DERIVING THE PLUCKING POINT LOCATION ALONG A GUITAR STRING FROM A LEAST-SQUARE ESTIMATION OF A COMB FILTER DELAY

Caroline Traube
Music Technology
Faculty of music, McGill University
Montréal, Québec, Canada
caroline.traube@umontreal.ca

Philippe Depalle
Music Technology
Faculty of music, McGill University
Montréal, Québec, Canada
depalle@music.mcgill.ca

Abstract

This paper focuses on the extraction of the excitation point location on a guitar string by an iterative estimation of the structural parameters of the spectral envelope. In a simple digital physical model of a plucked-string instrument, the resonant modes translate into an all-pole structure while the initial conditions (a triangular shape for the string and a zero-velocity at all points) result in a FIR comb filter structure. The delay D of the comb filter can be expressed as the product of the relative plucking position R and the fundamental period T_0. We propose a general method to estimate the plucking point location, working into two stages: starting from a measure related to the autocorrelation of the signal as a first approximation, a least-square estimation is used to refine the comb filter delay value to better fit the measured spectral envelope.

Keywords: *physical modeling of stringed instruments, spectral analysis, comb filter delay estimation*

1. INTRODUCTION

Recent years have seen great advances in physical model-based synthesis. In these endeavors, knowledge of the physics and acoustics of the instruments is a theoretical starting point for the modeling. Certain simplifications can make the models computationally efficient and they can then be implemented to run in real-time on a computer. Since implemented physical models are derived from the physics of the instruments, they result in the synthesis of particularly realistic instrumental sounds. But if the physical model running on a computer is intended to be *played*, then research must be extended to the performer's action in order to understand how to interact with the computer model.

For the particular case of the classical guitar, efficient string synthesis algorithms exist and are continually being improved [1, 2, 3, 4, 5]. For the analysis counterpart, research has been undertaken in an attempt to understand the relationships between timbre nuances and model [6], physical, expressive [7, 8] and psychoacoustical [9] parameters.

Among the parameters that contribute to the sound, the plucking point position on the string has a major influence on the timbre nuance.

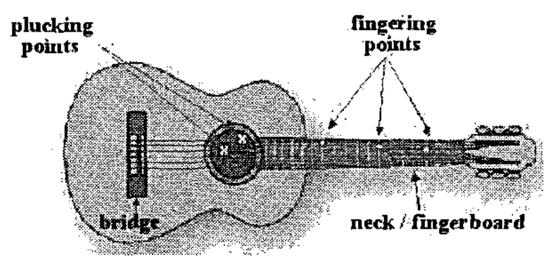

Fig. 1: Location of typical plucking and fingering points on a guitar.

Plucking a string close to the bridge produces a tone that is softer in volume, brighter and sharper. The sound is richer in high-frequency components. This happens when playing the guitar *sul ponticello*. The other extreme is playing *sul tasto*, near or over the fingerboard, closer to the midpoint of the string. In that case, the tone is louder, mellower, less rich in high frequency components. The neutral position of the right hand is just behind the sound hole.

CCECE 2003-CCGEI 2003, Montréal, May/mai 2003
0-7803-7781-8/03/$17.00 © 2003 IEEE

2. THEORETICAL CONSIDERATIONS

2.1. Plucking an Ideal String

The plucking excitation initiates wave components traveling independently in opposite directions. The resultant motion consists of two bends, one moving clockwise and the other counterclockwise around a parallelogram [10]. In the ideal cases, the output from the string (force at the bridge) lacks those harmonics that have a node at the plucking point.

The amplitude C_n of the nth mode of an ideal vibrating string of length l, with an initial vertical displacement h is given by [13]:

$$C_n = \frac{2h}{n^2\pi^2R(1-R)}\sin(n\pi R) \tag{1}$$

with $R = p/l$, the fraction of the string length from the point where the string was plucked to the bridge.

The digital signal processing interpretation of the physical phenomenon is the following: in a simple digital physical model of a plucked-string instrument, the resonant modes translate into an all-pole structure, while the initial conditions (a triangular shape for the string and a zero-velocity at all points) result in a FIR comb filter structure. The comb filtering effect is illustrated on Figure 2.

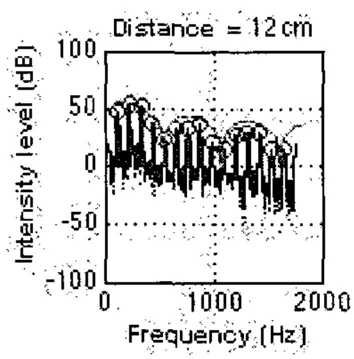

Fig. 2: Magniture spectrum of a guitar tone, plucked at 12 cm from the bridge on a 58 cm string, showing the effect of the comb filtering.

The delay D of the comb filter corresponds to the time the wave needs to travel from the plucking point to the fixed end of the string (the bridge or the nut) and back. It can be expressed as the product of the relative plucking position R and the fundamental period T_o.

$$D = R \times T_o$$

3. LEAST-SQUARE ESTIMATION OF A COMB FILTER DELAY

A simple way to estimate the plucking point location along the string from a recording could be to look for the missing harmonics in the spectrum ($C_n = 0$). However, the string is usually not plucked exactly at the node of any of the lowest harmonics. That is why we propose in this paper a more general method to estimate the plucking point location, working into two stages: starting from a measure related to the autocorrelation of the signal as a first approximation, a least-square estimation is used to refine the comb filter delay value to better fit the measured spectral envelope.

This work builds on other methods proposed previously and reported in [12], [13] and [14].

3.1. First Approximation of R

The autocorrelation function of a periodic signal $x(t)$ with period T_o can be expressed in terms of its Fourier coefficients C_n:

$$a(\tau) = C_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2\cos(\omega_o n\tau)$$

where $\omega_o = 2\pi/T_o$.

While the long-term features of the autocorrelation function can be very useful to estimate the fundamental frequency of a periodic signal (since it should show a peak at a lag corresponding to the fundamental period), its short-term evolution can probably tell us something about the plucking point position.

Figure 3 displays the plots of the autocorrelation function estimated for 12 guitar tones plucked at various distances from the bridge ranging from 4 cm to 17 cm. One can see that the autocorrelation takes on different shapes for different plucking positions but the information about the comb filter delay can not be extracted in an obvious way, directly from these graphs.

As we want to detect the low amplitude harmonics, we modify the structure of the autocorrelatiom function, by taking the log of the Fourier coefficients. This emphasizes the contribution of low amplitude harmonics (around the *valleys* in the comb filter spectral envelope) by introducing large negative weighing coefficients. The obtained *"log-correlation"* is expressed as follows :

$$r(\tau) = \sum_{n=1}^{\infty}\log(C_n^2)\cos(\omega_o n\tau)$$

Fig. 4 displays this measure for the same 12 guitar tones. Those plots reveal an interesting pattern: the minimum appears around the location of the lag correspond-

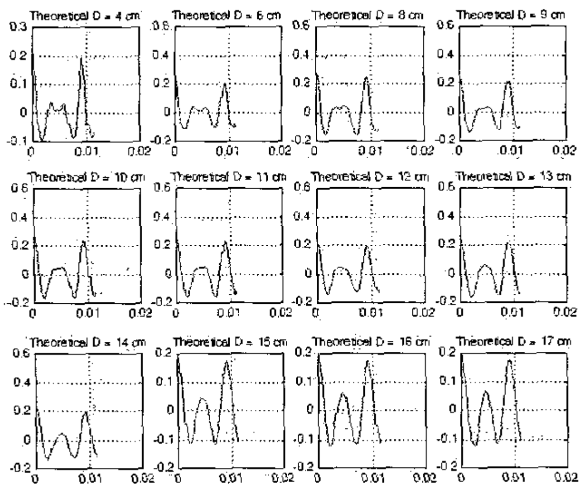

Fig. 3: Autocorrelation graphs for 12 guitar tones plucked at distances from the bridge ranging from 4 cm to 17 cm.

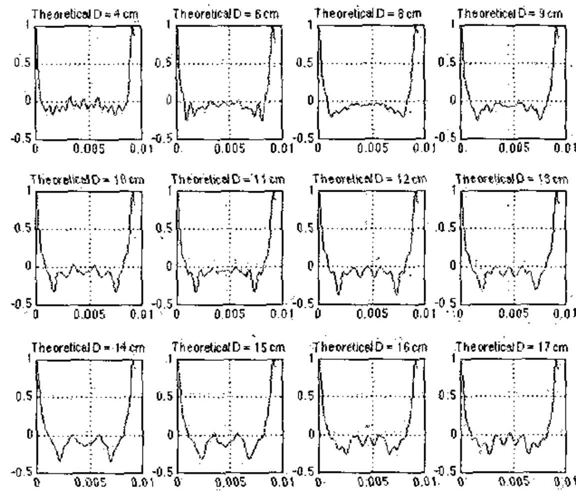

Fig. 4: Log-correlation graphs for 12 guitar tones plucked at distances from the bridge ranging from 4 cm to 17 cm.

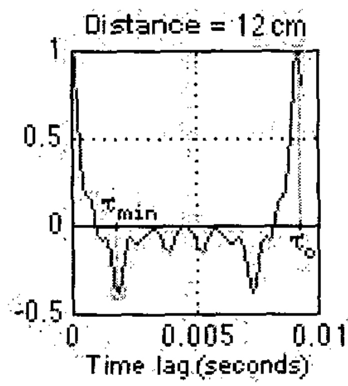

Fig. 5: Log-correlation for a guitar tone plucked 12 cm from the bridge.

ing to the plucking position. Therefore, we can conclude that

$$R \approx \frac{\tau_{min}}{\tau_o}$$

where R is the relative plucking point position (ratio of the plucking point distance with respect to the bridge and the length of the string or its vibrating portion), τ_{min} is the lag corresponding to the minimum in $r(\tau)$ and τ_o is the lag corresponding to the fundamental period, as illustrated on Fig. 5.

Figure 6 summarizes the estimation results for the data set of 12 tones. Except for a significant error for the first distance (4 cm), the estimation is accurate for all other distances (below 1 cm of accuracy).

3.2. Iterative Refinement of R Value Through Least-Square Estimation

The second stage of the estimation consists in finding the values of h and R that minimize the distance between the theoretical expression of the ideal string magnitude spectrum $C_n(h, R)$ (Eq. 1) and its observation. Although $C_n(h, R)$ is a non linear expression in terms of h et R, a least-square estimation technique can still be used after linearizing $C_n(h, R)$. The linearization is performed with a first order Taylor's series approximation about a first approximation of R and of the height h of the string displacement. Its leads to an expression of C_n as a linear combination of the two correcting values $\Delta h = h - h_o$ et $\Delta R = R - R_o$ (omitting the $2/\pi^2$ factor) :

$$C_n = D_n + \alpha_n \Delta h + \beta_n \Delta R \tag{2}$$

where

$$D_n = \frac{h_o \sin(n\pi R_o)}{n^2 R_o(1 - Ro)}$$

$$\alpha_n = \frac{\sin(n\pi R_o)}{n^2 R_o(1 - Ro)}$$

and

$$\beta_n = \frac{h_o \pi}{n R_o(1 - R_o)} \cos(n\pi R_o) + \frac{h_o(2R_o - 1)}{n^2 R_o^2(1 - R_o)^2} \sin(n\pi R_o)$$

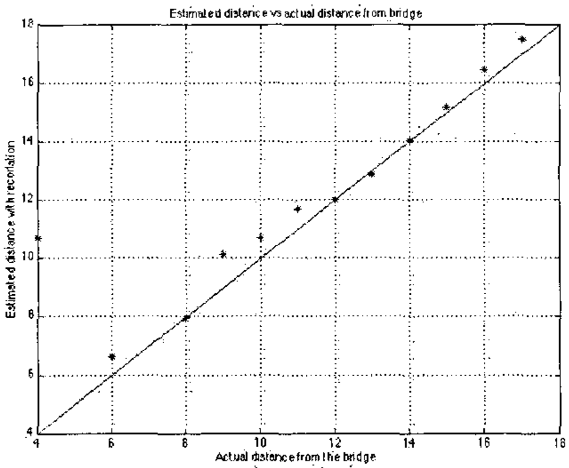

Fig. 6: Plucking point estimation with log-correlation

Once ΔR et Δh are obtained by solving the equation (2) in a least-square sense, the two parameters R and h are iteratively refined (using $R_o + \Delta R$ as second approximation and so on.)

4. CONCLUSION

This paper proposes an efficient method for the extraction of the excitation point location on a guitar string by an iterative estimation of the structural parameters of the spectral envelope.

Many applications can benefit from the algorithm, especially in the context of automatic tablature generation and sound synthesis. Beyond musical applications, this technique can be used to derive the value of the delay of any kind of comb filter, from the spectral peak parameters.

References

[1] Jaffe, D.A., and Smith, J.O., "Extensions of the Karplus-Strong Plucked-String Algorithm", Computer Music Journal, Vol. 7, No. 2, pp. 56-69, Summer 1983.

[2] Karjalainen, M., Valimaki, V. and Janosy, Z., "Towards High-Quality Sound Synthesis of the Guitar and String Instruments", Proc. International Computer Music Conference, pp. 56-63, 1993.

[3] Smith, J.O., "Efficient Synthesis of Stringed Musical Instruments", Proc. International Computer Music Conference, pp. 64-71, 1993.

[4] Valimaki, V., Huopaniemi, J., Karjalainen, M. and Janosy, Z., "Physical Modeling of Plucked String Instruments with Application to Real-Time Sound Synthesis", J. Audio Engineering Soc., Vol. 44, No. 5, pp. 331-353, May 1996.

[5] Karjalainen, M., Valimaki, V. and Tolonen, T., "Plucked-String Models: From the Karplus-Strong Algorithm to Digital Waveguides and Beyond", Computer Music Journal, 22:3, pp.17-32, Fall 1998.

[6] Tolonen, T. and Valimaki, V., "Automated Parameter Extraction for Plucked String Synthesis", Proc. International Symposium on Musical Acoustics, vol. 1, pp. 245-250, Scotland, 1997.

[7] Erkut, C., Valimaki, V., Karjalainen, M. and Laurson, M., "Extraction of Physical And Expressive Parameters for Model-based Sound Synthesis of The Classical Guitar", AES108th Convention, Paris, 2000

[8] Cuzzucoli, G. and Lombardo, V., "A physical Model of the Classical Guitar, Including the Player's Touch", Computer Music Journal, 23:2, pp. 52-69, Summer 1999.

[9] Orio, N., "The timbre space of the classical guitar and its relationship with the plucking techniques", Proc. International Computer Music Conference (ICMC'99), pp.391-394, China, 1999.

[10] Fletcher, N.H. and Rossing, T.D., The Physics of Musical Instruments, second edition, Springer-Verlag, 1998.

[11] Legge, K.A., and Fletcher, N.H., "Nonlinear generation of missing modes on a vibrating string", J. Acoust. Soc. Amer., Vol. 76(1), 1984.

[12] Bradley, K., Cheng, M.-H. and Stonick, V.L., "Automated Analysis and Computationally Efficient Synthesis of Acoustic Guitar Strings and Body", Proc. IEEE ASSP, 1995.

[13] Traube, Caroline and Smith, Julius O., "Estimating the plucking point on a guitar string", Proc. Conference on Digital Audio Effects (DAFX00), Verona, Italy, pp. 153-158, December 2000.

[14] Traube, Caroline and Smith, Julius O., "Extracting the fingering and the plucking points on a guitar string from a recording", Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA'01), Mohonk Mountain House, New Paltz, New York, 2001.