$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Step 1 program is A

$\mathbb{C}(q) \langle x_i^{\pm}\ y_1^{\pm}\ x_2^{\pm}\ y_2^{\pm} \rangle \Big/ \ y_i x_i = q x_i y_i$ else commute.

Step 2 tell computer our basis is

$1 \quad x_2 \quad \cdots \quad x_2^5$

$y_2 \quad x_2 y_2 \quad \cdots \quad x_2^5 y_2$

via rules:

2a monomial $x_1^{a_1} y_1^{a_2} x_2^{b_1} y_2^{b_2}$

$$[x_1^{a_1} y_1^{a_2},\ x_2^{b_1} y_2^{b_2}] = x_1^{a_1} y_1^{a_2} x_2^{b_1} y_2^{b_2} - x_2^{b_1} y_2^{b_2} (x_2^2 y_2)^{a_1} (x_2^3 y_2^2)^{a_2}$$

$\boxed{\text{Warning}}$: $(x_2^2 y_2)^2 = x_2^2 y_2 x_2^2 y_2 = q\, x_2^4 y_2^2$

$(x_2^2 y_2)^{-1} = y_2^{-1} x_2^{-2}$

2b given $x_2^a y_2^b = M$ express in

$1 \quad x_2 \quad \cdots \quad x_2^5$

$y_2 \quad x_2y_2 \quad \cdots \quad x_2^5 y_2$

Use

$8 y_2^2 . \underbrace{M} - x_1 x_2^2 y_2 \underbrace{M} \; y_2^{-1} x_2^{-2} x_1^{-1}$ (1)

$\underbrace{M} - y_2^{-2} y_1^{-1} x_2^3 \underbrace{M} x_2^3 y_1 y_2^2$ (2)

If $0 \le a < 6$ good. If not replace a w/ $a+6$, plug M into (2) or plug in $x_2^{a-6} y_2^b$ for M

If $0 \le b < 2$ good. Else replace b with $b \pm 2$ using (1)

Step 3 ~~take~~ $\mathrm{Hdc}(A; A_{g(\omega)})$ ← is 12 dim here, then

find subvector space $\mathrm{Im}(1-g)$

and quotient out.

$g \in C(2)$

$(1-g)$ span

$\{ x_1^{a_1} y_1^{b_1} x_2^{a_2} y_2^{b_2} - x_1^{a_2} y_1^{b_2} x_2^{a_1} y_2^{b_1} \}$ = subvector space

HH / subvector space. all $(a_1, b_1, a_2, b_2) \in \mathbb{Z}^4$