Adjudicating AE Ursae Majoris' Usefulness as a Standard Candle

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Abstract

Delta-Scuti-type variable stars are particularly interesting to analyze as standard candles due to the fact that they have well-defined, tested, and researched period-luminosity relation; furthermore, most Delta-Scuti are observable with relatively short pulsation periods. We observed the brightness and period of one such Delta-Scuti–AE Ursa Majoris (AE UMa). Over 4.5 hours, we obtained, solved, calibrated, and cataloged 166 V-band images of this target. After obtaining a light curve, we utilized a quadratic fit to local minima to obtain the period of pulsation; in turn, using a known Delta-Scuti period-luminosity relationship, we obtained the absolute magnitude of AE UMa. Performing differential photometry on the solved images using a reference star, we derived a measure of flux (and thus apparent magnitude) for the reference and target stars. Together the absolute and apparent magnitudes allowed us to derive the distance to AE UMa. We find that although our intermediate measurements of period and apparent magnitude were roughly in line with the literature, our period-luminosity relationship calculation resulted in a very inaccurate final distance measurement, likely due to the fact that our chosen relationship was not well-calibrated to AE Ursae Majoris specifically.

Key words: *Delta Scuti, SX Phoenicis, AE Ursae Majoris, Variable, Period-Luminosity, Flux, Standard Candle*

1 Introduction

Determining distances to and from interstellar objects is highly relevant for astrophysicists and amateur astronomers alike, prompting questions such as: how can one determine which stars are closer and which are further from the observer? How can one know whether two stars that appear adjacent to each other are actually proximal? The essence of the astrophysics that aims to tackle these issues is the study of *standard candles*–stellar objects whose observed brightness can be cross-referenced with their known luminosity to derive an interstellar distance scale [1]. Together, these standard candles form what is known as the *cosmic distance ladder*, a network of previous distance observations and calculations that allow astronomers to more easily extrapolate new distances, at least on a rough scale [1].

Traditionally, pulsating variable stars (stars who expand and contract somewhat radially) are intriguing and typical candle exemplars [1]. Such stars, like Cepheids and Delta-Scuti, have the advantage of having well-defined period-luminosity relations that make them especially useful as standard candles [1]. Recent studies, for example, have analyzed thousands of multi-mode Delta Scuti stars and established explicit period-luminosity relationships for both the fundamental modes and overtones of these stars [2, 3]. This continuity is highly insightful: given similar methods of measuring cosmic distances and systems, astronomers no longer have to shift models and paradigms, star-by-star. At best, this makes it much easier for scientists to identify where theoretical error might arise and propagate from in the cosmic distance ladder. The essence of our project is to test the feasibility of using one particular star, AE Ursae Majoris, as a standard candle by using its flux/time spectrum to derive a bound on the star's distance from Earth.

As shown in **Table 1**, our chosen target is AE Ursae Majoris (AE UMa). AE UMa lies in the Ursa Major constellation, which, based on our observation location at the Stanford Student Observatory, was deemed to be observable (see Appendix A) for extended periods during the night which we conducted observation (May 8, 2024). Our initial objective during our observation period can be stated simply as follows:

Objective: Based on the literature, AE UMa has a period of roughly two periods [4]. Can we obtain enough AE UMa flux data (over at least two periods, if not more) to verify this period?

The essence of our mathematical analysis are the following equations:

1. M_A = (-3.01 +/- 0.07) ln(P/d) - (1.40 +/- 0.07) [5]
2. D = 10^{(M_a - M_A + 5)/5 [6]

Equation 1 describes the absolute magnitude (M_A) of a Delta-Scuti star as a function of P/d (the period in days). This result was determined by regression from experimental data (see [5]) in 2022.

Equation 2 represents the heart of our analysis: here, M_a denotes the apparent magnitude (as calculated from our flux measurements) while M_A denotes the absolute magnitude; D represents the distance to the interstellar object at hand.

Inherent and implicit in our use of these above two equations are the following assumptions:

3. AE UMa, like most Delta Scuti, pulsates only radially. If this assumption breaks down, Equation 1 becomes useless (this is a key assumption of reference paper [5] as well)
4. Utilizing the regression generated by the authors of [5] is no less valid for our purposes than a new (and, perhaps, impractical) theoretical derivation of a Delta-Scuti period-luminosity relationship, assuming we explicitly account for error propagation in the coefficients of their equation.
5. AE UMa's observable pulsation is primarily in the fundamental mode.

Target #	Target name	RA	Dec	Inst	Exposure (h)	Constraints?
1	AE Ursae	$09^h\ 36^m\ 53.25^s$	+44° 04′ 09.8″	24-inch	4	Listed

	Majoris					Below

Table 1: Target Information

2 Observations

Our observations were performed over the course of 4 hours, and we took 170 images in the V-band. All but 3 of the images were solvable. We discarded an additional exposure due to visual star trailing (tracking error).

In addition to our target star AE Ursae Majoris, we included a reference star in our frames: TYC 2998-1166-1. This is a non-variable star with catalog data for its magnitude, allowing us to calibrate our data when necessary.

3 Data Reduction

After observations, we had 170 V-band images of our target (and reference) star. To perform data reduction, we calibrated all of our images by using bias, dark, and flat frames. Then, we solved each image in ASTAP, discarding the 3 images that could not be solved. In addition, we manually inspected each remaining image for noticeable artifacts – during this inspection we removed 1 more image that had visible light trails.

When creating and analyzing our light curve, we worked with the ratio of our target star flux to our reference star flux. We chose this method primarily because unmodeled systematic error was extremely prominent in our data; this may be partly due to the fact that we began taking exposures in the early evening and ended well past midnight. For this reason, (target flux / reference flux) helps reduce much of the systematic noise in our data by effectively normalizing the target star's flux to a known constant: the reference star's flux.

4 Analysis Methods

Looking back at our equation for distance, we require two statistics: absolute magnitude, which is derived from period, and apparent magnitude. The majority of our analysis involved the rigorous extraction of this information from the prepared data.

4.1 Absolute Magnitude

To extract absolute magnitude, we require some information on the period to take advantage of the literature derived relationship between period and absolute magnitude for Delta Scuti stars. This period analysis involved model-free and model-based methods operating on the light curve presented in Figure 1 below.

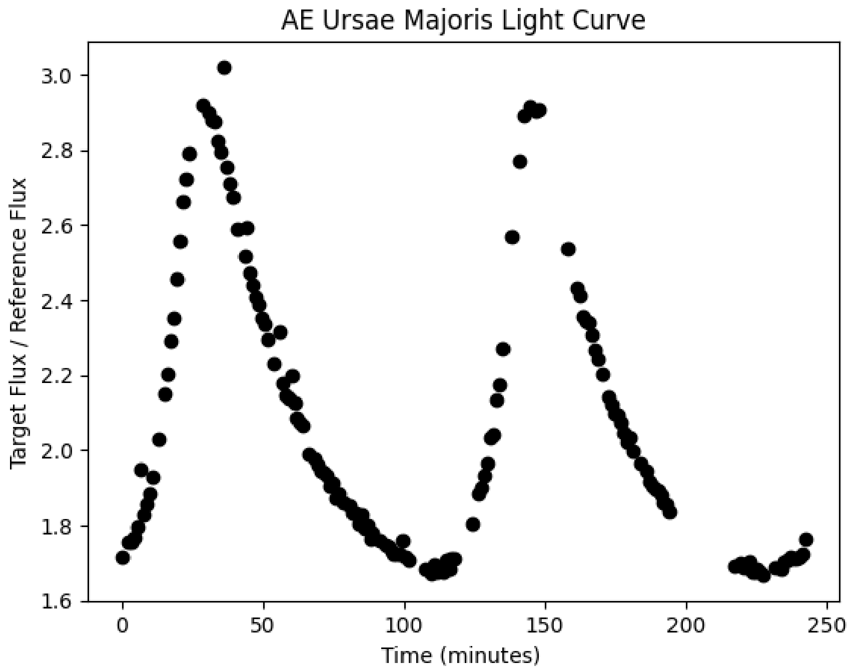

Figure 1: Light curve of AE Ursae Majoris based on fluxes from target star and reference star. Error bars on this light curve are present but indistinguishable due to their small magnitudes given by the high exposure time used and number of counts generated for each gathered image.

4.2 Period

4.2.1 Sine Curve fitting

The first method we try to obtain the period of our data is sine curve fitting. Specifically, we fit a sine curve to our data and then take the period of this sine curve as the period of our data.

Notably, we decided not to use this method for our final analyses because, given our virtually negligible error bars, all of our chi-square fit statistics were very bad. This is because our statistical error was very small, as our 60-second exposures captured many counts of our target star, while unmodeled systematic error likely dominated our “true” error, rendering chi-square analysis unilluminating.

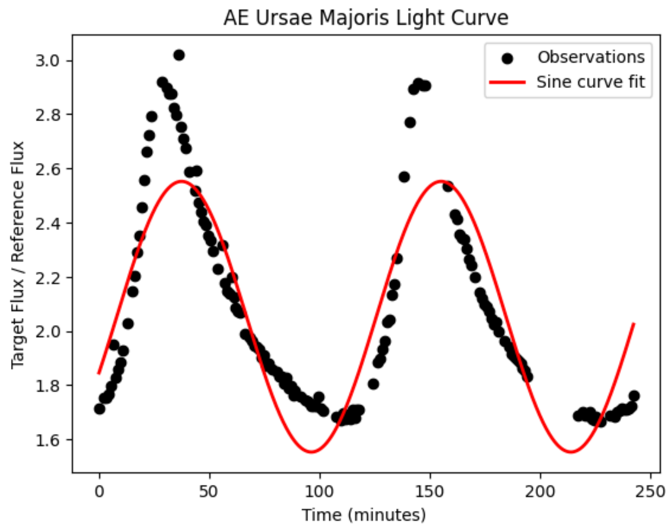

Figure 2: Depiction of our best-fit sine curve

4.2.2 Quadratic Minima/Maxima Fitting

Because sine curve fitting requires modeling our data completely and our negligible statistical error makes chi-square analysis virtually impossible (our generated curve would need to fit our data almost perfectly), we decide to use a more "model-free" approach to determining the period of our data. Specifically, instead of assuming that our entire data can be modeled by a sine curve, we only look at the distance between local minima/maxima and assume they take the shape of a quadratic. To do so, we take a representative set of points (ten points for maxima and forty points for minima) from each local extremum and fit quadratic functions to these.

With this fitted quadratic function, we can parse the vertex to more accurately parse a maximum and minimum value from these peaks and valleys rather than purely parsing maximum and minimum values from the data. These maximum and minimum values can then be converted to a period by calculating the distance between them. Notably, quadratic fitting generated two distinct period calculations: one period from the distance between maximum values (peaks) and one period from the distance between minimum values (trough). See **Figure 3** for a detailed representation of our quadratic fitter specifically applied to minimum values.

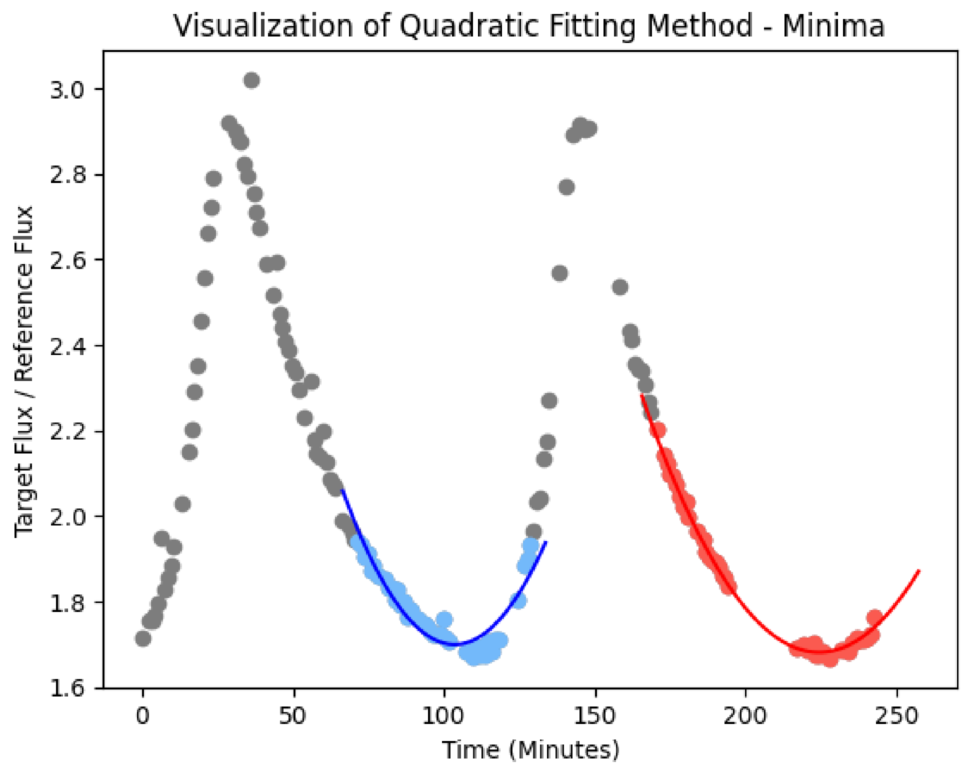

Figure 3: Quadratic fitting to find the period of AE Ursae Majoris. This specific case used the minimum instead of maximum values to parse the period.

4.2.3 Redundant Period Fitting Models

We additionally attempted to fit the period using three standard models in the literature [7]:

1. A sum of a sine curve and another sine curve with half the period.
2. A sum of periodic Gaussian functions.
3. A sum of seven Fourier terms.

All three methods (after performing a simple bootstrap analysis of 100 repetitions) provided median periods of around 120 minutes (roughly what we'd expect). However, none of the models provided an acceptable p-value. Indeed, this concurs with a qualitative analysis we made of our data: our error bars were too small for any reasonable fita, and, as such, any fit was too stringent in its assumptions on what curve our data might resemble. As a result, we proceeded with the model-free period methods elucidated above.

4.3 Apparent Magnitude

While the calculation of the apparent magnitude is less involved than the calculation of the absolute magnitude (due to the difficulty surrounding the period calculation), there are necessary considerations one needs to make before ultimately arriving at a parsed apparent magnitude. Since we can use a catalog to pinpoint the apparent magnitude of our reference star which is known to be held at a constant brightness, we can use this reference star as a guideline to remove any atmospheric disturbances that may plague the calculation of our apparent magnitude.

Given that AE Ursae Majoris is a variable star, we expect its apparent magnitude to fluctuate with time and thus need to decide on what metric best represents the apparent magnitude of AE Ursae Majoris. As is described in the literature and is common for variable stars, we often take the mean of the apparent magnitude over one period to be used as our apparent magnitude. It is important that only period be used in this calculation or else the apparent magnitude may be artificially skewed too high or too low. Thus, our calculation of apparent magnitude inherently relies on the accuracy of our period calculation, showcasing why so much of our energy was expended reducing error for that metric.

Apparent magnitude was calculated through co-adding images from 1 period, and then following procedures we used in Labs 1.3 and 1.4.

4.4 Error Propagation

As explained in the introduction, our explicit manipulation of our experimental data required three equations:

1. M_A = -3.01 +/- 0.07 ln(P/d) - (1.40 +/- 0.07)
2. D = 10^{(M_a - M_A + 5)/5
3. R = u / v

Here, equation 3. represents the ratio of fluxes, utilized when normalizing our target flux.

Each of these equations has an error propagation (in terms of standard deviance) that can be derived via the following equation:

1. For any function f(x_1, …., x_n), sigma_f^2 = sigma_{x_1}^2(\frac{\partial f}{\partial x_1})^2 + … + \sigma_{x_n})^2(\frac{\partial f}}{\partial x_n})^2, where additional covariance have been left out due to the assumed independence of x_i.

Applying this equations to the above three equations (where the coefficients in equation 1 have been replaced by A and B and replacing P/d with P), we obtain the following results:

1. **Equation 3:** sigma_{M_A}^2 = 2sigma_A^2\ln(P) + sigma_B^2 + sigma_P^2\frac{A^2}{P^2}
2. **Equation 4:** sigma_D^2 = ((D^2ln(10)^2)/25)(sigma_{M_A}^2 + sigma_{M_a}^2)
3. **Equation 5:** sigma_R^2 = sigma_u^2(1/v^2) + sigma_v^2(u^2/v^4)

5 Results

Our period analysis results in the following, described in **Table 2**.

	68% confidence interval	Best-fit period, in minutes (median)	Bootstrapped # iterations used (thousands)
Literature Results [8]	NaN	123.86457504	NaN
Quadratic fitting on minima (40 lowest points)	[117.601, 122.423]	120.380	100
Quadratic fitting on maxima (10 highest points)	[113.155, 119.616]	117.119	100
Quadratic fitting on both minima and maxima	[117.018, 121.117]	119.297	100

Table 2: Period estimates with confidence intervals

We obtained the following apparent magnitude: 11.410 ± 0.000105. Our statistical error is extremely small because we are co-adding ~90 images with ~50 million counts in our target source, leading to negligible poisson error. However, as a rough proxy for a confidence interval, we also try calculating apparent magnitude with the 68% confidence interval values of our period (i.e. taking 117 minutes worth of images and 121 minutes worth of images), yielding a rough interval estimate of [11.408, 11.412] or ± 0.002.

After using Equation 1 which expresses absolute magnitude as a function of period in days, we obtain a result of 6.097254 ± 0.0683813981107 for the absolute magnitude. As explained, we then employ Equation 2**,** which is a rearrangement of an equation given in class, with M_a reterring to apparent magnitude, M_A referring to absolute magnitude, and d to distance in parsecs.

We obtain from this a value of 115.49128141 ± 0.780792784899 parsecs for the distance. Here, we have used the general formula for propagating error of ratios (Equations 3, 4, 5) to derive variances on both the absolute magnitude and the distance in parsecs. By contrast, the literature provides AE UMa's distance as 740 ± 20 parsecs [8].

6 Conclusions

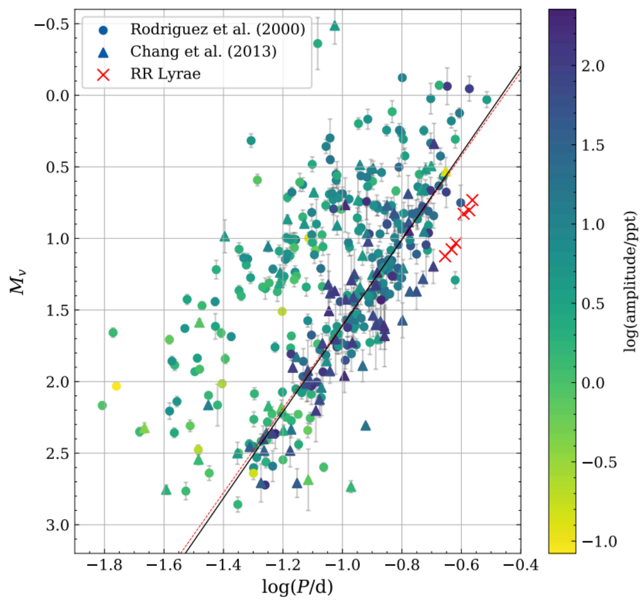

Figure 4: The variance amongst variable stars and the inability of a closed form period luminosity relationship to accurately model period relationships for all variable stars. [9]

Since our light curve clearly displays (with low error) the periodic nature of AE Ursae Majoris and considering that our statistical errors on our light curve are nearly non-existent, it's reasonable to assume that the period luminosity equations used from the literature that govern Delta Scuti stars poorly reflect AE Ursae Majoris (See **Figure 4**). Modern equations for period luminosity often incorporate much more than just period or magnitude, taking in terms related to the temperature, density, or even metallicity of the star. We believe that for tighter errors, more modern equations must be used and thus more and better detailed data must be collected. On top of this, our error analysis did not involve systematic errors, only statistical errors.

Future work involves using the B-band images to generate an HR diagram of AE Ursae Majoris for better understanding of the nature of AE. Generally, gathering more data to reduce error is a natural avenue for progress as well.

7 Acknowledgments

Sincere gratitude to the Physics 100 staff for their insightful contributions and advice to our project. Special thanks to Ben Dodge for his mentorship and guidance with regards to calculating our period. Data preparation methods were inspired from Physics 100 labs. Catalog data and solving functionality from ASTAP was vital throughout our research.

Appendix:

Appendix A: Observability of AE Ursae Majoris during the Observation Night

Figure 4: Altitude Map for May 8th, 2024

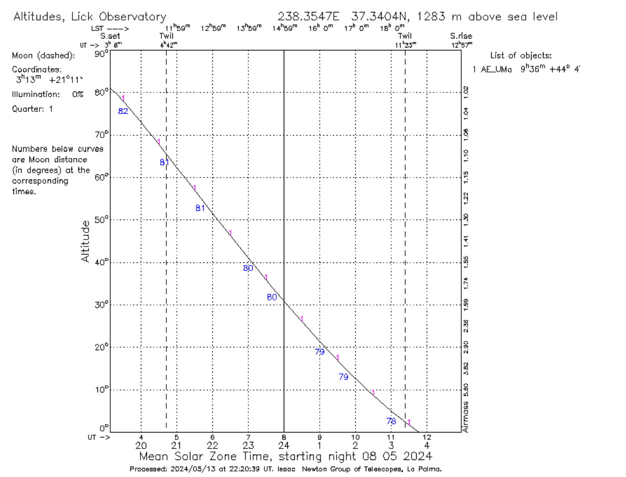

Figure generated from Isaac Newton Group of Telescopes:
http://catserver.ing.iac.es/staralt/index.php

References:

[1]: NASA Jet Propulsion Laboratory,
https://www.jpl.nasa.gov/news/cosmology-standard-candle-not-so-standard-after-all
[2]: Poleski, et. al, 2010, https://arxiv.org/pdf/2110.09806
[3]: OGLE Team, https://ogle.astrouw.edu.pl/atlas/delta_Sct.html
[4]: Hintz, E.; et al., 1997, "Time-Series Ensemble Photometry of SX Phoenicis Stars. II. AE Ursae Majoris", *Publications of the Astronomical Society of the Pacific*, **109**: 1073–1076,
[5]: Natascha Barac, Timothy R. Bedding, Simon J. Murphy, & Daniel R. Hey (2022). Revisiting bright delta Scuti stars and their period-luminosity relation with TESS and Gaia DR3.
[6]: Australia Telescope National Facility,
https://www.atnf.csiro.au/outreach/education/senior/astrophysics/photometry_magnitude.html
[7]: *Gaia Early Data Release 3: Summary of the contents and survey properties*,
https://arxiv.org/abs/2012.01533
[8]: Inno et. al, 2014, http://arxiv.org/pdf/1410.5460
[9]: Zhou, A. Period and amplitude variations in the high-amplitude δ Scuti star AE Ursae Majoris, 2001, https://www.aanda.org/articles/aa/pdf/2001/28/aa1303.pdf