Inferential Statistics and Probability, Segment 3

Empirical Rule

~68% of data within one standard deviation of mean

~95% of data within 1.96 standard deviations of mean

~99.7% of data within 3 standard deviations of mean

- Two key assumptions
  - The mean estimation error is zero
  - The distribution of the errors in the estimates is normal

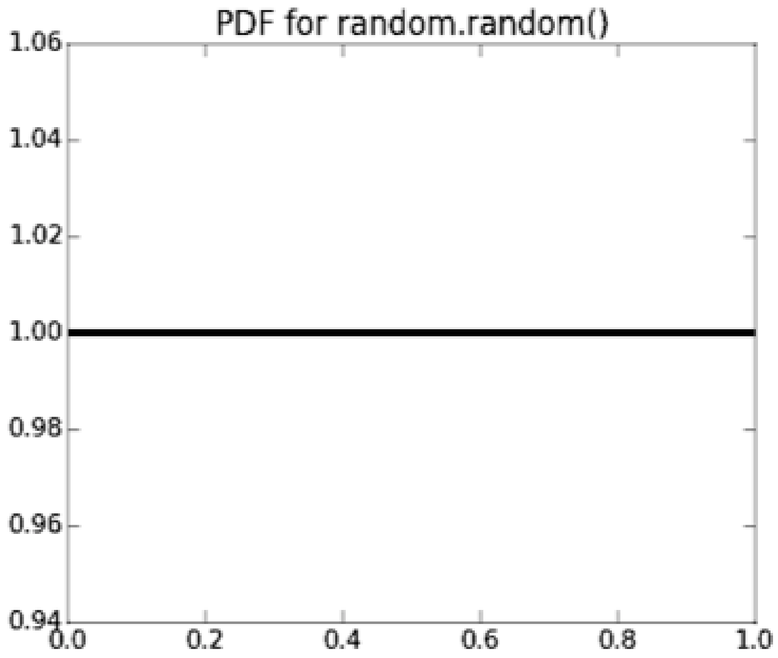
Defining Distributions

- Use a probability distribution
- Captures notion of relative frequency with which a random variable takes on certain values
  - Discrete random variables drawn from finite set of values
  - Continuous random variables drawn from reals between two numbers (i.e., infinite set of values)
- For discrete variable, simply list the probability of each value, must add up to 1
- Continuous case trickier, can’t enumerate probability for each of an infinite set of values

PDF’s

- Distributions defined by *probability density functions* (PDFs)
- Probability of a random variable lying between two values
- Defines a curve where the values on the x-axis lie between minimum and maximum value of the variable
- Area under curve between two points, is probability of example falling within that range

Two PDF's

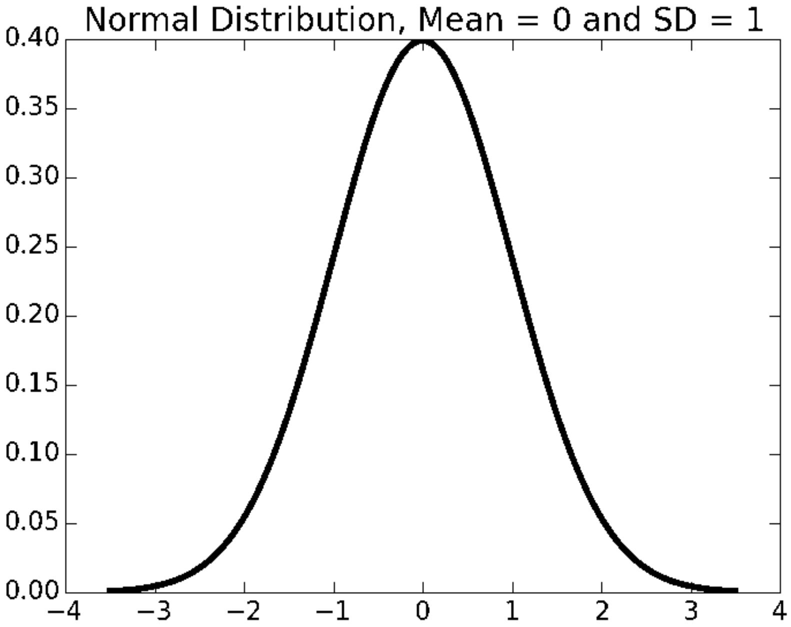

PDF for random.random()
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.0
0.2
0.4
0.6
0.8
1.0


PDF for random.random()+random.random()
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0

Normal Distributions

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} * e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$



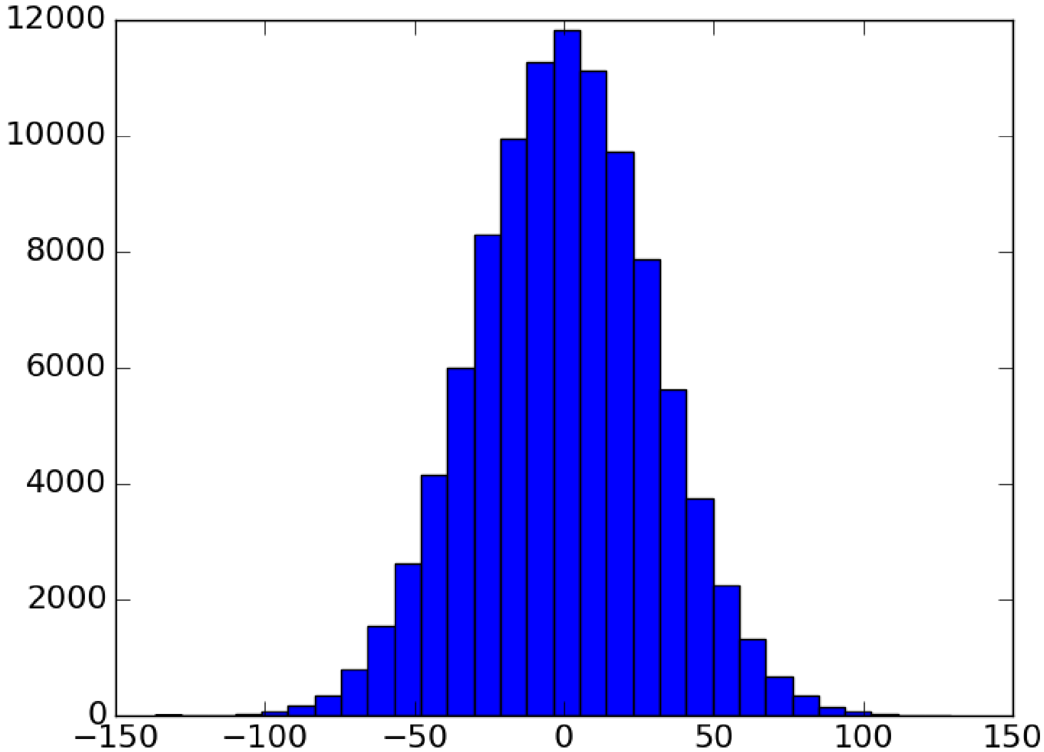
Why We Like Normal Distributions

- Nice mathematical properties
- Occur a lot!

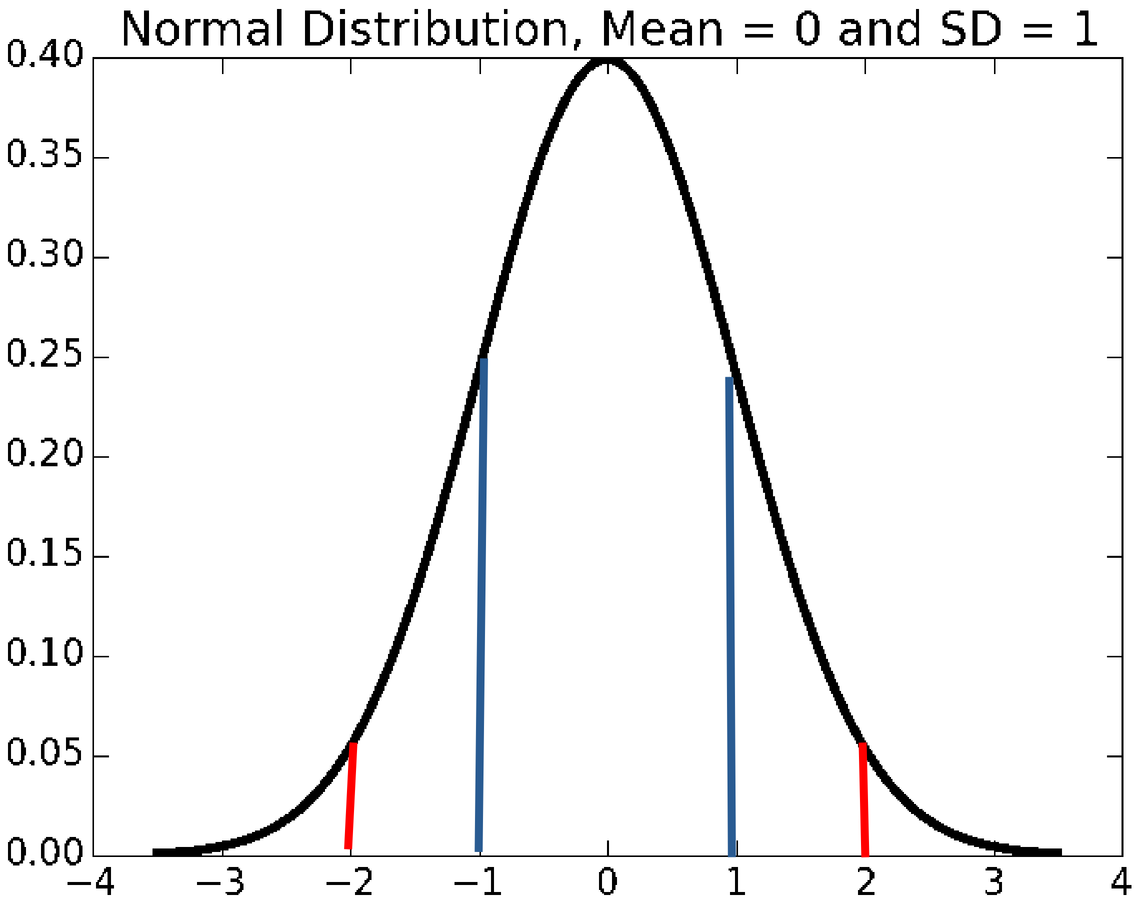
Generating Normal Distributions

```
dist = []
for i in range(100000):
    dist.append(random.gauss(0, 30))
pylab.hist(dist, 30)
```

Discrete Approximation



Empirical Rule



A Digression

- SciPy library contains my useful mathematical functions used by scientists and engineers
- scipy.integrate.quad has up to four arguments
  - a function or method to be integrated
  - a number representing the lower limit of the integration,
  - a number representing the upper limit of the integration, and
  - an optional tuple supplying values for all arguments, except the first, of the function to be integrated
- scipy.integrate.quad returns a tuple
  - Approximation to result
  - Estimate of absolute error

Checking Empirical Rule

```
import scipy.integrate

def gaussian(x, mu, sigma):
  factor1 = (1.0/(sigma*((2*pylab.pi)**0.5)))
  factor2 = pylab.e**-(((x-mu)**2)/(2*sigma**2))
  return factor1*factor2

def checkEmpirical(numTrials):
  for t in range(numTrials):
     mu = random.randint(-10, 10)
     sigma = random.randint(1, 10)
     print('For mu =', mu, 'and sigma =', sigma)
     for numStd in (1, 1.96, 3):
        area = scipy.integrate.quad(gaussian,
                                    mu-numStd*sigma,
                                    mu+numStd*sigma,
                                    (mu, sigma))[0]
        print(' Fraction within', numStd, 'std =', round(area, 4))
```

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} * e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Result

For mu = 9 and sigma = 6
  Fraction within 1 std = 0.6827
  Fraction within 1.96 std = 0.95
  Fraction within 3 std = 0.9973
For mu = -6 and sigma = 5
  Fraction within 1 std = 0.6827
  Fraction within 1.96 std = 0.95
  Fraction within 3 std = 0.9973
For mu = 2 and sigma = 6
  Fraction within 1 std = 0.6827
  Fraction within 1.96 std = 0.95
  Fraction within 3 std = 0.9973

But Not All Distribution Are Normal

- Empirical works for normal distributions
- But are the outcomes of spins of a roulette wheel normally distributed?
- No, they are uniformly distributed
  - Each outcome is equally probable
- So, why does empirical rule work?