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Also note $x_m \cdots x_3 x_2 x_1$ is clearly a fixed pt $= x^{\begin{bmatrix}1\\0\end{bmatrix}} x^{A\begin{bmatrix}1\\0\end{bmatrix}} x^{A^2\begin{bmatrix}1\\0\end{bmatrix}} \cdots x^{A^{m-1}\begin{bmatrix}1\\0\end{bmatrix}}$

$$= x^{(I+A+A^2+\cdots+A^{m-1})\begin{bmatrix}1\\0\end{bmatrix}}$$

Similar for $y_m \cdots y_2 y_1 = x^{(I+A+\cdots+A^{m-1})\begin{bmatrix}0\\1\end{bmatrix}}$

As $\det(A-I)\det(I+A+\cdots+A^{m-1}) = \det(A^m - I) = \dim HH_0$

it makes sense $\det(A-I)$ tells us how many distinct fixed pts A has mod $(A^m - I)$

If $r \mid m$ and we search for fixed pts of A^r $\quad m = rh$

we can construct same as $\underset{\underset{rh}{\shortparallel}}{X_m}\ \underset{\underset{r(h-1)}{\shortparallel}}{X_{m-r}} \cdots X_{2r} X_r$

$$= x^{\begin{bmatrix}1\\0\end{bmatrix}} x^{A^r\begin{bmatrix}1\\0\end{bmatrix}} x^{A^{2r}\begin{bmatrix}1\\0\end{bmatrix}} \cdots x^{A^{(h-1)r}\begin{bmatrix}1\\0\end{bmatrix}} = x^{(I+A^r+A^{2r}+\cdots+A^{(h-1)r})\begin{bmatrix}1\\0\end{bmatrix}}$$

and $\det(A^r - I)\det(I+A^r+\cdots+A^{(h-1)r}) = \det(A^{hr}-I) = \det(A^m - I)$

so I expect $\det(A^r - I)$ many fixed pts. ← have not really checked

If so, avg # f.p. are:

	I	A	A^r	$\cdots$
# elts w/ same order	1	$\phi(m)$	$\phi(m/r)$	
# f.p ??	$\det(A^m - I)$	$\det(A-I)$	$\det(A^r - I)$	

Average $\frac{1}{m} \sum_{\substack{r \mid m \\ \text{divisors}}} \phi\left(\frac{m}{r}\right) \det(A^r - I)$

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