2.3 Overview

In the third column the "rule" is always considered with respect to $\tilde{w}$, i.e. the partition without repeated $\lambda_i' s$. To get the dimension of $\mathrm{HH}_0(A, A_{\gamma w})_{C_W(w)}$, we know by Lemma 2.1 that it is enough to take the product of all the contributions corresponding to λ_i in $\tilde{w}$.

γ	$\mathrm{tr}(\gamma)$	"Rule" for $\dim(\mathrm{HH}_0(A, A_{\gamma w})_{C_W(w)})$	Generating function
S and $S^3 = -S$	0	$\lambda_i \rightsquigarrow 1$ if $\lambda_i \equiv 0 \mod 4$ $\lambda_i \rightsquigarrow 2$ else	$\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \mod 4 \\ 1 + \frac{2x^k}{1-x^k} & \text{else} \end{cases}$
$E_+ = TS$ and $E_- = (TS)^{-1}$	1	$\lambda_i \rightsquigarrow 1$ (NOTE: regular)	$\prod_{k=1}^{\infty} \frac{1}{1-x^k}$
$-E_+$ and $-E_-$	-1	$\lambda_i \rightsquigarrow 1$ if $\lambda_i \equiv 0 \mod 3$ $\lambda_i \rightsquigarrow 3$ else	$\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \mod 3 \\ 1 + \frac{3x^k}{1-x^k} & \text{else} \end{cases}$
$S^4 = \mathrm{Id}$	2	$\lambda_i \rightsquigarrow 1$	$\prod_{k=1}^{\infty} \frac{1}{1-x^k}$
$S^2 = -\mathrm{Id}$	-2	*odd* $\lambda_i \rightsquigarrow 4$ *even* $\lambda_i \rightsquigarrow 1$	$\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ \frac{1}{1-x^k} & \text{if } k \text{ is even} \end{cases}$
T^m for $m \in \mathbb{Z}_0$	2	$\lambda_i \rightsquigarrow \|m\|\lambda_i$	$\prod_{k=1}^{\infty} \left[1 + \frac{\|m\|kx^k}{1-x^k}\right]$
$-T^m$ for $m \in \mathbb{Z}_0$	-2	*odd* $\lambda_i \rightsquigarrow 4$ *even* $\lambda_i \rightsquigarrow 2$	$\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ 1 + \frac{2x^k}{1-x^k} & \text{if } k \text{ is even} \end{cases}$
regular	$\|\mathrm{tr}(\gamma)\| > 2$	$\lambda_i \rightsquigarrow \|\mathrm{tr}(\mathrm{Id} - \gamma)\|$	$\prod_{k=1}^{\infty} \left[1 + \frac{\|\mathrm{tr}(\mathrm{Id}-\gamma)\|x^k}{1-x^k}\right]$

Note that if you do not like case distinctions in the generating functions, you could also write:

$$\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \mod 3 \\ 1 + \frac{3x^k}{1-x^k} & \text{else} \end{cases} = \prod_{k=0}^{\infty} \left[\frac{1}{1-x^{3k}}(1 + \frac{3x^{3k+1}}{1-x^{3k+1}})(1 + \frac{3x^{3k+2}}{1-x^{3k+2}})\right]$$

$$\prod_{k=1}^{\infty} \begin{cases} \frac{1}{1-x^k} & \text{if } k \equiv 0 \mod 4 \\ 1 + \frac{2x^k}{1-x^k} & \text{else} \end{cases} = \prod_{k=0}^{\infty} \left[\frac{1}{1-x^{4k}}(1 + \frac{2x^{4k+1}}{1-x^{4k+1}})(1 + \frac{2x^{4k+1}}{1-x^{4k+2}})(1 + \frac{2x^{4k+1}}{1-x^{4k+3}})\right]$$

$$\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ \frac{1}{1-x^k} & \text{if } k \text{ is even} \end{cases} = \prod_{k=1}^{\infty} \left[1 + \frac{(2,5 - 1,5(-1)^k)x^k}{1-x^k}\right]$$

$$\prod_{k=1}^{\infty} \begin{cases} 1 + \frac{4x^k}{1-x^k} & \text{if } k \text{ is odd} \\ 1 + \frac{2x^k}{1-x^k} & \text{if } k \text{ is even} \end{cases} = \prod_{k=1}^{\infty} \left[1 + \frac{(3 - (-1)^k)x^k}{1-x^k}\right]$$