

Probability Examples

Example 1

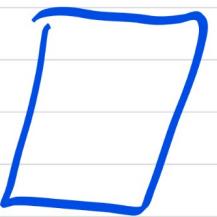
Roll a six-sided fair die. What is the probability of not rolling 2?

Solution:

Let A be the event that we roll 2. Then the complement $\underline{A^c}$

By the complement rule

$$P(\text{do not roll 2}) = P(A^c) = 1 - P(A)$$



Example 2

If you flip a fair coin 10 times, what is the probability of getting all heads

Solution:

Let A be the event that we get all heads. Observe that we can break down A as follows :

$A = \text{event first is heads} \quad \underline{\text{and}} \quad \text{event second is heads} \quad \underline{\text{and}} \quad \text{so on}$

each of these events are independent

therefore

$$P(A) = P(\text{first heads}) \cdot P(\text{second heads}), \text{ so } \cdot P(\text{tenth heads})$$

=



=

=

=



Note

$$\frac{1}{1024} \approx 0.001$$

Example 3

Suppose we flip a fair coin

10 times. What is the probability of getting at least one tails?

Solution

Let A be the event that we get at least one tails.

Observe that A^c is the event

_____. Equivalently, A^c is the event

_____. Therefore

$$\overline{P(\text{at least one tails})} = P(A) = 1 - P(A^c)$$

=



Example 4

In a multiple choice exam, there are 5 questions and 4 choices for each question (a, b, c, d). Suppose we randomly guess an answer for each question. What is the probability that the first question we get right is the 5th one?

Solution

Notice that if A is the event that the first question we get correct is the 5th one then we can break A down as

$$A =$$



these are all independent events

So

$$P(A) =$$

now, for any question

$$P(\text{question correct}) = \frac{1}{4}$$

So $P(\text{question wrong}) = 1 - P(\text{question correct}) =$ _____

therefore,

$$P(A) =$$

$$= \boxed{\quad} \approx \underline{\quad}$$

Example 5

Suppose 80% of people like peanut butter, 89% like jelly, and 78% like both.

Given that a randomly sampled person

likes peanut butter, what's the

Probability they also like jelly ?

Solution

Let $A =$ _____

$B =$ _____

$A \text{ and } B =$ _____

We want to compute $P(A | B)$

By the formula for conditional probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

So

$$P(A|B) =$$

$$= \boxed{\quad}$$

Example 6

The Behavioral Risk Factor Surveillance Survey (BRFSS) is an annual telephone survey designed to identify risk factors in the adult population and report emerging health trends. The following table displays the distribution of health status of respondents to this survey (excellent, very good, good, fair, poor) and whether or not they have health insurance.

Health Coverage		Health Status					Total
		Excellent	Very good	Good	Fair	Poor	
No		0.023	0.0364	0.0427	0.0182	0.005	0.1262
Yes		0.2099	0.3123	0.2410	0.0817	0.0289	0.8738
Total		0.2329	0.3486	0.2838	0.1009	0.0338	1.00

(a) Are being in excellent health and having health coverage mutually exclusive?

Solution to (a)

No, there are individuals who are both excellent in health and have health coverage.

(c) What is the probability that a randomly chosen individual has excellent health given that they have health coverage?

Solution to (c)

Let A = event individual has excellent health
 B = event individual has health coverage

then $P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \boxed{\frac{0.2099}{0.8738}}$

$$\approx 0.24$$

(e) Do having excellent health and having health coverage appear to be independent?

Solution to (e)

No, notice that $P(A|B) \approx 0.24$

but $P(A) = 0.2329$ and $P(A) = P(A|B)$

if A and B were independent.