Problem 1

X^{ℓ} - $N_\ell \times N_\ell \times F_\ell$ data tensor at layer ℓ, where the spatial coordinates are denoted by $i,j \in \{1,\dots,N_\ell\}$ and the feature dimension $f \in \{1,\dots,F_\ell\}$

Δ_ℓ - Filter half dimension at layer ℓ.

$W^{\ell}_{\epsilon,\delta,f,g}$ - Filter weights defined for $|\epsilon|, |\delta| \leq \Delta_\ell$, $f \in \{1,\dots,F_{\ell-1}\}$ and $g \in \{1,\dots,F_\ell\}$

S_ℓ - Filter Stride at layer ℓ.

b^{ℓ}_g - Bias at layer ℓ, feature dimension g

(a.) $$X^{\ell}_{i,j,g} = b^{\ell}_g + \sum_{\epsilon=-\Delta_\ell}^{\Delta_\ell} \sum_{\delta=-\Delta_\ell}^{\Delta_\ell} \sum_{f=1}^{F_{\ell-1}} W^{\ell}_{\substack{\epsilon+\Delta_\ell+1 \\ \delta+\Delta_\ell+1 \\ f \\ g}} X^{\ell-1}_{\substack{S_\ell(i-1)+\epsilon+1 \\ S_\ell(j-1)+\delta+1 \\ f}}$$

$$N_\ell = \frac{N_{\ell-1}}{S_\ell}$$

(b.) $$f\left(b^{\ell}_g + \sum_{\epsilon=-\Delta_\ell}^{\Delta_\ell} \sum_{\delta=-\Delta_\ell}^{\Delta_\ell} \sum_{f=1}^{F_{\ell-1}} W^{\ell}_{\substack{\epsilon+1+\Delta_\ell \\ \delta+1+\Delta_\ell \\ f \\ g}} X^{\ell-1}_{\substack{S_\ell(i-1)+\epsilon+1 \\ S_\ell(j-1)+\delta+1 \\ f}} \right)$$

(c) $$X^{\ell}_{i,j,g} = \max_{\epsilon,\delta \in [-\Delta_\ell, \Delta_\ell]} X^{\ell-1}_{\substack{S_\ell(i-1)+\epsilon+1 \\ S_\ell(j-1)+\delta+1 \\ g}}$$

Problem 2

$$\mathrm{Conv}_\Delta(X, W, S)_{i,j,g} = b_g + \sum_{\epsilon=-\Delta}^{\Delta} \sum_{\delta=-\Delta}^{\Delta} \sum_{f=1}^{F_1} W_{\substack{\epsilon+\Delta \\ \delta+\Delta \\ f \\ g}} X_{\substack{S(i-1)+\epsilon+1 \\ S(j-1)+\delta+1 \\ f}}$$

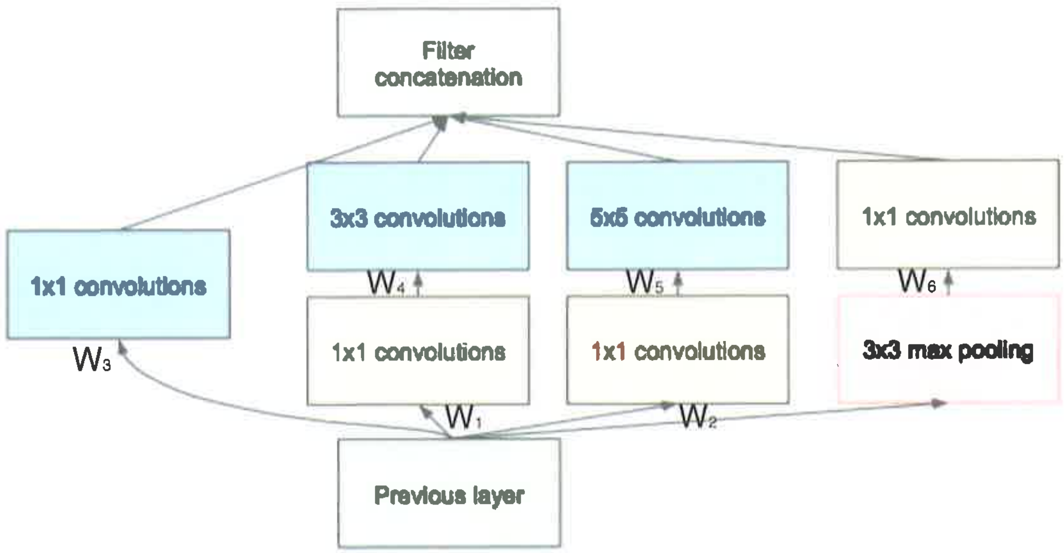
$$\mathrm{Relu}(X)_{ijg} = \max\left(X_{i,j,g}, 0\right)$$

$$\mathrm{MaxPool}(X, \Delta, S)_{i,j,g} = \max_{\epsilon,\delta \in [-\Delta, \Delta]} X_{\substack{S(i-1)+\epsilon+1 \\ S(j-1)+\delta+1 \\ g}}$$

$$\mathrm{Concat}(X^1, X^2)_{i,j,f} = \begin{cases} X^1_{i,j,f} & \text{if } f \in [1, \ldots, F_1] \\ X^2_{i,j,(f-F_1)} & \text{if } f > F_1 \end{cases}$$

$$\mathrm{Sum}(X^1, X^2)_{i,j,f} = X^1_{i,j,f} + X^2_{i,j,f}$$

Problem 3



function $\text{GoogleMod}(X, W_1, W_2, \dots, W_6)$

local $X_1 = \text{Relu}(\text{Conv}_\Delta(X, W_1, 1))$

local $X_2 = \text{Relu}(\text{Conv}_\Delta(X, W_2, 1))$

local $X_3 = \text{Relu}(\text{Conv}_\Delta(X, W_3, 1))$

local $X_4 = \text{Relu}(\text{Conv}_\Delta(X_1, W_4, 1))$

local $X_5 = \text{Relu}(\text{Conv}_\Delta(X_2, W_5, 1))$

local $X_6 = \text{Relu}(\text{Conv}_\Delta(\text{MaxPool}(X, 1, S), W_6, 1))$

local $X_7 = \text{Concat}(\text{Concat}(X_3; X_4), \text{Concat}(X_5, X_6))$

return X_7

Problem 3

```
function GoogleNet(X, k, W)
    if k=1
        return GoogleMod(X, W^{k,1}, ..., W^{k,6})

    else
        res = GoogleNet(X, k-1, W)
        return GoogleMod(res, W^{k,1}, ..., W^{k,6})
```

Problem 4

```
function ResMod (X, W¹, W², W³)
    local X₁ = Relu( Conv_Δ(X, W¹, 1) )
    local X₂ = Relu( Conv_Δ(X₁, W₂, 1) )
    local X₃ = Conv_Δ(X₂, W₃, 1)
    return sum (X, X₃)

function ResNet (X, K, W)
    if K=1
        return ResMod (X, W^{K,1}, ..., W^{K,3})
    else
        res = ResNet ( X, K-1, W)
        return ResMod (res, W^{K,1}, ..., W^{K,3})
```

Problem 5

Let $F(X)_{ijf} = M(\tilde{X}^{i,j})_f$ and let

$$S_\ell' = \prod_{i=0}^{\ell-1} S_i$$

$$F(X_{i,j})_f = \sum_{\epsilon=-\Delta_\ell}^{\Delta_\ell} \sum_{\delta=-\Delta_\ell}^{\Delta_\ell} \sum_{z=1}^{F_{\ell-1}} W^{\ell}_{\substack{\epsilon+1+\Delta_\ell \\ \delta+1+\Delta_\ell \\ z \\ f}} M^{\ell-1}\left(\tilde{X}^{i+S_\ell'\epsilon,\, j+S_\ell'\delta}\right)_z$$