The Visual Computer

# Computer generation of eroded valley and mountain terrains

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An improved physical erosion model for computer graphics is proposed. Valley and mountain terrains created with this model and based on the erosion of river flow, rainfall, and weathering were observed to be more realistic than fractional Brownian terrains. Earth layer structures such as the Grand Canyon were also formed on the surfaces of the valleys. Further, growing processes of eroded valleys and mountains have been simulated successfully. Terrains with differing surfaces were created easily by adjusting the erosion intensities of rainfall and thermal weathering.

**Key words:** Computer graphics – Simulation – Terrain models – Erosion models

## 1 Introduction

We use fractional Brownian motion (fBm) obtained mainly from Poisson faulting (Mandelbrot 1975, 1982; Voss 1985; Peitgen and Saupe 1988) and Fourier filtering (Mandelbrot 1982; Voss 1985; Peitgen and Saupe 1988) for creating fractal terrain models so that we can produce pictures of realistic terrains. Therefore, it is reasonable to use fBm when we represent a natural landscape such as a mountain terrain in computer graphics. However, the pictures created with fBm may appear unnatural, since they have no eroded surfaces that can be seen clearly in the natural landscape. The surfaces of the earth become eroded as time passes. If, therefore, we cannot observe eroded terrains in the pictures, it seems unnatural. From this point of view, the use of fBm and related fractal methods (Gardner 1985; Miller 1986; Mastin et al. 1987; Barnsley 1988; Anjo 1991) may not be effective for the representation of natural features of the earth, if they are used independently.

Mountains are one of the most dominant terrains on earth and are widely distributed all over the world. Natural mountains are created by various causes such as movements of the earth's crust and physical erosion, but the outlines and surfaces of the mountains in a rainy region are characterized mainly by physical erosion. If erosion due to river flow is dominant, the creation of mountains is related to the development of the valleys. The valleys can be created in some places by upheaval due to movement of the earth's crust. Natural valleys are created in the upheaved lands by water and thermal weathering erosion, so that they sometimes have peculiar appearances, such as are seen in the Grand Canyon. Consequently, if we need to simulate natural valleys more realistically, then the use of the physical erosion-models (Kelley et al. 1988; Musgrave et al. 1989; Roudier et al. 1993) may be most suitable. Musgrave et al. subdivided the modeling for imaging-eroded synthetic terrains into two steps, namely, terrain generation and erosion simulation. To generate the terrain model initially, they used an fBm algorithm and then erosion was simulated. In this method, the terrains with unique appearances such as layer structures cannot be represented, since initial terrain models are made with the fBm technique beforehand.

Roudier et al. (1993) propose a terrain simulation model for erosion and deposition, using geological parameters and laws from simplified geomorpho-



The Visual Computer (1997) 13:456–464

logical theories. These laws consist of gravity creep, detritus removal by running water, chemical dissolution, and alluvial deposition laws. Further, in their paper they only consider the processes that depend on running and infiltrated water. In general, it is difficult to simulate geological terrains precisely, since lots of geological parameters are needed and, in addition, characteristics of the geological structures between the different locations are frequently quite different. Therefore, appropriate selection and setting of the geological parameters for an eroded terrain simulation are extremely important.

In this paper, our aim is to show the terrains in a warm climate with much rain. Especially, we focus our attention on the representation of the valleys and mountains with earth layer patterns on their surfaces, so that a new algorithm that can represent the eroded terrain showing peculiar outlines must be proposed. The proposed method requires a minimum of geological parameters. Eroded terrains can be created automatically without additional operations such as fBm modeling and we can represent the peculiar layers. This algorithm is effective for the simulation of the valley and mountain evolution under the river flow erosion. In the first step, the initial river courses must be set. To draw the initial river courses, we use the midpoint displacement method (Fournier et al. 1982). If the river courses are varied, different terrains can be obtained. Of course, it is possible to set the initial river course artificially by using other methods. In this algorithm, the rainfall effect and weathering erosion were also considered in developing the valleys and mountains.

## 2 Valley creation

Valley creation in the earth were assumed as follows:

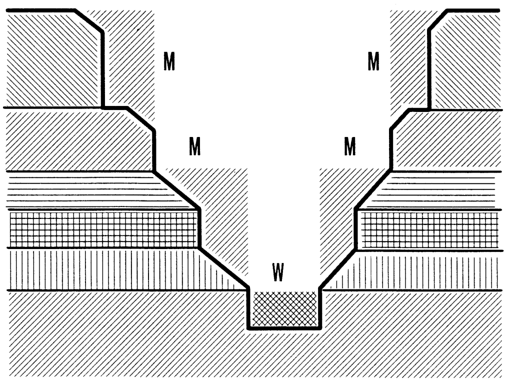
1. First, upheavals of land occur because of the movement of the earth's crust, and rain falls on the upheaved terrain.
2. Rain water gathers in the small undulations on the surface of the land and small puddles form. Further, the increasing number of small puddles combine to form a little-river. Finally, large rivers form from the small ones. Since the initial rivers are narrow and shallow, their streams are generally rapid. These rivers easily grow to large rivers with time. The rivers with rapid streams can scrape the soils from the river banks and beds, but the hydraulic erosion diminishes when the rivers flow slowly.
3. If the earth is repeatedly upheaved, the river banks and beds will be eroded by the flowing water wherever the land is sharply slanted, so that sheer cliffs will be generated by the erosion. The sheer cliffs vary to gentle slopes due to the continuous erosion of the thermal weathering and rainfall. This erosion can lead finally to the creation of mountainous terrain. We are to simulate the creation of the eroded valley and mountain terrains in the situations mentioned.

To make an initial curved river flow numerically by computer, we used the midpoint displacement method. The initial rivers obtained from the numerical calculations were placed as a series of coordinate points on a 2D plane. The various geological parameters and parallel multilayers were also assumed in the model. In the next step, the iterative cycles of erosion for creating the valleys was started.

If the perpendicular walls of the valleys created by the hydraulic erosion are crumbled down by physical forces such as thermal weathering or the shock of an earthquake, then the rivers become broader and the creation of the large valleys begins. The rough surfaces of initial valleys may become smooth if rain continues to fall. Our simulation took these circumstances into consideration. Obviously, hydraulic erosion based on the river flow is one of the most important factors in the creation of valleys. We, therefore, consider the hydraulic erosion in the rivers first of all.

There must be a difference of height between the upper and lower regions of the rivers in order for them to flow constantly, but if the heights of two points become equal, the river may stop flowing, which means the termination of the hydraulic erosion. For this reason, the rivers that we can simulate are limited to those with rapid flows.

Figure 1 shows the geographical features of the layer cross-section of the valley created by the physical erosion caused by hydraulic and thermal weathering. Hydraulic weathering consists of running water and rainfall. Thermal weathering caused by the physical phenomena of the atmo-

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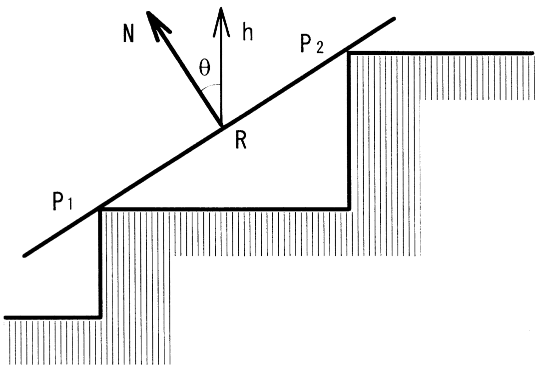


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**Fig. 1.** Schematic representation of a cross section of a valley eroded by hydraulic and thermal weathering

**Fig. 2.** Calculation of quantity fallen rock or transported soil due to thermal weathering or rainfall in the inclined plane

sphere, can indirectly level the steep inclines located on both sides of the rivers and can smooth the surfaces of the inclines, as does rainfall. Mass $M$ in Fig. 1 shows the rock or soil removed from the river banks by erosion.

First, hydraulic erosion begins and sheer cliffs are created on both sides of rivers. Second, the cliffs are eroded by thermal weathering and rainfall. If the land is heaved up, hydraulic erosion occurs again in the river bedding. Such iterative action by hydraulic and thermal weathering erosion continues till deep valleys are created. Thus, through these iterations, deep valleys with stepping layers are constructed along the river courses. If this erosion continues for a long time, then the valleys may give way to mountains or sand dunes.

Here we consider the hydraulic erosion theoretically in a limited area situated in an upriver district. Let $Wp^t$ be a volume of water at height $Hp$ and time $t$, and let $Wq^{t+\lambda}$ be a volume of water at height $Hq$ and time $t+\lambda$. Assuming that there is some suspended soil in the water that can reach the point $P$ from upriver, then the amount of suspended soil $Sp^t$, contained in $Wp^t$ is

$$Sp^t = K_0 Wp^t, \tag{1}$$

where $0<K_0<1$. Soil suspended in the water is deposited in the river bed or passes through point $P$. The deposition value of soil in the water can be expressed as $KdSp^t(0<Kd<1)$, so that the soil carried from point $P$ is $(1-Kd)Sp^t$. However, since the deposition of the soil generally cannot occur in a river with a rapid current, the effect of the deposition can be ignored. Nonetheless, the water picks up new soil produced by the hydraulic erosion and is conveyed down river. This soil $Spn^t$ is given as

$$Spn^t = Kp\, Wp^t, \tag{2}$$

where $Kp$ is a constant that corresponds to the magnitude of the hydraulic erosion.

Since the river with a rapid current at the upriver point does not deposit the soil suspended in the water in the river bed, the amount of the soil $Sps^t$ conveyed from the point $P$ down river is

$$Sps^t = K_0 Wp^t + Kp\, Wp^t = (K_0 + Kp)\, Wp^t. \tag{3}$$

If tributaries flow into the main river at several points between $P$ and $Q$, then an amount of soil $Sq^{t+\lambda}$ suspended in the water $Wq^{t+\lambda}$ at point $Q$ must include the soil conveyed from the tributary streams. Therefore,

$$Sq^{t+\lambda} = Sps^t + Kr\Delta W, \tag{4}$$

where $\Delta W$ is the volume of water added to the main river flow from tributary streams when the water reaches point $Q$. $Kr$ is a constant, and $0<Kr<1$. Similarly, hydraulic erosion also occurs at point $Q$. The amount of the soil generated by erosion $Sqs^{t+\lambda}$ is

$$Sqs^{t+\lambda} = Kq\, Wq^{t+\lambda}, \tag{5}$$

where $Kq$ is a constant. If there is no sediment at point $Q$, the amount of the soil $Sqc^{t+\lambda}$ carried from point $Q$ is the sum of Eqs. 3–5. This value is expressed as

$$Sqc^{t+\lambda} = Sps^t + Kr\,\Delta W + Sqs^{t+\lambda}$$
$$= (Ko + Kp)\,Wp^t + Kr\,\Delta W + Kq\,Wq^{t+\lambda}. \quad (6)$$

If the river has no small streams flowing into the main stream, then $\Delta W$=0 and $Wq^{t+\lambda}$=$Wp^t$. With this assumption, Eq. 6 becomes,

$$Sqc^{t+\lambda} = (Ko + Kp + Kq)\,Wp^t. \quad (7)$$

Since the soil suspended in a river with a rapid current does not accumulate in the river bed, the condition of $Kp$ and $Kq$>0 is required. In other words, hydraulic erosion usually occurs with the condition $Kp$ and $Kq$>0. In order to simulate the creation of steep valleys, $Kp$=$Kq$>0 was assumed in the present model since the steep valleys are created upriver with rapid current, and the river never deposits soil there. These values were varied for different layers of soil. In fact, the cliffs having partially different layers were obtained with different values of $Kp$ and $Kq$ proportional to the stiffness of the layer under the river bed.

The soil of the river bed is carried by the current and, therefore, the height of the river bed above sea level is decreased by the iterative hydraulic erosion. If we represent the magnitude of the erosion by a depth, then the increased depth $H_i$ at time lapse $\lambda$ is

$$H_i = H_{t+\lambda} - H_t = R_H \lambda, \quad (8)$$

where $H_t$ and $H_{t+\lambda}$ are the depths of the river at time $t$ and at time $t$+$\lambda$, respectively. $R_H$ is a coefficient proportional to the stiffness of the layer of the river bed and corresponds to the value of $Kp$ or $Kq$. These considerations are concerned chiefly with the river having a single flow, but rivers with complex networks of water courses can also be considered with the same method as is used for a single river.

Another value required for modeling is the total value of fallen rocks or lost soil resulting from erosion by thermal weathering and rainfall. Assuming that the amount of the lost material due to erosion is proportional to the angle of inclination between two points on the surface, then this value can be calculated by using the normal vector to the surface over both sides of the inclined plane, as shown in Fig. 2.

The magnitude of the normal vector at point $R$ was estimated by the height values around the points $P_1$ and $P_2$. The normal vectors were calculated at all points along the water course where erosion may occur. If, of course, more eroded large valleys are needed, then the calculation areas in the 2D plane extended to parts distant from the river points and, consequently, computer calculation time is increased. Next, we discuss the method for calculating the amount of fallen rock and lost soil.

Let $N$ be the surface normal vector perpendicular to the plane through points $P_1$ and $P_2$ as shown in Fig. 2. Then the element of $N$ in the direction of the $y$-axis (height) $h$ is $N \cos \theta$. Therefore, the intensity of the erosion $F$ in the vertical direction $h$, is given by

$$F = A \cdot N(1 - \cos\theta), \quad (9)$$

where $A$ is a constant that varies with the materials of the eroded surfaces, and $\theta$ is the angle between $N$ and $h$. The value of $F$ in Eq. 9 is proportional to the amount of deposited soil and rocks. However, whether a large quantity of the soil and rock is deposited depends on the incline between two points. We therefore assume that dry soils at the foot of the valley accumulates above an angle of 60º, and completely wet soil, above 30º. In this case, the condition of 60º applies to simulations in which weathering is dominant. However, when the rain is heavy, the condition of 30º is used. The intensities of rainfall and thermal weathering can be adjusted by varying the constant $A$ in Eq. 9.

The height values of $P_1$ and $P_2$ are obtained from the height mean values of the several points around $P_1$ and $P_2$, respectively, since these points are chosen at an interval of several pixels. This must be calculated over the 2D plane.

If rain continues to fall on the stepped cliffs created by erosion, rain that falls on the surfaces of the inclines begins to erode the surfaces again. As a result of precipitation, the slopes with rough surfaces become gentle. Since the magnitude of the erosion of rainfall fluctuates irregularly, weighted random numbers were used in the computation as an erosion constant of rainfall. To display the results obtained from the simulation, a ray-casting method with a rectangular array of height values for terrain representation was used, and Phong shading was used to shade the polygons (Phong 1975).

## 3 Algorithms

The simplified algorithm to create the eroded terrains is as follows:

/*height_value[$x$, $z$]:Array of height values computed through erosion processes. Height axis corresponds to $y$-axis in 3D space*/
Set initial values
    Viewpoint, light source, iterative number of operations, various erosion coefficients, earth layer structure coefficients such as thickness and stiffness.
Calculate the location ($x_s$, $z_s$) of the initial river on the ($x$, $z$)-plane, using the midpoint displacement technique and set the initial river height values to height_value[$x_s$] [$z_s$]
/*Suppose that the direction of the positive $z$-axis is upriver and the opposite direction is downriver and height values upriver are a little larger than those downriver*/
/*Seach the points ($x$, $z$) that lie along the initial river course*/
Repeat the erosion process{
/*Height value is diminished by iterative operations*/
for ($x$=0; $x$<$n$; $x$++) /*The plane size is $n$×$m$*/
    {for ($z$=0; $z$<$m$; $z$++)
    Calculate the height value at the current point ($x$, $z$) from erosion intensity of river flow
    /*Height value is proportional to erosion intensity*/
    Calculate the amount of eroded soil from the angle of inclination between two points and transfer it to height value
    Change the height values at the points around the current point.
    /*We assume that the eroded land exists upriver and that soil is all conveyed downriver with the river flow*/
    If the height value reaches the point that the property of the earth layer varies
        {Calculate the erosion intensities again}
        }
    }
Render the height fields by using the ray-casting technique

## 4 Experimental results

Since an accurate selection of geological parameters is considerably difficult, the values of the parameters were varied mostly by viewing the pictures resulting from the iterative operations of erosion. In terrain synthesis by computer, we assume that eroded terrains are situated upriver and that soil from erosion is all conveyed downriver, so that sedimentation of the soil does not occur. One of our intentions is to represent the layer patterns on the terrains. For this purpose, several parallel layers were assumed to be underground in the initial land.

Assuming that the heights (depths) of the layers from the surface are $h_1$, $h_2$ … $h_n$, and that the layers have the stiffnesses $s_1$, $s_2$ … $s_n$, then the erosion value $h$ is calculated by using $s_1$ till the height of the eroded soil reaches $h_2$. If the erosion continues to layer $h_2$, next, the stiffness value $s_2$ of layer $h_2$ is used in the computation. We continue to repeat the computation as long as $h<h_n$. Since the initial rivers formed by the connection of small water pools may have complicatedly curved streams, the randomly curved initial river courses were generated with the midpoint displacement method. In addition, if we use this method, it is also possible to create the river courses with flows from the positive direction of the $z$-axis to the origin. The initial land surface is divided by a 800×640 pixel plane, but if more detailed landscapes are required, larger 2D planes can be prepared. Simulation was accomplished with an SGI Indy R5000 workstation.

Figure 3 shows simulations of eroded valleys. Figure 3a is an initial valley without water. The river width is adjusted randomly with Eq. 9 along the numerically computed river courses. Figure 3b, c are the valleys created by erosion. To show clearly the bases of valleys and mountains, the rivers are not visualized in all figures. Figure 3c is a scene of a great valley from a side viewpoint. All valleys shown in Fig. 3 are created from a single river, but the valley shown in Fig. 3c is more eroded than that in Fig. 3b. When the erosion greatly increases, large valleys are created (Fig. 3c). Here the stepped layers can be seen on the surfaces of the slopes of the valley. The valleys with stepped lay-

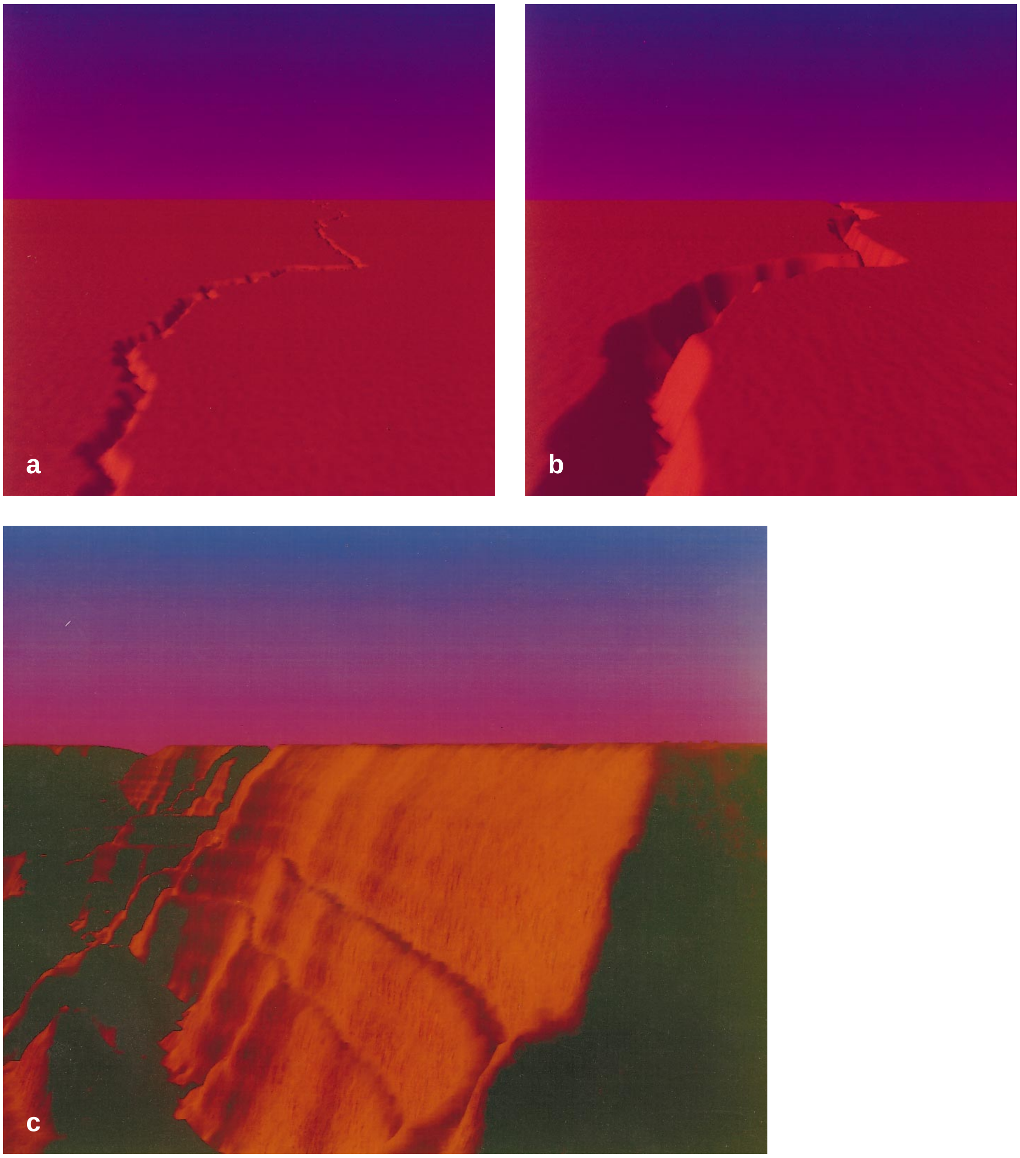
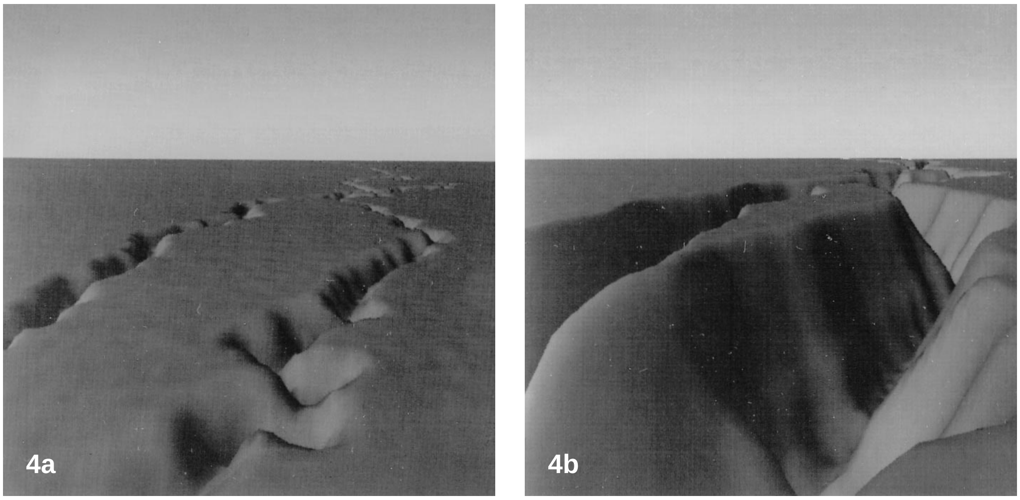



**Fig. 3a–c.** Generation of valleys (numbers of iterative operations are 5 in **b** and 10 in **c**, respectively): **a** an initial river course without water; **b** valleys created by hydraulic erosion and thermal weathering; **c** side view of a great valley with several earth layers

ers can easily be obtained by varying the stiffness of the layers. The valley shown in Fig. 3c was made by using the various stiffness values of the layers in the vertical direction. A single valley can be created from a single river, but if complex valleys are required, then multiriver courses must be used. The complex river networks are useful for generating complicated valleys and eroded mountains.

Figure 4a shows two initial river courses and was obtained with the same method as Fig. 3a. Figure 4b shows the mountain created by the erosion of the river flow. If several river courses such as that of Fig. 5a are used, rugged mountain terrains (Fig. 5b) are created by thermal weathering and rainfall. These figures were produced with the idea that the theories effective for a single river are also applicable to complicated river networks.

Appearances of the valleys with several rivers are varied with the magnitudes of hydraulic and thermal weathering erosion. The eroded valleys can finally be changed into smooth mountains or hills.



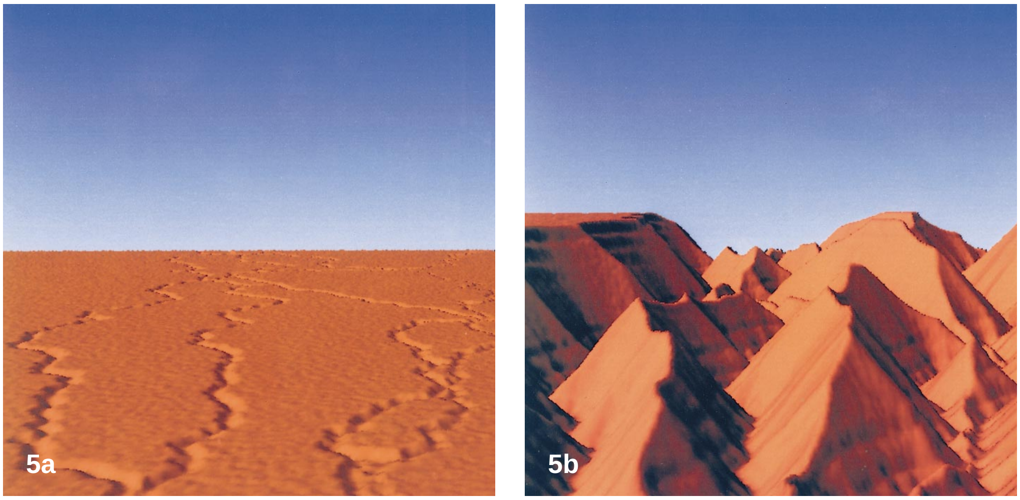




**Fig. 4a–b.** Mountain created from eroded valleys (number of iterative operations is 10 in **b**): **a** initial valleys; **b** the mountain created from the eroded valleys

**Fig. 5a–b.** Creation of valleys and mountains with rugged surfaces (number of iterative operations is 10 in **b**): **a** initial river networks **b** valley and mountain terrains with rugged surfaces created from the river networks

An example of smooth mountains and/or hills created by erosion is shown in Fig. 6a.

In the early years of mountain creation, the mountains had relatively sharp outlines such as Fig. 5b, but continuous rain and thermal weathering erosion change the mountains to gentle hills like the sand dune shown in Fig. 6b. In Fig. 6b, the rough surfaces vanish, and the external shapes of the eroded mountains become completely smooth. However, if the rainfall is extremely heavy, then intricate terrains with rugged surfaces appear (Fig. 7). It is clear from Figs. 3–7 that the figures obtained by using the present method are different from those created with fBm.

The terrains with heavy thermal weathering and rainfall were created by using Eqs. 7–9 and the smoothing operation for the height values of 4×4. In the case of extremely heavy rainfall only, the computation was achieved by varying the value of $A$ in Eq. 9. For the experiments, it is difficult to know the detailed terrain structures beforehand, since the computation for terrain formation is iter-

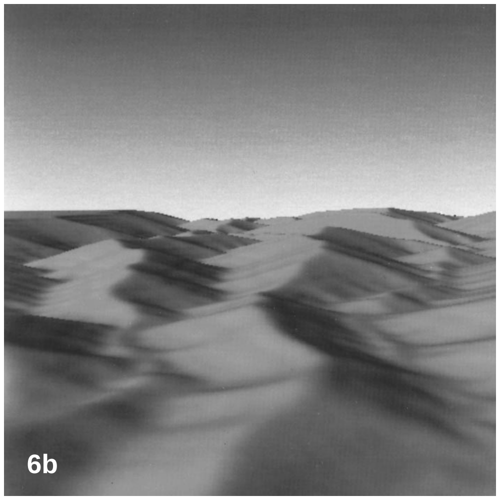





**Fig. 6a–c.** Eroded terrains created by thermal weathering and rainfall (numbers of iterative operations are 25 in **b** and 50 in **c**): **a** an example created from the terrains shown in Fig. 5b by addition of thermal weathering and rainfall; **b** terrains created by more increased thermal weathering and rainfall than those in the case of **a**

**Fig. 7.** Example of the terrains created when extremely heavy rain continues to fall on the land shown in Fig. 5a for a long time (number of iterative operations is 80)

ated against all the planes and by varying many parameters.

The sights in the foreground of the figures seem somewhat out of focus. This is because the figures have the same pixel number in both foreground and background, so that the foreground sights are lengthened by perspective effect.

The total CPU time for computing is approximately 3 min for 20 iterative operations. Computing time depends mainly on array sizes and the number of iterative cycles.

## 5 Conclusion

Simulation of the terrains with earth layer patterns on their surfaces, created by the physical erosion of river flow, precipitation, and thermal weathering were conducted. From the simulation experiments, valleys and mountains with layer traces were clearly seen in the pictures.

In this case, each of the multilayers is parallel to others, but if there are variously curved layers underground, then different traces may be obtained. If, however, precise terrain representation is required in the erosion of the curved multilayer, more geological parameters may be needed than in the present case. The natural valleys and mountain terrains with multilayer structures can be created by using the model based on physical erosion without the fractal terrain model. Obviously, the erosion model used in this experiment was suitable for representing the valleys realistically, and various kinds of valleys were constructed easily. Furthermore, the appearance of valleys with the earth layer structures could be varied by varying the stiffness of the layers.

The evolution from valleys to mountains was also simulated successfully. In the experiment, different combinations of thermal weathering and rainfall created the various terrains that have simple or complex valleys and mountains. Therefore, when the representation of unique terrains such as eroded valleys and mountains with layer traces on the surfaces are required in computer graphics, the use of the erosion model proposed here may be more desirable than those of the fBm and related models.

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