

### 2.3 Overview

In the third column the “rule” is always considered with respect to  $\tilde{w}$ , i.e. the partition without repeated  $\lambda_i$ 's. To get the dimension of  $\text{HH}_0(A, A_{\gamma w})_{C_W(w)}$  one takes the product of all the contributions corresponding to  $\lambda_i$  in  $\tilde{w}$ .

$\gamma$	$\text{tr}(\gamma)$	“Rule” for $\dim(\text{HH}_0(A, A_{\gamma w}))$	Generating function
$S$ and $S^3 = -S$	0	$\lambda_i \rightsquigarrow 1$ if $\lambda_i \equiv 0 \pmod{4}$ $\lambda_i \rightsquigarrow 2$ else	$\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \pmod{4} \\ 1 + \frac{2x^k}{1-x^k} & \text{else} \end{cases}$
$E_+ = TS$ and $E_- = (TS)^{-1}$	1	$\lambda_i \rightsquigarrow 1$ (NOTE: regular)	$\prod_{k=1}^{\infty} \frac{1}{1-x^k}$
$-E_+$ and $-E_-$	-1	$\lambda_i \rightsquigarrow 1$ if $\lambda_i \equiv 0 \pmod{3}$ $\lambda_i \rightsquigarrow 3$ else	$\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \pmod{3} \\ 1 + \frac{3x^k}{1-x^k} & \text{else} \end{cases}$
$S^4 = \text{Id}$	2	$\lambda_i \rightsquigarrow 1$	$\prod_{k=1}^{\infty} \frac{1}{1-x^k}$
$S^2 = -\text{Id}$	-2	<i>odd</i> $\lambda_i \rightsquigarrow 4$ <i>even</i> $\lambda_i \rightsquigarrow 1$	$\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ \frac{1}{1-x^k} & \text{if } k \text{ is even} \end{cases}$
$T^m$ for $m \in \mathbb{Z}_0$	2	$\lambda_i \rightsquigarrow  m \lambda_i$	$\prod_{k=1}^{\infty} \left[ 1 + \frac{ m kx^k}{1-x^k} \right]$
$-T^m$ for $m \in \mathbb{Z}_0$	-2	<i>odd</i> $\lambda_i \rightsquigarrow 4$ <i>even</i> $\lambda_i \rightsquigarrow 2$	$\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ 1 + \frac{2x^k}{1-x^k} & \text{if } k \text{ is even} \end{cases}$
<i>regular</i>	$ \text{tr}(\gamma)  > 2$	$\lambda_i \rightsquigarrow  \text{tr}(\text{Id} - \gamma) $	$\prod_{k=1}^{\infty} \left[ 1 + \frac{ \text{tr}(\text{Id} - \gamma) x^k}{1-x^k} \right]$

Note that if you do not like case distinctions in the generating functions, you could also write:

$$\begin{aligned}
\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \pmod{3} \\ 1 + \frac{3x^k}{1-x^k} & \text{else} \end{cases} &= \prod_{k=0}^{\infty} \left[ \frac{1}{1-x^{3k}} \left( 1 + \frac{3x^{3k+1}}{1-x^{3k+1}} \right) \left( 1 + \frac{3x^{3k+2}}{1-x^{3k+2}} \right) \right] \\
\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \pmod{4} \\ 1 + \frac{2x^k}{1-x^k} & \text{else} \end{cases} &= \prod_{k=0}^{\infty} \left[ \frac{1}{1-x^{4k}} \left( 1 + \frac{2x^{4k+1}}{1-x^{4k+1}} \right) \left( 1 + \frac{2x^{4k+2}}{1-x^{4k+2}} \right) \left( 1 + \frac{2x^{4k+3}}{1-x^{4k+3}} \right) \right] \\
\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ \frac{1}{1-x^k} & \text{if } k \text{ is even} \end{cases} &= \prod_{k=1}^{\infty} \left[ 1 + \frac{(2, 5-1, 5(-1)^k)x^k}{1-x^k} \right] \\
\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ 1 + \frac{2x^k}{1-x^k} & \text{if } k \text{ is even} \end{cases} &= \prod_{k=1}^{\infty} \left[ 1 + \frac{(3 - (-1)^k)x^k}{1-x^k} \right]
\end{aligned}$$