# Automatic Guitar Tablature Transcription from Audio using Inharmonicity Regressions and Bayesian Classification

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July 2017

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Submitted in partial fulfillment of the requirements for the degree of Master of Science in Music and Technology

#### Abstract

We propose and evaluate two new methods to classify guitar strings with the goal of developing automated tablature transcription. The classifiers use only audio input and are based on the measured inharmonicity of the sounds produced by the strings. While previous inharmonicity-based systems were able to obtain good performance classifying guitar strings based on sample data produced by individual guitars, we attempt to develop a system that can obtain improved string-identification performance for guitars on which the system wasn't individually trained, using training data aggregated from multiple guitars of the same type. We also derive and evaluate an inharmonicity compensation feature to allow for tablature transcription of arbitrarily-tuned guitars, provided the tuning is known.

The first method linearly regresses guitar strings' log-inharmonicities against their pitches to obtain characteristic trajectories, then assigns unseen notes to the strings whose trajectories' means and variances maximize the probability of the notes' measured inharmonicities. The second method, developed as a baseline against which to compare the first and which we refer to as simple Bayesian classification, characterizes the inharmonicity distribution of each fretboard position as a normal probability density, then similarly assigns unseen notes to the fretboard positions that maximize the likelihood of their observed inharmonicities. Results from classical and acoustic guitars in the standard Real World Corpus of guitar recordings show that both methods rival performance of the existing inharmonicity-based system for guitars on which the system wasn't individually trained, as well as show that leveraging the linear structure of log-inharmonicity trajectories increases overall performance compared to the baseline method. Also, results from personally-recorded acoustic and electric guitars in various alternate tunings show that our tuning compensation is successful at predicting tuning-related inharmonicity changes and improving transcription performance.

#### Acknowledgments

My sincere gratitude goes to Rich Stern for advising me every step of this journey. I'd never before tackled a research project of this scope, but Rich's guidance, patience, and insight somehow made this thesis happen. It's been an immeasurably rewarding experience, and I owe that to him. I'll miss our weekly one-on-one meetings that evolved into friendly chats about music, family, and life.

I'm indebted to Tom Sullivan for lending his qualified examination of this thesis. I'm also indebted to him in the literal sense as he welcomed me to Pittsburgh by treating me to lunch. His hospitality and accessibility helped me know I was in the right place when I arrived two years ago.

I'd like to thank Matthieu Hodgkinson for sharing with me his MAT algorithm code, as well as Jacob Abesser and Isabel Barbancho for their helpful correspondence.

My peers Che-yuan Liang and Raymond Xia have both, of their own accord, contributed meaningfully to this work through our many brainstorming and discussion sessions, and for them I'm grateful. Many thanks to Nick Pourazima, whose Fender Telecaster we recorded for experiments.

I need to thank the remainder of my CMU graduate music technology cohort – Devansh Zurale, Steven Krenn, Chen Liang, Mutian Fu, Hao Huang, Anirudh Mani, Garrett Osborn, and Nick Pourazima – for the support role they played by simply being the fun constructive community that they were.

Finally, I acknowledge my mom and dad, to whom all success is due. I'd also like to thank my brother Nick Michelson solely for persuading me to abandon writing a grandiose Acknowledgments section, and definitely nothing else.

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## **Chapter 1: Introduction**

The guitar is a popular musical instrument whose family comprises a diverse collection of stringed instruments. Its most prominent classes are the acoustic, classical, and electric guitars, and intraclass variation is high in characteristics such as body shape, string material, and amplification modality. Despite this diversity, the majority of guitar configurations exhibit some unifying features, like having six strings tuned to E2 (82Hz), A2 (110Hz), D3 (147Hz), G3 (196Hz), B3 (247Hz), and E4 (330Hz), for example. Performance is relatively constant as well; to play, a performer presses these strings with one hand against the neck's metal frets, each of which increase the string's pitch by one half-step, while strumming or plucking them with the other hand. Guitars typically have upwards of 20 frets, allowing for versatile musical passage realizations along the fretboard.

A consequence of the strings' open-tunings and extensive fretting locations is liberal pitch overlap between neighboring strings; multiple fretboard locations can usually be selected to produce a given pitch. Conventional music scores that represent passages as notes and chords therefore fail to communicate fretboard position. This ambiguity renders scores unfit for guitar students trying to learn more skillful placement of scales and arpeggios, or for enthusiasts trying to decrypt the fretboard positions used on recordings of virtuosic players' riffs. Figure 1.1 illustrates this ambiguity.

Tablature, on the other hand, is an alternative music notation that doesn't suffer from the oneto-many mapping of scores. The staff, instead of representing pitch as in classical scores, depicts a birds-eye view of the guitar neck from the performer's perspective. Each of the six horizontal lines signify a corresponding string on the guitar, and numbers on each line specify the fret to be fingered. Time progresses from left to right as in scores.



Figure 1.1: Musical score and two identical realizations of that score in tablature notation. Figure taken from [4].

Tablature (abbr. tab) is widely more popular than scores for a number of likely reasons [21]. Its accessibility is attractive to those without formal musical training; interpreting a pictorial representation of the fretboard poses a smaller learning curve than studying music theory and deciphering score notation. Additionally, the lines and numbers in tablature that represent the strings and frets are easily represented with ASCII characters, rendering it an exceptionally portable mode of communication. Indeed, one can create, modify, and transfer tablature files using simply a text editor.

Accurate tablature transcription is a fairly laborious task, however. Dependable tabs containing few errors typically require a seasoned musician's careful listening to an audio recording, and possibly video recordings for additional consultation. Reliable automation of this transcription task would benefit the guitar student community by expediting quality tab generation and unearthing fretboard positions of guitar riffs in songs for which accurate tablature doesn't yet exist. Recent technologies proposed to encourage proliferation of accessible tab transcription systems, such as web framework Robotaba [6], demonstrate a desire among MIR researchers to see this theoretically mature but practically nascent field mature into student-benefitting applications.

An important feature in systems that transcribe tablature from audio input is inharmonicity. Briefly, inharmonicity is a phenomenon that skews a vibrating string's harmonic partials upwards in frequency and whose variation trend is somewhat unique per string. Because of this, it is highly discriminative and has successfully been used as the sole feature in a transcription system [4] which we refer to as the existing inharmonicity-based method. This is discussed further in Chapter 2, but for now is introduced simply for context.

This work builds on the existing method. We propose an alternative procedure to exploit the inharmonicity of an unseen note for string classification: we assign to it the string whose precomputed log-inharmonicity trajectory most probably predicts the note's estimated log-inharmonicity. To contextualize its performance as well as that of its predecessor, we also introduce a baseline Bayesian classifier of fretboard position given an unseen note. To the author's knowledge, no results for strictly inharmonicity-based systems have yet been reported from a simple classifier for the sake of benchmarking. The measured inharmonicities of each fretboard position are collected and fit to a normal distribution, and are used to return the string that maximizes the probability of having observed the unseen note's measured inharmonicity.

We evaluate these two systems' performances in three scenarios, comparing their performance to that of the existing method in [4]: when the systems have been trained on recordings of only the test guitar, when the systems have been trained on recordings of multiple guitars including the test guitar (all of which were in the same classical, acoustic, or electric class), and when the systems have been trained and tested on separate sets of guitars. Results for classical and acoustic guitars show that these new systems perform about as well as the existing system. The results also show that our leveraging of the linear log-inharmonicity string trajectories yields slight performance gains over the baseline fretboard-position classifier.

Additionally, this work introduces an inharmonicity compensation factor that allows for successful transcription of guitars whose tunings differ from those of the training set, provided that the alternate-tunings are known. Experiments on a personally recorded Fender Telecaster and a Takamine G-series in various tunings reveal that our compensation factor can restore

an arbitrarily-tuned guitar's transcription performance to that of its more accurately transcribed standard-tuned trial.

In the next chapter, we survey previous work in the field of automatic tablature transcription, highlighting the existing inharmonicity-based method. This is followed by a more substantial motivation of inharmonicity's suitability for the string discrimination task and a review of the literature on inharmonicity estimation. In Chapter 3, we introduce both systems in detail. Chapter 4 reports the results of our experiments, and Chapter 5 contains analysis of the results, limitations of significance, and future work. We summarize this thesis in Chapter 6.

# **Chapter 2: Previous Work**

Music transcription is a rich research topic with many contributors. We begin with a brief review of various teams who have focused specifically on tablature transcription. We then introduce a feature of guitar audio referred to as inharmonicity, which has been used exclusively and successfully in one system, whose detailed summary we defer to Chapter 3. We close this chapter with a survey of the inharmonicity estimation literature.

## 2.1 Automatic Tablature Transcription

#### 2.1.1 Graph Search Methods

A popular framework for the stringed-instrument transcription task, introduced by Sayegh [27] for both transcription and fingering problems, is that of weighted directed graphs that capture all plausible sequences of fretboard positions for a given musical passage. Edge weights in the graph are typically informed by mechanical or musical desirability of the fretboard sequences implied by the vertex transitions. Radicioni [24] used this graph paradigm to obtain optimal fingerings from music scores. Yazawa [31] extracted pitch and melody information from guitar recordings, and computed an optimal fingering based on plausible multipitch estimation results. Burlet [6] developed a polyphonic extension to monophonic optimal search algorithms. Burlet [7] also introduced a guitar-specific deep belief network whose polyphonic transcription output was arranged into tablature with a graph search routine. Radisavljevic [25] proposed a method to learn the optimal weights on the graph given optimal tablature.

#### 2.1.2 Machine Learning Solutions

A number of teams have used machine learning techniques to ascertain fretboard position or related parameters. Gagnon [12] proposed using neural networks on scores to deduce general fretboard region position and number of strings playing, while Tuohy [30] used a genetic algorithm on scores to obtain tablature. Barbancho [3] attempted guitar string classification with a Fisher linear discriminant using a multitude of time and frequency-domain features. Barbancho [2] used HMMs on polyphonic recordings of common guitar chord sequences to transcribe their fret position and fingering. Abesser [1] and Kehling [19] achieve good string transcription performance after training an SVM on standard-tuning guitar audio and using a slew of partials-related features, while Dittmar [9] adapted [1] into a real-time implementation.

#### 2.1.3 Multimedia Approaches

Some researchers have explored usage of media other than audio or musical information (i.e., scores). O'grady [22] exploited hexaphonic pickups and matrix factorization of their output to record fretboard positions. Paleari [23], Burns [8], and Kerdvibulvich [20] extract tablature from video recordings using image processing techniques.

## 2.1.4 The Partial Coincidence Tally (PCT) Method

One particular system inspired this work. Barbancho [4] realized the discriminative potential of a singular feature known as inharmonicity, and devised an algorithm that exclusively used inharmonicity to achieve impressive transcription results using only audio as input. We discuss inharmonicity in detail and methods for its estimation in the remainder of this chapter. Their method, which we refer to as the partial coincidence tally (PCT) method, is also given a more thorough treatment later in Chapter 3.

We were particularly interested in the PCT method for a number of reasons. Firstly, its input

is audio. Among the most interesting use cases for an automatic tab transcription system is direct audio-to-tablature capability, as audio is the natural output of the object we're interested in transcribing. Other inputs, like scores or video, are to an extent already a form of transcription that detracts from the impressiveness of a tablature transcriber. Secondly, because PCT examines notes' underlying physical signatures, namely their inharmonicities, it can theoretically decode the fretboard position of any note despite how unexpected its location may be. Other methods, like graph searches, rely on optimization of cost functions that are generalizations of tab playability rules, and therefore aren't as theoretically well-suited to correctly transcribe aberrant note sequences. Lastly, their method exclusively uses inharmonicity. Other systems that use inharmonicity [1, 3, 9, 19] exploit additional features that aid in string discriminability. We were captivated by the idea of singularly using inharmonicity to achieve impressive transcription accuracies.

### 2.2 Inharmonicity Motivation

To understand the motivation for this work, we first need to address the motivation behind inharmonicity as a feature itself. Inharmonicity is key in many string discrimination systems [1, 3, 4, 9, 19]. When an ideal string fixed at both ends is displaced, the restoring force that causes it to oscillate is its tension. However, for a real string with stiffness, its elasticity contributes to this restoring force [11] which causes the frequencies of the modes of vibration to no longer be integer multiples of the fundamental. Instead, the solutions to the one-dimensional wave equation become:

$$f_k = k f_0 \sqrt{1 + \beta k^2} \tag{2.1}$$

where  $f_k$  is the kth harmonic of fundamental  $f_0$  and  $\beta$  is the string's inharmonicity, defined by

$$\beta = \frac{\pi^3 Q d^4}{64T l^2}. (2.2)$$

In words, the inharmonicity  $\beta$  of a vibrating string is a dimensionless quantity that depends on the string's Young's modulus Q, diameter d, tension T, and vibrating length l, and which skews the string's kth partial upwards in frequency according to equation (2.1). See Figure 2.1. Note that for an ideally harmonic string,  $\beta = 0$  and (2.1) reduces to  $f_k = k f_0$ , aligning with the ideal integer-multiple relationship between a fundamental and its harmonics' locations.

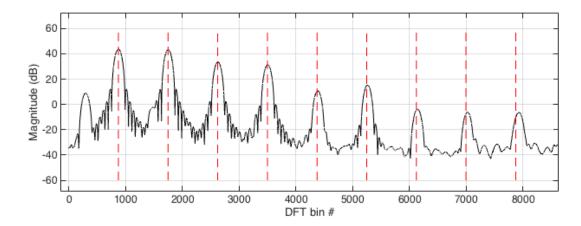


Figure 2.1: Power spectrum of note D2 plucked on an electric guitar at string 5 and fret 5. Red dashed lines are drawn at integer multiples of the fundamental. The partial peaks begin visibly skewing rightward after the fifth harmonic.

Inharmonicity's suitability for the string discrimination task can be understood through equation (2.2). The six strings of any guitar necessarily exhibit different combinations of Q, d, and T due to varying thicknesses, material, and tuning. Each string therefore has an "inharmonic signature" which presumably distinguishes it from the others. Capturing the pattern of a string's inharmonicity variation along its frets should thus reveal sufficiently identifying information.

## 2.3 Inharmonicity Estimation

Empirical estimation of inharmonicity is itself a topic of research which merits a brief review. Using pitch extraction techniques to perform estimation was first attempted by Galembo in 1979 and 1986 [13, 14]. Several additional methods have been proposed since then. In 1994, Galembo [15] hypothesized a connection between partials-based fundamental pitch estimates and the degree of

inharmonicity in a spectrum. They specifically investigated cepstral analysis and a variant of the harmonic product spectrum method applied to both synthetic tones and recorded piano notes. They found that, because these two techniques leverage periodicity in the frequency domain to produce their pitch estimates, inharmonicity influenced the quality of their estimates in a deterministic manner: more inharmonicity produced wider and less focused pitch estimate lobes, and from quantifying this relation an inharmonicity estimate could be obtained.

The same authors in [16] introduced another method in 1999, dubbed the inharmonic comb filter (ICF) method. To estimate a note's inharmonicity, they first applied sets of comb filters to the note's spectrum. The locations of the comb filters' notches were parametrized with a range of inharmonicity values that sufficiently sampled the note's expected inharmonicity range. The ICF whose inharmonicity best matched that of the note would produce the output with lowest spectral energy, since its notches would most align with the inharmonic note partials. The inharmonicity parameter associated with this output-minimizing ICF was thus selected as the estimate.

Rauhala [26] introduced an efficient procedure in 2007 that iteratively catalogued estimates of the partial frequencies' deviations and returned increasingly better estimates of the inharmonicity. The algorithm, known as the partial frequency deviations (PDF) method, is initialized with a reasonable first estimate of inharmonicity, and then the note's spectrum is searched for partial peaks. Differences are measured between the locations of these discovered peaks and those of the expected peaks, and the aggregate deviation trend is used to refine the inharmonicity estimate – a majority of positive differences implies the inharmonicity should be reduced, and a majority of negative differences implies the opposite.

In 2009, Hodgkinson [18] proposed a yet more efficient and accurate algorithm, named median-adjustive trajectories (MAT). Their routine exploits the fact that inharmonicity can be estimated with any two partials and their corresponding indices in the harmonic series. Pairs of low-index partials are first considered, since they're minimally affected by inharmonic skew and therefore have high location reliability. With these initial inharmonicity estimates, additional partials are collected and used to refine the previous inharmonicity estimates, with which more

partials are located, so on and so forth. MAT was the inharmonicity estimation routine employed in this work, and is discussed at length in Chapter 3.

Three years later, Barbancho [4] introduced another PFD-based approach as part of a guitar tablature transcription system. They started by cataloguing the first 10 partials' deviations, then fitting a polynomial to the PFD curve. They derived the relation between the polynomial's coefficients and the inharmonicity, and returned an initial inharmonicity estimate according to this derivation. Then using this inharmonicity to aid in their search for higher-index partials, they catalogued a larger number of partials' deviations than that of the first iteration, and performed the same polynomial fit and inharmonicity derivation routine. This would refine the inharmonicity estimate, allowing for increasing numbers of partials for which to search.

Also in 2012, Abesser [1] used parametric spectral modeling, again in the context of a guitar tablature transcription system. To estimate the inharmonicity of a plucked guitar note, they modeled each audio frame with the autoregressive (AR) filter that best approximated its spectral content. They obtained the poles' locations and the ideal harmonics' locations, and like Barnbancho [4] fit the PFD curve with a polynomial from whose coefficients an inharmonicity estimate could be derived.

The MAT algorithm was chosen for its efficiency and accuracy over the algorithms that preceded it. In [18], benchmark tests are run against [26]'s PDF method (which itself is more efficient than [16]'s earlier ICF method) and results shows that MAT is about an order of magnitude quicker and more precise on the test set of synthesized inharmonic tones. To our knowledge, no comparisons exist between MAT and the two methods (modified PFT [4] and AR filter modeling [1]) developed after it as part of tablature transcription systems. Though since we were having trouble replicating the two more recent systems and the author of [18] was willing to share the algorithm's code, we decided to use MAT for this work.

# **Chapter 3: Proposed Transcription Methods**

In this chapter, we present both our inharmonicity trajectory method and our baseline method in detail. More time was spent innovating and developing the former, so we appropriately give it a lengthier treatment. We begin with the trajectory method's motivation, then we present its various components: the specific inharmonicity estimation algorithm we used, transformation of inharmonicity to log-inharmonicity, learning log-inharmonicity trajectories, string classification using the learned trajectories, and the final tablature conversion and refinement process. Next, we discuss the baseline classifier's motivation, its inharmonicity estimation algorithm, and its fretboard-position classification mechanism. We finally introduce our feature that allows compensation for arbitrary tuning before concluding with overviews of the two presented systems.

## 3.1 Inharmonicity Trajectories

#### 3.1.1 Motivation

Barbancho [4] derived that inharmonicity varies in a deterministic manner with respect to fret positions along a given string. Specifically, they showed that the inharmonicity  $\beta(s,n)$  of a note produced by string s at fret number n could be expressed in terms of the inharmonicity of the open-string note  $\beta(s,0)$  according to:

$$\beta(s,n) = \beta(s,0)2^{\frac{n}{6}} \tag{3.1}$$

In other words, the variation in inharmonicity along a given string can be expressed simply in terms of its open-note inharmonicity, effectively defining a restricted trajectory along which the inharmonicity must vary. Figure 3.1 illustrates these trajectories.

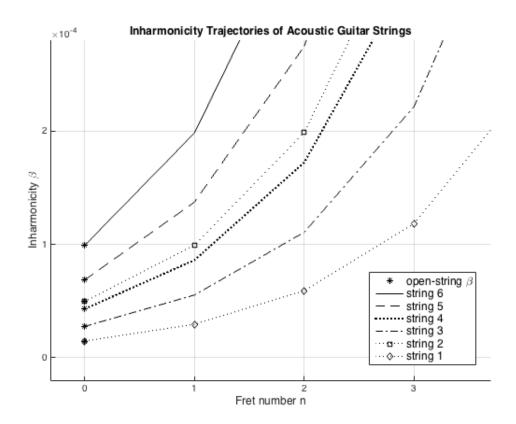


Figure 3.1: Example inharmonicity trajectories of acoustic guitar strings, calculated from measured open-string inharmonicities. Only frets 0 (open-string) through 3 are shown. Observe the restriction to the exponential trajectory defined by equation (3.1).

Recognizing this, they were able to successfully transcribe string and fret number of an unknown guitar note. Their system's essential mechanism was a similarity-determination routine  $A(X, \hat{\beta}_s)$  that quantified the resemblance between an unknown note's spectrum X and the expected inharmonicities  $\hat{\beta}_s$  of candidate strings s from which the note could have originated. First, to estimate an open-note's inharmonicity they followed the procedure described in section 2.3 in which they estimated its fundamental, then catalogued its partials' deviations from their ideal harmonic locations, and fit to them a polynomial from whose coefficients they derived the inhar-

monicity estimate. Next, they inferred the corresponding fret on which the note could be played for each candidate string, and used these string-fret candidate numbers in equation (3.1) to obtain expected inharmonicity values of the unknown note for each candidate string. In other words, they followed each candidate string's theoretical inharmonicity trajectory to arrive at an expected inharmonicity value at the appropriate fret. Lastly, they applied their similarity determination algorithm  $A(X, \hat{\beta}_s)$ , which went as follows. Given each candidate fretboard position's expected inharmonicity, they obtained the corresponding expected skewed frequency locations at which to find the inharmonic peaks, and searched the unseen note's spectrum for them using search windows centered at those expected locations. They performed this search for a set number of peaks, and catalogued their deviations from the search window centers. Their motivation was that the expected inharmonicity of the true fretboard position would produce a spectrum with inharmonic peaks that were measurably more coincident with the inharmonic peaks of the unknown note than were any of the imposter strings. They found that, indeed, the theoretical inharmonicity of the fretboard position on which the note was truly plucked would reliably yield the largest number of coincident peaks located inside the spectral search window bounds, hence our name "partials coincidence tally" for their algorithm. Their work demonstrated the sufficiency of inharmonicity as a discriminating feature.

This method capitalizes further on their trajectory realization. Rather than zooming into fret-board positions' spectra to evaluate their candidacy, we propose characterizing each candidate string by its inharmonicity trajectory and classifying a note as simply belonging to the string whose inharmonicity trajectory evaluated at an unseen note's fundamental best approximates the note's inharmonicity. In other words, our similarity routine is rather some likelihood calculation  $A'(\beta, \hat{\beta}_s)$ , discussed in detail in the next sections, between the note's measured inharmonicity  $\beta$  and each candidate fretboard positions' predicted inharmonicity  $\hat{\beta}_s$  for the note. This is an algorithmically simple approach, requiring merely inharmonicity estimation of the unknown note, inharmonicity trends of training guitars' strings, and a likelihood measure. Additionally, this method is governed by the intuitive spatial constraint of inharmonicity trajectory: notes played

on a given string can exhibit only a set of inharmonicities, so classifying based on a spatially-informed metric, like proximity to a trajectory line, makes physical sense.

#### 3.1.2 Inharmonicity Estimation

The first step in our system is estimation of the inharmonicity of an unseen note. The algorithm we employed was the median adjustive trajectories (MAT) method [18] proposed by Hodgkinson in 2009, and we present it here. At the core of this method is a recasting of the definition of inharmonicity in terms of any two partials and their indices. We can rearrange equation (2.1) to express the fundamental  $f_0$  in terms of the frequency  $f_m$  of the mth partial and its index m as

$$f_0 = \frac{f_m}{m\sqrt{1+\beta m^2}}. (3.2)$$

Next, we can substitute (3.2) back into  $f_0$  in equation (2.1) to obtain

$$f_k = \frac{kf_m}{m\sqrt{1+\beta m^2}}\sqrt{1+\beta k^2}.$$
 (3.3)

Solving for  $\beta$  yields

$$\beta = \frac{(f_k \frac{m}{k})^2 - f_m^2}{k^2 f_m^2 - m^2 (f_k \frac{m}{k})^2},\tag{3.4}$$

which defines the inharmonicity of a note in terms of an arbitrary two partials k and m and their corresponding frequencies  $f_k$  and  $f_m$ .

The MAT algorithm first pre-processes a note with the Complex Spectral Phase Evolution (CSPE) method [28] to refine the subsequent frequency estimates. The inharmonicity routine then begins by locating partials 1 and 2 in this refined spectrum based on a user-input estimate of the fundamental (slight mis-tuning aren't a problem, since the algorithm refines the  $f_0$  estimate with each iteration as will be seen). Searching for these partials is straightforward, since the skew effect of inharmonicity is low for small indices. From these locations, (3.4) is used for every combination of the located partials (which for this first iteration, is only one) to obtain an array

of  $\beta$  estimates. The median of the  $\beta$  array is returned as the composite  $\beta$  estimate of this iteration, and this median together with the partials' locations are used in (3.2) to obtain an array of revised  $f_0$  estimates that contains as many elements as partials under consideration. Finally, the median of this  $f_0$  array and the median of the  $\beta$  array are used in equation (2.1) to predict the frequency of the next partial for which to search, which is the third in this case. At succeeding iterations, the lengths of these  $f_0$  and  $\beta$  arrays grow longer (since both the number of partials, and therefore the number of combinations of partials pairs, increases), and the estimates which they contain grow increasingly more accurate. The algorithm is summarized in the flowchart in Figure 3.1.2, taken from [18]. MAT determines the ideal number of partials for which to search by finding the highest partial index that lies above the average level of the magnitude spectrum, and terminates once this partial index is reached. The result is an efficient and accurate inharmonicity estimation routine.

We apply the MAT estimation procedure to five consecutive 100ms windows of audio, starting from detection of a note onset. This yields five inharmonicity estimates which we aggregate into one effective result using the frame aggregation process described in Section 3.1.5.

### 3.1.3 Log-inharmonicity

Next in our pipeline, we apply a log transformation to the estimated inharmonicity to make its variation with respect to MIDI pitch linear. This has the advantage of using simpler regression models for classification at later stages. If we reconsider equation (3.1) in terms of MIDI pitch number m instead of fret number n, we obtain

$$\beta(s,m) = \beta(s,m_{os})2^{\frac{m-m_{os}}{6}},$$
(3.5)

where  $m_{os}$  is the open-string pitch of string s. The exponential trajectory is maintained since incrementing both fret number and MIDI pitch number accomplishes an equivalent increase in fundamental frequency. Figure 3.3 illustrates the trajectories in terms of MIDI pitches.

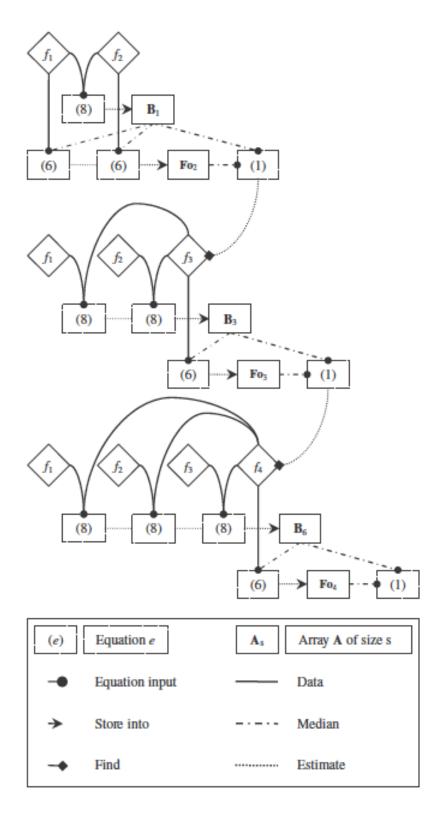


Figure 3.2: Median-adjustive-trajectory (MAT) method flowchart.

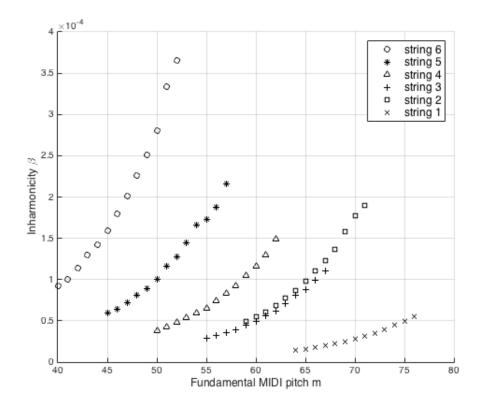


Figure 3.3: Inharmonicity estimates for frets 0-12 on strings 1-6 of RWC acoustic guitar 111AG. Note the exponential trajectory as in Fig. 3.1 and the increased segregation since notes and their inharmonicities are now plotted against their fundamental instead of their fret number.

Now if we consider the log-inharmonicity  $\beta_l(m)$ , we see that

$$\beta_l(m) = \log_2 \beta(s, m) = \log_2 [\beta(m_{os}) 2^{\frac{m - m_{os}}{6}}]$$
 (3.6)

$$\beta_l(m) = \log_2 \beta(m_{os}) + \frac{m - m_{os}}{6}$$
 (3.7)

$$\beta_l(m) = (\log_2 \beta(m_{os}) - \frac{m_{os}}{6}) + (\frac{1}{6})m, \tag{3.8}$$

where string s has been dropped from the notation since we're looking only at variation along a fixed string. Substituting  $w_0$  for  $\log_2 \beta(m_{os}) - \frac{m_{os}}{6}$  and letting  $w_1 = \frac{1}{6}$ , we obtain the familiar linear form

$$\beta_l(m) = w_0 + w_1 m,\tag{3.9}$$

showing us that the log-inharmonicity trajectory of a given string varies linearly with respect to MIDI pitch. See Figure 3.4. This relationship also holds for fret number, but fundamental pitch is a more likely and generic feature to have been estimated, and consequently is more relevant to consider here.

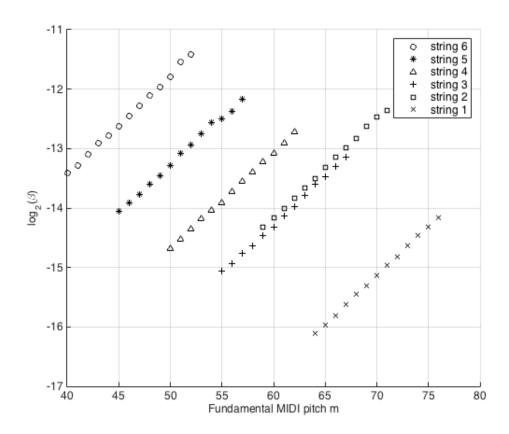


Figure 3.4: Log-inharmonicity estimates for same frets, strings, and guitar as Fig. 3.3

### 3.1.4 Learning Inharmonicity Trajectories

We next need to train our system to understand strings' typical log-inharmonicity trajectories. There are two methods we use to do this. The first, which we call *theoretical* trajectory determination, is essentially the method employed by the existing PCT method [4]. For each guitar, we collect the six notes labeled with fret 0 (i.e. played open-string) from its recordings, and estimate their inharmonicities. Our line trajectories are then simply those defined by the weights

in equation (3.9) in terms of the open-string inharmonicities. The "learning" in this case is just the estimation of open-string inharmonicities; the lines themselves are simply extrapolations of the expected theoretical relationship between a string's log-inharmonicity and its pitch.

The other method we use is linear regressions for obtaining *empirical* log-inharmonicity trajectories. Regressing log-inharmonicities of notes against their pitches could capture a more fitting trajectory than the strictly theoretical trajectories obtained from equation (3.9). In this method, we collect all notes with common string labels in our training data, and we perform linear regression of their log-inharmonicities  $\beta$  against their fundamental pitches m (in MIDI note number format). We do this for each string label. Let  $s \in \{1, 2, 3, 4, 5, 6\}$  be the string label to which each training note is assigned, and  $N_s$  be the number of notes we have belonging to string label s. If we let  $\mathbf{x}_s^{(i)} = [1, m_s^{(i)}]^T$  represent the ith note with fundamental pitch  $m^{(i)}$  belonging to string s, and  $\beta_s^{(i)}$  be the log-inharmonicity of the ith note belonging to string s, we can solve

$$\mathbf{w}_s = \underset{\mathbf{w}}{\operatorname{arg\,min}} \sum_{i=1}^{N_s} (\beta_s^{(i)} - \mathbf{w}^T \mathbf{x}_s^{(i)})^2$$
(3.10)

which yields the weight vector  $\mathbf{w}_s$  that minimizes the sum of squared error between the measured and predicted inharmonicities of the notes belonging to string s. Figure 3.5 illustrates the differences between the trajectory and regression approaches.

Occasional estimates from the inharmonicity measurement routine would be outliers that deviated substantially from the linear trajectory model, and which disproportionately skewed the regressions. For this reason, we rather performed a bisquare weighted linear regression, instead of the standard unweighted regression in (3.10), to empirically approximate the log-inharmonicities. Bisquare weighting is a robust linear regression procedure that minimizes a weighted sum of squares which de-emphasizes outliers. This provided us with higher-quality regressions that were more robust to stray inharmonicity measurements. The iterative procedure, implemented using Matlab's linear-model fitting routine 'fitlm', works as follows and is taken from the routine's oneline documentation [29]. First, a regular least squares fit is obtained. Then,

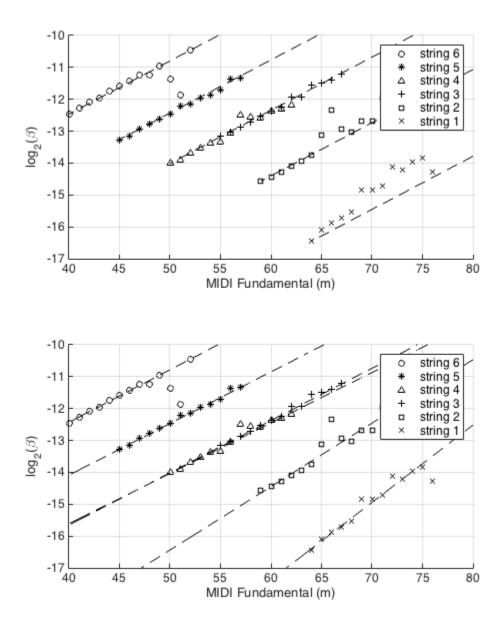


Figure 3.5: Comparison of string inharmonicity characterization approaches for our recorded Fender Telecaster. Top: *theoretical* trajectories. Note how each line begins precisely at the first marker of each string (i.e. the open-string note) and continues along a trajectory that ignores empirical inharmonicity estimates. String 1 most clearly shows this. Additionally, the trajectories for strings 3 and 4 are effectively co-linear and therefore indiscriminable. Bottom: regressions, or *empirical* trajectories. Each regression was obtained by minimizing the bisquare weighted error of the string's residuals, so the lines more closely follow the empirical inharmonicity scatterplot estimates.

the standardized adjusted residuals are computed according to

$$u = \frac{r_i}{Ks\sqrt{1 - h_i}},\tag{3.11}$$

where  $r_i$  are the plain residuals,  $h_i$  are the leverages, K=4.685 is a tuning constant, and s is the robust variance, which is equal to M/0.6745 where M is the median absolute deviation of the residuals. Next, the robust weights are computed according to

$$w_i = \begin{cases} (1 - u_i^2)^2, & |u_i| < 1\\ 0, & |u_i| \ge 1 \end{cases}$$
 (3.12)

The process is repeated, iteratively improving the weights until the fit converges.

#### 3.1.5 String Classification

The next step is to use these learned string trajectories to classify unknown notes. Since these trajectories tell us how to expect different strings' inharmonicities to vary with respect to pitch, determining the trajectory which best approximates the unseen note's inharmonicity estimate seems a fitting procedure to ascertain the originating string. Specifically, we impose Gaussian distributions centered at the trajectories over each strings' inharmonicity measurements, and evaluate them at unseen notes' inharmonicities to determine the most likely string from which it came.

We can think of each inharmonicity trajectory as rather a trajectory of one-dimensional Gaussian log-inharmonicity distributions, whose means are the log-inharmonicity values of points on the line, and whose variances are the regression's residuals' variance  $\sigma_s^2$ . In this manner, the probability distribution that characterizes the inharmonicity trajectory  $\mathbf{w}_s$  of string s at the fundamental pitch  $m_0^{(i)}$  of the ith note  $\mathbf{x}^{(i)} = [1, m_0^{(i)}]^T$  is defined as the normal distribution  $\mathcal{N}(\mu, \sigma^2)$ , with mean  $\mu = \mathbf{w}_s^T \mathbf{x}^{(i)}$  and variance  $\sigma^2 = \sigma_s^2$ . Now to classify the nth unseen note  $\mathbf{x}^{(n)}$ , we

simply measure its log-inharmonicity  $\beta^{(n)}$  and obtain the probability that this log-inharmonicity was observed given each string s according to

$$P(\mathbf{x}^{(n)}, \beta^{(n)}|s) = \mathcal{N}(\beta^{(n)}|\mathbf{w}_{s}^{T}\mathbf{x}^{(n)}, \sigma_{s}^{2}), \tag{3.13}$$

and return the predicted string  $\hat{s}^{(n)}$  that maximizes this probability,

$$\hat{s}^{(n)} = \arg\max_{s} P(\mathbf{x}^{(n)}, \beta^{(n)}|s). \tag{3.14}$$

This classification procedure is illustrated in Figure ??

Another conceivable decision function is classification based on an unseen note's residual distance to the string trajectories. Instead of selecting the string whose trajectory maximizes the probability of inharmonicity observation, one could select the string whose trajectory exhibits minimum residual with the note's inharmonicity measurement. Indeed, this worked well, though experiments comparing this "hard" classifier versus the aforementioned "soft" probabilistic classifier found that the probabilistic method performed slightly better, so we used this for our experiments in Chapter 4.

Occasionally, outlier inharmonicity estimates would throw off this classification scheme. To combat this, we employed frame aggregation, a refinement technique used by [1]. In frame aggregation, inharmonicities of multiple audio windows are estimated and classified individually, then the multiple results are aggregated into a compound final decision. In essence, it's a form of averaging that relies on the theoretically low probability that consecutive inharmonicity estimates will all be outliers; the more probable stable estimates will contribute to a more reasonable aggregate string decision. For each frame r in an R-frame aggregation of a note, we obtain the string-wise probabilities  $\mathbf{p_r} = [p_1, p_2, p_3, p_4, p_5, p_6]^T$  of its log-inharmonicity estimate and all possible strings s. Then, we simply sum the probabilities  $\mathbf{p_r}$  over all frames and select the string

whose aggregate likelihood is maximal:

$$\hat{s} = \underset{s \in \{1, 2, 3, 4, 5, 6\}}{\operatorname{arg max}} \sum_{r=1}^{R} \mathbf{p_r}.$$
(3.15)

The result is a slight boost in classification accuracy. For our system, we used 5 abutting frames, each 100ms long, which start at the detected onset of a note. Onsets were obtained through automated detection for convenience, the details of which are beyond the scope of this work but are cited for reference [5, 10]. This frame-aggregation procedure is possible only because the RWC evaluate notes are all at least 500ms long, and this is of course a limitation – two notes per second amounts to quarter notes at 120bpm, which is an unrealistic requirement of guitar audio, notorious for featuring licks played with rapidity that exceeds eighth and sixteenth notes at considerably higher tempos.

#### 3.1.6 Tablature Conversion and Refinement

The final step is transformation of the string classifier output into tablature notation. Because the classifier outputs are string numbers, we still need to infer notes' fret positions. This is a trivial task provided we know the tuning  $\mathbf{t} = [m_1, m_2, m_3, m_4, m_5, m_6]^T$  of the unknown guitar, where  $m_s$  is the MIDI pitch number of the open note on string s. Because we already have an estimate of the nth unknown note's MIDI pitch  $m^{(n)}$ , simply taking the difference between  $m^{(n)}$  and its assigned string's open pitch  $m_s$  yields the fret on which the note was played; each fret on a guitar increases the string's pitch by one half-step, or equivalently the MIDI pitch number by one integer.

We further refine the fretboard estimates with "plausbility filtering" [1] in which we reject and reassign those string decisions that are implausible given the constraints and assumptions of the data. If our systems assigns to a note a string on which the corresponding fretted position is negative, a mistake has clearly been made since frets can't possibly be lower than 0 (the openstring). Similarly, if our system assigns to a note a string on which the corresponding fret is

greater than 12, we know this is an error as well because our guitar recordings feature only frets 0-12. When either of these situations arise, we reject the string classification decision and select the next closest string. If the succeeding string is also implausible, the process is iterated until all six strings have been considered.

## 3.2 Baseline Bayesian Classifier

#### 3.2.1 Motivation

The hypothesis of our first method (inharmonicity trajectories) is that exploitation of the linear trajectories of log-inharmonicities is helpful for transcription, but in order to objectively evaluate this we need for comparison the results of a baseline classifier which *doesn't* leverage the inharmonicity trajectories. This method, which we're calling simply Bayesian classification of fretboard position, serves as this benchmark requirement.

#### 3.2.2 Inharmonicity Estimation

As before, the first step in this method is measurement of the inharmonicities of notes. We again use median-adjustive-trajectories (MAT) to estimate this crucial feature. MAT is explained in detail in Section 3.1.2.

#### 3.2.3 Fretboard Position Inharmonicity Distributions

Next, we need to capture the inharmonicity distribution of each of the F=78 fretboard positions f (13 fretting options for each of the 6 strings). For each position, we calculate the median  $\tilde{\beta}_f$  and variance  $\sigma_f^2$  of its inharmonicities, and use these to parameterize a Gaussian probability density  $\mathcal{N}_f(\tilde{\beta}_f, \sigma_f^2)$ . The median was chosen to lessen the influence of outliers, and was found to produce slightly better results.

#### 3.2.4 Fretboard Position Classification

Now that each fretboard position f has been characterized as a probability distribution  $\mathcal{N}_f$  of inharmonicities, we can classify an unseen note. With the interpretation of inharmonicity as a random variable, we measure the inharmonicity  $\beta^{(i)}$  of the ith unseen note  $\mathbf{x} = [1, m_0^{(i)}]^T$  with fundamental MIDI pitch  $m_0^{(i)}$  and simply evaluate the probability  $P_f(\beta^{(i)}|f) = \mathcal{N}_f(\beta^{(i)}|\tilde{\beta}_f, \sigma_f^2)$  of it having been generated by the Gaussian distribution  $\mathcal{N}_f$ . We narrow our search space  $f \in \{1, 2, 3, ..., F - 1, F\}$  of possible fretboard positions by considering just the positions  $f \in \{f_{m,1}, f_{m,2}, ...\}$  that agree with the unseen note's fundamental frequency. For standard-tuning, this is a maximum of only three positions. To classify a note, we select the candidate fretboard position which maximizes the likelihood of the note's measured inharmonicity, just as in equation (3.14):

$$\hat{f} = \underset{f \in \{f_{m,1}, f_{m,2}...\}}{\arg \max} P(\beta^{(i)}|f). \tag{3.16}$$

We also perform frame aggregation with this classification method, using the same specifications as discussed in Section 3.1.5: five abutting 100ms windows following the onset of a test note are individually analyzed to produce five inharmonicity measurements. Each measurement is used in equation (3.16), and the individual probabilities for candidate fretboard positions are summed across the five measurements. The position with maximum aggregate probability is selected as the classification output. Again, the unrealistic tempo limitation of this procedure must be recognized – quarter notes at 120bpm is an atypically slow bound for a guitarist's performance.

## 3.3 Tuning Compensation Feature

After transforming our data into log-inharmonicity space in the first method, and after estimating the probability distributions of each fretboard positions' inharmonicities in the second method, we can apply a tuning compensation feature step if the test guitar is known to be in alternate-tuning. In this section, we derive the straightforward adjustment and explain how it is incorpo-

rated into our system, focusing on the context of the inharmonicity trajectory method.

Though standard tuning is the most common pitch configuration of six-string guitars, there exist numerous other tunings in which performers often play. Aside from altering the musicality of the instrument, alternate tunings complicate the transcription process by introducing uncertainty about the open-string pitches. They distort inharmonicity-based methods, since the tensions T of alternately-tuned guitar strings differ from those in standard-tuned strings, thereby affecting estimation of the inharmonicity.

Current string classification systems don't address this degree of freedom. PCT [4] is technically tuning-invariant, but requires access to recordings of the alternately-tuned test guitar's open strings and so isn't considered a classifier here. Other supervised learning systems [1, 9, 19] are trained and evaluated only on standard-tuning guitars.

A possible approach to augment inharmonicity-based string classifiers with tuning-invariant performance is to simply introduce a scaling factor on the expected inharmonicity, or equivalently an additive factor on the expected log-inharmonicity. The fundamental frequency  $f_0$  of an ideal vibrating string is related to its tension T according to

$$f_0 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \tag{3.17}$$

where L is string length and  $\mu$  is its density. From this, we can see that

$$T \propto f_0^2. \tag{3.18}$$

Recognizing that for a change in fundamental pitch of  $\Delta m$  semitones, the equivalent change in frequency is  $2^{\frac{\Delta m}{12}}$ , we see that the proportional change in tension (with all other factors constant) is

$$T \propto 2^{\frac{\Delta m}{6}} f_0^2$$
. (3.19)

The resulting log-inharmonicity of this new open-string note with pitch  $m_{os}'=m_{os}+\Delta m$ , with

original open-string pitch  $m_{os}$ , is therefore

$$\log_2 \beta(m'_{os}) = \log_2 \left[ \frac{\pi^3 Q d^4}{64T l^2} (2^{-\frac{\Delta m}{6}}) \right]. \tag{3.20}$$

$$\log_2 \beta(m'_{os}) = \log_2 \frac{\pi^3 Q d^4}{64T l^2} - \frac{\Delta m}{6}.$$
 (3.21)

Substituting (3.21) into  $w_0$  from equation (3.9) to obtain the new intercept term  $w'_0$ , we get

$$w_0' = \log_2 \beta(m_{os}') - \frac{m_{os}'}{6} \tag{3.22}$$

$$w_0' = \left(\log_2 \frac{\pi^3 Q d^4}{64T l^2} - \frac{\Delta m}{6}\right) - \frac{m_{os} + \Delta m}{6}$$
(3.23)

$$w_0' = \log_2 \frac{\pi^3 Q d^4}{64T l^2} - \frac{m_{os}}{6} - \frac{\Delta m}{3}$$
 (3.24)

$$w_0' = w_0 - \frac{\Delta m}{3}. ag{3.25}$$

The slope coefficient  $w_1 = -\frac{m}{6}$  is independent from  $\Delta m$ , so the affected log-inharmonicity trajectory of this alternately-tuned string is therefore

$$\beta'(m) = w_0' + w_1' m \tag{3.26}$$

$$\beta'(m) = w_0 + w_1 m - \frac{\Delta m}{3}. (3.27)$$

In words, equation (3.27) tells us that the effect of a string's alternate tuning is simply addition of a bias term  $\frac{-\Delta m}{3}$  to the log-inharmonicity, where  $\Delta m$  is the semitone deviation from standard tuning. We should thus be able to compensate our predictive trajectories to arbitrarily-tuned guitars, provided we're given the alternate tuning in which the unseen notes are being performed. As can be seen in Figure 3.3 when one string on the standard-tuned electric guitar is modified using equation (3.27), the predictive alternate-tuning regression aptly captures the trend of the alternate-tuning inharmonicity. This is a straightforward modification for any inharmonicity-

exploiting classifier, and we report our results with this modification in the next chapter.

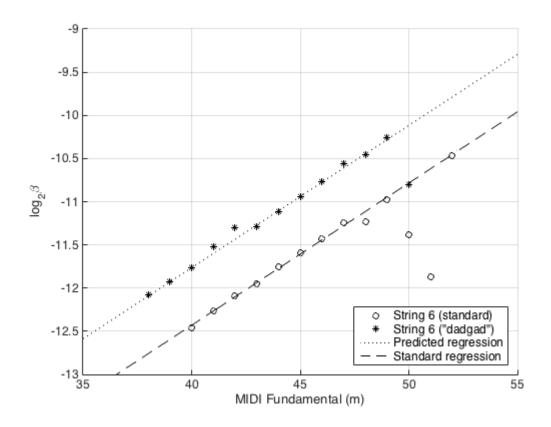


Figure 3.6: Effect of alternate tuning on an electric guitar's log-inharmonicity trajectories. The circles are log-inharmonicity estimates of frets 0-12 on string 6 of RWC electric guitar EG131 in standard tuning, and the dashed line is their bisquare-weighted regression. The asterisks are log-inharmonicity estimates for the same frets, string, and guitar recorded two semitones down. The dotted line passing through them is the *predicted* regression obtained by applying equation (3.27) to the dashed line.

## 3.4 System Overviews

Below in Figure 3.7, we summarize the operation of our inharmonicity trajectory system. Inharmonicity estimates are obtained for our training guitars' notes, then we represent these estimates in log-inharmonicity space with respect to MIDI pitch to obtain linear trends. We then check if the evaluation guitars' tunings are non-standard, and if so, we appropriately compensate the

weight vectors that define the linear trajectories. Next, we perform classification on test recordings by assigning to unknown notes the strings whose Gaussian log-inharmonicity trajectories maximize the likelihood of the notes' log-inharmonicities. Finally, tablature conversion and refinement is performed with plausibility and constraint filtering.

Our baseline Bayesian fretboard-position classifier is summarized in Figure 3.8. It crucially differs from the inharmonicity trajectory method in that it doesn't exploit any aspect of the trajectory at all; it simply performs likelihood-maximization using the individual fretboard inharmonicity distributions, providing a relevant benchmark of transcription performance. First, inharmonicity estimates for the training guitar notes are obtained, and normal distributions are fitted to each fretboard positions' inharmonicity variation. Then, if the test guitar's tuning is non-standard, the distributions can be compensated appropriately. Notes from the test guitar are classified by measuring their inharmonicities and, as before, selecting the fretboard positions that maximize the likelihood of their observation. Tablature refinement is built-in by considering as candidate fretboard positions only those whose fundamental pitches match the unseen notes'.

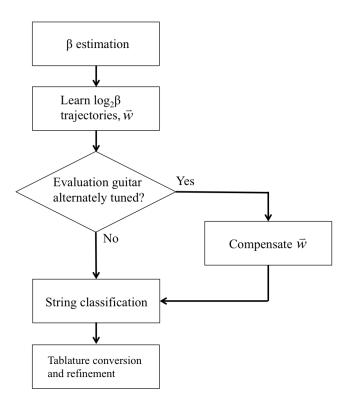


Figure 3.7: System overview of tablature transcription with log-inharmonicity trajectories using probabilistic classification.

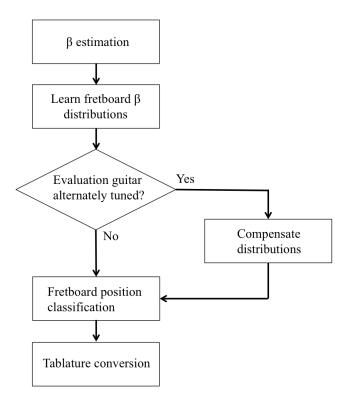


Figure 3.8: System overview of tablature transcription with Bayesian fretboard-position classification.

# **Chapter 4: Experiments and Results**

#### 4.1 RWC Evaluation

We used subsets of the Real World Corpus Music Instrument Database (RWC-MDB-I) [17] and personal recordings of an electric and acoustic guitar to conduct our experiments.

Our RWC subset comprised nine guitar recordings (three classical, three acoustic, and three electric), each of which were performed twelve times exhibiting various permutations of particular musicality parameters (playing style, dynamic level, pickup selection). For our work, we used only six of the twelve recordings for each of the electric guitars, as the omitted ones featured musicality parameters like vibrato and palm-muting that weren't featured in the classical and acoustic sets and which made inharmonicity estimation difficult. At the time of this thesis' writing, we're not certain whether [4] also used this smaller subset of electric guitars, so we can't fairly compare our electric guitar results to theirs; we accordingly draw comparisons for electric guitar results only between our baseline and our regression method. The RWC recording performances themselves were simply clean, isolated, monophonic enumeration of every fret (from open-string to 12th fret) on every string (from string 6 to string 1), constituting 78 total note plucks per audio file. The resolution is 16 bits per sample, at 44.1kHz. Labels of strings and frets are thus obtained by their location of occurrence in the recording. The classical guitars recorded were a Stafford, a Sakurai Kohno Professional-J, and a Yuichi Imai YJ-II; the three acoustic guitars captured were an Ovation, a Yamaha APX, and Yairi WY1; the electric guitars featured were a Fender Stratocaster, an Aria PE, and an Ibanez Artcore.

We conduct our experiments on all nine RWC guitars at three levels of context, which comprise the headings of Table 4.1: guitar-specific, guitar-averaged, and guitar-independent. For guitar-specific trials, we train and test the system on the same guitar using three-fold cross-validation with test folds that are one-third of the guitar's recordings. The folds' results are averaged together for composite transcription errors. For guitar-averaged trials, we train the system on all recordings of the two guitars in the same class as the test guitar, as well as a two-thirds training portion of the test guitar, and then evaluate on the remaining third of the test guitar recordings. Again, three-fold cross-validation is used to iterate through the portions of the test recordings. For guitar-independent trials, the system never sees any of the test guitar's recordings before evaluation. We train the system on two of the guitars in the same class as the test guitar, and test on all recordings of the remaining one. In all cases, Table 4.1 reports error probabilities, as this was the transcription metric used in [4].

In the "Bayes" columns of each heading in Table 4.1, we report transcription results using the baseline Bayesian classifier we developed. To review, Gaussian distributions were fit to the training data's inharmonicity measurements of every fretboard position. Then, notes were classified by selecting the fretboard positions that most probably produced their measured inharmonicities, according to the learned Gaussian likelihoods.

We next report string identification results (taken from [4]) using the PCT method under the "PCT" columns of Table 4.1. As discussed in Chapter 2, this method uses estimates of the openstring inharmonicities of the training audio's guitar(s) to predict *expected* inharmonicities of an unknown note's candidate fretboard positions. The degree of similarity between the unknown note's measured inharmonicity and that of the candidate fretboard positions is assessed by tallying their coincident partials. The candidate fretboard position that produces the highest tally is returned as the transcribed tablature. Note that no results for the "Guitar-independent" column exist for this method, as no results were reported in [4] for this scenario.

Finally, we evaluate our trajectory classification method, reporting error probabilities using empirical trajectories (i.e. regressions) under the "Regr." columns of Table 4.1. We performed

bisquare weighted regression of the training guitars' log-inharmonicity estimates against their pitches. String classification was performed on the test notes by selecting the regressions whose imposed Gaussian PDFs maximized the likelihood of the notes' inharmonicities, as discussed in Chapter 3. We also evaluated our *theoretical* trajectory method, in which regressions aren't computed but likelihood maximization is still used for classification. However, error probabilities were similar to those of our *empirical* trajectory (i.e. regression) method, so we omitted them from Table 4.1 for conciseness.

Transcription Error Probabilities										
	Guit	tar-spec	eific	Guita	ar-avera	aged	Guitar-independent			
Guitar	Bayes	PCT	Regr.	Bayes	PCT	Regr.	Bayes	PCT	Regr.	
Classical CG091	0.02	0.04	0.03	0.13	0.09	0.03	0.14	_	0.03	
Classical CG092	0.15	0.00	0.06	0.16	0.01	0.06	0.13	_	0.03	
Classical CG093	0.08	0.01	0.04	0.14	0.00	0.06	0.14	_	0.06	
Overall CG:	0.08	0.02	0.04	0.14	0.03	0.05	0.14	_	0.05	
Acoustic AG111	0.00	0.00	0.03	0.01	0.06	0.03	0.02	_	0.03	
Acoustic AG112	0.02	0.00	0.03	0.03	0.22	0.06	0.13	_	0.13	
Acoustic AG113	0.03	0.00	0.04	0.06	0.04	0.06	0.10	_	0.07	
Overall AG:	0.02	0.00	0.03	0.03	0.11	0.05	0.08	_	0.08	
Electric EG131	0.22	_	0.18	0.21	_	0.18	0.24	_	0.23	
Electric EG132	0.15	_	0.10	0.15	_	0.17	0.17	_	0.21	
Electric EG133	0.06	_	0.09	0.20	_	0.19	0.22	_	0.27	
Overall EG:	0.14	_	0.12	0.19	-	0.18	0.21	_	0.24	

Table 4.1: Error probability comparisons between the baseline classifier (Bayes), the existing method's partial coincidence tally (PCT), and our proposed probabilistic regression classifier (Regr.). PCT results are taken from [4]; we've excluded error due to  $f_0$  estimation in their results. Note that PCT results for electrics are intentionally excluded to avoid potentially unfair comparison, while PCT results for the "Guitar-independent" scenario aren't available.

We point the reader's attention to the "Overall..." rows in Table 4.1 to highlight the central results of this work. For the columns under the "Guitar-specific" column, the existing PCT method excels for every guitar. Clearly, tallying the coincident partials using inharmonicity estimates from the exact guitar on which tests are conducted outperforms our baseline and our regression classification method. Though for the "Guitar-averaged" scenario, both our baseline and our

regression method outperform PCT for acoustic guitars, though we examine the statistical significance of this observation in the next paragraph. And finally, we see that our regression-based scheme outperforms our baseline scheme in overall average performance (the significance of which we test below as well), suggesting that there is indeed merit in leveraging the structure of the inharmonicity trajectories compared to the structure-agnostic baseline method.

We performed two-tailed matched-pair t-tests to assess whether differences in transcription error between the existing PCT method and each of our novel methods were statistically significant at the  $\alpha=0.05$  level. Because we didn't have access to PCT's trial-by-trial transcription results, we couldn't perform a contingency-table-based test like the McNemar test we used between our novel methods. For each evaluation guitar, we obtained an approximation of the mean of the distribution of the differences between the PCT method and our comparison method (either Bayes or Regression) as  $\mu_d = p$ , where p is the success parameter obtained by differencing the PCT method's and comparison method's error probabilities. Then to obtain the standard deviation  $\sigma_d$  of the differences' distribution, we used the square root of a Bernoulli approximation of variance  $\sigma_d^2=p(1-p)$  where p is the success parameter as before. We calculated each comparison's t-score according to  $t = \mu_d/(\sigma_d/\sqrt{n})$ , with n = 12 for the number recordings over which the error probabilities were averaged (and n = 36 for the overall error measures), then summed the right and left tails of the c.d.f of the student's t-distribution at  $\pm t$ . This gives us the probability, or p-value, of observing the classifiers' performance differences due to chance given that the null hypothesis of zero-mean performance differences is true. In Table 4.2, we highlight transcription results from t-tests which yielded p-values less than  $\alpha = 0.05$ , allowing us to reject the null hypothesis in favor of the alternative that the classifiers' performance differences don't have zero-mean, i.e. that the highlighted results differ significantly from PCT. The only significant difference was the baseline's worse performance in the "Guitar-averaged" case of CG3, suggesting that in general our baseline and regression's performance don't differ much from PCT's.

We next performed two-tailed McNemar tests on our regression and baseline methods' results

Transcription Error Probabilities - Significance Comparison w/ PCT										
	Guit	tar-spec	eific	Guit	Guitar-averaged			Guitar-independent		
Guitar	Bayes	PCT	Regr.	Bayes	PCT	Regr.	Bayes	PCT	Regr.	
Classical CG091	0.02	0.04	0.03	0.13	0.09	0.03	_	_	_	
Classical CG092	0.15	0.00	0.06	0.16	0.01	0.06	_	_	-	
Classical CG093	0.08	0.01	0.04	0.14	0.00	0.06	_	_	-	
Overall CG:	0.08	0.02	0.04	0.14	0.03	0.05	_	_	_	
Acoustic AG111	0.00	0.00	0.03	0.01	0.06	0.03	_	_	_	
Acoustic AG112	0.02	0.00	0.03	0.03	0.22	0.06	_	_	-	
Acoustic AG113	0.03	0.00	0.04	0.06	0.04	0.06	_	_	-	
Overall AG:	0.02	0.00	0.03	0.03	0.11	0.05	_	_	_	

Table 4.2: Error probability comparisons. Results from our baseline ("Bayes") and regression ("Regr.") methods which differ from PCT to a statistically-significant ( $\alpha=0.05$ ) degree are shaded in gray. Electric guitar trials and "Guitar-independent" scenarios are omitted since comparisons in those cases aren't available.

to assess the validity of the null hypotheses that both classifiers performed equally well for each RWC evaluation guitar at the  $\alpha=0.05$  significance level. (See Table 4.3). Transcripts of our two classifiers' 78 notes  $\times 12$  recordings = 936 string assignments (468 for the electrics' fewer recordings) for each evaluation guitar along with its true labels were examined with Matlab's 'testcholdout' function to provide p-values. For "Overall..." rows, we concatenated the three guitars' assignments. We found that for the "Guitar-specific" scenario, our regressions improved on CG2, CG3, and EG2, while our baseline performed better on AG1. All other differences under this heading were insignificant with p-values greater than 0.05. For the "Guitar-averaged" scenario, regressions excelled at all CGs, the baseline excelled at AG1 and AG2, and all other differences were insignificant. Finally for the "Guitar-independent" scenario, the regression's improvements for CG1, CG2, CG3, and AG3 as well as the baseline's superiority for AG1, EG2, and EG3 were all statistically significant.

Also displayed in Tables 4.4, 4.5, and 4.6 are the *string-wise* F1-scores using our regression-based classifier for the "Guitar-averaged" scenario. The F1-score is the harmonic mean of precision and recall, and is a commonly used classification metric elsewhere, so we use it here and wherever comparison with [4] is absent. These tables' left-hand columns denote the string num-

Transcription Error Probabilities - Novel Methods Significance Comparison											
	Guit	tar-spec	eific	Guit	Guitar-averaged			Guitar-independent			
Guitar	Bayes	PCT	Regr.	Bayes	PCT	Regr.	Bayes	PCT	Regr.		
Classical CG091	0.02	_	0.03	0.13	_	0.03	0.14	_	0.03		
Classical CG092	0.15	_	0.06	0.16	_	0.06	0.13	_	0.03		
Classical CG093	0.08	_	0.04	0.14	_	0.06	0.14	_	0.06		
Overall CG:	0.08	_	0.04	0.14	_	0.05	0.14	_	0.05		
Acoustic AG111	0.00	_	0.03	0.01	_	0.03	0.02	_	0.03		
Acoustic AG112	0.02	_	0.03	0.03	_	0.06	0.13	_	0.13		
Acoustic AG113	0.03	_	0.04	0.06	_	0.06	0.10	_	0.07		
Overall AG:	0.02	_	0.03	0.03	_	0.05	0.08	_	0.08		
Electric EG131	0.22	_	0.18	0.21	_	0.18	0.24	_	0.23		
Electric EG132	0.15	_	0.10	0.15	_	0.17	0.17	_	0.21		
Electric EG133	0.06	_	0.09	0.20	_	0.19	0.22	_	0.27		
Overall EG:	0.14	_	0.12	0.19	_	0.18	0.21	_	0.24		

Table 4.3: Error probability comparisons. Superior results in trials with statistically-significant ( $\alpha=0.05$ ) performance differences are shaded in gray. PCT results omitted to highlight performance differences between our baseline and regression methods.

ber of the rows' F1-scores, and the right-hand columns are the row-wise average of the three guitars in the class. We present this information to highlight the individual strings' contributions to the overall transcription accuracy.

Transcription F1-scores									
String No.	CG1	CG2	CG3	Overall CG					
1	0.96	0.93	0.96	0.95					
2	0.92	0.90	0.91	0.91					
3	1.00	0.95	0.96	0.97					
4	0.96	0.93	0.91	0.93					
5	1.00	0.96	0.93	0.96					
6	1.00	0.98	1.00	0.99					

Table 4.4: String-wise F1-scores for the RWC classical guitars, using regression classification for the "Guitar-averaged" scenario.

Transcription F1-scores									
String No.	AG1	AG2	AG3	Overall AG					
1	0.96	0.96	0.96	0.96					
2	0.94	0.80	0.87	0.87					
3	0.91	0.89	0.85	0.88					
4	1.00	1.00	0.99	1.00					
5	1.00	1.00	0.99	1.00					
6	1.00	1.00	1.00	1.00					

Table 4.5: String-wise F1-scores for the RWC acoustic guitars, using regression classification for the "Guitar-averaged" scenario.

Transcription F1-scores									
String No.	EG1	EG2	EG3	Overall EG					
1	0.96	0.94	0.96	0.95					
2	0.94	0.95	0.72	0.87					
3	0.47	0.42	0.42	0.44					
4	0.66	0.72	0.79	0.72					
5	0.85	0.92	1.00	0.92					
6	0.96	0.98	1.00	0.98					

Table 4.6: String-wise F1-scores for the RWC electric guitars, using regression classification for the "Guitar-averaged" scenario.

## 4.2 Tuning Compensation

For the next experiment, we also recorded a Fender Telecaster (electric guitar) and a Takamine G-series (acoustic guitar) at 16 bits and 44.1kHz in the same vein as the RWC database: string and fret enumeration, totaling 78 notes per recording. The acoustic's string gauge was 0.12, while the electric's was unknown. We captured various tunings: standard, "DADGAD" (in which the open pitches of strings 6 through 1 are given by the respective letters in name), "WSU" (wholestep up), and "WSD" (whole-step down). The Fender, which was recorded directly into our audio interface, was performed with a thick plectrum at center pickup orientation (both bridge and neck pickups on) and at similar moderate dynamic levels throughout. We recorded the Takamine in an isolated acoustic chamber with a large diaphragm condenser microphone (an MXL 990) placed 12 inches away from fret 12, and we also played it with a thick plectrum and at constant dynamic levels. Analog-to-digital hardware included a Focusrite Saffire Pro 24 and a 2011 Macbook Pro running Logic. No other musicality parameters were captured; we focused on tuning variance here.

In standard tuning, strings 6 through 1 are tuned to E2, A2, D3, G3, B3, and E4. In so-called "DADGAD" tuning, the open-string pitches are made to be D2, A2, D3, G3, A3, and D4 as these open pitches concatenated together form its name. Whole-step up and whole-step down are tunings in which each of the standard-tuned strings are respectively incremented or decremented by two semitones. For whole-step down, this results in D2, G2, C3, F3, A3, and D4. For whole-step up, we have F#2, B2, E3, A3, C#4, and F#4.

We learned inharmonicity regressions from the recordings of the standard-tuned RWC electric and acoustic guitar recordings, and predicted string classes on our alternate-tuned Fender Telecaster and Takamine G-series recordings, respectively. We evaluated string-wise F1-score performance both with and without the tuning compensation factor in our system discussed in Chapter 3. Tuning compensation results are denoted by the "comp." column, and original results

without compensation are denoted by the "orig." column. See Table 4.7. For reference, we also report transcription performance on our personal guitars in standard tuning, in the second column from the left. We highlight in gray the results which are statistically significant at the  $\alpha=0.05$  level differences, determined by a McNemar test. Highlighted cells under 'orig.' columns indicate that the performance degradation due to the corresponding tuning is significantly different from the standard-tuned performance ('Standard' left-most heading). Highlighted cells under 'comp.' columns indicate that the performance gain achieved by our compensation system is significantly different than its un-compensated counterpart.

Alternate-tuning Transcription F1-scores (Electric)										
	Standard	"DADGAD"		"WSU"		"WSD"				
String No.	orig.	orig.	comp.	orig.	comp.	orig.	comp.			
1	0.96	0.96	0.96	0.87	0.96	0.96	0.96			
2	0.96	0.92	0.92	0.70	0.96	0.96	0.96			
3	0.50	0.48	0.43	0.65	0.50	0.50	0.50			
4	0.76	0.76	0.74	0.38	0.76	0.76	0.76			
5	1.00	1.00	0.87	0.56	1.00	1.00	1.00			
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00			
Overall:	0.86	0.85	0.82	0.69	0.86	0.86	0.86			

Table 4.7: String-wise F1-scores for our electric Fender Telecaster at various tunings. Regressions trained on recordings of all three RWC electric guitars. WSU: whole-step up (from standard); WSD: whole-step down (from standard); orig.: no tuning compensation used in this trial; comp.: tuning compensation used in this trial. The "Overall" row displays average F1-score of each column. Statistically significant ( $\alpha=0.05$ ) results highlighted in gray.

Alternate-tuning Transcription F1-scores (Acoustic)									
	Standard	"DADGAD"		"WSU"		"WSD"			
String No.	orig.	orig.	comp.	orig.	comp.	orig.	comp.		
1	0.96	0.87	0.92	0.96	0.96	0.92	0.96		
2	0.82	0.70	0.63	0.44	0.82	0.67	0.92		
3	0.96	0.93	1.00	0.74	0.96	0.00	0.96		
4	0.87	0.79	0.79	0.90	0.87	0.44	0.93		
5	1.00	1.00	1.00	0.82	1.00	0.81	1.00		
6	1.00	1.00	1.00	0.96	1.00	1.00	1.00		
Overall:	0.93	0.88	0.89	0.80	0.93	0.64	0.96		

Table 4.8: String-wise F1-scores for our acoustic Takamine G-series at various tunings. Regressions trained on all recordings of all three RWC acoustic guitars. WSU: whole-step up (from standard); WSD: whole-step down (from standard); orig.: no tuning compensation used in this trial; comp.: tuning compensation used in this trial. The "Overall" row displays average F1-score of each column. Statistically significant ( $\alpha=0.05$ ) results highlighted in gray.

# **Chapter 5: Discussion**

In this chapter, we begin with an analysis of the previous chapter's results, followed by consideration of factors that potentially limit the significance of our findings. We close with a discussion about future work.

## 5.1 Analysis

Comparison of our regression-based classifier results ("Regr." columns) and our baseline classifier results ("Bayes" columns) in Table 4.3 revealed that, of the cases in which statistical significance at the  $\alpha=0.05$  level was achieved, leveraging the linear structure of log-inharmonicities more often improved transcription accuracy over the baseline than vice versa. Interestingly, our regression method overall outperformed the baseline on classical guitars for all headings, while the reverse was true in general for acoustics. Performance on electrics was fairly evenly split, with the regression method excelling under the "Guitar-specific" heading, the baseline excelling under the "Guitar-independent" heading, and no significant difference under the "Guitar-averaged heading."

Differences between our methods and the existing PCT method weren't nearly as dramatic. According to our statistical significance testing, effectively none of the performance differences observed in Table 4.2 were significant at the  $\alpha=0.05$  level, suggesting that there's considerably high probability that these observed accuracy differences arose from random chance and that the null hypothesis – the mean of the differences between our and the existing methods' results is

zero – is true. Of course, the one exception appears to be that our baseline performs significantly worse than PCT for the "Guitar-averaged" scenario.

The alternate-tuning experiments validated our tuning compensation feature; we see in Tables 4.7 and 4.8 that the effect of appropriately scaling the inharmonicity regressions for tunings which significantly degraded standard-tuned performance is to restore transcription accuracy towards that of the standard tuning. Interestingly, the effect of various tunings ("DADGAD" in both electric and acoustic, "WSD" in the electric) on our guitars' transcriptions is statistically insignificant, suggesting that we may have overestimated how problematic some alternate-tunings are for inharmonicity-based systems. Also surprisingly, we learned that the "Standard" and "WSD" accuracies for our Telecaster are identical both with and without tuning compensation, and that the tuning compensation effect for "DADGAD" on our Telecaster is, though statistically insignificant, detrimental. These were perplexing results which we couldn't answer.

To begin our investigation into the factors that degrade transcription, we plotted every guitars' six inharmonicity regressions side-by-side in Figure 5.1. The informative plots reveal a number of things. The most prominent descriptors of these plots are the variability of the inharmonicity measurements, and the compactness or proximity of the regressions. Correlate these descriptors with the performance scores in Table 4.1 is insightful. Both acoustics and classicals performed well, despite their differing regression qualities: the acoustics' more stable measurements and separated lines versus the classical regressions' less stable and closer locations. This can be explained by the fact that the classicals' strings whose regressions nearly overlap or are nearly co-linear occupy distinct pitch regions. The top left CG1 plot illustrates this best. The two left-most parallel regressions belong to nicely separated clusters of inharmonicity measurements, so classification confusion between them is unlikely because of dataset constraints that confine possible fret choices to be less than 12 and greater than 0. So inharmonicity variance and regression nearness don't seem to heavily impact transcription results if the concerned regressions belong to distinguishable inharmonicity measurement clusters. For the electric guitars, however, the strings' inharmonicity variances appear even larger, and the regressions' proximities exhibit

multiple overlaps (see bottom left EG1 plot). This, coupled with the fact that the concerned strings are neighbors and therefore share a higher number of plausible fretting locations, caused great difficulty for our classifier.

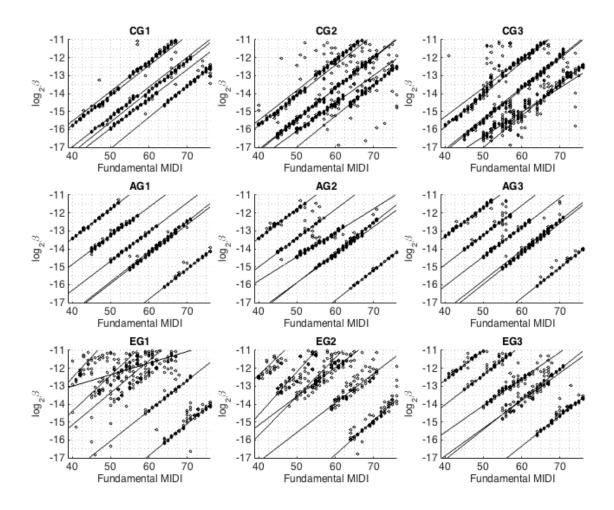


Figure 5.1: Log-inharmonicity regressions learned from each acoustics' and classicals' 12 recordings and each electrics' 6 recordings, superimposed over the recordings' inharmonicity measurements.

We zoomed into Figure 5.1 to overlay the guitars' strings to get a closer reading on how each strings' regressions vary with guitar. As can be seen in Figures 5.2, 5.3, and 5.4, the classical and acoustic guitars' inharmonicity regressions remain fairly stable from guitar to guitar, whereas

	Classification									
Truth	1	2	3	4	5	6				
1	1.00	0	0	0	0	0				
2	0	0.77	0.23	0	0	0				
3	0	0.08	0.62	0.38	0	0				
4	0	0	0.23	0.46	0.31	0				
5	0	0.08	0	0	0.77	0.15				
6	0	0	0	0	0	1.00				

Table 5.1: Confusion matrix for one classification trial with EG131. Strings 3 and 4, as they are here, were frequently misclassified because of their regressions' poor discriminability.

those of the electric guitars clearly exhibit more variability, making likelihood-based classification more difficult. Electric guitar regressions belonging to strings 3 and 4 overlap considerably, which is a behavior not absent in the classicals (strings 1 and 4, 2 and 5, 3 and 6) and acoustics (strings 2 and 3) with some strings but which manifests in less error due to their regressions' more consistent placements than those of their electric counterparts. Indeed, the electric guitars' confusion matrices support this, and one which highlights this error trend is shown in Table 5.1.

We wondered to what extent our inharmonicity measurement variability was simply due to intrinsic variability in our inharmonicity measurement routine. To try to answer this, we recorded 25 uniform plucks of all strings' open notes on both our personal acoustic (Takamine G-series) and another electric (Ibanez RG) guitars, and analyzed statistics of their estimated log-inharmonicities. The string gauges for the acoustic and electric were 0.12 and 0.10 respectively. Both were played with a thick plectrum. Boxplots displaying the median, lower and upper quartile, data range, and any outliers of each guitar are shown in Figure 5.5 and Figure 5.6. The acoustic's and electric's data spans are on the order of one-hundredths of a unit of log-inharmonicity, which are orders of magnitude smaller than the variation we see in Figures ??, ??, and ??. It seems our inharmonicity measurements are fairly sound, and can't solely account for the larger measurement variance we see, particularly in the electrics.

We examined the quality of fit of string regressions obtained from all recordings of each of the 3 guitars from each of the 3 types, and found that the classicals' and acoustics' range

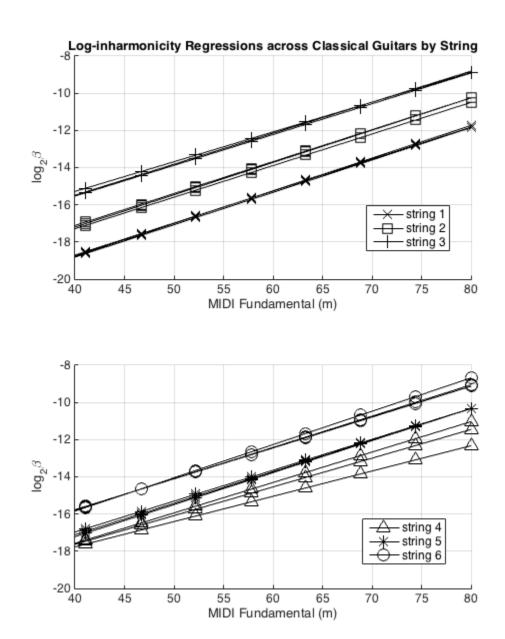


Figure 5.2: Superimposed string-wise regressions of all three RWC classical guitars. Top: strings 1, 2, and 3. Bottom: strings 4, 5, and 6. Plots were separated for visibility, and axes scales were kept identical for comparison. Observe the consistent intercept and slope despite varying guitar (with the only exception being string 6).

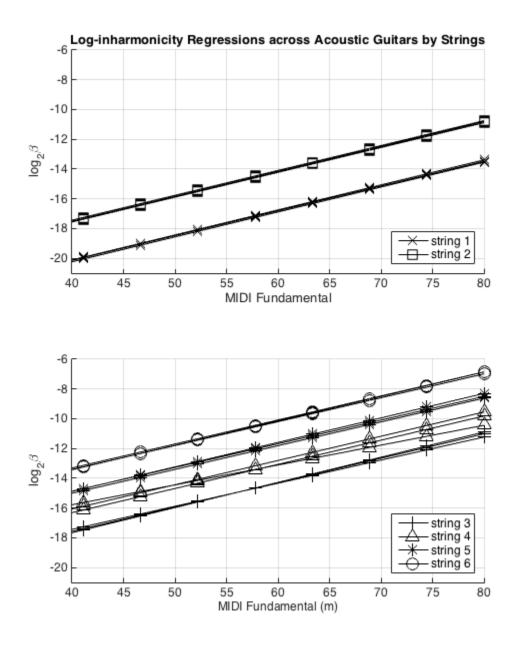


Figure 5.3: Superimposed string-wise regressions of all three RWC acoustic guitars. Top: strings 1 and 2. Bottom: strings 3, 4, 5, and 6. Plots were separated for visibility, and axes scales were kept identical for comparison. Observe the consistent intercept and slope despite varying guitar.

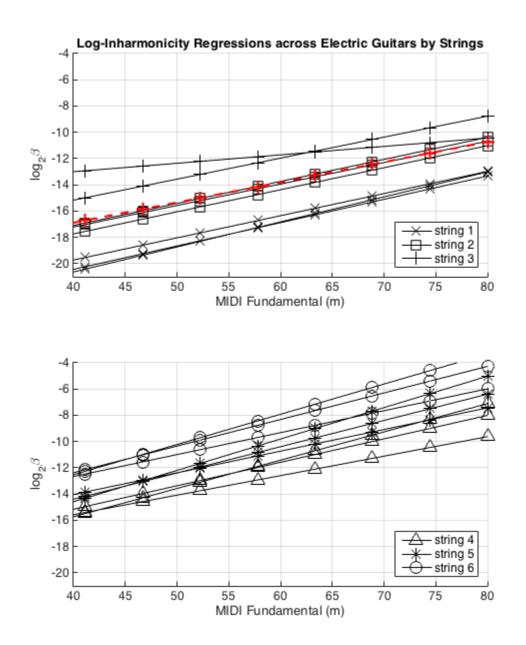


Figure 5.4: Superimposed string-wise regressions of all three RWC electric guitars. Top: strings 1, 2, and 3. Bottom: strings 4, 5, and 6. Plots were separated for visibility, and axes scales were kept identical for comparison. Observe the *inconsistent* intercept and slope. In particular, note the overlap between one of string 3's regressions (dashed red) and string 2's regressions. Also observe the overlapping slope and intercept ranges for strings 3 and 4 across the two plots.

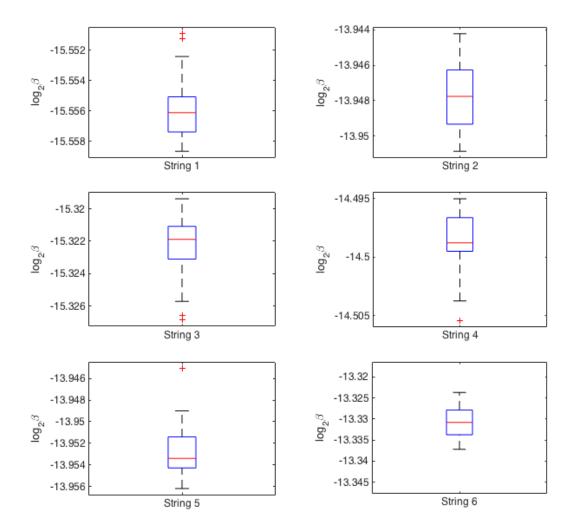


Figure 5.5: Boxplots of log-inharmonicity estimates for 25 uniform plucks of our acoustic guitar's (Takamine G-series) six open-string notes. Horizontal red lines are medians, horizontal blue lines are upper and lower quartiles, whiskers denote data range excluding outliers, red "+"s are outliers.

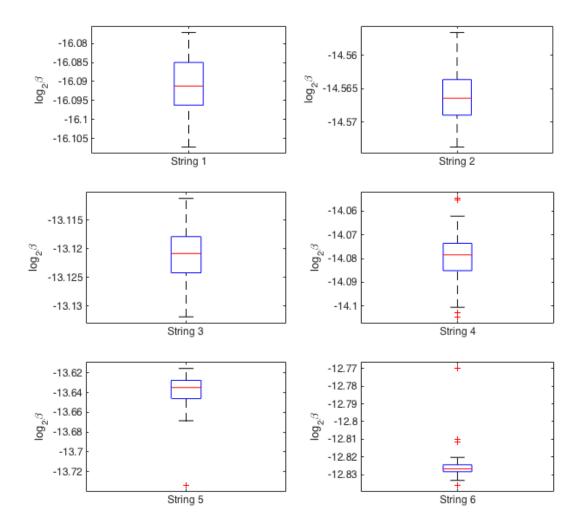


Figure 5.6: Boxplots of log-inharmonicity estimates for 25 uniform plucks of our electric guitar's (Ibanez RG) six open-string notes. Horizontal red lines are medians, horizontal blue lines are upper and lower quartiles, whiskers denote data range excluding outliers, red "+"s are outliers.

of determination coefficients was substantially smaller than the electrics'. Highest and lowest quality regressions for CG091, AG111, and EG131 are shown in Figure 5.7 as an example of this; CG091's and AG111's best and worst strings'  $r^2$  coefficients differ by only about one-hundredth, but the  $r^2$  difference for EG131 is an enormous 0.87. The bottom-row of this figure suggests that, though our inharmonicity estimate is reliable, something about the RWC electric guitar EG131 is causing inconsistent measurements.

We wondered if the transduction of the EG pickups somehow played a role in impeding reliable intra-guitar inharmonicity estimation – either through their transfer functions, possible noise introduction, or some non-linear characteristic. If this were the case, it would follow that higher-intensity performances would elicit this impeding phenomenon to a greater extent and produce measurably more variable inharmonicity estimates than would lower-intensity performances. Since one of the RWC musicality parameters that varies for each featured guitar is dynamic level of the performance, we were able to separate higher- and lower-intensity performances of the same guitar and analyze this interaction between degree of pickup transduction and inharmonicity estimation variability. For each guitar, and for each string, we collected note performances by common dynamic levels (piano, mezzo, and forte) and regressed their log-inharmonicities against their fundamentals, as usual. We then plotted statistics of the loginharmonicities' residuals against their played performance intensity. Figure 5.8 illustrates these statistics for EG131; other electric guitars' boxplots were omitted because their statistics were similar. We expected that variability would increase with dynamics, but the data didn't support this hypothesis. Interquartile ranges didn't exhibit any clear trend with respect to performance intensity, nor was the magnitude of their variation noteworthy.

Interestingly, both our inharmonicity estimation reliability and performance dynamic level don't explain the large variance we're seeing in inharmonicity estimates in the one electric guitar in Figure 5.7. Plucking with the finger vs. with a plectrum wasn't investigated in this work, so it remains unclear why such large variance persists in measurements on a single guitar. Possible other factors are numerous: fret buzz, plectrum thickness, variability in the gripping strength of

#### Best-Worst Fit Comparisons for Log-Inharmonicity Regressions

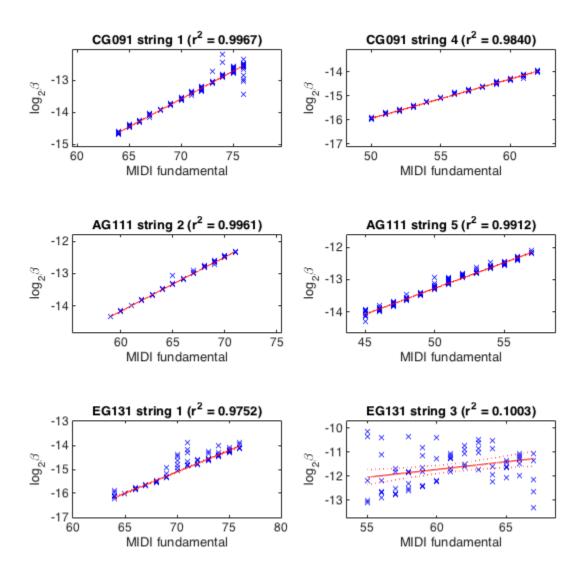


Figure 5.7: Best and worst bisquare log-inharmonicity regression fits, as measured by  $r^2$ , for representative guitars from each of the three classes. Top row: classical. Middle row: acoustic. Bottom row: electric. Left and right columns are best and worst fits, respectively. Dashed red lines, only visible on the bottom-right plot, denote 95% confidence intervals.

# Distribution of $\log_2 \beta$ Residuals for EG131

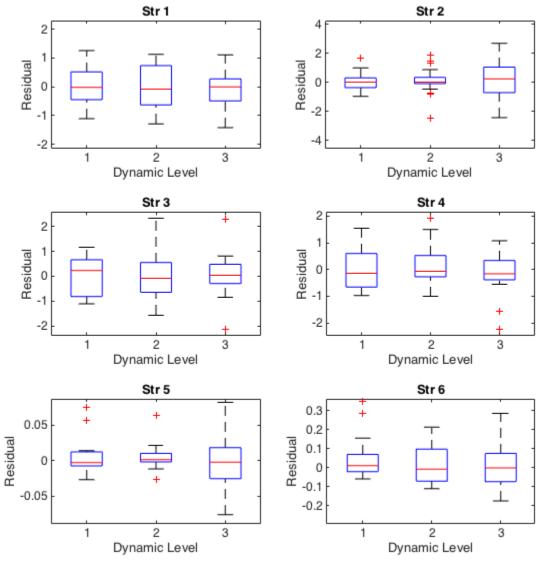


Figure 5.8: Boxplots of log-inharmonicity regression residuals for EG131 by dynamic level. Group 1: piano, Group 2: mezzo, Group 3: forte. Red center horizontal lines are medians; top and bottom edges of blue boxes represent upper and lower quartiles; whiskers depict full range of data excluding outliers; red plus signs denote outliers. Vertical axes not constant throughout plots; we're highlighting relative distributions of the boxplots here.

the fret-depressing hand, etc.

Many more factors could also be contributing to the *inter-guitar* inharmonicity measurement variance. Differences in the physical properties of the various RWC guitars' strings (like gauge, wound vs. unwound, length due to intonation adjustments, stiffness due to age) could be vast, and each of these factors are indeed components in the definition of inharmonicity in equation (2.2). Details regarding these instrument-specific factors aren't recorded in the RWC annotations, so it's plausible that our system's electric guitar transcription difficulty is exacerbated by unknown combinations of these confounding factors. To get an idea of the range of inharmonicity changes imparted by different values of these factors, we performed the following experiment. We recorded 0.09, 0.10, and 0.11 set gauges of the unwound third string of a personal electric guitar (Ibanez RG), as well as another take of a 0.11 set with a nickel-wound third string. We also had two sets of 0.10 set gauges, one of which was old and the other which was new, in an attempt to examine the effect of its possible age-related stiffness. Finally, we recorded each of the previous settings at two lengths: the owner's default 64cm intonation length and a 0.5cm intonation extension. Boxplots displaying the variation in inharmonicity measurements due to these factors are shown in Figure 5.9.

The plots in Figure 5.9 tell us that these different guitar geometry factors – string gauge, winding, stiffness, and intonation length – indeed affect, to varying degrees, our inharmonicity measurements. This agrees with the definition of inharmonicity in equation (2.2), which was a function of string diameter d, Young's modulus Q, and vibrating length l. Most of these factors impart log-inharmonicity changes on the order of only hundredths or tenths, which don't seem large enough to perturb the regressions in Figure 5.4 such that significant increases in transcription error occur; the stable regressions (belonging to strings 1 and 6, for example) are separated from their neighboring regressions by about 2 units, so it seems unlikely that inharmonicity biases due to age-related string stiffness or intonation-motivated length differences would compromise the performance of our system. However, the effect of string winding (top row, middle plot of Figure 5.9) is substantial: inharmonicity measurements for a nickel-wound third string

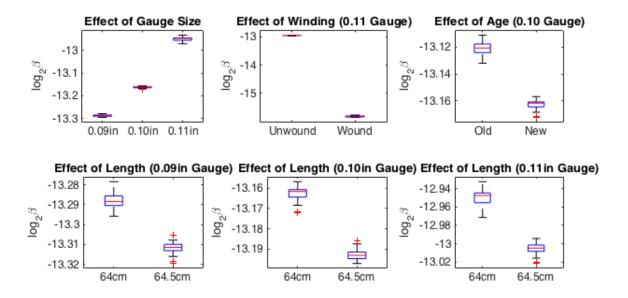


Figure 5.9: Effects on log-inharmonicity measurements of various electric guitar geometry parameters. In all boxplots, measurements are from 25 uniformly-played open notes (G3) on string 3 in standard tuning. Horizontal red lines are medians, horizontal blue lines are upper and lower quartiles, whiskers denote data range excluding outliers, red "+"s are outliers. Note the changes in scale on the vertical axes.

were about 3 whole units less than its unwound counterpart. It therefore seems clear that winding has the potential to considerably alter the placement of strings' regressions and compromise their predictive performance. This could also shed light on the outsized variance exhibited by the third electric guitar string's regressions in the top plot of Figure 5.4. The magnitude of the third string's deviations across guitars is within the range of winding-related inharmonicity changes, so it's plausible that this is the case in some of the RWC electric guitars.

Lastly, we examined the consequences of an overdrive effect on inharmonicity estimation. Though not immediately concerned with this work (the RWC dataset we used is entirely clean), it's a relevant question that seeks further insight into inharmonicity-based transcription systems' practicality in real-world guitar recordings. Briefly, overdrive is a phenomenon that occurs when guitarists play at gain levels that saturate the vacuum tubes in their amplifiers. The tubes non-linearly clip the guitar signal, producing harmonics and overtones that lend a warm and increasingly "broken-up" tone to their instrument. This analog phenomenon is an iconic facet of the

electric guitar's sound, and software plug-ins and pedals that attempt to simulate the effect without vacuum tubes are abundant. We applied two levels (12dB and 24dB) of Logic Pro's stock Overdrive plug-in to the 25 open notes of our Ibanez RG's third string. The tone knob was fixed to 980Hz and the output level was set to 0dB. For each trial, we measured log-inharmonicities as before and plotted their distributions in Figure 5.10.

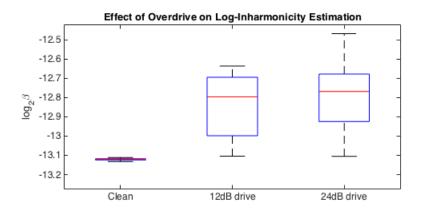


Figure 5.10: Boxplots of

We see that the effect of Logic Pro's overdrive plug-in on log-inharmonicity is to shift the median estimates upward by a few tenths and to increase the data range by an order of magnitude. Interestingly, the 24dB trial produces a more focused interquartile range of measurements, though the data range increases further still. Not shown beyond the bounds of this plot were two outliers for each of the 12dB and 24dB trials – these were omitted for greater boxplot visibility. It seems that the relative degradation of inharmonicity measurement quality with the introduction of a digital overdrive is appreciable, though we're unsure of the degree to which this might affect inharmonicity-based transcription results in practice.

#### 5.2 Limitations

It's important to highlight factors that potentially limit the significance of our findings. Our system currently operates only on clean (i.e. no effects processing), isolated, monophonic guitar

recordings, which are obviously not representative of the majority of guitar audio. Multiple layers of effects processing are commonly used by performers of the instrument to achieve distinctive, creative sounds. Guitarists often play in concert with other musicians, producing complex soundscapes instead of being featured in isolation. Moreover, guitar passages are rarely in their entirety strictly monophonic. These are large obstacles that separate our RWC subset from actual guitar performances, and thus the implications of our transcription results should be considerably tempered. Noteworthy though, is the extent to which other teams have overcome some of these factors. Certain tablature transcription systems [1, 4, 9, 19] have implemented polyphonic capabilities into their systems with good success, so polyphony is perhaps a smaller barrier than the others. As part of a multipitch estimation system in [31], Yazawa deduced tablature from RWC recordings that featured guitar passages playing over backing tracks, though the tablature accuracy is not reported.

Additionally, we assumed perfect fundamental pitch estimation in this work. Our decision to simply use pitch labels of the RWC recordings stemmed from convenience and scope; the focus of this work was simply evaluation of a specific approach to examining notes' inharmonicities for tablature transcription, so not concerning ourselves with the reliability of a pitch detection routine was a luxury that expedited this thesis. That being said, it should be acknowledged that the reliability of one's pitch estimator is a sure factor in the translation of this work's results to the real world.

Furthermore, the MAT inharmonicity estimation routine we used was different from Barbancho's. The credibility of the comparison between our results and theirs is thus somewhat compromised. It's quite possible that any performance gains observed in our implementation are due in part to potentially superior inharmonicity extraction. This is, unfortunately, a factor we didn't control for, and if time had allowed we should have replicated Barbancho's system and substituted their inharmonicity estimation step with MAT to produce results that were as comparable as possible.

#### **5.3** Future Work

Inharmonicity-based tablature is a fascinating topic, and future researchers should advance some of the preliminary findings in this thesis. In the previous section, we confirmed that inter-guitar geometry parameters played a significant role in inter-guitar inharmonicity measurement variability. This makes clear that understanding and accounting for the factors that perturb a system's inharmonicity measurement is key for good performance. More work should be done on characterizing these perturbations given a particular change in string gauge, length, winding, or stiffness. Perhaps learning and exploiting separate inharmonicity trends for various commonly-encountered combinations of these factors would make inharmonicity-based systems more viable. There's also much we didn't solve about our intra-guitar inharmonicity variability, and answers should be sought to why inharmonicity variability remains high on just one RWC guitar (i.e., eliminating inter-guitar variability) in some cases.

We also need to more deeply understand how inharmonicity measurements are affected by the complex alterations of guitar output that are common in recordings of real-world guitar performances. Creative effects processing with distortion, reverb and delay, for example, are a ubiquitous feature of the guitar's performance, and inharmonicity-based systems need to show that reliable measurement amidst this processing is attainable if they're to claim any practical merit. Thorough characterization of the consequences of string-bending, neck-bending, palmmuting, vibrato, and other performance variations is also needed, since these too are common features of guitar performances that most likely obscure the inharmonicity estimation process.

Future work should also consider ways to improve performance on the system level. One potential area for run-time performance enhancement is the use of musical context. Musical passages on the guitar tend to exhibit continuity; it's usually the case that a riff is played in a localized area of the fretboard, for example, instead of jumping around the length of the guitar neck. Transcription systems could apply these performance-borne simplifications to their output

to potentially improve accuracy, say, when a stray note is transcribed a plausible yet questionable twelve frets away from the rest of those in the riff. A measure of objective confidence in the transcription is required to do this. Without it, there'd be no way to determine whether it was the stray note or the majority of the riff which was in fact correctly transcribed. To this end, we investigated the amenability of our regression classification scheme to reliable confidence measurements, and found encouraging results. Figure 5.11 displays the probability, for each of the nine RWC guitars, of our system producing a correct fretboard transcription given increasingly wide windows in which we consider a note's inharmonicity estimate to have fallen. The inverse trend between residual window width and correctness probability shows that we can reliably infer some degree of confidence in a note's transcription given its distance from its classified line. This should be leveraged in a post-processing stage in later iterations of this and others' work to further refine tablature with isolated erroneous, yet plausible, transcriptions.

Many changes specific to this thesis can be explored in further work as well. Considering the good performance of the baseline Bayes classifier, experimentation should be continued with other standard classifiers. Perhaps a different well-studied classification routine that we've overlooked is even better-suited to the structure of the inharmonicity data, and would outperform our regression-based likelihood approach. Additionally, varying the regression polynomial could be an interesting experiment. Perhaps the system would perform differently if the log inharmonicity transformation was forgone and an exponential fit was explored instead. Fitting our regression system with polyphonic capability and observing the extent to which performance changes would also be a worthwhile study.

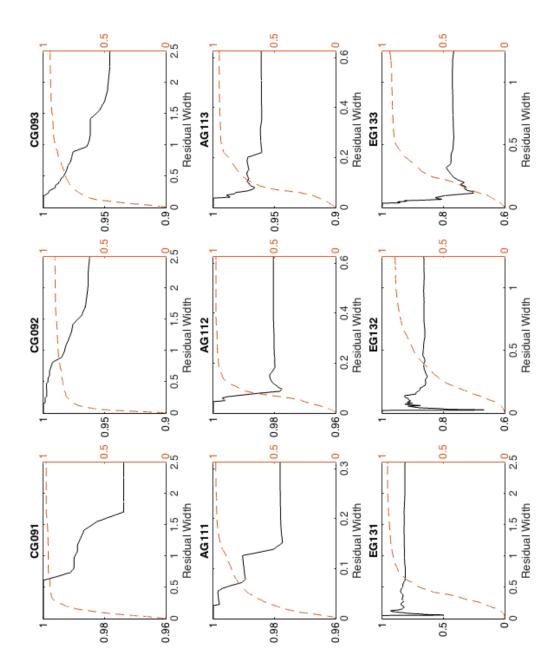


Figure 5.11: Solid black lines and left vertical axis: probability of correctly transcribing a note given the residual width under which the note's log-inharmonicity falls. Dashed orange lines and right vertical axis: CDF of the log-inharmonicity residuals, so one can get a sense of the proportion of data represented by the horizontal axis.

# **Chapter 6: Summary**

We began by introducing the guitar and an essential component of its musicality: the overlapping pitch ranges of its strings. We related this to tablature and its ability to uniquely specify fretboard positions for musical passages, then highlighted its popularity and its tedious manual annotation process. This led us to introduce automatic transcription systems and our goals for this work as an extension to the topic, which we reviewed in the following chapter.

We paid special attention to the partial coincidence tally (PCT) transcription method introduced by Barbancho in [4], which exploited only one feature known as inharmonicity in development of a successful system using only audio. We explained how inharmonicity affects the degree of upward skew of a note's partials, then discussed the intuition behind its discriminative power before introducing our novel approaches.

After reiterating one of Barbancho's key realizations – that inharmonicity along a given string was deterministic – we showed that the logarithm of these trajectories was linear with respect to MIDI pitch. We consequently proposed characterizing guitar strings as their log-inharmonicity trajectories and performing classification (and therefore implicitly tablature transcription) by probabilistically assigning to unknown notes the strings whose lines best approximated the notes' log-inharmonicities. Following this, we proposed a baseline Bayesian method against which we could evaluate the utility of our regression innovation, and which similarly operated by assigning to an unknown note the fretboard position which maximized the probability of having observed its measured inharmonicity. Lastly, we derived the effect of a tuning change on our regression method, and arrived at a general tuning-compensation feature applicable to any inharmonicity-

based system.

Next, we reported results from experiments on a subset of the RWC music instruments database and a couple personal guitar recordings. Comparisons of our baseline and regression approaches against the PCT method revealed similar accuracies overall, though our regression approach exhibits statistically-significant better performance than our baseline, arguing that leveraging inharmonicity trajectories is a worthwhile pursuit. We also validated our inharmonicity compensation feature, reporting accuracy increases on personal recordings of alternately-tuned electric and acoustic guitars.

A cursory investigation of the cause for the poor electric guitar transcription performance revealed that inharmonicity estimation for the instruments is less reliable, and both the electric guitars' pickups and the intrinsic variability of the measurement routine aren't to blame. It was found that factors related to the guitar geometry (string gauge, length, stiffness, winding) certainly induce changes in inharmonicity, which helps explain high inter-guitar inharmonicity variance, though it's still unclear why intra-guitar inharmonicity variance remains high for some of the RWC guitars' strings. We discussed limitations of this work, cautioning against misinformed application of our reported results to real-world guitar scenarios, and closed with future work, highlighting further experiments into inharmonicity's variation with respect to guitar geometry and effects processing, as well as potential system-level improvements and additions.

In conclusion, characterizing guitar strings as log-inharmonicity trajectories against pitch and classifying notes by maximizing the trajectories' observation probabilities is an effective approach to tablature transcription. It outperforms the benchmark probabilistic classifier introduced in this work and rivals performance of the existing inharmonicity-based method on the RWC guitar dataset. Furthermore, adjusting inharmonicity estimates with the compensation factor introduced in this work is effective for improving transcription accuracy on alternately-tuned guitars.

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