A Communication-focused Approach to Building Path Diagrams for Multilevel Models

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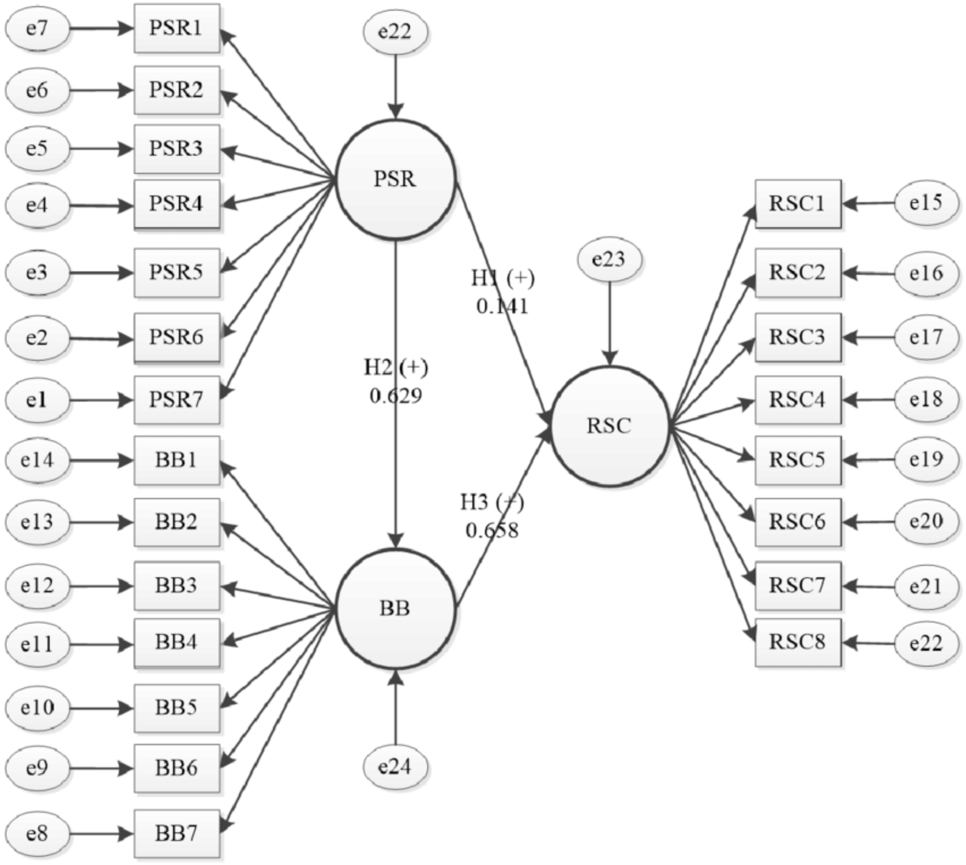
What are the benefits of path diagrams?

Path diagrams visually communicate **model structure** (and, optionally, **results**)

They are ubiquitous in path modeling, factor analysis, and others forms of SEM

They are helpful and attractive additions to posters, slideshows, and manuscripts

They are invaluable when explaining, teaching, and even designing models



Are there path diagrams for MLMs?

- Curran & Bauer (2007)*
  - Designed for multilevel regression (LMM) models specifically
- Muthén (1994) and Muthén & Muthén (1998-2021)
  - Designed for multilevel structural equation (MSEM) models
  - Can be adapted to multilevel regression (LMM) models
- Mulder & Hamaker (2021)
  - Adaptation / extension of Muthén & Muthén approach
  - Designed for random-intercept cross-lagged panel (RI-CLPM) models
  - Can be adapted to multilevel regression (LMM) models

Why aren’t path diagrams used in MLM?

- Existing approaches have emphasized *completeness*
  - All parameters are depicted, which is great for model reproducibility
  - But, as a result, they quickly become visually complex and cramped
  - They can be intimidating and overwhelming to creators and viewers
- A communication-focused approach would emphasize *clarity*
  - We can simplify by omitting certain (e.g., non-focal) parameters
  - We can highlight the defining trait of MLM: its use of discrete levels
  - Adoption will increase if they are easier to create and understand

What is my proposed approach?

- I continue the tradition of Curran & Bauer (2007) in making an LLM-focused approach
- But I integrate techniques from MSEM (Muthén) and RI-CLPM (Mulder & Hamaker) diagrams
- I also introduce some new principles and innovations to emphasize communication clarity

1. Clearly **separate each level** using dashed lines and **colorblind-friendly colors**
2. Organize **nested levels vertically** and **crossed levels horizontally** with clear labels
3. Emphasize the decomposition of the outcome variable into **level intercepts**
4. **Omit non-focal parameters** (residuals, random effect means, random effect covariances)
5. Extend approach to accommodate **three-level models** and **cross-classified models**

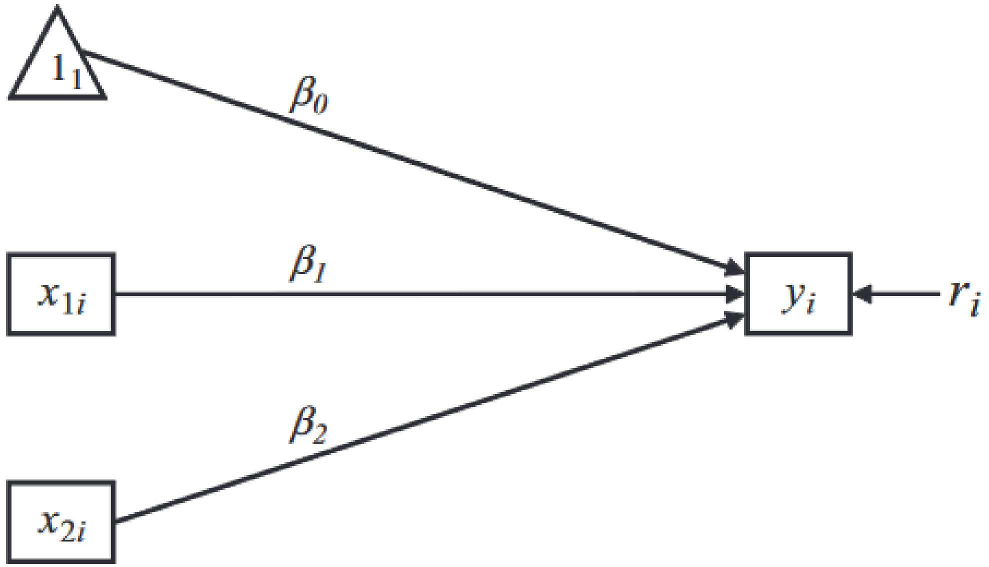
Curran-Bauer Approach

Psychological Methods (2007)

Multiple Regression (CB)

Level 1 Model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + r_i$$



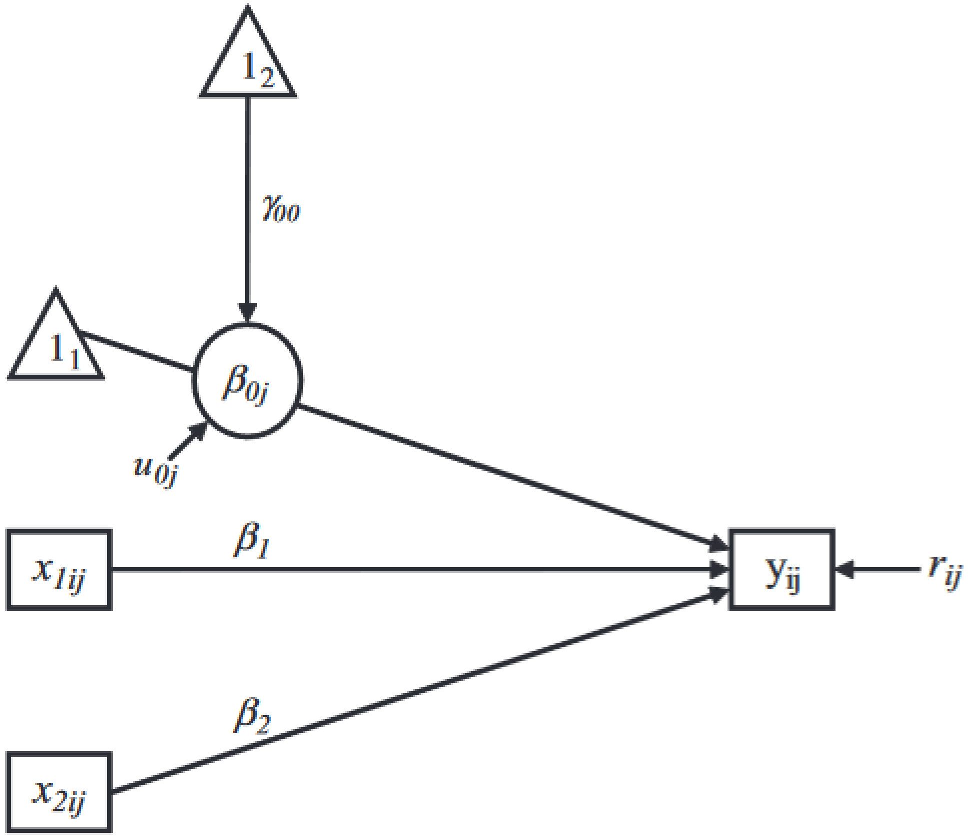
Random Intercepts (CB)

Level 1 Model

$$y_{ij} = \beta_{0j} + \beta_1 x_{1ij} + \beta_2 x_{2ij} + r_{ij}$$

Level 2 Model

$$\beta_{0j} = \gamma_{00} + u_{0j}$$



Random Slopes (CB)

Level 1 Model

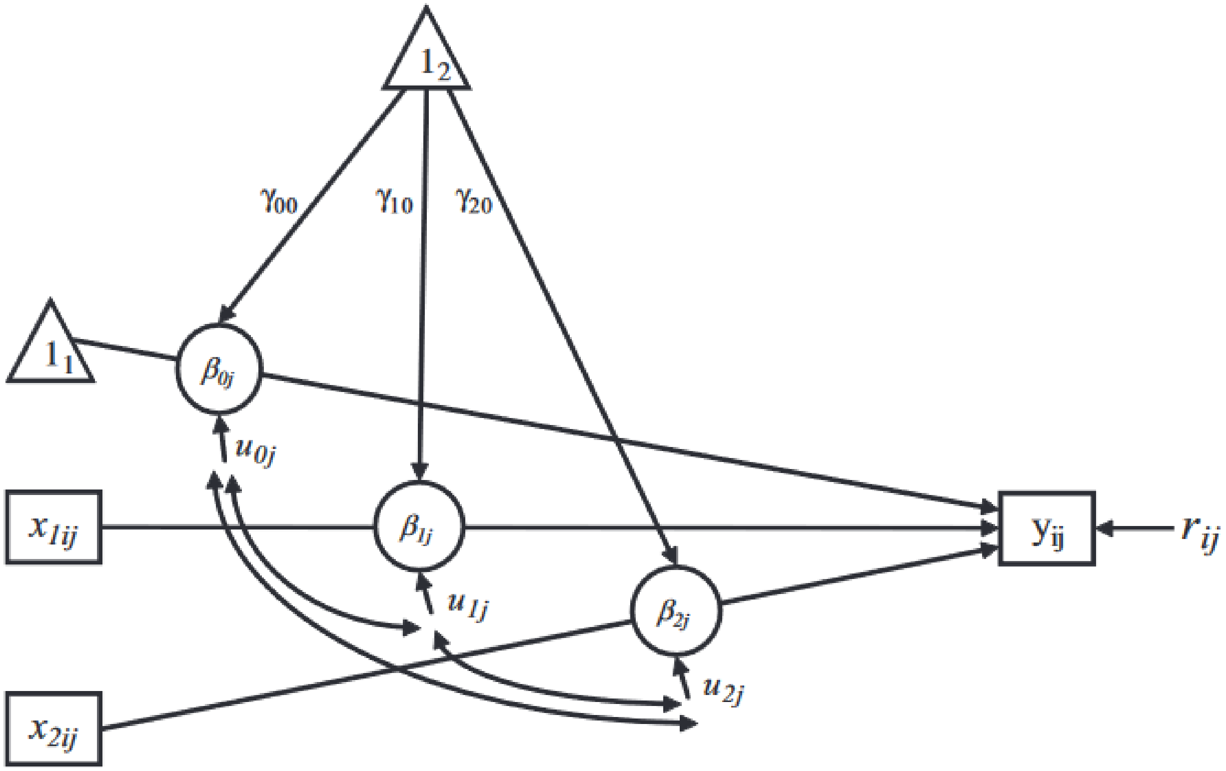
$$y_{ij} = \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + r_{ij}$$

Level 2 Model

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + u_{2j}$$



Cross-level Interactions (CB)

Level 1 Model

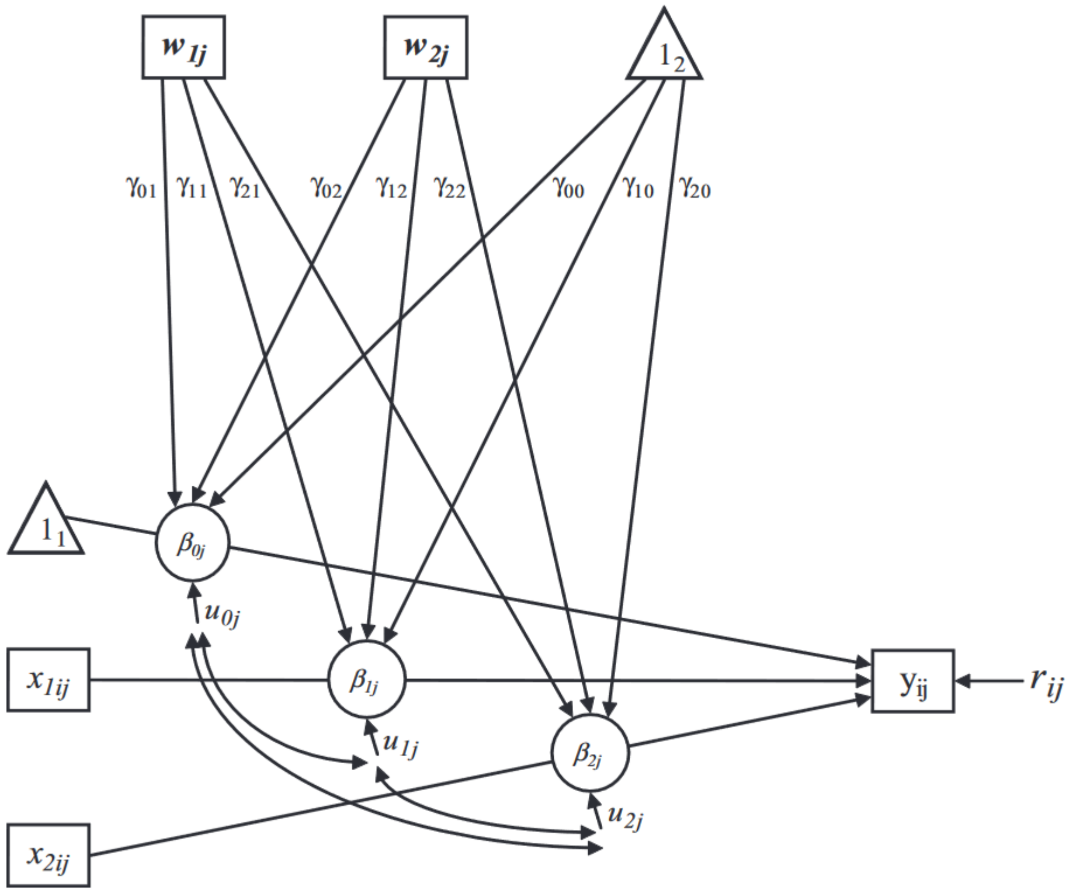
$$y_{ij} = \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + r_{ij}$$

Level 2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}w_{1j} + \gamma_{02}w_{2j} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}w_{1j} + \gamma_{12}w_{2j} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}w_{1j} + \gamma_{22}w_{2j} + u_{2j}$$



Nested Levels (CB)

Level 1 Model

$$y_{ijk} = \pi_{0jk} + \pi_{1jk}a_{1ijk} + \pi_{2jk}a_{2ijk} + e_{ijk}$$

Level 2 Model

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}x_{1jk} + \beta_{02k}x_{2jk} + r_{0jk}$$

$$\pi_{1jk} = \beta_{10k} + \beta_{11k}x_{1jk} + \beta_{12k}x_{2jk} + r_{1jk}$$

$$\pi_{2jk} = \beta_{20k}$$

Level 3 Model

$$\beta_{00k} = \gamma_{000} + \gamma_{001}w_{1k} + \gamma_{002}w_{2k} + u_{00k}$$

$$\beta_{01k} = \gamma_{010}$$

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$$\beta_{10k} = \gamma_{100} + \gamma_{101}w_{1k} + \gamma_{102}w_{2k} + u_{10k}$$

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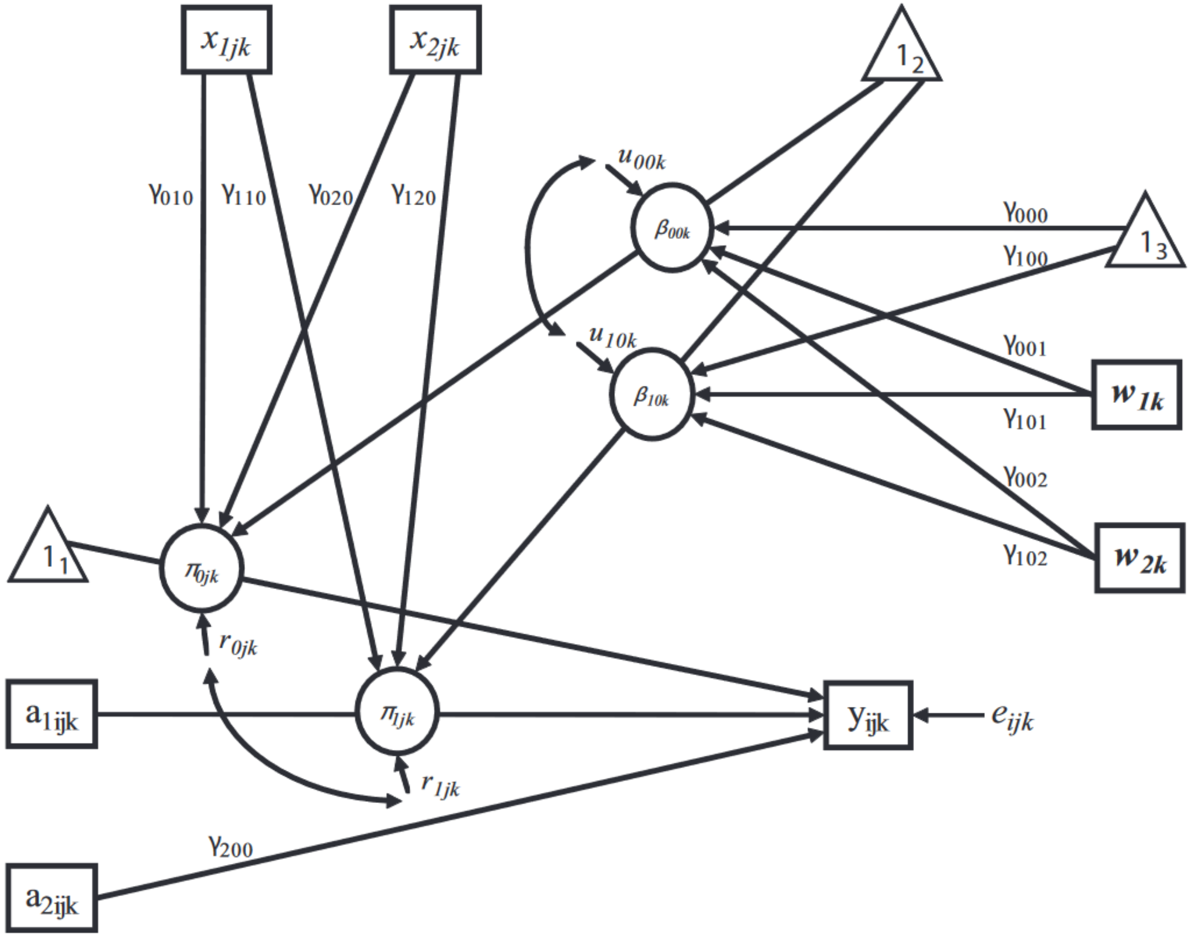


Cross-classified Levels (CB)

Level 1 Model

$$y_{i(jk)} = \beta_{0(jk)} + \beta_{1(jk)}a_{ijk} + e_{i(jk)}$$

Level 2 Model (Generic)

$$\beta_{0(jk)} = \gamma_{00} + \beta_{0j} + \beta_{0k}$$

$$\beta_{1(jk)} = \gamma_{10} + \beta_{1j} + \beta_{1k}$$

Level 2a Model (Specific)

$$\beta_{0j} = \gamma_{01}x_j + u_{0j}$$

$$\beta_{1j} = \gamma_{11}x_j + u_{1j}$$

Level 2b Model (Specific)

$$\beta_{0k} = \gamma_{02}w_k + v_{0k}$$

$$\beta_{1k} = \gamma_{12}w_k + v_{1k}$$

Not discussed in Curran & Bauer (2007)

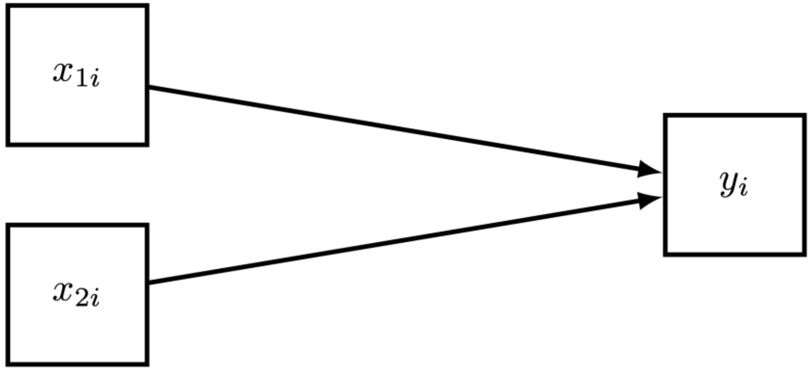
Adapted Mulder-Hamaker Approach

Structural Equation Modeling (2021)

Multiple Regression (MH)

Level 1 Model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + r_i$$



Random Intercepts (MH)

Level 1 Model

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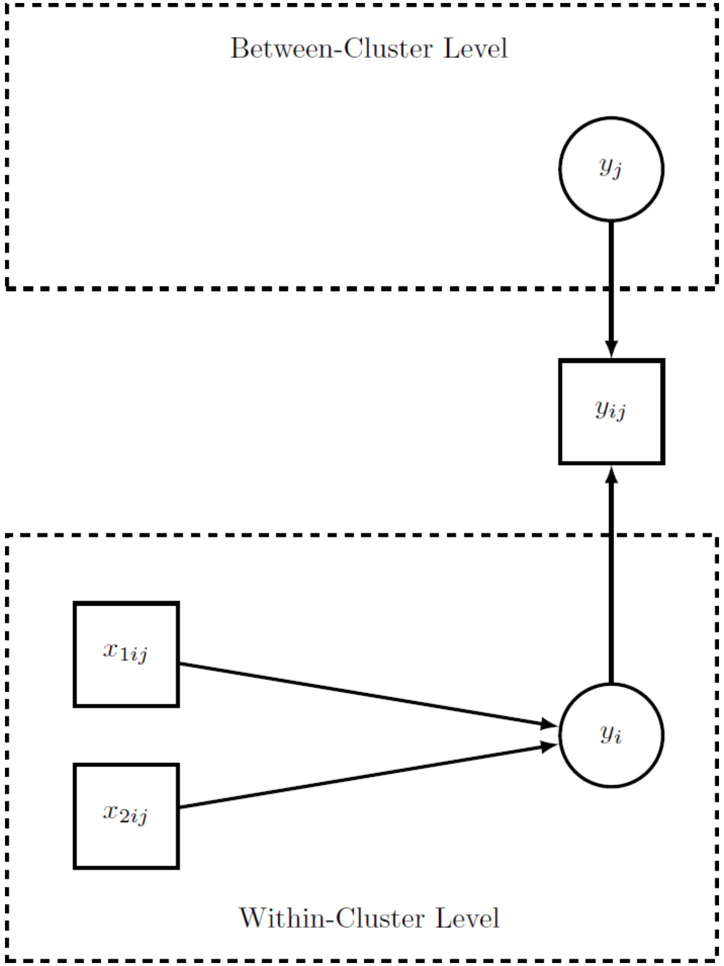

Random Slopes (MH)

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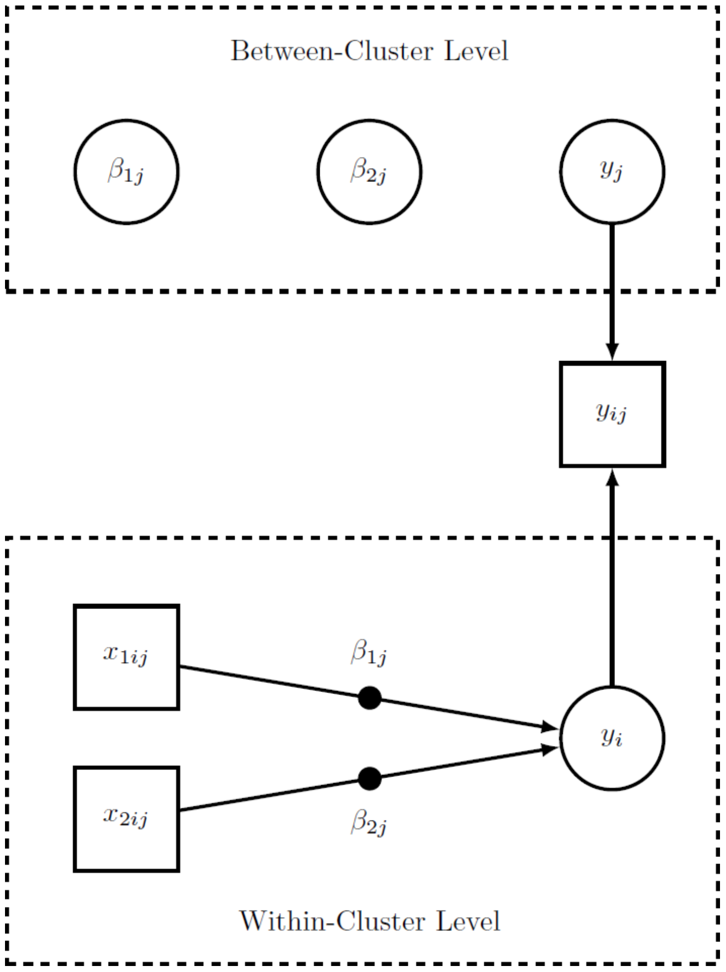

Cross-level Interactions (MH)

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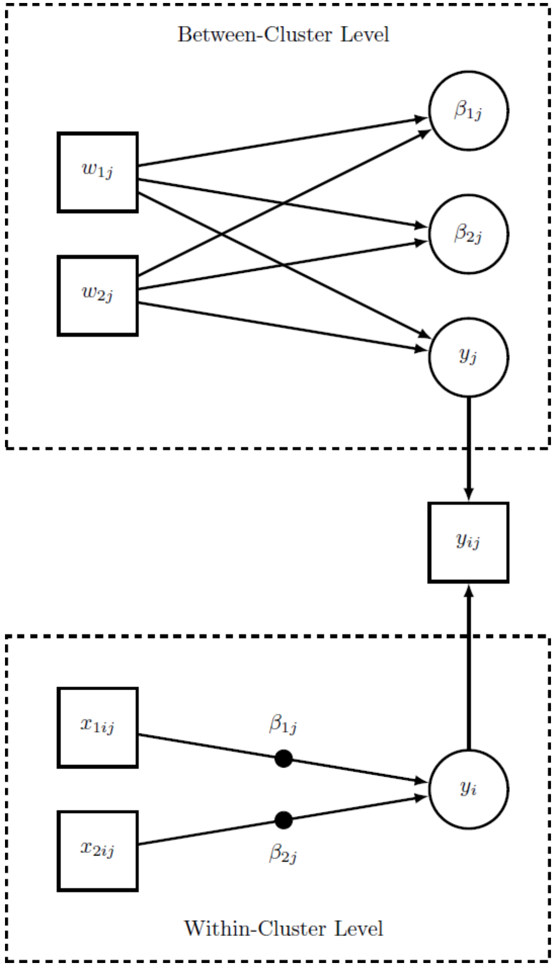

Nested Levels (MH)

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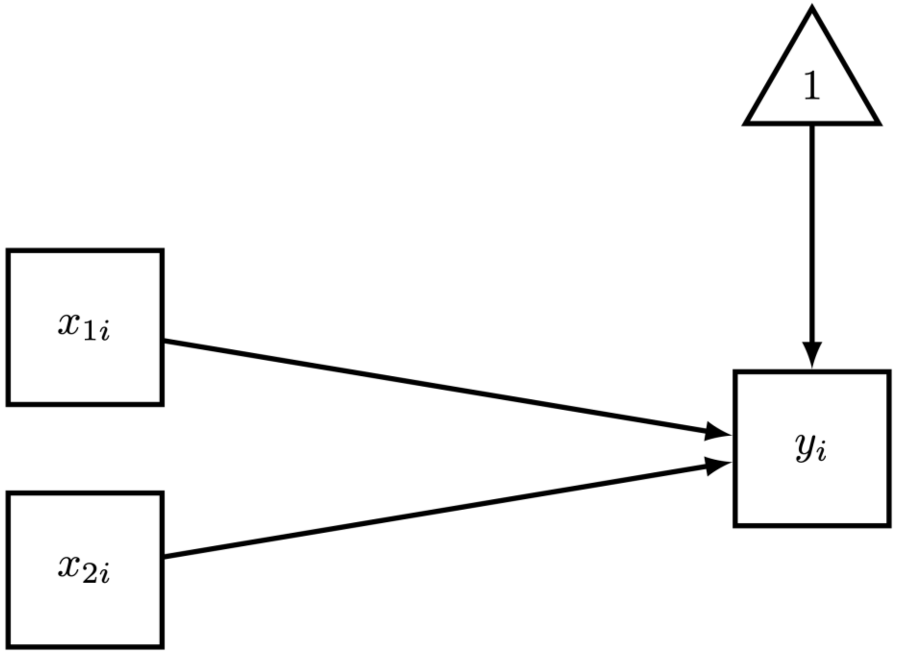
My Proposed Approach

Manuscript in preparation

Multiple Regression (G)

Level 1 Model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + r_i$$



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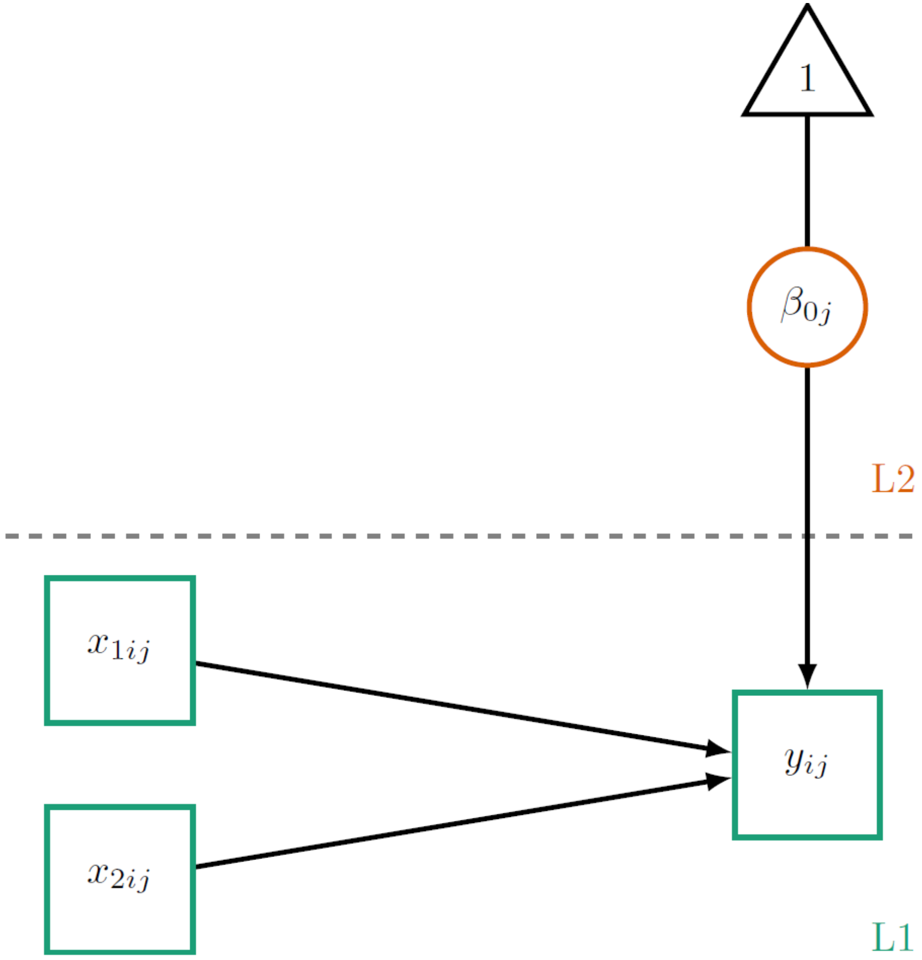

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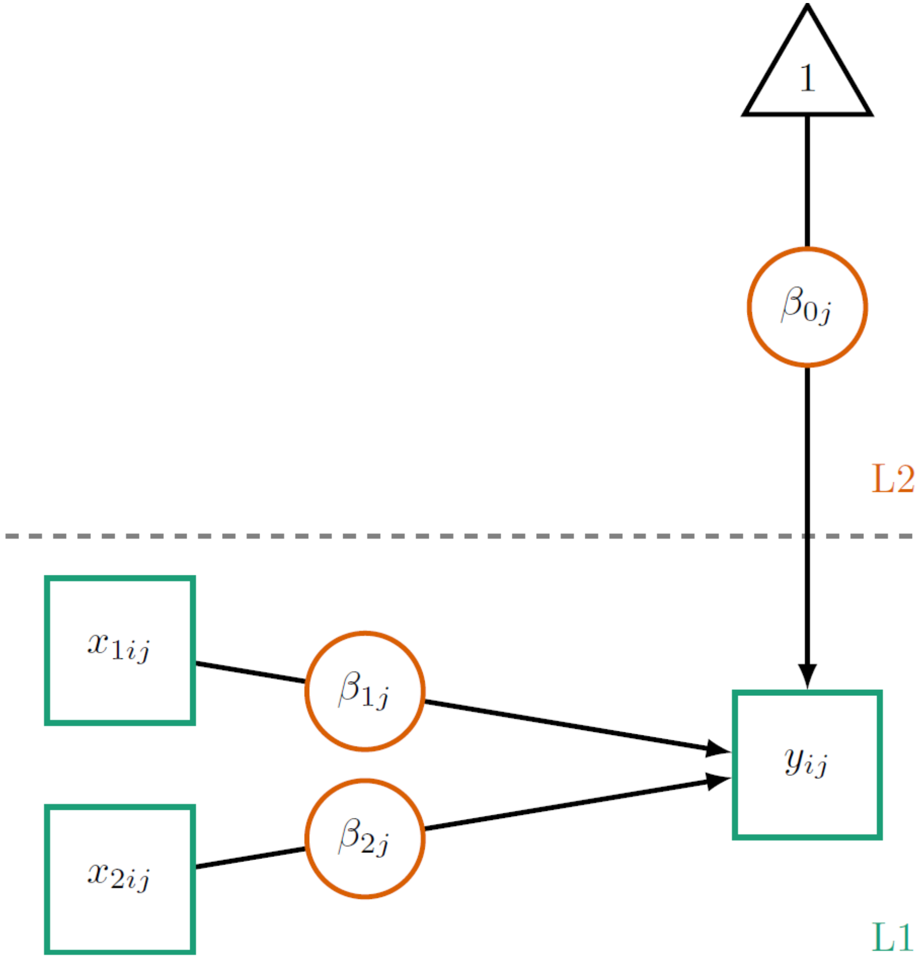

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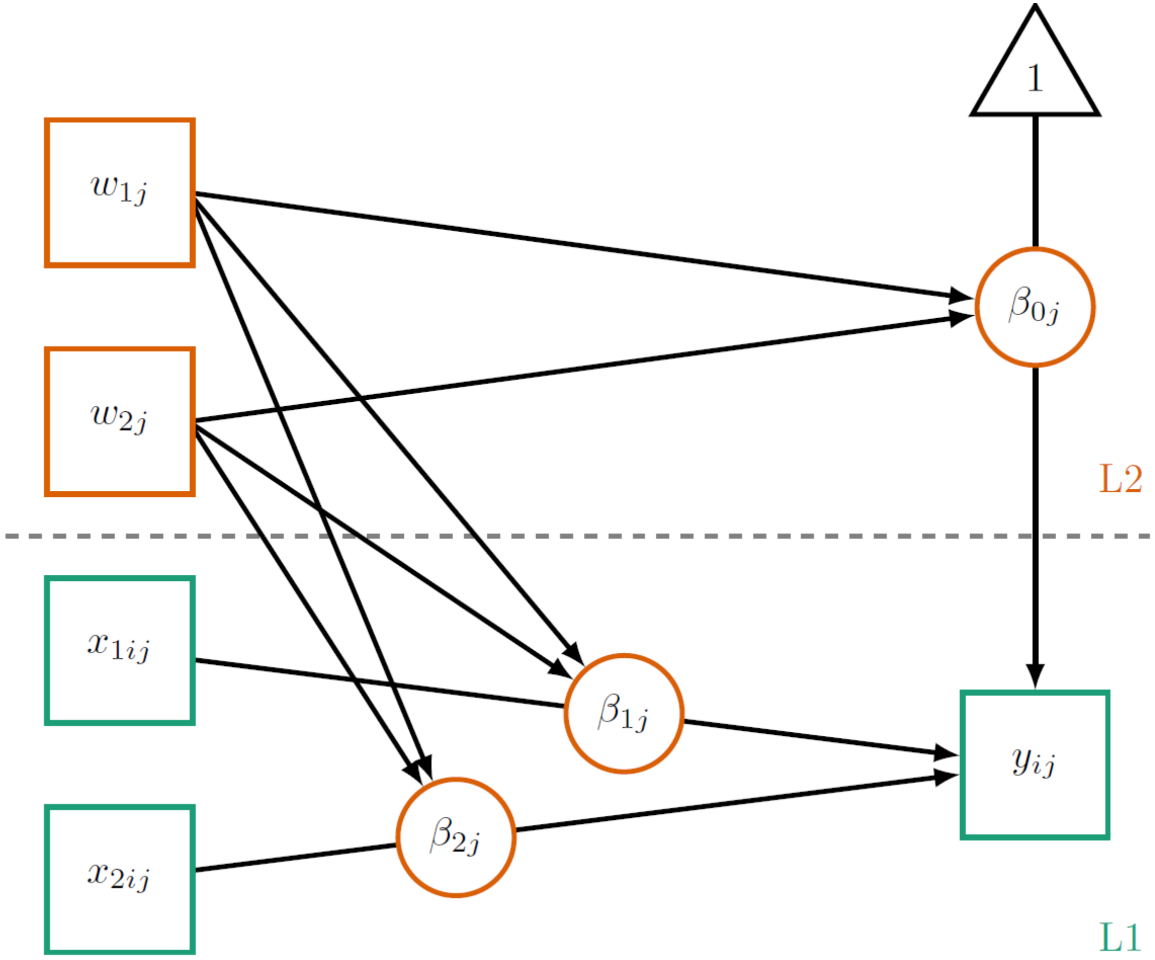

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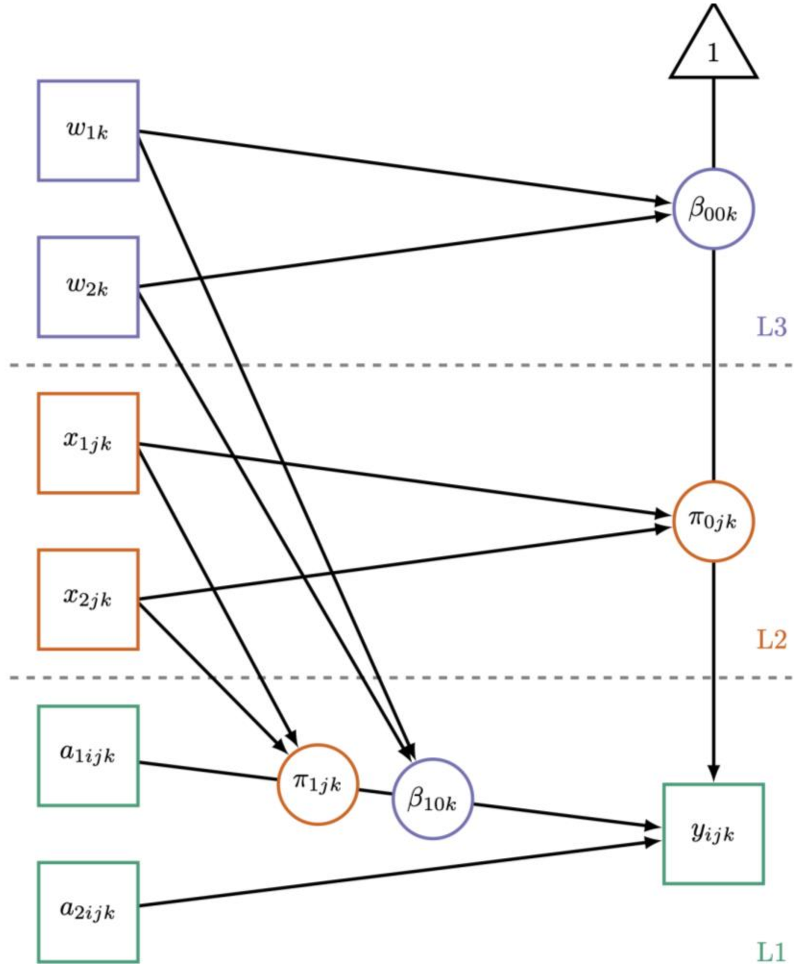

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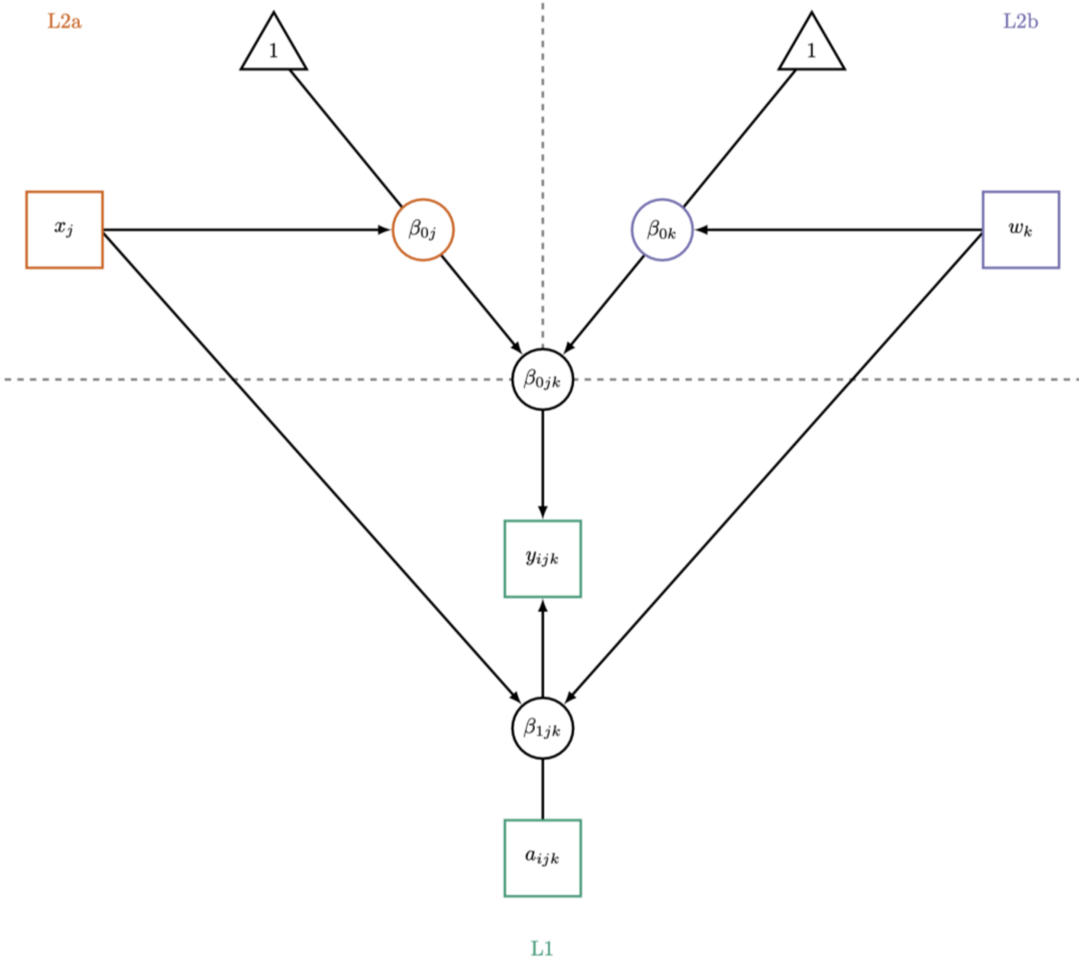
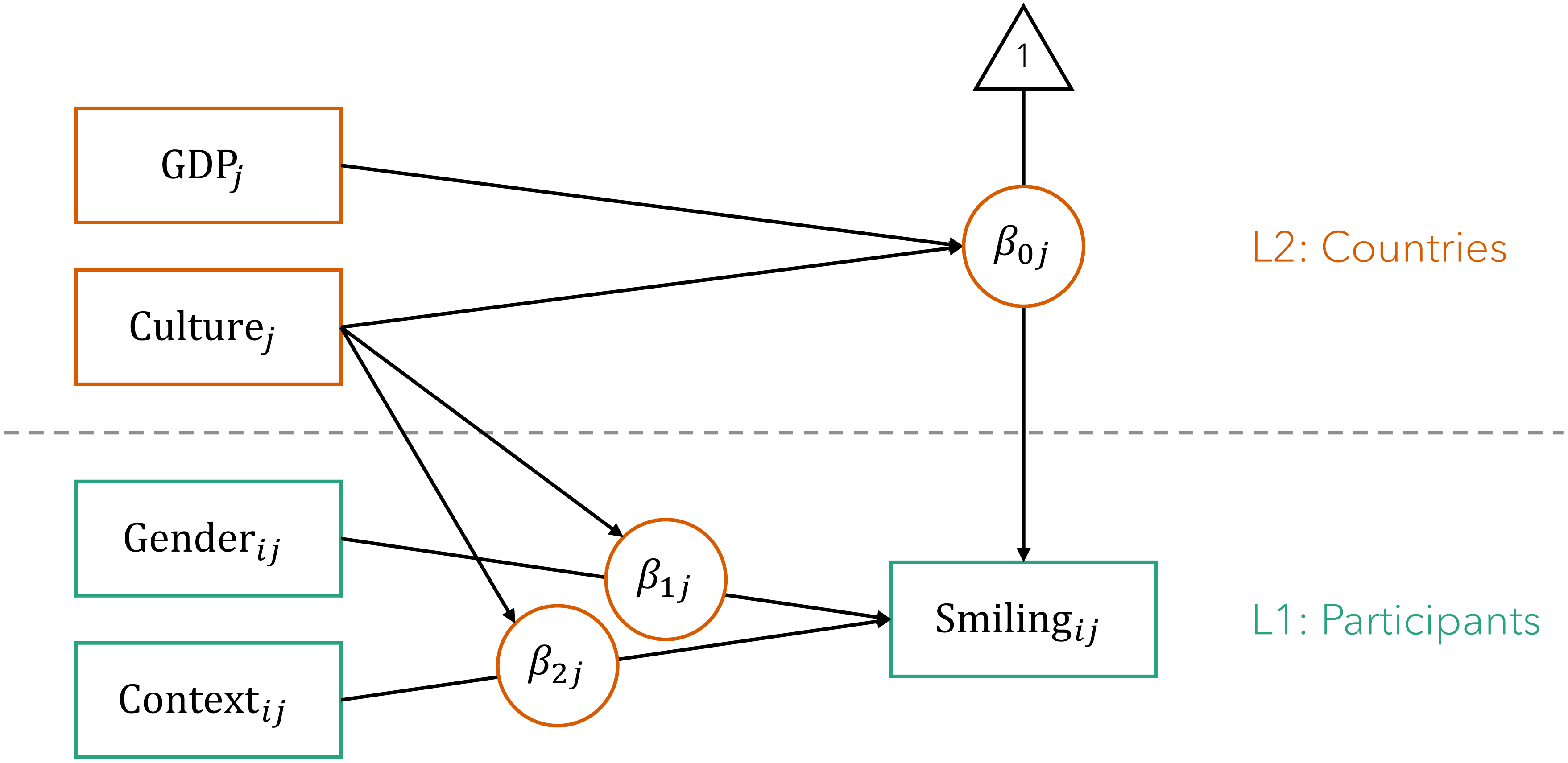


Applied Examples

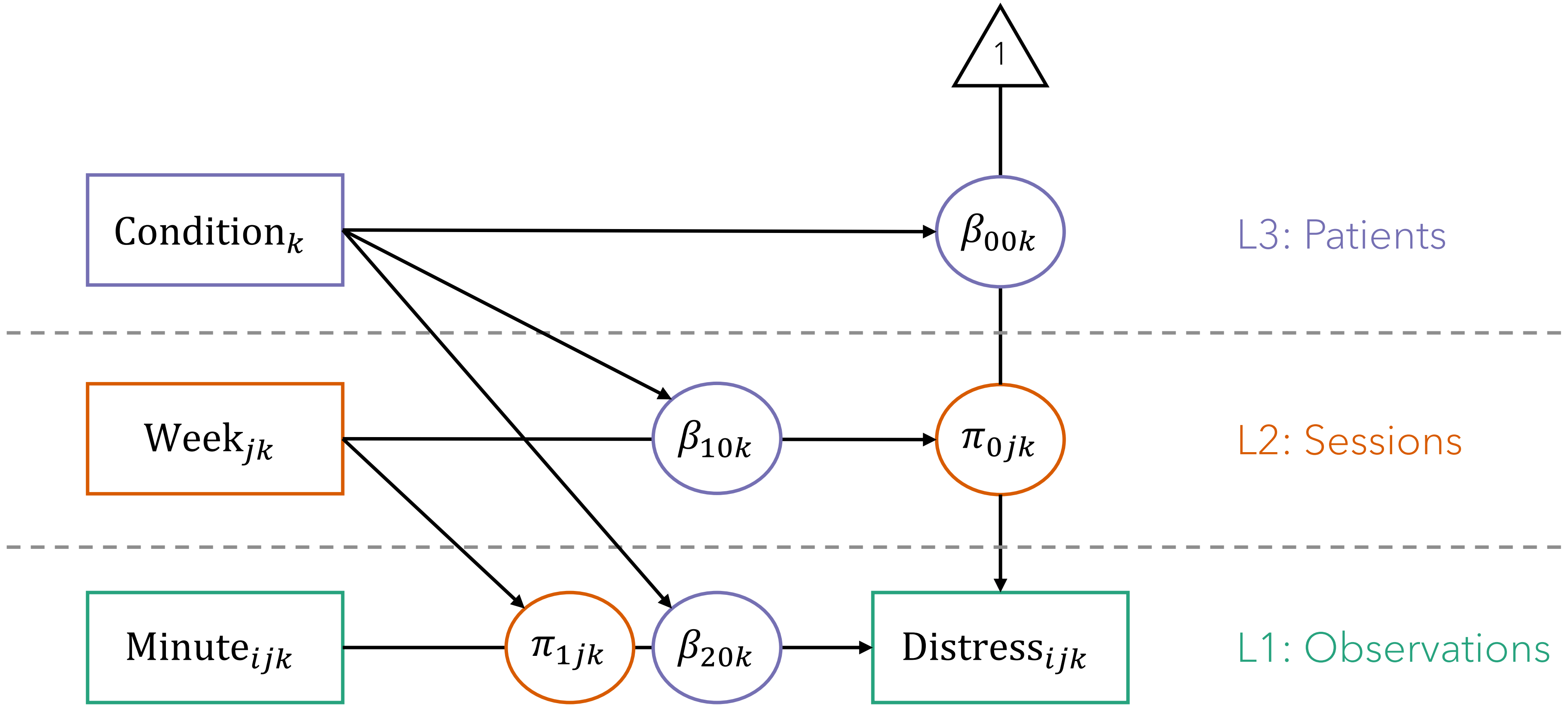
From My Research

Example #1: Culture and Gender



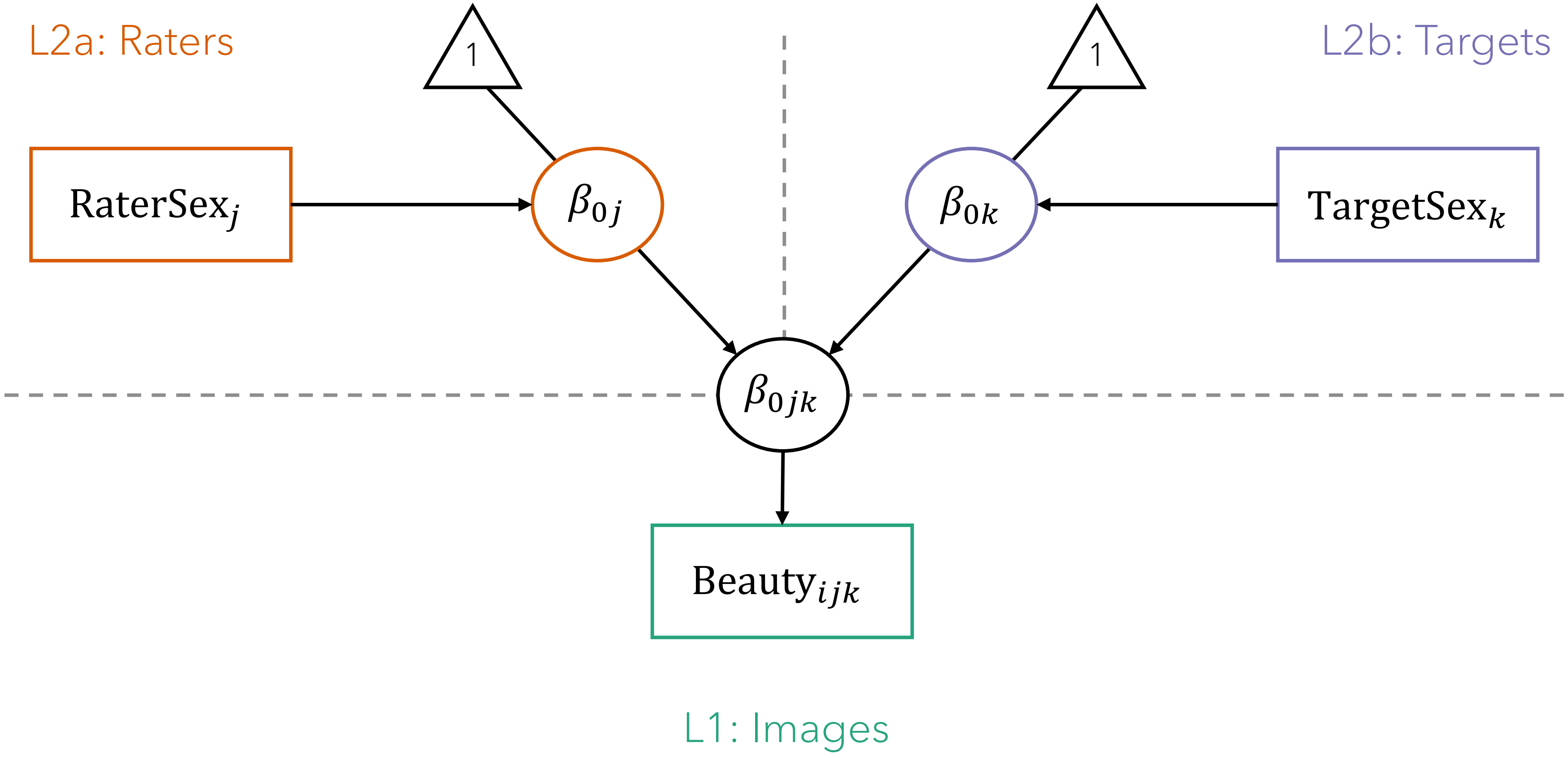
McDuff & Girard (2019)

Example #2: Multilevel Habituation



Butler, Christian, Girard, Vanzhula, & Levinson (in press)

Example #3: Perceived Attractiveness



Bowdring, Sayette, Girard, & Woods (2021)

Remaining Challenges

- Add more (4+) nested levels – pretty easy
- Finalize approach for cross-classification – pretty close
- Mixing nested and crossed levels – will it all fit?
- Incorporating results/estimates – still thinking…
  - Numbers on lines (slopes)
  - Numbers in ovals (intercepts or variances?)
- Visualize families and link functions (GLMM) – developing…
  - Oval (latent version) pointing to manifest non-normal variable?
  - Oval is what the level 1 predictors and level 2+ intercepts point to

Questions?
Suggestions?

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