Probability Examples

Example 1

Roll a six-sided fair die. What is the probability of not rolling 2?

Solution:

Let A be the event that we roll 2. Then the complement A^c ________

____

By the complement rule

$$P(\text{do not roll } 2) = P(A^c) = 1 - P(A)$$

Example 2

If you flip a fair coin 10 times, What is the probability of getting all heads

Solution:

Let A be the event that we get all heads. Observe that we can break down A as follows:

$$A = \underbrace{\begin{matrix}\text{event first} \\ \text{is heads}\end{matrix} \ \underline{\text{and}} \ \begin{matrix}\text{event second} \\ \text{is heads}\end{matrix} \ \underline{\text{and}} \ \text{so on}}_{\text{each of these events are independent}}$$

each of these events are <u>independent</u>

therefore

$$P(A) = P\begin{pmatrix}\text{first} \\ \text{heads}\end{pmatrix} \cdot P\begin{pmatrix}\text{second} \\ \text{heads}\end{pmatrix} \cdot \begin{matrix}\text{so} \\ \text{on}\end{matrix} \cdot P\begin{pmatrix}\text{tenth} \\ \text{heads}\end{pmatrix}$$

$=$

$= \qquad = \qquad = \boxed{}$

note $\frac{1}{1024} \approx 0.001$

Example 3

Suppose we flip a fair coin

10 times. What is the probability of getting at least one tails?

Solution

Let A be the event that we get at least on tails.

Observe that A^c is the event ____________________ ______. Equivalently, A^c is the event ____________________ ______. Therefore

$$P\begin{pmatrix}\text{at least}\\ \text{one tails}\end{pmatrix} = P(A) = 1 - P(A^c)$$

$= $ ______________________________

$= \boxed{} \approx$ ____________________

Example 4

In a multiple choice exam, there are 5 questions and 4 choices for each question (a, b, c, d). Suppose we randomly guess an answer for each question. What is the probability that the first question we get right is the 5^{th} one?

Solution

Notice that if A is the event that the first question we get correct is the 5^{th} one the we can break A down as

$A =$

these are all independent events

So

$P(A) =$

now, for any question

$$P\left(\text{question correct}\right) = \frac{1}{4}$$

So $P\left(\text{question wrong}\right) = 1 - P\left(\text{question correct}\right) =$ ______

therefore,

$$P(A) = \frac{\qquad\qquad}{\qquad\qquad} = \frac{\boxed{\qquad}}{\qquad} \approx \frac{\qquad}{\qquad}$$

Example 5

Suppose 80% of people like peanut butter, 89% like jelly, and 78% like both. Given that a randomly sampled person likes peanut butter, what's the probability they also like jelly?

Solution

Let A = ______

B = ______

A and B = ______

we want to compute $P(A \mid B)$

By the formula for conditional probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

so

$$P(A \mid B) = \underline{\qquad\qquad} = \boxed{\qquad\qquad}$$

Example 6

The Behavioral Risk Factor Surveillance Survey (BRFSS) is an annual telephone survey designed to identify risk factors in the adult population and report emerging health trends. The following table displays the distribution of health status of respondents to this survey (excellent, ver good, good, fair, poor) and whether or not they have health insurance.

Health Coverage	Health Status: Excellent	Very good	Good	Fair	Poor	Total
no	0.023	0.0364	0.0427	0.0192	0.005	0.1262
yes	0.2099	0.3123	0.2410	0.0817	0.0289	0.8738
Total	0.2329	0.3486	0.2838	0.1009	0.0338	1.00

(a) Are being in excellent health and having health coverage mutually exclusive?

Solution to (a)

No, there are individuals who are both excellent in health and have health coverage.

(c) What is the probability that a randomly chosen individual has excellent health given that they have health coverage?

Solution to (c)

Let A = event individual has excellent health
B = event individual has health coverage

then $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \boxed{\dfrac{0.2099}{0.8738}}$

≈ 0.24

(e) Do having excellent health and having health coverage appear to be independent?

Solution to (e)

No, notice that $P(A|B) \approx 0.24$

but $P(A) = 0.2329$ and $P(A) = P(A|B)$

if A and B were independent.