

# Smart Residential Energy Scheduling Utilizing Two Stage Mixed Integer Linear Programming

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**Abstract**—In this paper, we design and evaluate the feasibility of a system which minimizes residential electricity cost of individual homes by shifting demand over a daily forecast price cycle. Ideally, our system will accept use-time preferences from consumers and optimize their appliances' operation around those given patterns. However, using the system to *recommend* optimum use-times to consumers is also possible by accepting ideal use preferences from an external load manager and computing the cost savings of these preferences relative to the cost of the consumer's current use patterns.

We implement the optimization problem in two stages by using Mixed Integer Linear Programming (MILP). In the first stage, we obtain the optimum scheduling for appliances to be connected to the outlet; the output of this stage only shows the hours that we can use the appliance and does not reflect the actual consumption hours of each appliance. In the second stage, we model the random behavior of users via Monte Carlo simulation by running the appliances within the times they receive power, specified in the first stage. Finally, we evaluate the effectiveness of the proposed model in terms of cost savings by considering three appliances and four pricing schemes.

## I. INTRODUCTION

Residential electricity consumption varies on an hourly basis each day, as well as a seasonal basis each year. In recent years, increasing power demand has led to higher peak loads. Furthermore, environmentally-friendly and economical operation of the future power grid strongly depends on load management at the residential level.

### A. Background

In "Homeostatic Utility Control" [1], published in 1980, Schweppe summarizes the source of one significant power grid problem which remains today. He states that "Today's regulated electric utility system was built and is operated under a 'supply follows demand' philosophy". Consumers can request an arbitrary amount of power and expect to receive it from the utility company at a basically flat rate. According to [5], we can delay the need for infrastructure upgrades by devising incentive systems to reduce peak loads. In this vein, Joo and Ilić propose a brokering system which maintains near real time balance between power supply and demand and incentives power use at off-peak hours [2]. According to [2], adaptive load management (ALM) is a load management system which can integrate large scale demand response efficiently. In [3],

[4], several methods have been introduced for the integration of price-responsive demand. Additionally, [6] introduces home area networks (HAN's) as a novel means of facilitating such customer-level dynamic pricing. In [7], the problem of optimal smart home appliance scheduling to minimize electricity cost is considered using mixed integer linear programming (MILP).

In this paper, we are not only considering electricity costs but also taking the customers' preferences into account. Prior studies, such as [8], [9], have represented the impact of a time-varying electricity tariff on the load management in the power system in terms of peak demand reduction; however they have not considered the customers' willingness to participate in load management programs.

### B. Motivation

Current end-user energy management systems (EMS) are typically deployed in large buildings and utilize wireless communication network and Internet protocols to remove gross inefficiencies in energy consumption [6]. These designs, known as building automation systems (BAS) [10], require significant scale to be cost effective, deterring home owners from investing in similar energy-saving technology.

Our challenge is to design a cost-effective, automated framework for residential use which incentivizes both utilities and end-users to adopt technologies dynamically optimizing energy demand based on periodic pricing signals. The system will allow homeowners to optimize their energy demand without immediately upgrading their appliances. In this paper, we propose a two stage approach to find an efficient ON/OFF scheduling for end-user appliances. In the first stage, we obtain the optimum hourly schedule for appliances to be connected to the outlet. In the second stage, we model the behavior of users by running the appliances at random subsets of hours each receives power, as determined by the first stage. Fig. 1 shows the general framework of the proposed approach.

### C. Organization of the paper

The rest of the paper is organized as follows: Section II explains the general framework of the proposed approach. Section III introduces the different pricing schemes. Section IV is devoted to customer preferences, as well as manufacturer specifications, of each appliance. Section V includes discussions about our test system and implementation of the approach. Finally, in Section VI, we give conclusions.

\*M.H. Amini and J. Frye contributed equally to this work.

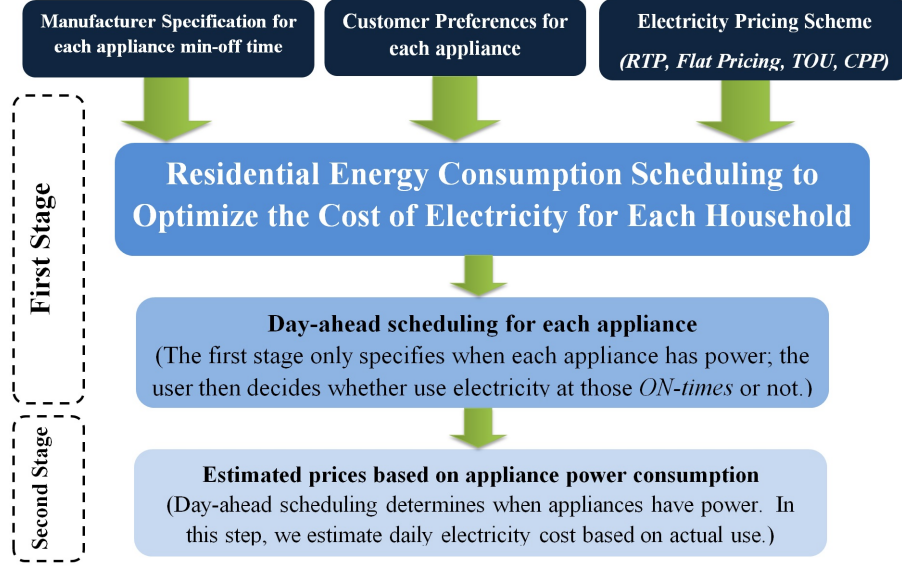


Fig. 1: General Framework of the Proposed Energy Scheduling System

## II. PROPOSED RESIDENTIAL ENERGY SCHEDULING APPROACH

The main purpose of this paper is to design a system which minimizes residential electricity cost of individual homes by shifting demand over an  $H = 24$ -hour forecast price cycle. Our approach is mixed integer linear programming. The outline for the objective function expressing this problem is shown in (1).

$$\begin{aligned}
 & \underset{U}{\text{minimize}} && \sum_{t=1}^H D_t P_t \\
 & \text{subject to} && \text{Home appliances' requirements :} \\
 & && \begin{aligned}
 & \bullet \text{ Customer preferences} \\
 & \bullet \text{ Appliance power use} \\
 & \bullet \text{ Manufacturer requirements} \\
 & \bullet \text{ Electricity tariffs}
 \end{aligned}
 \end{aligned} \tag{1}$$

where  $D_t$  is the individual home's electrical demand and  $P_t$  is the grid electricity price at the  $t^{\text{th}}$  hour of day.

We differentiate between the appliance states of *powered* and *running* in our solution. An appliance is *powered* if it is electrically-connected to its outlet (and thus the power grid), and it is *running* if it is currently utilizing power to perform its intended task. Thus, appliances can be powered but not running, and appliances must be powered to be running.

Let  $A$  be the set of appliances to optimize represented by their indices. We label the home's appliances 1 through  $n$  such that the members of  $A$  are  $\{1, 2, \dots, n\}$ .

The output of our optimization problem is a set of  $u_i$  vectors (one per appliance), where we define  $u_i$  as a length- $n$  vector of binary variables with  $u_{i,t}$  equal to 1 if appliance  $i$  is scheduled to be powered in time period  $t$  and 0 otherwise (not powered).

We are given that each appliance  $i$  has power demand  $d_i$  when it is running, and, for the context of solving the optimization problem, we say an appliance is running whenever it

is powered. This simplification is appropriate under our model because we include no information on probable use patterns. Thus, we must simulate the worst possible cost; namely, the user runs an appliance whenever it has power. Our goal is to minimize this cost. After solving the optimization problem of finding the best times to power the appliance, we will return to differentiating between *powered* and *running* appliances via Monte Carlo simulation.

Using the variables defined above, we can write the objective function more precisely:

$$\begin{aligned}
 \min_U \left( \sum_{t=1}^H D_t P_t \right) &= \min_U \left( \sum_{t=1}^H \sum_{i=1}^N (d_i \cdot P_t) \cdot u_{i,t} \right) \\
 &= \min_U \left( \sum_{t=1}^H P_t \left( \sum_{i=1}^N d_i \cdot u_{i,t} \right) \right)
 \end{aligned}$$

The above equations result from the observation that the home's total demand at time  $t$  is the sum of its appliances' demand, and the cost of energy per appliance is the hourly electricity demand of the appliance  $d_i$  (MWh) multiplied by the cost of electricity at the current timestep  $P_t$  (\$/MWh).

We now proceed to define our problem's constraints. Each appliance has three primary quantities forming its constraints: (1) the minimum powered time ( $T_{\text{on}}^{\min}$ ), (2) the maximum off (not powered) time ( $T_{\text{off}}^{\max}$ ), and (3) the required powered times (length  $H$  binary vector  $v_i$ ).

- 1) The minimum powered time  $T_{\text{on}}^{\min}$  represents the number of hours per  $H$ -hour period for which the appliance must be powered.
- 2) The maximum off time represents the maximum number of consecutive hours the appliance can be off (not powered) during the  $H$ -hour period.
- 3) The required powered times  $v_i$  represent the specified hour indices for which the appliance must be powered, allowing users or external preference monitors to specify personal preferences (e.g., the refrigerator must

be on at 8 am when I open it to make breakfast). Like  $u_i$ , the  $v_i$  vector is 1 where the appliance is powered and 0 at all other hours.

Now we can write the mathematical constraints corresponding to these quantities:

- 1) For the minimum powered time requirement, we have the following requirement that the sum of elements in each appliance's binary  $u$  vector be at least its min on time (Each 1 in the vector represents 1 on hour). Thus, we have one constraint equation for each appliance, resulting in  $N$  total constraint equations in this category.

$$\left( \sum_{t=1}^H u_{i,t} \right) \geq T_{\text{on}}^{\min}, \quad \forall i \in A$$

- 2) For the maximum off time requirement, we form the constraints using a sliding window approach. For each window with size (max off time + 1), there must be at least one hour when the appliance is powered. (Otherwise, there are more than  $T_{\text{off}}^{\max}$  consecutive hours the appliance is not powered, violating the constraint.) Thus, for each appliance, there are  $H - T_{\text{off}}^{\max}$  sliding window constraint equations. Considering all  $N$  appliances, the total number of constraint equations in this category is then  $N \cdot \sum_{i=1}^N (H - (T_{\text{off}}^{\max})_i)$ .

$$\left( \sum_{j=t}^{t+T_{\text{off}}^{\max}+1} u_{i,j} \right) \geq 1, \quad \forall i \in A, \forall 1 \leq t \leq H$$

- 3) For each appliance, the required power time vector  $v_i$  is the lower bound on the entries of the  $u_i$  vector we output, i.e.  $u_{i,t} \geq v_{i,t}$ ,  $\forall i \in A, \forall 1 \leq t \leq H$ . (Every  $u$  and  $v$  vector entry is 0 or 1, and if for some  $1 \leq t \leq H$ ,  $v_{i,t} = 1$ , then  $u_{i,t}$  must be 1 also because we must power the appliance whenever the user directly orders us to.)

We provide two separate implementations of the optimization problem described above: One uses MATLAB's standard optimization toolbox, and the other uses a third-party optimizer called YALMIP [18]. YALMIP is a modeling language for advanced modeling and solution of convex and nonconvex optimization problems. It is implemented as a free access toolbox for MATLAB and provides a cross-check for our MATLAB optimizer [19]. Both implementations yield same cost schedules within roundoff error; however, it is interesting to note that these optimizers sometimes propose different power schedules for the same input, indicating more than one optimal solution exists. In most of the pricing schemes, we have the same electricity price for several hours of the day, resulting in multiple optimum schedules for each appliance. See the included MATLAB code for additional details.

The optimizer gives us a length  $H$  for  $u$  vector of each appliance, indicating the times that appliance is powered. We now want to determine the expected use cost and savings for each appliance given its powered schedules. We employ Monte Carlo simulation to approximate these costs.

We first establish an expected unoptimized use cost as a baseline and then derive the expected optimized use cost imposing our power schedule for comparison. In both cases, the user runs the appliance for a given number of use hours  $h_{\text{use}}$ . We assume the user distributes these use hours uniformly across times the appliance is powered, noting that in the optimized schedule the appliance is only powered at times there is a reasonable chance the user will run it.

1) For the **unoptimized case**, the appliance is always powered; therefore, the run times are uniformly distributed across  $1 \leq t \leq H$ . After randomly placing the running hours, we compute the appliance's resulting electricity cost over one period based on (1) its energy consumption in the given season and (2) electricity prices under the given pricing scheme. We run this experiment (uniformly distributing use hour and computing cost) repeatedly to find the expected base cost.

2) For the **optimized case**, we have from the optimizer the vector of hours  $u_i$  the appliance receives power. Then, rather than randomly place the use hours in  $1 \leq t \leq H$ , we instead place them randomly within only the subset of hours the appliance actually receives power (corresponding to 1 entries in  $u_i$ ).<sup>1</sup> We do not know when the user will turn on the appliance when it has power.

We can now compare the estimated unoptimized and optimized costs to find the expected savings of our optimization scheme. We discuss these results under various season and pricing scheme conditions in the discussion section.

### III. ELECTRICITY PRICING SCHEMES

#### A. Seasonal Effect on the Electricity Prices

In order to consider the effect of electricity prices at different seasons on the total cost, we select a specific day in approximately the middle of each season. Real time hourly electricity prices have been extracted from [12]. Fig. 2 represents the electricity prices for four specific days in Chicago. As

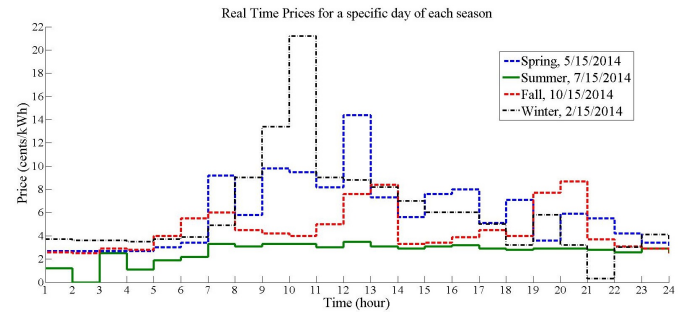


Fig. 2: Hourly Electricity Price for four days of 2014

this figure shows, generally the electricity price in summer is lower than electricity price in winter, in Chicago. Furthermore, spring and fall have the same overall price level.

#### B. Time-varying Electricity Tariffs

In addition to the real-time prices for each season, we have considered three different tariffs [13].

<sup>1</sup>Note that we choose  $T_{\text{on}}^{\min} \geq h_{\text{use}}$  when formulating the optimization problem to ensure this placement is possible.

1) *Real Time Pricing*: In this study, real time pricing (RTP) scheme refers to a situation where electricity tariffs reflect the electricity market price. Electricity prices are not pre-defined and are normally subject to hourly changes. In this context, intensive data exchange, requirement for new billing procedures and short term price forecasting made this pricing scheme difficult to handle; for instance a fixed quantity of energy can be sold at time-of-use rates, the costs beyond the time-of-use (TOU) value being the only ones charged according to the RTP scheme. This pricing scheme reduces the effect of price volatility on the customers. Additionally, it may offer higher revenue stability to utilities. Some utilities offer financial risk management to their customers so that the customers handle their energy costs when they expose price changes, for instance customers can register for forward contracts to reduce their sensitivity to electricity price volatilities. There are some examples of voluntary RTP in the United States and the effect of using this tariff on load shifting in [14].

2) *Flat rates*: In this case, we assume that the price of electricity is fixed during 24 hours of a day. In other words, customers monthly bill is fixed based on their predicted demand. For instance, the *FlatBill* tariff for Georgia Power has been implemented based on the following requirements:

- Applicable to residential customers who have been in their current residence over the previous 12 months, have had their electricity priced on the “Residential Service” or “FlatBill” tariff over the previous 12 months and are currently in good financial standing with Georgia Power.
- Service hereunder shall be for a period of one year. There is no true-up in customers bills at the end of the contract period. All eligible “FlatBill” offers will be updated with their previous year consumption, and contracts will automatically renew for the following year, unless the customer notifies the Company otherwise.

More details about Georgia Power *FlatBill* can be found in [15].

3) *Time-of-Use (TOU) rates* : The Time-of-Use (TOU) tariffs are the most commonly used time-varying pricing schemes. In this scheme, days are split to two to 5 intervals and each time interval has a fixed electricity price. The process of splitting days and determining prices is defined monthly. Utilities consider a fixed price for each interval based on the customers’ predicted demand. TOU tariffs have two important features: 1) They partially reflect the electricity market prices (in the intervals shorter than a day); and 2) Generally, TOU is not as dynamic as real time prices and we can consider electricity price fixed at short intervals so that the customers, residential or industrial, can schedule their consumption in the TOU time blocks.

4) *Critical Peak Pricing (CPP)*: In this pricing scheme, generally one of TOU or RTP schemes is valid for most of the days in one year. Nevertheless, a few days in each year face price change at specific hours that correspond to hours of peak electricity demand. In these periods utilities’ generation capacity could not provide the demand sufficiently without changing the pricing scheme. They can either use real time market prices or pre-determined peak demand electricity prices

at these periods [16]. Fig. 3 represent different tariffs which are considered for the case study in Spring at Chicago.

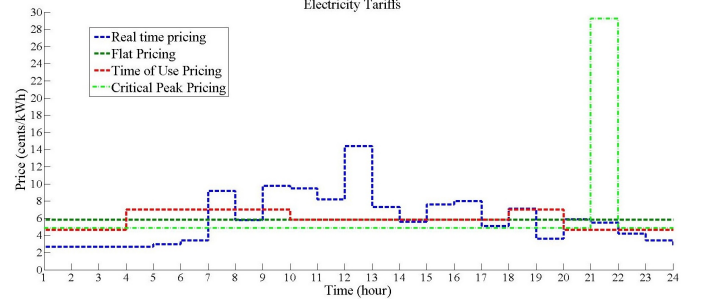


Fig. 3: Different Electricity Tariffs

In addition to considering the potential benefit of optimization under the above pricing schemes, we also estimated the possible cost savings of several distinct sets of customer appliance-use preferences.

#### IV. CUSTOMER PREFERENCES AND MANUFACTURER SPECIFICATIONS FOR EACH APPLIANCE

Each appliance has two constraints: minimum on time  $T_i^{on}$  and max number of consecutive off hours  $T_i^{off}$ . Users directly specify  $T_i^{off}$  for appliance  $i$ . We derive an appliance’s  $T_i^{on}$  as the max of the specified manufacturer and user’s minimum on time. For instance, the manufacturer may specify its refrigerator have power  $H/2$  hours per  $H$ -hour optimization cycle, and the user specifies the refrigerator should have power  $3H/4$  hours per cycle. Thus  $T_{refrigerator}^{on}$  is  $\max(\frac{H}{2}, \frac{3H}{4}) = \frac{3H}{4}$ . One of the inputs of our optimizer devoted to the above-mentioned customers’ preferences. Each customer can define specific hours for the appliance as MUST-ON-TIME hours. These preferences change each season. In the linear programming, we modeled this as a  $1 \times 24$  binary vector for each appliance which is specified by user. The customer preferences for Spring, for the AC are shown in (2). It shows that the customers prefer to turn on the AC at noon for maximum 4 hours.

$$\text{Prefs} = [\text{zeros}(1,11) \quad 1 \quad 1 \quad 1 \quad 1 \quad \text{zeros}(1,9)] \quad (2)$$

Table I represents the value of seasonal minimum on/off time as well as power consumption of each appliance.

Notice that  $N_i^{powered} \geq N_i^{running}$ , where  $N_i^{powered}$  shows the times that we provide electricity to the appliance based on first stage optimization (linear programming) and  $N_i^{running}$  is the average time of utilization for each appliance at a specific season based on real appliances’ usage data.

#### V. DISCUSSION

Figure 4 compares the total optimized and base electricity costs across the four seasons. This graph represents the sum of energy costs of all three of our appliances (refrigerator, air heater, air conditioner). For each season, we compare the optimized costs and base costs of our four pricing schemes: real-time (RTP), flat, time of use (TOU), and critical peak (CPP).

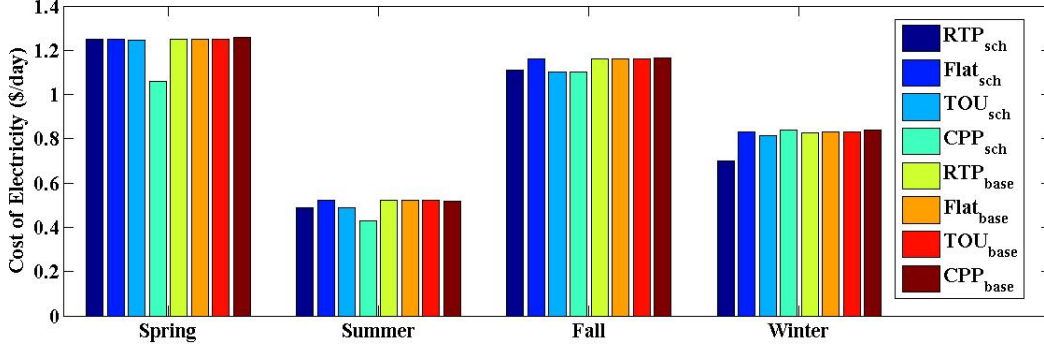


Fig. 4: Total Electricity Cost for Base and Optimized Scenario

TABLE I: Power Consumption ( $P$ ), Minimum on-time  $T_{season}^{on}$ , and Maximum off-time  $T_{season}^{off}$  of Appliances [17]

Appliance	Electric Air Heater	Refrigerator	Air Conditioner
$P$ (kW)	1.5	0.134	3
$T_{Spring}^{on}$	12	16	4
$T_{Spring}^{off}$	2	6	8
$T_{Summer}^{on}$	0	20	18
$T_{Summer}^{off}$	24	1	2
$T_{Fall}^{on}$	16	16	4
$T_{Fall}^{off}$	2	1	10
$T_{Winter}^{on}$	20	14	0
$T_{Winter}^{off}$	1	2	24

We can draw several conclusions from this graph.

- 1) Possibly the graph's most obvious trend is that the cost of electricity in the summer is significantly lower than the electricity cost in other seasons. This pricing difference results from the fact electricity is cheaper in Chicago during the summer than other seasons and has no reflection on the performance of our optimization. This price reduction during summer might be a result of using electricity in winter.
- 2) A second feature is that for each season, the price of electricity across the different pricing schemes is essentially equivalent. The purpose of different electricity pricing schemes is to encourage end users to change their behavior, i.e. when and how much they use appliances. Since we simulated uniform use patterns under all pricing schemes, it is sensible that all pricing schemes yield nearly (but not exactly) the same costs.
- 3) Third, comparing the cost for each season and for each pricing scheme across the base and optimized schedules, we see that the optimized schedule provides little cost saving in most cases. While the optimized critical peak pricing schedules appear to yield the largest relative savings, even these are relatively small (under 10% of the base cost). Furthermore, as we mentioned in the critical peak pricing section, utilities use this pricing scheme only a few days of the year, during times of anticipated heavy load when they could not otherwise

provide for demand. The real-time pricing cost, which is essentially equivalent across the base and optimized schedules, is more indicative of our results.

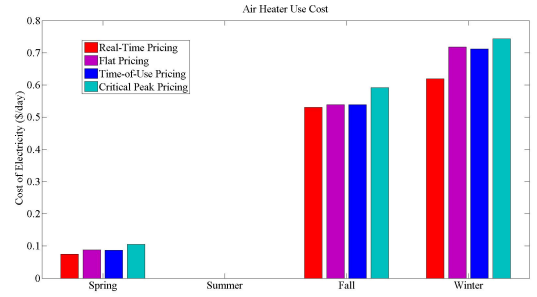


Fig. 5: Optimized Electricity Cost of Heater

We now break down these total electricity costs per appliance to evaluate the optimizer's performance in greater detail.

Figure 5 illustrates the possible electricity cost savings of just the air heater. First note the cost is 0 in summer simply because the heater is turned off. Otherwise, we see the trend ( $\text{Cost}_{\text{RTP}} \leq \text{Cost}_{\text{Flat}} \approx \text{Cost}_{\text{TOU}} \leq \text{Cost}_{\text{CPP}}$ ). We can speculate on the reason for this trend as follows: We can slightly reduce cost by powering the heater in local minimum of the RTP scheme. However, the cost of TOU pricing is basically equivalent to flat pricing because we use the heater across all the TOU intervals, and the cost across all intervals approximately averages to the flat price. The CPP scheme actually costs us more probably because we power the heater at the critical peak times. (When everyone else needs their heaters, we probably do as well. Realistically, powering the heater at such times is necessary for consumer comfort.)

Figure 6 contains the the simulated electricity cost savings of just the refrigerator. We first observe the cost savings is minimized in summer, probably because the refrigerator must run the most frequently in summer, requiring us to schedule it at less than optimal times. Next, the price trend across seasons is generally ( $\text{Cost}_{\text{CPP}} \leq \text{Cost}_{\text{RTP}} \leq \text{Cost}_{\text{TOU}} \leq \text{Cost}_{\text{Flat}}$ ) with minor violations. This trend is sensible because the refrigerator has some flexibility in when it must run, allowing us to avoid the critical peak prices in scheduling and ensure significant savings. (Though the refrigerator must



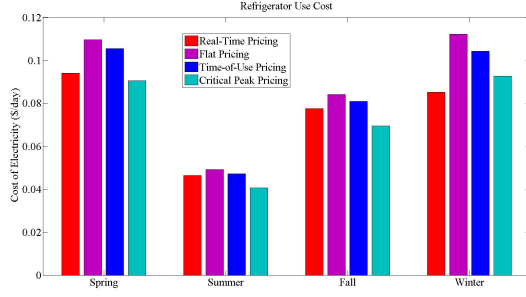


Fig. 6: Optimized Electricity Cost of Refrigerator

run a significant fraction of the day, there are fewer specific times it must be running.) Then,  $\text{Cost}_{\text{RTP}} \leq \text{Cost}_{\text{TOU}}$  because the refrigerator has a small maximum off time, meaning it must run throughout the day, covering all time intervals in the TOU pricing scheme and bringing the average price of this scheme close to the flat price. In contrast, the scheduler can pocket the refrigerator's operation into local minimum occurring throughout the day with the RTP scheme.

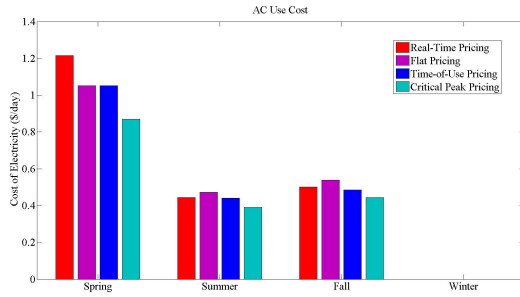


Fig. 7: Optimized Electricity Cost of Air Conditioner

Figure 7 demonstrates the anticipated electricity cost savings for the air conditioner. We first observe that the air conditioner is off and thus has 0 cost in the winter. Otherwise, we see that TOU and CPP may offer minor but relatively insignificant cost savings over flat pricing, which is to some degree expected as the air conditioner (like the air heater) must both run throughout the day and at many specific hours to maintain a comfortable environment. The RTP scheme actually costs notably more than the others in spring, probably because the time we specified the AC must be on correspond to relative maximum points in the real-time price. Overall, no pricing scheme offers significant savings with the air conditioner.

## VI. CONCLUSION

In this paper, we used two stage Mixed Integer Linear Programming (MILP) to solve the residential energy scheduling optimization problem, considering the customer preferences and appliances' specifications. We solved this scheduling problem under all seasons and four unique pricing schemes to evaluate the potential for optimizing the total cost. We found that while some appliances offer marginal cost savings under optimization, the total reduction in electricity costs is around ten percent of base operating cost in all cases. Large-scale

utilization of the proposed approach will lead to significant savings. Additionally, the results show that without motivating customers to change their use patterns, we can only facilitate minimal cost reduction under any pricing scheme. Changing consumers' use patterns is a prerequisite to achieving significant cost reductions. To this end, our proposed framework is capable of computing the monetary benefit of optimum use schedules derived from external load managers, such as *ALM*.

This work can be expanded to consider the means of data transfer between home appliances and our proposed energy management system (EMS), considering two major Home Area Network (HAN) categories: wireless communication network and power line communication (PLC).

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