Runge – Kutta de Orden Superior

La mayor exactitud se ve afectada por un excesivo trabajo computacional así como de complejidad.

$k_1 = h\, V_n$
$m_1 = h\,[\pm a\, V_n \pm b\, U_n, q_n]$
$k_2 = h\,(V_n + m_1)$
$m_2 = h\,[\pm a\,(V_n + m_1) \pm b\,(U_n + k_1), q_n + h]$

$y_{n+1} = y_n + ½\,(k_1 + k_2)$
$y´_{n+1} = y´_n + ½\,(m_1 + m_2)$

$V_n = y´$ $\quad U_n = y$ $\quad q_n = t$

Ejemplo.

$y´´ - y´t - y = 0 \quad y_0 = 1 \quad h = 0.5 \quad y´_0 = 2$

Entonces

$y´´ = y´t + y$

Por lo tanto.

$a = 1,\ b = 1,\ V_n = y´_0 = 2,\ U_n = y_0 = 1,\ q_n = t = 0$

$k_1 = h\,(y´_0)$ $\quad k_1 = h\,(V_n)$

$k_1 = 0.5\,(2)$

$\boxed{k_1 = 1}$

$m_1 = h\left\{y´t + y\right\}$ $\quad m_1 = h\left\{V_n\, q_n + U_n\right\}$

$m_1 = 0.5\left\{\left((2)\,(0)\right) + 1\right\}$

$\boxed{m_1 = 0.5}$

$k_2 = h\,(y´_0 + m_1)$ $\quad k_2 = h\,(V_n + m_1)$

$k_2 = 0.5\left(2 + (0.5)\right) = 0.5\,(2.5)$ $\quad \boxed{k_2 = 1.25}$

$$m_2 = h \left\{ a\,(y'_0 + m_1)\,(t + h) + b\,(y_0 + k_1) \right\}$$

$$m_2 = h \left\{ a\,(V_n + m_1)\,(q_n + h) + b\,(U_n + k_1) \right\}$$

$$m_2 = 0.5 \left\{ \left[(1)\,(2 + 0.5)\,(0+0.5) \right] + (1)\,(1 + 1) \right\}$$

$$\boxed{m_2 = 1.625}$$

$y_1 = y_0 + 1/2\,(k_1 + k_2)$ $\qquad$ $y_1 = U_n + 1/2\,(k_1 + k_2)$

$y_1 = 1 + 1/2\,(1 + 1.25)$

$$\boxed{y_1 = 2.125}$$

$y'_1 = y_0' + 1/2\,(m_1 + m_2)$ $\qquad$ $y'_1 = V_n + 1/2\,(m_1 + m_2)$

$$y'_1 = 2 + 1/2\,\left[(0.5) + 1.625 \right]$$

$$\boxed{y'_1 = 3.0625}$$

Encontrar: y_2, y'_2