Lecture 06:
Active Contours & Intelligent Scissors

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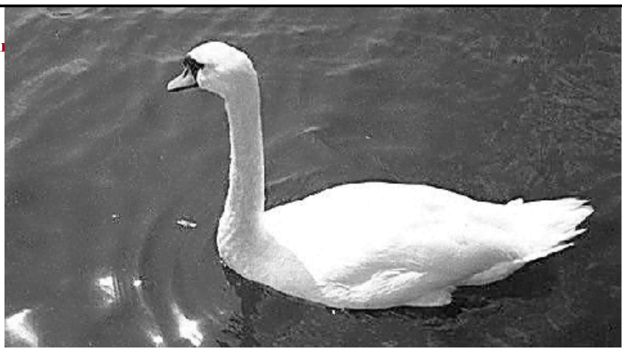
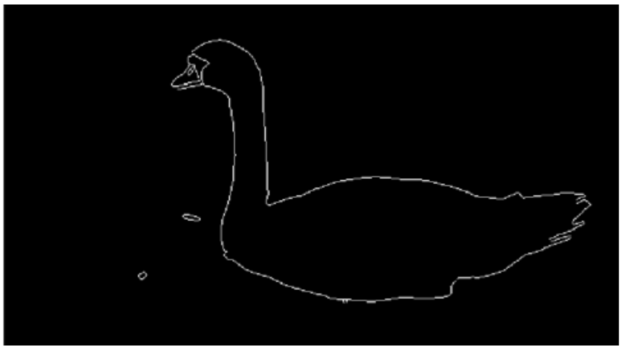
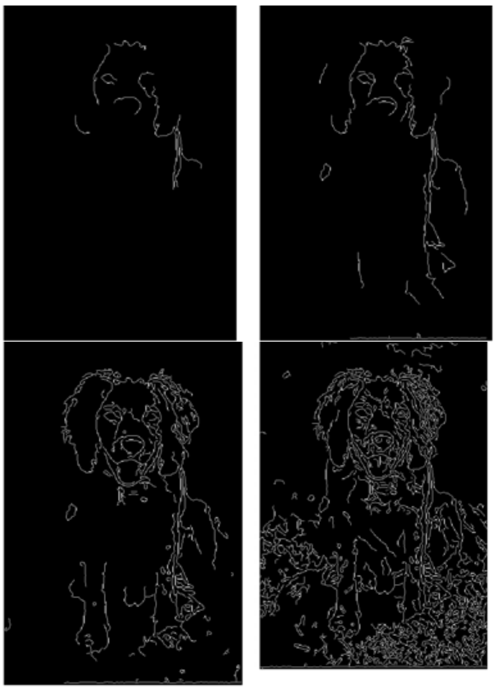
Active Contours

- Raises level of image feature description from edges to boundaries.
- Edge is strong change in pixel intensity.
- Boundary is boundary of an *object.*
  - – Smooth (more or less)
  - – Closed





Sometimes edge detectors find the boundary pretty well.

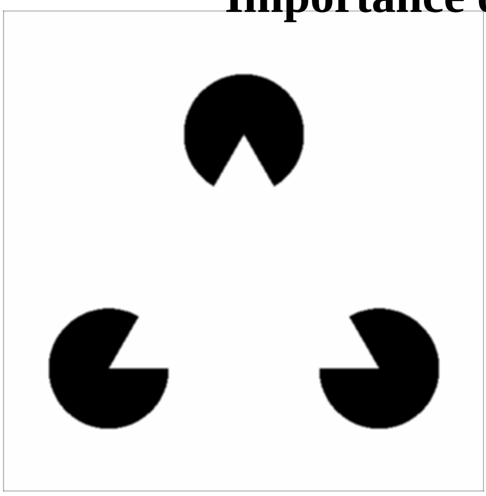


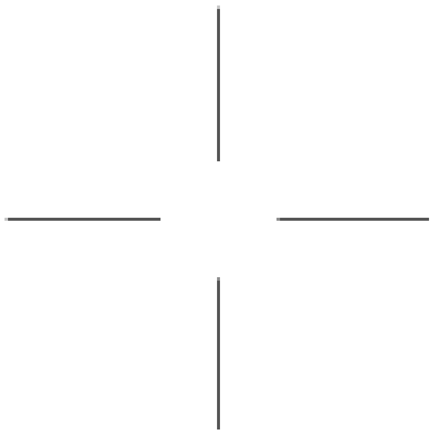
Sometimes they don’t

Improved Boundary Detection

- Integrate information over distance.
- Use shape cues
  - Smoothness
  - Closure
- Get User to Help.

Importance of Continuity



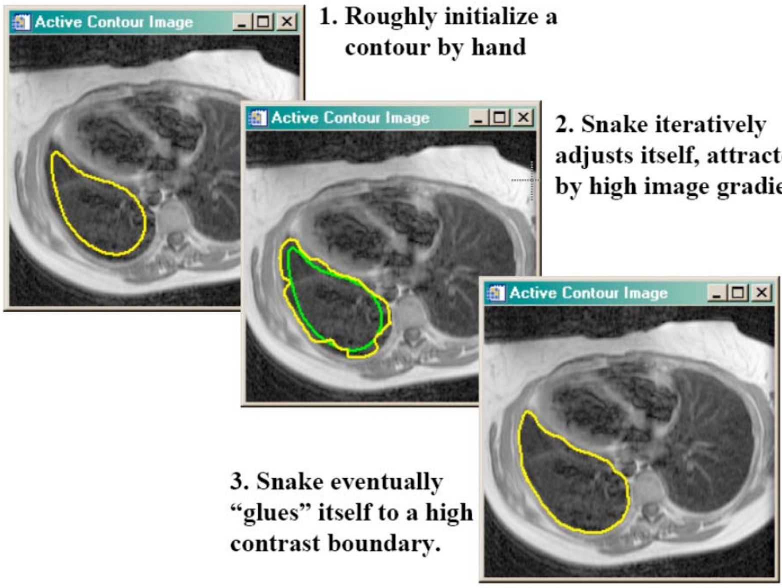


Human perception is good at integrating contour continuity information into boundary detection process

Active Contours

- They are also called
  - Snakes
  - Deformable Contours
- Think of a snake as an elastic band:
  - of arbitrary shape
  - sensitive to image gradient
  - that can “wiggle” in the image
  - represented as a sequential list of points

Main Idea:



1. Roughly initialize a contour by hand

2. Snake iteratively adjusts itself, attracted by high image gradients

3. Snake eventually “glues” itself to a high contrast boundary.

The Energy Functional

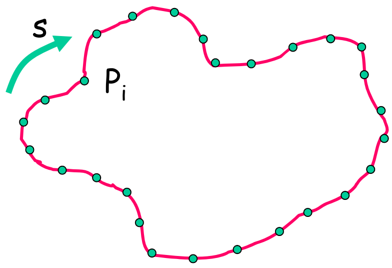
- Associate to each possible shape and location of the snake a value E.
  - Values should be s.t. the image contour to be detected has the *minimum* value.
  - E is called the energy of the snake.
- Iteratively adjust points on the snake to achieve a smaller energy E

Energy Functional Design

- We need a function that given a snake state, associates to it an Energy value.
- The function should be designed so that the snake moves towards the contour that we are seeking!

Snake "state": Contour Parametrization

Consider a contour parametrization c=c(s)
where s is the "arc length"



Each point P_i on the contour
has coordinates $(x_i(s),y_i(s))$

What moves the snake?

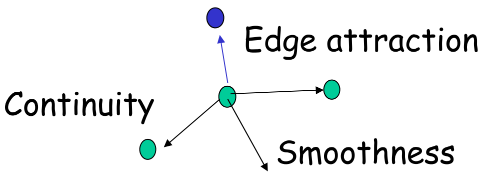
Forces applied to its points

External Forces:

- boundaries in the image (gradients)
- term associated with the DATA

Internal Forces:

- continuity and curvature
- terms associated with the CURVE



Forces moving the snake (External)

- It needs to be attracted to contours:
  - Edge pixels “pull” the snake points.
  - The stronger the edge, the stronger the pull.
  - The force is proportional to $|\nabla I|$

Edgeness Term

Given a snake with N points $p_1, p_2, \dots, p_N$

Define the edgeness term of the Energy Functional:

$$E_g(p_i) = -\|\nabla I(p_i)\|$$

Magnitude of the gradient should be LARGE
(which will make this term SMALL (very negative)

Forces preserving the snake (Internal)

- The snake should not break apart!
  - Points on the snake must stay close to each other
  - The farther the neighbor, the stronger the force to pull them back together
  - The force is proportional to the distance $|P_i - P_{i-1}|$

Continuity Term

Given a snake with N points $p_1, p_2, \ldots, p_N$
Let d be the average distance between points

Define the continuity term of the Energy Functional:

$$E_c(p_i) = (d - \underbrace{\| p_i - p_{i-1} \|})^2$$

Distance between points should be kept close to average

Forces preserving the snake (Internal)

The snake contour should be "smooth"

- – Penalize high curvature.
- – Force proportional to snake curvature

Smoothness Term

Given a snake with N points $p_1, p_2, \ldots, p_N$
Curvature should be kept small

Define the smoothness term of the Energy Functional:

$$E_s(p_i) = \underbrace{\left\| p_{i-1} - 2p_i + p_{i+1} \right\|^2}_{\text{Second derivative measures curvature}}$$

Snake Energy Functional

Given a snake with N points $p_1,p_2,\ldots,p_N$

Define the following Energy Functional:

$$E=\sum_{i=1}^{N} a_i E_c(p_i)+b_i E_s(p_i)+c_i E_g(p_i)$$

Where:

E_c "Continuity"

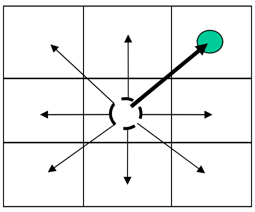
E_s "Smoothness"

E_g "Edgeness"

a_i,b_i,c_i are "weights" to control influence

Greedy Algorithm

Minimize energy one point at a time.
For each point, consider a finite set of moves
in a small window around it



Compute the new energy for each candidate location
Move the point to the one with the minimum value

Implementation Considerations

- To avoid numerical problems, the terms of the energy function should be normalized.
  - E_c and E_s are normalized by their maximum in the neighborhood
  - E_g is normalized as $|\nabla I - m| / (M - m)$
    - M and m are the *max* and *min* value of the gradient magnitude in the neighborhood

That is, want all terms scaled from 0 to 1 so they are treated equally

Implementation Considerations

Keeping high-curvature corners

- Before starting a new iteration:
  - Search for “corners”:
    - max curvature
    - large gradient
  - Corner points should not contribute to the energy (set $b_i = 0$)

Snake Algorithm

- Input:
  - gray scale image I
  - a chain of points $p_1, p_2, \ldots, p_N$
- f is the fraction of points that must move to start a new iteration
- U(p) is a neighborhood around p
- d is the average distance between snake points (computed from the list of points).

Snake Algorithm

While the fraction of moved points > f

1. For i=1,2,…,N
   1. find a point in $U(p_i)$ s.t. the energy E is minimum,
   2. move p_i to this location
2. For i=1,2,…,N
   1. Estimate the curvature $k=|p_{i-1}-2p_i+p_{i+1}|$
   2. Look for local max, and set $b_{max} = 0$
3. Update d

Where

$$E=\sum_{i=1}^{N} a_i E_c(p_i)+b_i E_s(p_i)+c_i E_g(p_i)$$

Yin, MST
Spring, 2014

Issue with this Algorithm

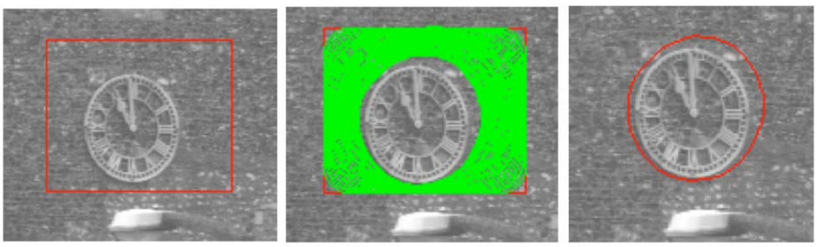
This algorithm is not guaranteed to find the "best" curve, in the sense of lowest cost.

Why? Greedy algorithms do not explore the space of all curves

Yin, MST
Spring, 2014

Non-Optimality

Typical snake behavior is far from optimal



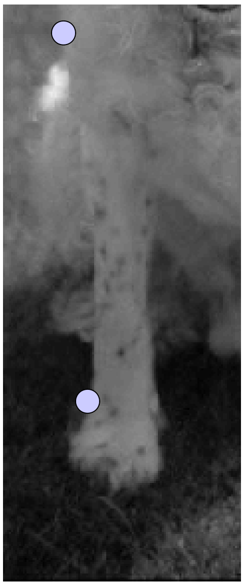
Snake movement gets "hung up" on high contrast stone in wall.

There have been a *lot* of papers written about how to make snakes work robustly: different energy functions, different optimization methods…

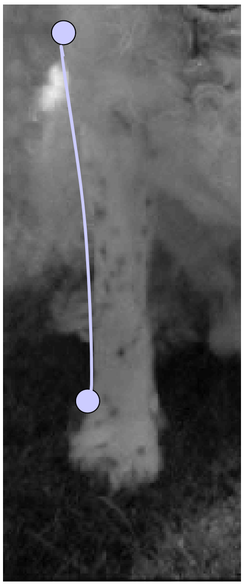
A More Optimal Strategy

- Given a start and end point, use dynamic programming to determine "best" path from start to end location.
  - Need to determine what is a good path?
  - Need procedure to find best path

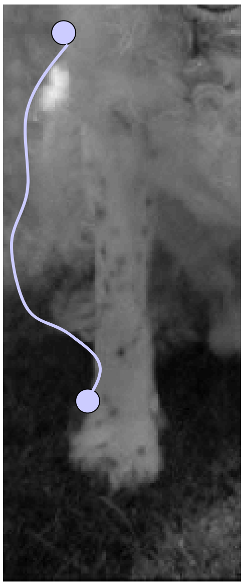
Algorithm we will discuss now is from
E. N. Mortensen and W. A. Barrett,
"Intelligent Scissors for Image Composition,"
in ACM Computer Graphics (SIGGRAPH `95),
pp. 191-198, 1995



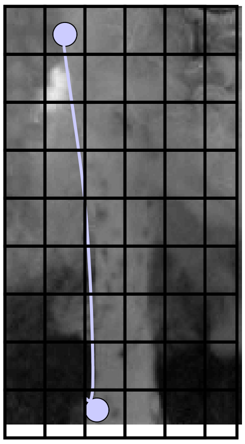
We'll do something easier than finding the whole boundary. Finding the best path between two boundary points.



How do we decide how good a path is? Which of these two paths is better?

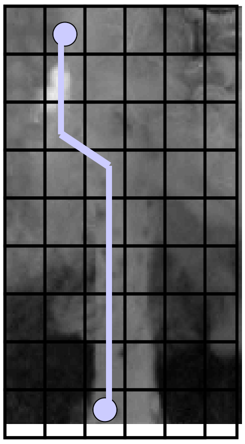


Discrete Grid



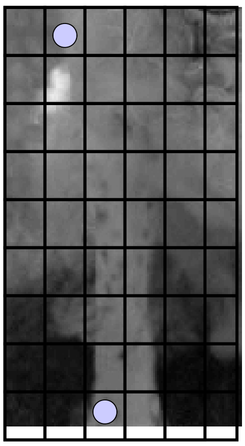
- Contour should be near edge.
  - Strength of gradient.
- Contour should be smooth (good continuation).
  - Low curvature
  - Low change of direction of gradient.

To start: contour near edge



- For each step from one pixel to another, we measure edge strength (change in intensity across edge).
- Find path with biggest total edge strength.

So How do we find the best Path?



Dynamic programming

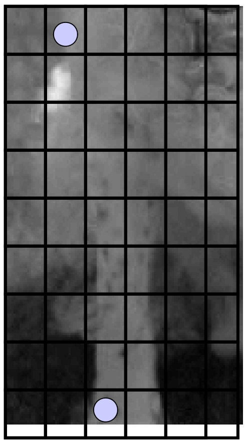
A Curve is a path through the grid.

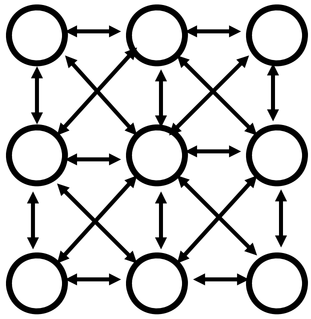
Cost depends on each step of the path.

We want to minimize cost.

Incrementally determine best path, starting from end state

Map problem to Graph

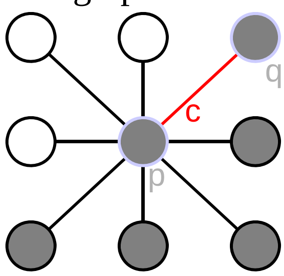




Weight represents cost of going from one pixel to another. Next term in sum.

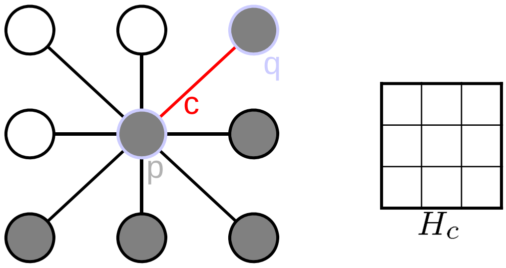
Defining the costs

- Treat the image as a graph



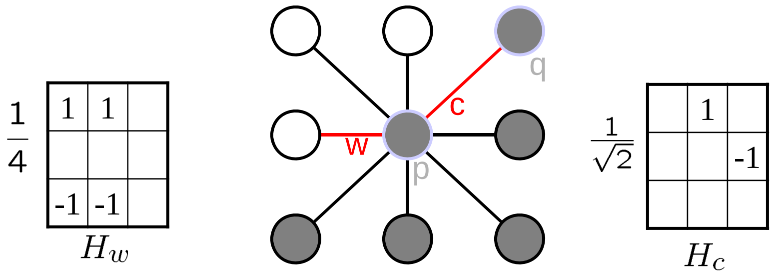
- Want to hug image edges: how to define cost of a link?
  - the link should follow the intensity edge
    - want intensity to change rapidly orthogonal to the link
  - c ≈ - |difference of intensity orthogonal to link|

Defining the costs



- First, smooth the image to reduce noise.
- c can be computed using a cross-correlation filter
  - assume it is centered at p
- Also typically scale c by it’s length
  - set c = (max-|filter response|) * length(c)
    - where max = maximum |filter response| over all pixels in the image

Defining the costs



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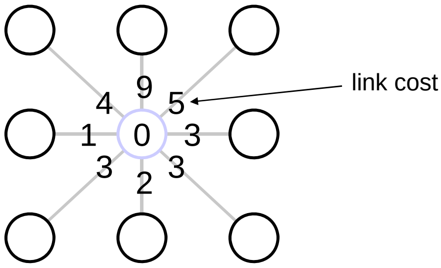
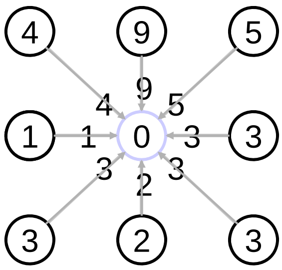
Dijkstra’s shortest path algorithm



- Algorithm
  1. init node costs to ∞, set p = seed point, cost(p) = 0
  2. expand p as follows:

     for each of p’s neighbors q that are not expanded
     - set cost(q) = min(cost(p) + c_{pq}, cost(q))

Dijkstra’s shortest path algorithm



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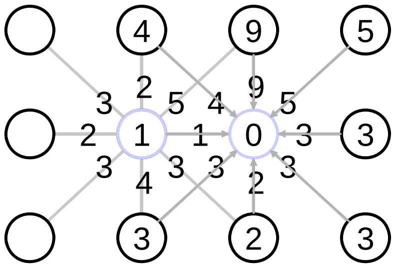
Dijkstra's shortest path algorithm



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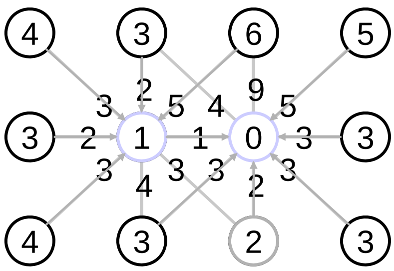
  4. repeat Step 2 for p = r.

Dijkstra's shortest path algorithm



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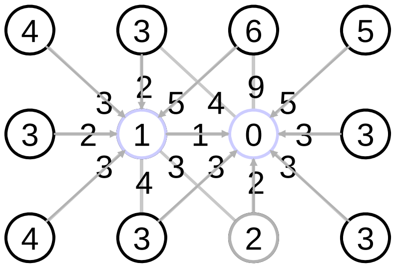

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  4. repeat Step 2 for p = r
  5. Stop when next point to expand is goal point. Read off shortest path.

Application: Intelligent Scissors

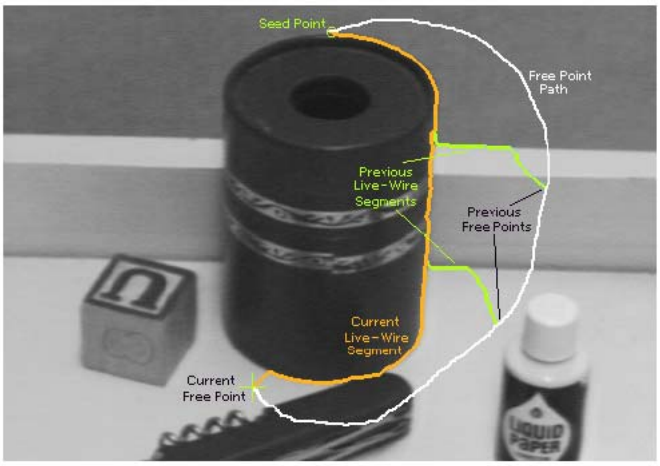

Figure 2: *Image demonstrating how the live-wire segment adapts and snaps to an object boundary as the free point moves (via cursor movement). The path of the free point is shown in white. Live-wire segments from previous free point positions (t_0, t_1, and t_2) are shown in green.*

Intelligent Scissors and Image Composition

Lessons Learned

- Perceptual organization: contour continuity constraints needed for boundary detection.
- Fully automatic methods for boundary finding (snakes, active contours) are not yet good enough
- Formulate desired solution as a cost function, then optimize it
  - greedy methods, easy but suboptimal
  - dynamic programming --> optimal solution
    - when the method is applicable