1}(n) - x_{i-3n} + 3x_{i-2n} - 3x_{i-n} + x_i\right)^2\right]} \quad (12)$$

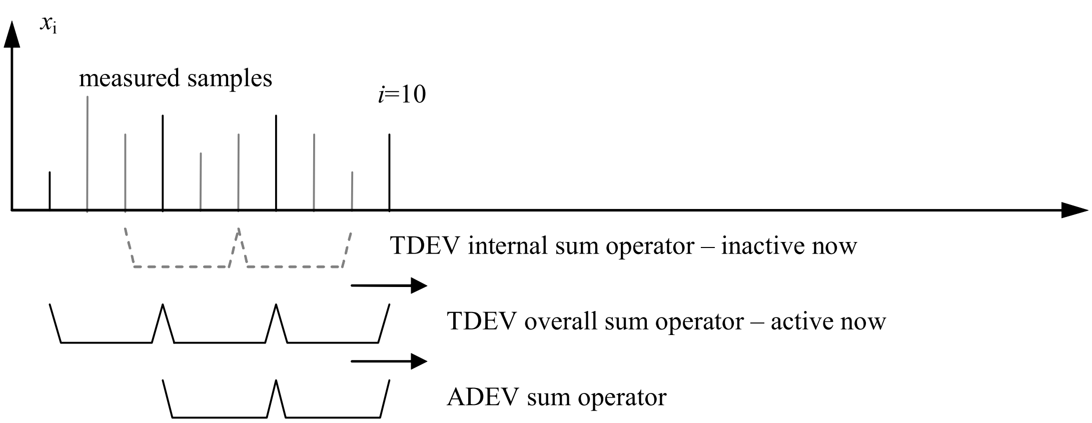

Figure 4. Real-time TDEV and ADEV computation for observation interval $3\tau_0$, sample no. 10 have been measured, ADEV sum operator is active, TDEV internal sum operator is inactive, TDEV overall sum operator is active

IV. Real-Time Computation of Dynamic Parameters

The computation methods presented in the previous section can be useful for the real-time assessment of dynamic parameters. The final result of computation of dynamic ADEV or dynamic TDEV contains the results of computation of parameter performed for particular data segments. In the case of *off-line* computation (performed after the measurement of time error series), we can consider the analysis of each data segment independently regardless of whether we are dealing with overlapping or non-overlapping data segments. When computing in the real time simultaneously for several observation intervals and several data segments, all necessary operations related with the time error sample just measured should be performed in the time period between two sampling instants, i.e. during the sampling interval τ_0. Therefore the computations for non-overlapping and overlapping data sequences will be considered separately.

A. Computation for non-overlapping data segments

We consider non-overlapping data segments when the time shift t_s between the initial points of successive segments is equal to or greater than the length of the segments T_s. In this case each segment can be analyzed independently during the real-time computation process of dynamic parameters. This process will run in the same way as the real-time computation process of usual Allan deviation or time deviation regarding that for each segment a new line of the three-dimensional graph is obtained. Only one set of observation intervals is analyzed for one sampling instant. The scheme of the real-time computation is presented in Fig. 5. The data segments (grey boxes) are successively analyzed. The operators of ADEV and TDEV sums (black and white lines) run along the incrementing data sequence within analyzed data segment (dark grey box).

B. Computation for overlapping data segments

The arrangement with overlapping data segments appears when the time shift t_s between the initial points of successive segments is shorter than the length of the segments T_s. In this case more than one set of observation intervals have to be analyzed for one sampling instant – each set is related with different data segment. The necessary operations performed within one sampling interval can be executed in different ways. First method can be described as "data segment first". The computations are performed for all observation intervals within one data segment. Then the next overlapping data segments are considered independently. The configuration of this order of computation is presented in Fig. 6. Detailed procedure for this method will be as follows:

1. Read TE sample from the TE meter and store it in the data file.
2. Read TE samples measured n, $2n$ or $3n$ sampling intervals earlier from data file.
3. Compute parameters' values according to (6) and (12) for first data segment.
4. Execute Steps 2 and 3 for successive longer observation intervals (greater n).
5. Execute Steps 2-4 for successive overlapping data segments.

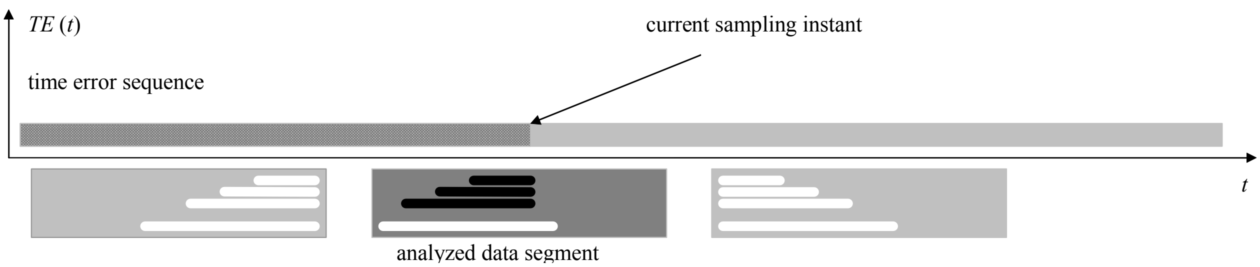

Figure 5. Real-time computation for non-overlapping data segments; dark grey box – analyzed data segment, black and white lines – ADEV and TDEV sum operators (active – black line, inactive – white line)

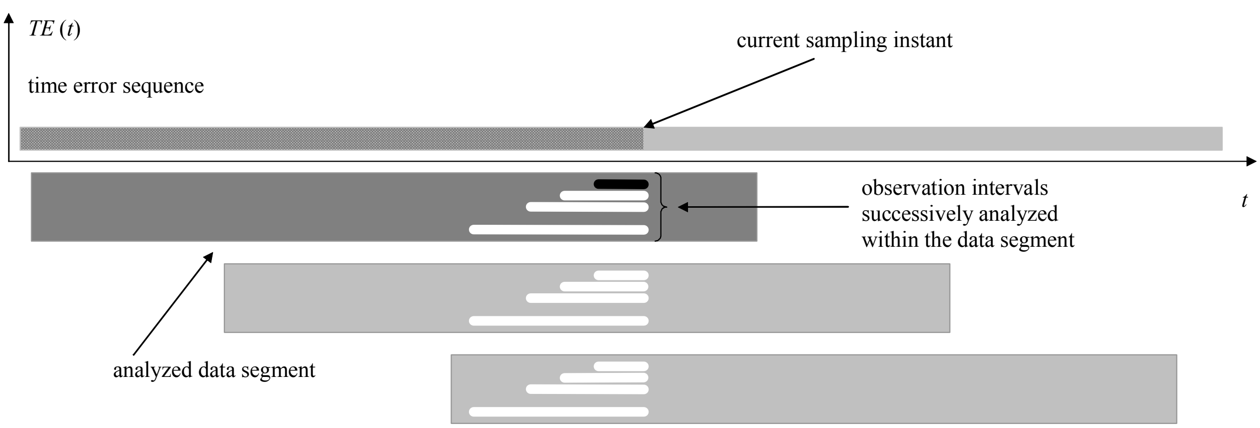

Figure 6. Real-time computation for overlapping data segments according to the rule "segment first"; dark grey box – analyzed data segment, black and white lines – ADEV and TDEV sum operators (analyzed operator – black line, waiting for analysis – white line)

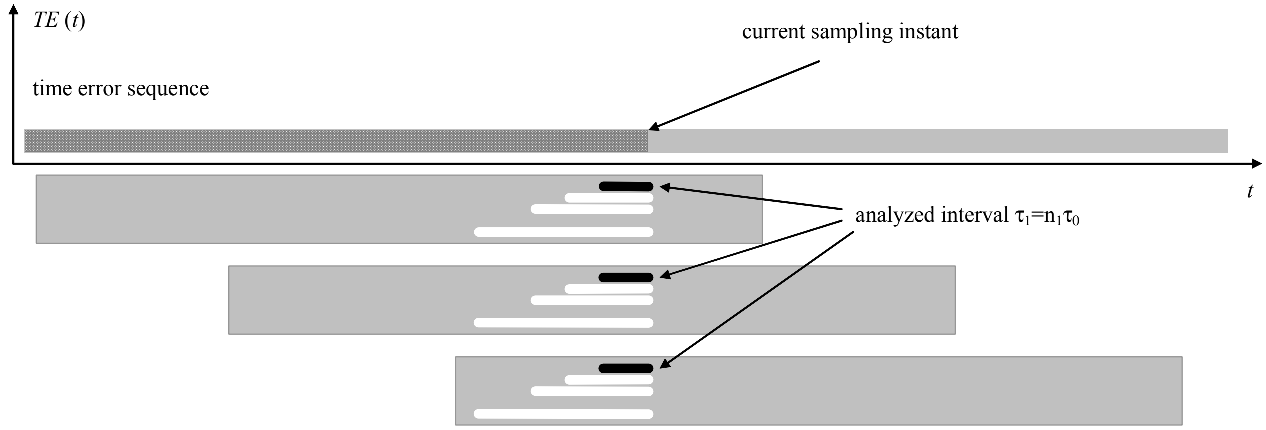

Figure 7. Real-time computation for overlapping data segments according to the rule "observation interval first"; black and white lines – ADEV and TDEV sum operators (analyzed operator – black line, waiting for analysis – white line)

Second method can be described as "observation interval first". The computations for the shortest observation interval τ_{min} for all segments (overlapping for this sampling instant) are performed first. Then successive longer observation intervals (for a greater n) are analyzed. The configuration of this order of computation is presented in Fig. 7. Detailed procedure for this method will be as follows:

1. Read TE sample from the TE meter and store it in the data file
2. Read TE samples measured n, $2n$ or $3n$ sampling intervals earlier from the data file.
3. Compute parameters' values according to (6) and (12) successively for each overlapping data segment for current n.
4. Execute Steps 2 and 3 for successively longer observation intervals (greater n).

The analysis of the observation interval $\tau=n\tau_0$ for a current sampling instant i according to the formula (6) and (12) requires the time error samples just measured and the samples measured n, $2n$, and $3n$ sampling instants earlier and stored in the equipment memory. If a personal computer is used to control of measurement process, the time error samples are stored in the form of data file on the hard drive. The time used for the data access is the critical issue in the real-time computation process. The second method of computation (interval first) gives us an advantage, because it allows to use the same samples (read using one procedure for a given n) for many overlapping data segments. Another solution, that effectively saves the time of data access, is creation of a buffer in the memory in order to store recently measured time error samples. All computational operations according to (6) and (12) will be performed on the samples stored in this buffer, which will reduce the time of data access. The buffer should have the length of $3n_{max}$ (where $\tau_{max}=n_{max}\tau_0$ is the maximum observation interval considered), in order to store all samples necessary to computation.

V. RESULTS OF EXPERIMENT

The methods of real-time computation of dynamic Allan deviation and dynamic time deviation were tested in the experiment. The experiment was realized similarly as the tests of real-time ADEV and TDEV computation methods presented in [10, 11]. The calculations were performed *off-line* with the imitation of *on-line* work. The data sequence contains time error samples taken with the sampling interval τ_0=1/30 s during the time of 20000 s.

The calculations were performed for 41 observation intervals (10 intervals per decade), arranged in the logarithmic scale in a range between τ_{min}=0.1 s (n=3) and τ_{max}=1000 s (n=30000).

Three arrangements of overlapping data segments were considered:

A. 21 segments having the length T_s=4000 s with the shift 800 s (maximum 5 overlapping segments analyzed simultaneously);
B. 21 segments having the length T_s=8000 s with the shift 600 s (maximum 12 overlapping segments analyzed simultaneously);
C. 21 segments having the length T_s=10000 s with the shift 500 s (maximum 20 overlapping segments analyzed simultaneously).

Three personal computers with Intel Pentium IV 3.0 GHz, Intel Core 2 Quad 2.83 GHz, and Intel Core i7 3.2 GHz microprocessors were used in the experimental tests. The maximum time used for calculation within one sampling interval was the observed quantity. We have assumed that this time cannot exceed the length of sampling interval τ_0=1/30 s = 33.3… ms.

The computations were performed using the rules "segment first" and "observation interval first", as well as the method with buffering of time error samples. The results of experimental tests are presented in the Table I. The best results (the shortest computation time) were obtained using the method with buffering of the time error samples. The method "observation interval first" have proved their advantage over the method "segment first". However, the time results were satisfactory for all cases considered. The maximum time spent for computation within one sampling interval does not exceed the length of considered sampling interval 1/30 s.

TABLE I. MAXIMUM TIME OF COMPUTATION FOR ONE SAMPLING INSTANT (IN MILLISECONDS)

Segments arrangement	**Type of computer**								
	Pentium IV			**Core2 Quad**			**Core I7**		
	segment first	***interval first***	***buffering***	***segment first***	***interval first***	***buffering***	***segment first***	***interval first***	***buffering***
A	2.39	0.65	0.06	0.99	0.28	0.04	0.96	0.27	0.03
B	6.83	0.70	0.11	2.79	0.31	0.07	2.77	0.30	0.06
C	10.06	0.72	0.15	4.17	0.32	0.09	4.14	0.31	0.07

The time results obtained in the experiment show that there is some reserve of computational power of the tested equipment. One can expect good results (maximum time not exceeding the length of sampling interval) for wider range and greater number of observation intervals, as well as for greater number of overlapping data segments than the quantities considered in the experiment.

VI. CONCLUSIONS

The results of the experimental tests have proved the ability of joint real-time computation of the dynamic Allan deviation and dynamic time deviation.

The real-time computation of the dynamic parameters can be performed jointly simultaneously for numerous series and wide range of observation intervals for different arrangements of data segments: for non-overlapping as well as for overlapping data segments.

The methods of real-time computation of the dynamic Allan deviation and dynamic time deviation can be very useful tools for analysis of the behavior of synchronization signals.

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