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Quality of Piano Tones

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A synthesizer was constructed to produce simultaneously 100 pure tones with means for controlling the intensity and frequency of each one of them. The piano tones were analyzed by conventional apparatus and methods and the analysis set into the synthesizer. The analysis was considered correct only when a jury of eight listeners could not tell which were real and which were synthetic tones. Various kinds of synthetic tones were presented to the jury for comparison with real tones. A number of these were judged to have better quality than the real tones. According to these tests synthesized piano-like tones were produced when the attack time was less than 0.01 sec. The decay can be as long as 20 sec for the lower notes and be less than 1 sec for the very high ones. The best quality is produced when the partials decrease in level at the rate of 2 db per 100-cps increase in the frequency of the partial. The partials below middle C must be inharmonic in frequency to be piano-like.

INTRODUCTION

THIS paper is a report of our efforts to find an objective description of the quality of piano tones as understood by musicians, and also to try to find synthetic tones which are considered by them to be better than real-piano tones.

The usual statement found in text books is that the pitch of a tone is determined by the frequency of vibration, the loudness by the intensity of the vibration, and the quality by the waveform. This picture is far too simple for any of these three subjective aspects of a tone. Pitch and loudness have received very extensive study. In this paper an attempt has been made to throw some additional light upon the quality of a piano tone.

It is true that the quality depends upon the waveform. But it also depends upon the pitch, the loudness, the decay and attack time, the variation with time of the intensity of the partials, the impact noise of the hammer, the noise of the damping pedal, and also the characteristic ending of the tone by the damping felt, etc.

In order to study the relative importance of these various factors, the following laboratory equipment and room facilities have been developed, namely, (1) anechoic chamber, (2) loudspeaker system, (3) tone synthesizer, and (4) the frequency changer. To these facilities have been added, a sonograph, an analyzer, a single-track tape recorder, a 5-track tape recorder, and other apparatus usually available in electronic research laboratories. A block diagram of the arrangement is shown in Fig. 1.

EQUIPMENT

1. Anechoic Chamber

An anechoic chamber was constructed for use as a listening room. It was built according to the architect's drawings loaned to us by the Bell Telephone Laboratories. Therefore, it is a copy of the one at those laboratories.

It consists of a rectangular block of cement with inside dimensions of $40\times30\times30$ ft. The block rests on sand and gravel, and is completely separate from the rest of the Eyring Science Center building. The room was treated with 6-ft acoustical wedges on each side, thus reducing the size 12 ft in each direction. A wire-mesh floor was constructed by stretching steel wires across the steel I-beams on the sides. These wires were separated by 2 in., and there were two sets at right angles to each other. This resulted in meshes 2 in. square. The floor is 10 ft below the ceiling edge of the

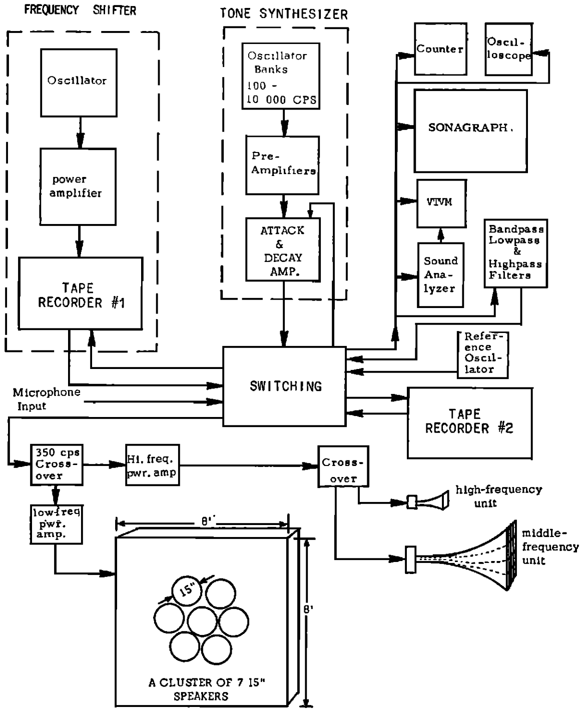

FIG. 1. Block diagram of equipment.

acoustical wedges and 8 ft above the lower set of wedges. It will support considerable weight but has little or no reflection for any of the sounds in the audible range. Chairs were supported on this wire floor at one end of the chamber for the jury of listeners. The loudspeaker system was installed at the opposite end.

2. Loudspeaker System

This consists of a three-channel system, each channel transmitting, respectively, the bands of frequency 20 to 400 cps, 400 to 4000 cps, and 4000 to 16 000 cps. The low-frequency channel consists of seven Altec 803 B drivers mounted close together in a circular arrangement as shown in Fig. 1, with a baffle which was 8 ft square. This baffle was mounted so it could turn about a horizontal axis. At certain angles from the vertical, a more uniform response for the very low frequencies was obtained.

As indicated in Fig. 1, the medium-range loudspeaker was a sectional-type horn, and the high-frequency loudspeaker was of a tweeter type. The response of this system is given in Fig. 2.

3. Tone Synthesizer

The tone synthesizer used to produce the synthetic tones consisted of five major parts, namely, a power supply, audio-frequency oscillators, a white noise source and filter set, attenuators and preamplifiers, and an attack and decay amplifier. A block diagram is shown in Fig. 3.

Power Supply

The requirements placed upon the power supply consisted of (a) supplying adequate power at the appropriate voltages and (b) maintenance of a low level of noise and hum. The second of the two requirements was the most difficult to satisfy, and was especially difficult in this case because of the surges of power involved in the operation of the attack and decay amplifier. To obtain this low noise level, a 6-v dc battery was used for the attack and decay amplifier vacuum-tube grid and filament supplies, and for the transistor oscillators' power requirements. The use of this simple battery eliminates the inherent problems posed by alternating current supplies. To obtain the required plate voltage for the attack and decay amplifier, a General Radio high-voltage supply was used in conjunction with specially designed filters. These filters consisted of *R-C* filter sections and voltage regulator tubes; it produced a constant voltage output with a noise or "ripple" level 80 db below the output signal level.

Audio-Frequency Oscillators

Since 100 oscillators were required it was necessary to select a design which was compact and which gave a stable oscillation. This was accomplished by using transistors with printed circuits. The circuit elements were mounted on a panel board 5 in. long and $1\frac{1}{2}$ in. wide. The inductance element can be varied by turning a key which can be inserted in a small hole through the face of the panel. In this way each oscillator can be turned to any desired frequency within about 1-octave range. The 100 oscillators can be tuned to cover a range from 50 to 15 000 cps. These 100 small panels supporting the oscillators were stacked together into three large panels. Two of these large panels are shown at the bottom half of Fig. 4. The other panel is on the opposite side of the portable table carrying the synthesizer.

Attenuators and Preamplifiers

The output of each oscillator is sent through an attenuator and then to its preamplifier. The 100 attenuators were arranged in a compact form at the back of a black panel board. White knobs (100 of them) which were projecting through the black panel board and which were connected to the sliding contact of the attenuator could be moved up and down in vertical slots to control the amount of attenuation introduced into each oscillating circuit. Each attenuator covered a range of 50 db. They were constructed so that the downward movement of the knob produced an attenuation

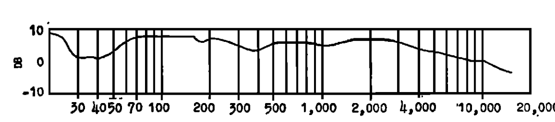

FIG. 2. Relative response of loudspeaker system.

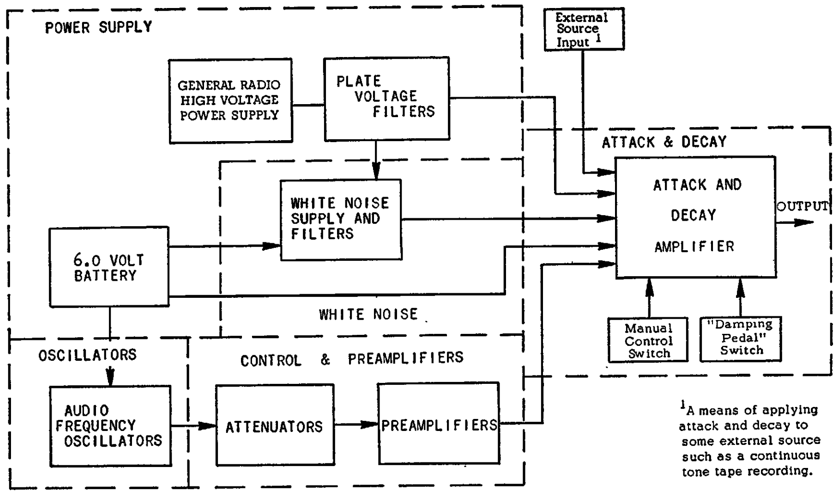

FIG. 3. Block diagram for tone synthesizer.

in db which was proportional to the distance it was moved. A db scale was engraved on the face of this panel. It is thus seen that the relative positions of these white knobs in vertical level give the relative levels in db of the current going from the oscillators to the preamplifiers and shows graphically the structure of the partials of the tone being synthesized.

In the upper part of Fig. 4 this panel is shown. The knobs in the picture are set to produce the synthetic piano tone A'''', the lowest note on the piano.

Attack and Decay Amplifier

The attack and decay amplifier[1] functions just as the name implies—it gives a beginning and an ending or attack and decay to any constant-level input. The specific requirements placed upon the amplifier, are listed as follows: (a) Frequency response: ± 2 db from 50 to 15 000 cps. (b) Noise and switching transients: 50 db below the signal level. (c) Intermodulation distortion: 5%. (d) Harmonic distortion: 2%. (e) Time rate of attack and decay continuously variable from 4,000–6 db/sec and very nearly exponential. (f) The attack and decay amplifier must operate over a range of 70 db (from maximum to minimum output signal). Considerable developmental work was necessary before a circuit was obtained which would fulfill these requirements, particularly the last one. The circuit which was finally used is given in Fig. 5. The concept involved is that of a two-stage push-pull amplifier in which the grids of both stages (4 vacuum tubes) are biased in accordance with the attack and decay required. The grid bias voltage comes from a resistance–capacitance–battery network which will thus provide an exponentially increasing or decreasing voltage depending upon whether the capacitor is charging or discharging. The function (attack or decay) of the amplifier is determined by connecting the battery voltage through a resistor and capacitor series circuit or shorting through the same capacitor but different resistors. This function is controlled by a push-button switch called the manual control switch. The output signal will increase when the button is pushed, build up to its maximum value, and remain at that point until the button is released. Upon release, the decay circuit is in use and the decaying rate is applied to the signal. The rate of attack and decay is determined by the adjustment of the resistance used in the *R-C* circuits. See Fig. 5.

Another control used is the "damping pedal" control which places a resistor in parallel with (by means of a pushbutton) the decay resistor. Thus the rate of decay can be changed to a more rapid one during the decay time and thus simulate the action of the damping pedal on the piano.

One serious limitation of the synthesizer is that the decay rate is constant and the same for all frequencies. This means that a curve showing level in db vs the time in seconds will be a straight line. The time in seconds to decay 20 db will be called the decay time. This is the time for the current in the attack and decay amplifier to decrease to 0.1 of its maximum value. Likewise, the

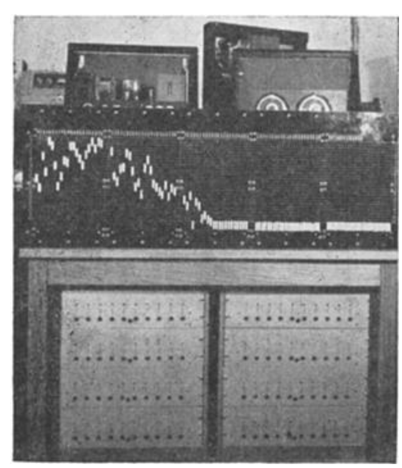

FIG. 4. Photograph of synthesizer.

[1] R. N. Christensen, "An Attack and Decay Amplifier Suitable for use in a Tone Synthesizer," thesis (unpublished), Brigham Young University, Provo, Utah (1959).

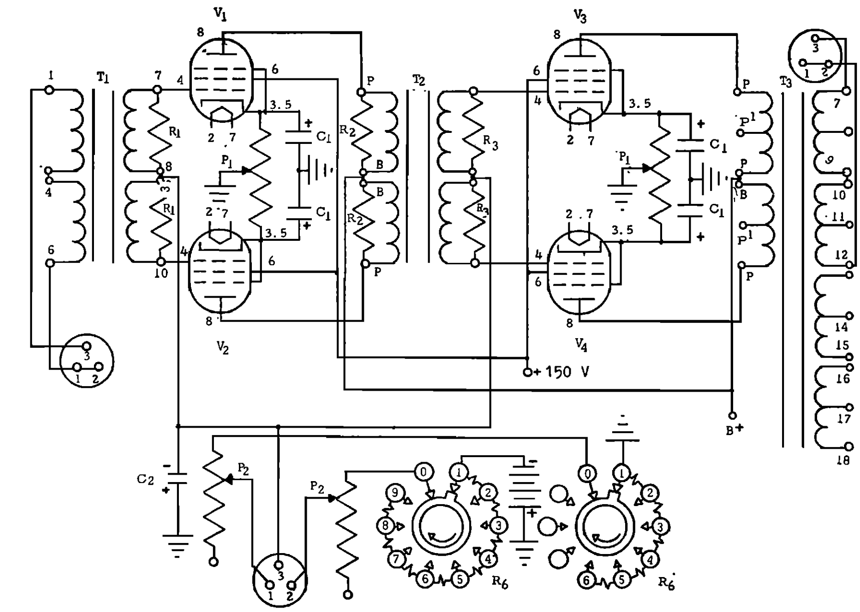

Fig. 5. Attack and decay amplifier circuit.

attack time is the time in seconds for the current to reach 0.9 of its maximum value.

4. Frequency Shifter

The frequency shifter consisted of a tape recorder with a synchronous motor which was driven by a combination of an oscillator and a power amplifier. By varying the frequency of the oscillator, the speed of the tape recorder could be increased to $\frac{4}{3}$ normal speed and decreased to $\frac{1}{4}$ normal speed or to any speed within these limits. This made it possible to set up any partial structure on the synthesizer, record it on the frequency shifter, and then shift to any desired fundamental frequency.

Method of Analyzing the Piano Tone

During the first part of our analysis work a sonograph was used. This instrument has been described in the literature and is in common use. The tone to be analyzed is recorded on a rotating cylinder. The sectioner is switched on and then the instrument draws horizontal lines whose lengths are proportional to the relative levels of the partials. The position of a peg on the top of the rotating cylinder determines the time during the duration of tone that corresponds to the partial structure being measured. By moving the peg on the rotating drum the chosen time can be changed in 0.04-sec intervals. The sonograph will record only sounds having a duration less than 2.4 sec. Thus the partial structure can be measured at 60 different times during the duration of such a tone. The approximate frequency can be read from this graph.

In the later part of our work a conventional analyzer was used which passed only a narrow band of frequency (approximately 4 cps). The output was read directly in db on a level meter. This procedure would be straightforward if the tone were steady. But a piano tone is continually varying so the piano tone was recorded on a continuous loop so that the tone could be continually repeated. The analyzer was set to give a maximum response for each partial near the beginning of the tone. Then a pure tone from an oscillator was sent through the analyzer with this setting and the frequency adjusted to give maximum response. The frequency of this pure tone was then measured on the electronic counter which will measure frequency with an accuracy of about 0.1%. In this way the frequencies and the relative levels of the partials at the beginning of the tone were determined. The variation of the partials with time can be inferred from the sonograph measurements.

In a third method of obtaining the partial structure and its variation with time, a level recorder was used. The complex tone from the loop was sent through the analyzer to the recorder. All of these methods have their advantages and disadvantages, and one method serves to check the others. In our general study of musical sounds all of these methods have been used. The general arrangement of these and other standard instruments is shown in Fig. 1.

As shown in Fig. 1, the current from the oscillators goes into the preamplifiers and then into the attack and decay amplifier. From here it may be switched to the loudspeaker system or to any of the other instruments shown.

With this instrumentation we can record the tones from the piano. From this recorded tone we are able to find the partial structure and how it varies with time. Also the attack and decay times can be measured. This analysis is then used to produce a synthetic tone, which may be compared to the real tone by judgment tests. Also new tones having any partial structure and any attack and decay times can be created for judgment uses.

EXPERIMENTAL

It is known that in a real-piano tone the partial structure is varying; that is, the decay curves of the partial tones will not be straight lines. It is also known[2] that the partials coming from a struck string are not strictly harmonic. At the impact of the hammer, a sound like that of hitting a board is superimposed on the tone from the string. This sound is particularly noticeable in the upper three octaves of the piano. When the piano tone is damped with the pedal it stops first the low tones and finally the higher ones giving a characteristic ending of the tone. The pedal itself produces a noise which is readily recognized as due to its movement. If one wanted to reproduce all these complicated sounds, one would make a tape recording of the actual piano tone. But we are interested in knowing the relative importance of these various factors.

This paper describes experiments which were designed to increase our understanding of these and other factors which govern the quality of piano tones. To begin the investigation, tones from a Baldwin grand piano were recorded. The piano was in a studio room of about the usual characteristics.

The tones recorded were designated thus:

C'''	Three octaves below middle C.
C''	Two octaves below middle C.
C'	One octave below middle C.
C	Middle C, frequency 261.6 cps.
C_1	One octave above middle C.
C_2	Two octaves above middle C.
C_3	Three octaves above middle C.

The same designations were used on the G-pitched, or any other tones, using middle G as the G a fifth above middle C.

About 2 sec after the key was struck, the damping pedal was used to dampen the tone. These tones were analyzed by use of the sonograph; three or four samples being taken at different times. The average of these four samples was taken as the partial structure. The results of these measurements are given in Fig. 6.

A careful measurement of the spacing on the frequency scale of the tracings made on the sonograph indicated that, within the observational error, the partials were approximately harmonic except for those of C''' and C''. The frequencies of the partials of these two low-pitched tones were found to be definitely higher than the harmonic frequencies. For example, for C''' the 30th partial frequency was found to be 1105 cps which is 134 cps greater than the harmonic frequency. It will be seen later that the partials of any piano tone are

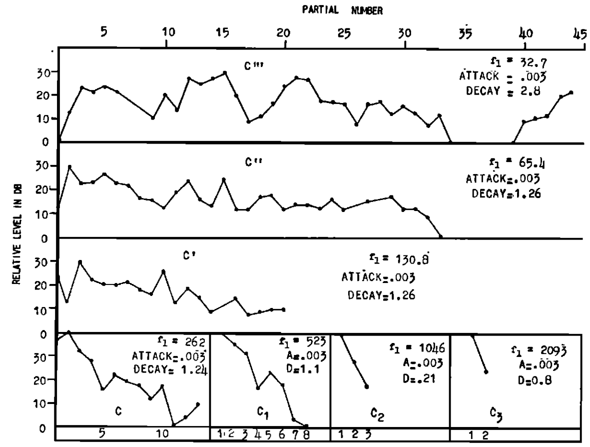

FIG. 6. Average partial structure of the studio piano.

inharmonic and can be calculated by the formula given by Young[2] and others.

Preliminary Judgments Tests

To obtain a first approximation of the relative importance of the various factors influencing the quality of the piano tone the following identification test was made. Synthetic tones were created having partial structures in accordance with those shown in Fig. 6. From these synthetic and the original piano tones, the following program was recorded. [The letter in the parentheses (R) indicates it was a real tone. Similarly the letter (S) indicates it was a synthetic tone.]

Test (1) $C(S)—C(S)—C(R)—C(S)—C(R)$.

Test (2) $C'(R)—C'(R)—C'(S)—C'(S)—C'(S)$.

Test (3) $C''(S)—C''(S)—C''(S)—C''(S)—C''(R)$.

Test (4) $C'''(R)—C'''(S)—C'''(S)—C'''(R)$.

Test (5) $C_1(S)—C_1(R)—C_1(S)—C_1(S)—C_1(R)$.

Test (6) $C_2(S)—C_2(S)—C_2(R)—C_2(R)—C_2(R)$.

Test (7) $C_3(S)—C_3(S)—C_3(S)—C_3(R)—C_3(S)$.

Test (8) $C''(S)—C'(R)—C''(R)—C(R)—C'''(S)—C(R)—C_1(S)—C_2(S)—C_3(R)$.

A jury of four musicians was asked to check which they considered were the real-piano tones. A second jury of laymen also took the test. The musicians identified 90% correctly and the laymen 86%. Both teams scored less than 75% on these tests for identifying middle C as a real-piano tone, showing the synthetic C tone was a better match for this than for the others.

From listening to the above program of tests and talking to members of these two juries, it was obvious that there were a number of clues for identifying the real-piano tones. Some of these are (1) the noise of the piano hammer striking the string, (2) the noise of the pedal dampening the tone, (3) higher background noise for the piano tones, (4) reverberation effects of the room

[2] R. W. Young, J. Acoust. Soc. Am. 24, 267–273 (1952).

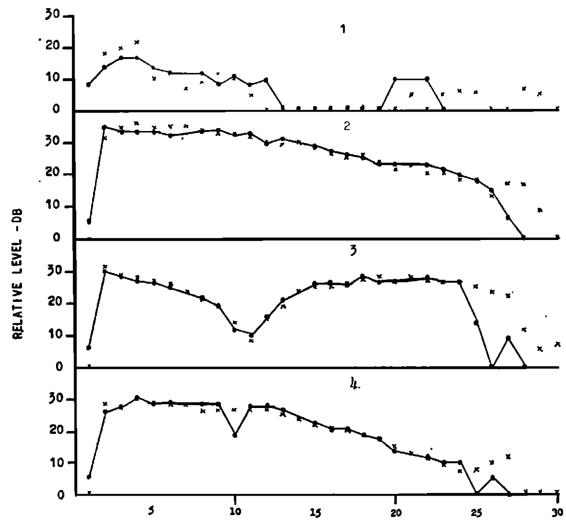

FIG. 7. Curves showing the changing level vs time for the partials 1, 2, 3, and 4 of the tone C‴.

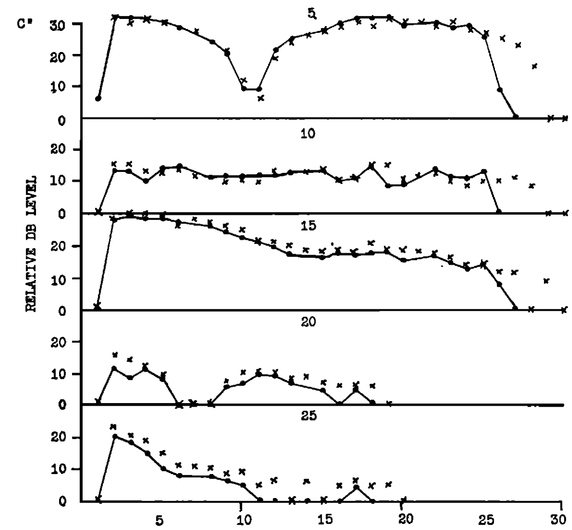

FIG. 8. Curves showing the changing level vs time for the partials 5, 10, 15, 20, and 25 of the tone C‴.

for the real-piano tones which were absent for the synthetic tones.

The electrical circuit was arranged so that the original beginning and ending of the tones were eliminated. This removed the clues associated with the starting and stopping of the tones. Then the musicians correctly identified 74% of the tones and the laymen 75%. In Test 8, where the tones are arranged haphazardly as to frequency, the scores were much lower, namely 63% correct. It will be remembered that a 50% score means that the observer is guessing. It should also be remembered that in these tests the frequency of each partial in the synthetic tone was an exact multiple of the fundamental and the rate of decay was constant and the same for all partials.

Partial Level vs Time

The preliminary tests showed more clearly how to proceed to match real-piano tones with synthetic tones. Before improving the synthetic tones, it was decided to make a more careful study of real-piano tones. So the C″ was chosen for a more critical study to see how the various partials change with time. By means of the sonograph, at a time interval of 0.08 sec, 30 observations were made on each of the 30 measurable components. Samples of these results are shown in Figs. 7 and 8. In Fig. 7 the decay curves for the first 4 partials are given and in Fig. 8 the curves for partials numbered 5, 10, 15, 20, and 25 are given.

These results are rather surprising although somewhat similar results have been observed before. To be sure there was no artifact, the key for C‴ was struck a second time and a separate analysis made. The two sets of points in the figures represent the two sets of data. It is seen that there is good agreement. The discrepancy at the end of the tones means simply that one tone was damped quicker than the other.

It will be seen that some of these curves exhibit a fairly uniform rate of decay such as for partial 2, 4, and 15. However, most of the others show very irregular decay time characteristics. It is obvious from these curves that the partial structure is continually changing as the tone dies away. In Fig. 9, similar data on the piano tone C is given for the first six partials. It is obvious from these data that the partials of the piano tone do not even approximately decay at the same rate. They sometimes increase in intensity rather than decrease. Thus to give the decay rate as $\times$db per sec,

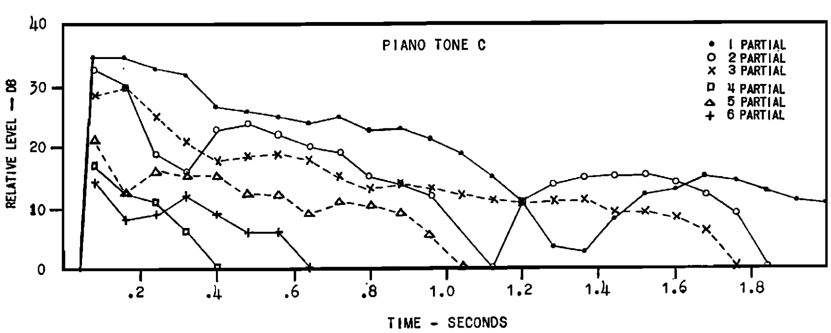

FIG. 9. Relative level vs time for the first six partials of the tone C.

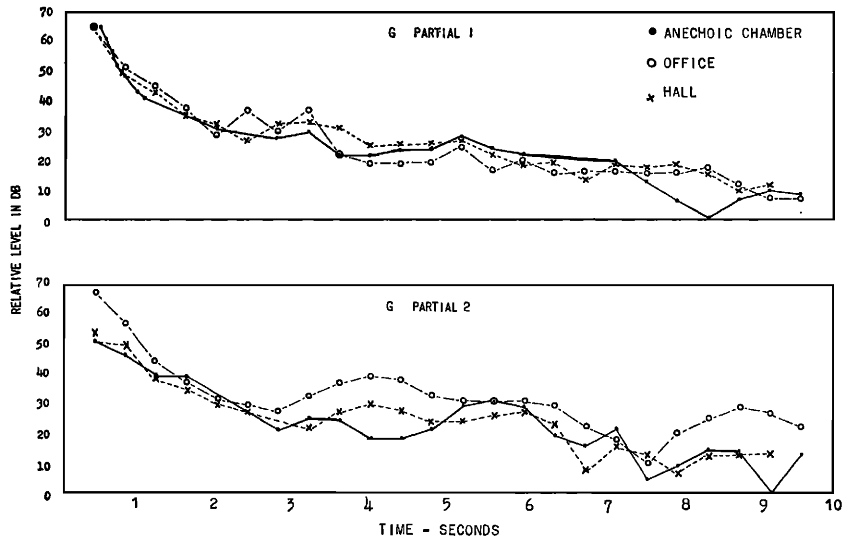

FIG. 10. Room effect on the level vs time curves for the partials 1 and 2 of the tone G.

as some authors still do, gives a rather erroneous picture.

Since these tones were recorded in a live music studio, it was decided to bring a piano into our anechoic chamber to see if these irregular variations with time were due to the room or were characteristic of the piano.

A Hamilton upright piano was taken into our laboratory where there were three rooms of different reverberation characteristics available in which the piano could be played. Room O had a reverberation time for speech of 0.6 sec (studio type). In Room H the reverberation time was 2.2 sec, that is, very reverberant. Room A was the anechoic chamber and the reverberation time was very close to zero. It could not be measured with the instruments available.

The same piano selection was recorded in each of the three rooms. Judgment tests were made first by a jury of 8 musicians and then by a jury of 12 laymen to determine which room was preferable for listening to the music. The musicians voted 1 for Room O (studio), 1 for Room H (reverberant), and 6 for Room A (anechoic). The laymen voted 1 for Room O, 5 for Room H, and 6 for Room A. Thus the anechoic chamber was definitely considered best for listening. This confirmed the conclusion reached some time ago that musicians prefer to listen to music in a nonreverberant room. However the player always prefers to play in a reverberant room.

While the piano was in each of these rooms, the tones produced by playing the white keys on the piano were also recorded on the tape recorder. The result of tests taken with the sonograph on the piano tone G played in these three rooms are given in Figs. 10 and 11. It is seen that the irregularities in the decay pattern exist in all three rooms. In the acoustically live room (hall) they are no greater than in the acoustically dead room.

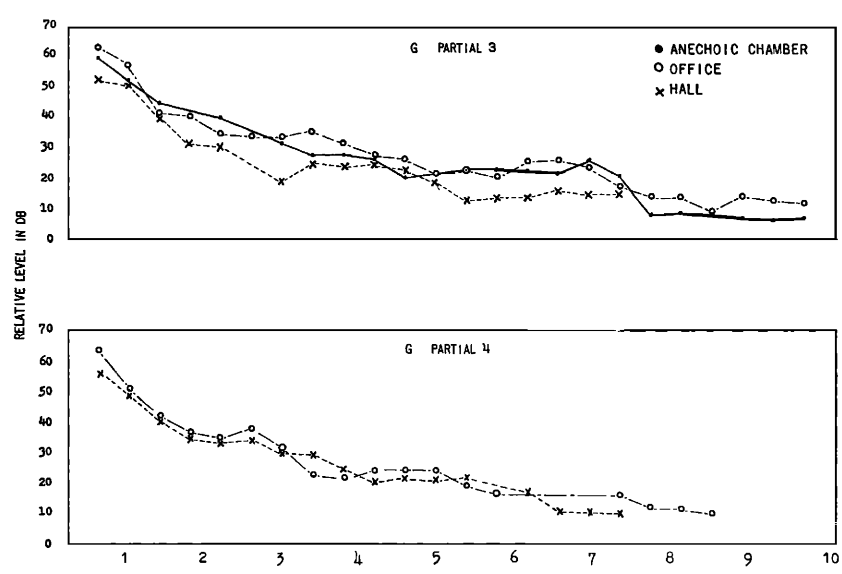

FIG. 11. Same as Fig. 10 but for partials 3 and 4 of the tone G.

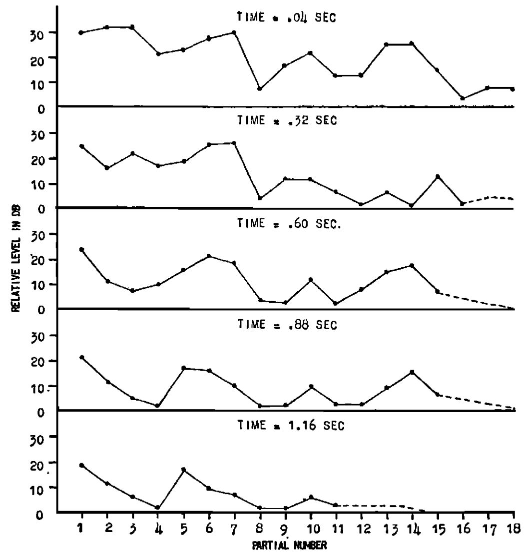

FIG. 12. Partial structures at different times for the tone G″.

It can also be noted that the decay rate is faster at the beginning and becomes slower after the first two sec. If the decay rate at the beginning persisted, the tone would go below hearing threshold in about 3 sec, but actually the tone can be heard for about 20 sec.

Measurements on G_1 and G_2 gave similar results. The curves in Fig. 12 show how the partial structure of the piano tone G″ changes with time. It is obvious that to match these real-piano tones with synthetic tones is a very complicated process. Of course the easiest way to do this is to make a tape recording of the tone. Then all these variations are preserved and can be reproduced.

TABLE I. Observed frequencies of the partials of piano tone A⁗, as well as the frequencies calculated for $B=0.00053$.

n	$27.5n$	obs f_n	calc f_n	db	n	$27.5n$	obs f_n	calc f_n	db
1	27.5	27.5	27.51	−23	26	715	835	833	−17
2	55	55	55.06	−29	27	742.5	876	874	−16
3	82.5	83	82.2	−9	28	770	917	916	−7
4	110	111	110.5	−19	29	797.5	959	959	−9
5	137.5	138	138.4	0	30	825	1003	1003	−18
6	165	166	166.5	−3	31	852.5	1049	1047	−23
7	192.5	193	195	−15	32	880	1094	1093	−25
8	220	222	224	−28	33	907.5	1144	1140	−33
9	247.5	251	253	−22	34	935	1189	1187	−16
10	275	281	282	−12	35	962	1237	1236	−11
11	302.5	311	312	−13	36	990	1287	1286	−17
12	330	340	342	−3	37	1017.5	1336	1337	−26
13	357.5	371	373	−3	38	1045	1386	1388	−28
14	385	402	404	−7	39	1072.5	1440	1441	−31
15	412.5	435	436	−11	40	1100	1495	1495	−33
16	440	467	469	−15	41	1127.5	1550	1550	−27
17	467.5	501	502	−8	42	1155	1602	1607	−34
18	495	534	535	−4	43	1182.5	1659	1664	−29
19	522.5	569	570	0	44	1210	1716	1722	−31
20	550	605	605	+4	45	1237.5	1776	1782	−36
21	577.5	641	641	0	46	1265	1834	1842	−38
22	605	677	678	−1	47	1292.5	1884	1904	−33
23	623.5	715	716	−6	48	1320	1940	1967	···
24	660	753	754	−21	49	1347.5	1997	2031	−37
25	687.5	793	793	−25					

But are these variations necessary for a good quality tone or are they just accidental?

To help answer this question, it is necessary to determine if a tone having a constant harmonic content and a constant decay rate would be indistinguishable by most observers from the piano tone if the duration of the tone was about 1 sec, corresponding to the time for a half or whole note depending on the tempo. It will be seen from the figures that during this time the decay curve is almost a straight line, corresponding to a logarithmic decay.

For matching the harmonic content with the synthesizer, we were not sure that the sonograph was sufficiently reliable for the higher partials, particularly for the low pitched tones. So the analyzer was used for obtaining the harmonic content at the beginning of the tone.

Warmth as a Factor in Piano Quality

Synthetic tones were constructed with partial structures thus determined. For the tones below middle C, there was still something lacking in the quality of the tone. The musicians said the tones lacked live-ness, or warmth. The warmth is probably due to rapid variation of the partial structure. We will use this term "warmth" for indicating this factor of the quality of a musical tone. To imitate exactly the varying intensity among the partials would require a rather complicated control mechanism. It probably could be built. However, it would give only a tone similar to one recorded directly from the piano.

Is it possible to warm the tone by some simple process rather than trying to follow the variations of 30 or 40 partials? A method for doing this was suggested by the comment of musicians that four or five violins playing in unison produces a much warmer tone than that from a single violin. One way of implementing this suggestion is as follows. Set up the desired partial structure on the synthesizer. With these settings create a continuous

TABLE II. Frequencies of partials of piano tone G‴ for $B=0.00028$.

n	$48.6n$	obs f_n	calc f_n	db	n	$48.6n$	obs f_n	calc f_n	db
1	48.6	49	48.6	−26	18	874.8	912	914	−36
2	97.2	97	97.3	−17	19	923.4	967	969	−23
3	145.8	145	146	0	20	972	1024	1026	−26
4	194.4	194	195	−15	21	1020.6	1082	1083	−33
5	243	244	244	−18	22	1069.2	1140	1139	−21
6	291.6	292	293	−20	23	1117.8	1198	1197	−21
7	340.2	340	342	−14	24	1166.4	1249	1257	−36
8	388.8	391	392	−29	25	1214	1318	1315	−40
9	437.4	440	442	−20	26	1263.6	1379	1378	−26
10	486	491	492	−9	27	1312.2	1441	1441	−26
11	534.6	542	544	−13	28	1360.8	1504	1504	−32
12	583.2	593	595	−7	29	1409.4	1569	1565	−39
13	631.8	644	647	−10	30	1458.0	1634	1632	−39
14	680.4	698	699	−4	31	1506.6	1701	1697	−39
15	729	750	752	−12	32	1552.2	1763	1767	−45
16	777.6	798	804	−23	33	1603.8	1832	1832	−43
17	826.2	···	856	···	34	1652.4	1902	1905	−45

tone and record on the frequency shifter. The tone from this is then recorded on the tape recorder. The speed of reproducing on the frequency shifter is then slightly changed and a second tone from it recorded on top of the first one. In the same way a third tone is superimposed. When these superimposed tones were reproduced from the tape recorder (2) it was found that the continuous tone was much warmer than a single tone. This warm continuous tone was sent through the attack and decay amplifier to give the synthetic tone the desired attack and decay times.

Later in this work a five channel tape recorder was added to the other instruments shown in Fig. 1. This made possible a second method. The tone from the synthesizer was recorded on the five channels, the frequency on each channel being slightly different. An attenuator in each channel made it possible to reproduce the five tones with any desired relative level. In this way a large range of warmth values was obtained.

A third method of warming the tone was used particularly for tones above middle C. Two adjacent oscillators were adjusted to nearly the same frequency as that of the partial in order that beats would occur. The loudness of the beats which depends upon the relative level of these three tones, could be controlled by raising and lowering the knobs on the synthesizer. This method is similar to that used sometimes in tuning the piano where three strings are provided for each note. The strings are not tuned to vibrate exactly in unison but to slightly different frequencies.

Tones warmed in this way were used in identification tests. The judgment tests indicated that the jury of musicians made three times as many errors in identifying real piano tones when warm tones were used instead of unwarmed ones. Similarly the jury of nonmusicians made twice as many errors with such tones.

Accurate Determination of the Frequency and Level of the Partials of Piano Tones

At this point it was surmised that perhaps the inharmonicity of the partials particularly for the lower range of pitches might be the cause of the warmth and be one of the factors for good quality rather than the reverse. So a very careful measurement of both the frequency and also the level of each of the partials in the tones A'''', G''', G'', G', G, G_1, and G_2 were made. These tones were produced on a good upright piano which was placed in the anechoic chamber. The analyzer was used to determine the partial structure at the beginning of the decay of the tone by the method described earlier in this paper.

The data are given in Tables I–VII. It is seen that the inharmonicity is very large especially for the strings in the lower frequency range. Young[2] and others found that this inharmonicity could be explained by Lord Rayleigh's equation for strings having stiffness. The magnitude of this effect depends upon the relative amount of the two factors contributing to the restoring force of a displaced piano string, namely, that due to the tension compared to that due to the stiffness. In free solid rods this latter effect produces the entire restoring force and produces partials which are nonharmonic. In strings without stiffness the restoring force is entirely due to the tension and the partials are harmonics if the ends are fixed rigidly. Such an analysis gives the following equations.

$$fn = nf_0(1+Bn^2)^{\frac{1}{2}}, \quad (1)$$

$$B = \pi^3 Q d^4 / 64\, l^2 T. \quad (2)$$

Q is Young's modulus, d is the diameter of the piano wire, l its length, and T its tension. The inharmonicity constant b as defined by Young[2] is related to B by

$$b = 865B. \quad (3)$$

For A'''' and G''' the hammer strikes a single string, which is a large gauge piano wire which has a smaller gauge wire wrapped around it. The G'' strings are con-

TABLE III. Frequencies of partials of piano tone G'' for $B=0.00015$.

n	$98n$	obs f_n	calc f_n	db	n	$98n$	obs f_n	calc f_n	db
1	98	98	98.01	−1	15	1470	1494	1494	−21
2	196	196	196.06	0	16	1568	1599	1598	−38
3	294	294	294.2	0	17	1666	1701	1701	−26
4	392	392	392.4	−10	18	1764	1805	1807	−27
5	490	492	490.9	−6	19	1862	1913	1912	−31
6	588	590	589.6	−2	20	1960	2019	2019	−41
7	686	689	688.5	−1	21	2058	2128	2126	−33
8	784	788	787.4	−25	22	2156	2334	2234	−26
9	882	889	887.4	−18	23	2252	2344	2344	−36
10	980	989	987.3	−12	24	2352	2453	2453	−39
11	1078	1087	1087	−20	25	2450	2571	2565	−41
12	1176	1189	1189	−25	26	2548	2669	2675	−38
13	1274	1291	1290	−11	27	2646	2778	2684	−44
14	1372	1392	1392	−10					

TABLE IV. Frequencies of partials of piano tone G' for $B=0.00005$.

n	$193.5n$	obs f_n	calc f_n	db	n	$193.5n$	obs f_n	calc f_n	db
1	193.5	193	194	−7	8	1552	1553	1553	−28
2	387	388	388	0	9	1731.5	1748	1748	−28
3	580.5	583	582	−2	10	1935	1942	1942	−33
4	774	776	776.1	−15	11	2182.5	2137	2137	−34
5	967.5	970	970.2	−16	12	2322	2328	2331	−37
6	1163	1167	1164.4	−14	13	2515.5	2525	2526	−38
7	1354.5	1362	1358.7	−17	14	2709	2720	2721	−43
					15	2902.5	2917	2917	−44

TABLE V. Frequencies of partials of piano tone G for $B=0.0004$.

n	$393n$	obs f_n	calc f_n	db	n	$393n$	obs f_n	calc f_n	db
1	393	393	393	0	5	1965	1976	1976	−21
2	786	785	786.7	−19	6	2358	2380	2376	−30
3	1179	1180	1181	−13	7	2751	2798	2780	−26
4	1572	1577	1577	−30	8	3144	3174	3187	−44

TABLE VI. Frequencies of partials of piano tone G_1 for $B=0.0002$.

n	$779n$	obs f_n	calc f_n	db	n	$779n$	obs f_n	calc f_n	db
1	779	779	779.1	0	4	3116	3123	3121	−36
2	1558	1562	1558.6	−22	5	3895	3897	3905	−31
3	2337	2337	2339	−24					

TABLE VII. Frequencies of partials of piano tone G_2 for $B=0.0002$.

n	$1568n$	obs f_n	calc f_n	db
1	1568	1568	1568.2	0
2	3136	3134	3137	−38
3	4704	4707	4709	−28

structed similarly but the hammer strikes two strings. Although Eq. (1) was developed for a single bare piano wire it was found that it would fit the data for the entire range of the piano provided the single constant B was obtained from the observed data rather than from Q, d, l, and T.

The value of B is given at the top of each table. The number of the partial n is given in the first column. In the second, third, fourth, and fifth columns are given, respectively, the harmonic frequency nf_0, the observed frequency f_n, the frequency f_n calculated from Eq. (1), and the relative level in db of the partial.

It will be seen that the calculated frequencies are in good agreement with the observed ones. Consider the data for A'''' in Table I. It is the first note on the piano. It can be observed that the 16th partial tone is a semitone sharp from the harmonic series. The 23rd partial is more than a whole tone sharp, the 33rd partial more than two tones sharp, and the 49th partial is 7.3 semitones sharper than the corresponding 49th harmonic. Similarly the 30th partial for G''' is two semitones sharper and the 27th partial for G'' is a semitone sharper than the corresponding harmonic series. For the tone G' and for the tones of higher frequency, Eq. (1) explains the departures from the harmonic series within the observational error of measuring these partial frequencies. However, the partial number is small and so the departures from the harmonic series are small. It was found that for these tones a good quality match could be made with a harmonic series of frequencies.

The partial structure as given in Table I was set up on the synthesizer. The various oscillators were tuned to the observed frequencies and the levels set according to those given in this table. The result was a very good match for the piano tone. No warming was necessary. It is obvious that the warmth is due to the inharmonicity of the partials. This warmth gives the piano tone its distinctive piano quality. With these facts in mind new identification tests were made using synthetic tones with partial structures corresponding to the observed ones in Tables I–VII. The oscillators in the synthesizer were tuned to these frequencies.

Final Judgment Tests

Synthetic tones made in this way were arranged with real tones according to the program shown in Table VIII. The members of the jury were asked to judge which were real and which were synthetic. Under M the percent of correct judgments by the musicians jury are given and under NM the percent of correct judgments by the nonmusicians. The letter S or R in parentheses after the notation of the note, indicates whether synthetic or real.

Before discussing the data in Table VIII, the A-B preference tests will be presented so that these tests and the identification tests can be discussed together. It will be remembered that in the preference A-B test, a tone designated A is produced and then a tone designated B of the same frequency but of different quality is produced. The observer is asked to decide which he prefers. These A-B judgment tests used the following synthetic tones. For Tone A'''', seven different qualities were considered.

(0) The tone was taken directly from a tape recording of the piano.
(1) This was a synthetic tone with partials having the same frequencies and levels as found for the piano and given in Table I.
(2) The partial levels were the same as 1, but the frequencies were made harmonic.
(3) This tone was the same as 1 except the fundamental was raised 38 db, that is, 15 db higher level than partial 5.
(4) The partial frequencies were the same as 1 but the levels were adjusted so that as the partial frequency changed 100 cps the level decreased 2 db.

These same notations were also used for the quality of the tones at other frequencies used in this test.

(5) The same partial frequencies as in 1 were used but the levels were adjusted as follows:

Number of partial	1	2	3	4	5	6	7	8	9	35 80.
Relative level, db	10	7	4	1	0	3	6	9	10	10.

From partial 9 to 35, the levels were at −10 db, and from 35 to 80 they decreased 2 db per 100 cps change in partial structure.

(6) The same partial frequencies were used as in 1. The levels were adjusted as follows. The first five partial levels were −18, −14, −10, −6, and −2. The levels of the partials from 6 to 35 were all at zero level. The partial levels above 35 were the same as 5.
(7) The same partial frequencies as 1 were used. The partial levels started at −12 db for number 1, then rose to 0 for number 6, then dropped to −18 at number 14, rose to 0 at number 22, dropped

again to −26 for number 33, rose again to −10 at number 40, and finally dropped again to −34 at number 48.

For the tone G‴, the qualities 1, 2, 3, and 4 have the same significance as for the similar qualities for A⁗ except the fundamental was at 3 db above the level of third partial instead of 15 db above it. Qualities 5 and 6 were special, as follows:

(5) For this quality of G‴, the levels of the components were:

Number of partial	1	2	3	4	5	6	7 etc.
Level, db	−14	−7	0	−1	−2	−3	−4.

Above the fourth partial, the level dropped 1 db per partial.

(6) For this quality, the partial structure was as follows:

Number of partial	1	2	3	4	5	6	7	8	9	10.
Level, db	−18	−15	−12	−9	−16	−3	0	−1	−2	−3.

Above the eighth partial, the level dropped 1 db per partial.

For the tone G″, the qualities described for A⁗ were used except for quality 5 where f_1 was +4 db. A fifth quality was used with partial structure as follows:

Number of partial	1	2	3	4	5	6	7	8	9	10.
Level, db	−10	−8	−6	−4	−2	0	−2	−4	−6	−8.

Above the eighth partial, the levels decreased 2 db per component.

For the tone G′, the four qualities described for A⁗ were used.

For tones G_2 and G_3, only qualities 1 and 4 were used, but the noise of the striking hammer was not considered part of the piano tone and was not in the synthetic tones.

Program for Real vs Synthetic Tones

The program of the A–B test was recorded as indicated in Table IX. Let us consider these data from the identification tests in Table VIII and A–B tests in Table IX. First consider the judgment data for A⁗, the first note on the piano. To a layman, the sound of this note is very much like a noise without any pitch. So it was thought that two improvements might be made. The first was to make the frequencies of the partials have harmonic ratios rather than those with inharmonic ratios.

None of the synthetic qualities was preferred to the piano quality. Qualities 1 and 3 were considered nearly equal to the piano quality. The identification tests (shown in Table VIII, tests 1, 12, and 31) also confirm

TABLE VIII. Synthetic and real tones used in identification tests and percent of correct judgments.

Tone no.	Note	Percent correct M	Percent correct NM	Tone no.	Note	Percent correct M	Percent correct NM
1	A‴(S)	50	23	19	G_1(S)	63	92
2	G′(R)	75	92	20	G′(R)	100	85
3	A⁗(R)	88	92	21	G(S)	88	77
4	G(R)	50	54	22	G″(R)	100	85
5	G_1(R)	100	69	23	G′(S)	63	69
6	G″(R)	100	85	24	A⁗(R)	88	100
7	G′(R)	63	69	25	G_1(R)	100	69
8	G‴(S)	37	77	26	G‴(R)	50	46
9	G″(S)	50	67	27	G_1(S)	75	85
10	G′(S)	25	54	28	G″(S)	88	62
11	G(S)	75	92	29	G_1(R)	75	85
12	A⁗(S)	75	31	30	G‴(R)	63	92
13	G(R)	63	69	31	A⁗(S)	100	77
14	G″(S)	63	67	32	G‴(S)	30	54
15	G(S)	88	67	33	G_1(S)	100	85
16	A⁗(R)	88	92	34	G′(S)	88	54
17	G‴(S)	63	54	35	G″(R)	88	85
18	G(R)	75	77	36	G‴(R)	50	77

TABLE IX. Preference test on piano tones.

	Test No.	Quality A	vs B	Preference A	Preference B		Test No.	Quality A	vs B	Preference A	Preference B
A⁗	1	0	1	16	5	G′	37	1	0	17	4
	2	2	0	2	19		38	0	2	14	7
	3	3	0	7	14		39	3	0	13	8
	4	0	4	18	3		40	4	0	14	7
	5	1	2	20	1		41	2	1	13	8
	6	3	1	6	15		42	1	3	8	13
	7	4	1	3	18		43	1	4	10	11
	8	3	2	10	11		44	3	2	12	9
	9	2	4	16	5		45	2	4	7	14
	10	4	3	5	16		46	3	4	10	11
	11	5	1	5	16						
	12	1	6	18	3	G	47	1	0	16	5
	13	1	7	16	6		48	0	2	6	15
							49	0	4	6	15
G‴	14	1	0	12	9		50	1	2	17	4
	15	0	2	13	8		51	4	1	12	9
	16	0	3	15	6		52	2	4	12	9
	17	4	0	7	14						
	18	2	1	9	12	G_1	53	0	1	13	8
	19	1	3	13	8		54	2	0	4	17
	20	4	1	7	14		55	4	0	4	17
	21	2	3	9	12		56	1	2	16	5
	22	4	2	9	12		57	4	1	11	10
	23	4	3	9	12		58	2	4	6	15
	24	1	5	15	6						
	25	1	6	19	2	G_2	59	0	4	15	6
							60	0	4	14	7
G″	26	1	0	9	12		61	0	5	16	5
	27	0	2	13	8		62	5	4	12	9
	28	3	0	10	11						
	29	0	4	14	7	G_3	63	0	4–5 db	14	7
	30	1	2	11	10		64	0	4–10 db	14	7
	31	1	3	7	14		65	0	5–10 db	13	8
	32	4	1	5	16						
	33	2	3	5	16						
	34	4	2	9	12						
	35	3	4	15	6						
	36	1	5	20	1						

this conclusion as the synthetic tone of quality 1 was mistaken for the real tone about half of the time.

The A–B on G‴ (see Table IX) indicated that no synthetic tone was definitely preferred over the real piano tone. However, quality 1 and possibly quality 2 were considered to be about equal in quality. The tests also show that quality 1 was preferred above any of the other qualities.

The identification tests (Table VIII, tests 8, 17, 26, 30, 32, and 36) show that the synthetic quality 1 of G‴ could not be distinguished from the real piano tone. These two sets of preference tests (for A⁗ and G‴) seem to indicate that any quality that departs much from real piano quality is discriminated against by either jury.

Consider now G″ and we will see that qualities 1 and 3 were about equal to the real piano tone (Table VIII). Also this synthetic tone of quality 1 could not be identified from the real piano tone as indicated in the data in Table VIII. Although quality 1 was preferred about equally with the piano, quality 3 was judged to be somewhat better (see Table VIII, tests 26, 28, and 31.)

Next, consider G′. Here, where we would least expect it, there was a preference for qualities 1, 3, and 4 over the real piano tone. Of these, 3 and 4 were preferred over 1 and also over 2. In the identification tests (Table VIII, tests 10, 23, and 34) the synthetic tone could not be identified from the real tone.

For middle G the qualities 1, 2, and 4 were preferred to the real piano tone. The other results of the A–B tests were inconclusive. For instance, test 50 (Table IX) indicates that 1 is better than 2, but tests 51 and 52 indicate the opposite. Thus it would be safe to say that qualities 1, 2, and 4 are nearly equally preferred and all are preferred over the piano. However, the identification tests (Table VIII) 11, 15, and 21 clearly show that the synthetic tone of quality 1 could be identified from the real piano tone.

For G_1 no synthetic tone was preferred to a real piano tone but quality 1 was close. It was preferred above the other two qualities tried. Due to the absence of the hammer noise in the synthetic tone, it was identified correctly 85% of the time. Nevertheless, in the identification tests the jury missed the real piano tone 74% of the time. To our great surprise the jury preferred the real piano tones with the high-impact noise for G_2 and G_3 rather than any synthetic tones without the impact noise.

These seem to indicate that most persons are satisfied with the quality of piano tones and that any large departures from this quality seem to be disliked. Comments indicated that some of these tones would be interesting if they were to come from a musical instrument different from the piano, but not as a piano tone.

We will now try to describe in an objective way what is meant when one says the tone sounds "piano-like."

RANGE OF ATTACK AND DECAY FOR PIANO-LIKE TONES

Judgments of the attack time were made from synthesized G″, G, and G_2 tones. A decay time was given each tone somewhat near the decay time of the piano tones having the same pitch. The attack time was then varied.

The attack for G″ tones must be between the limits of 0 to 0.09 sec to be piano-like, 0.09 to 0.14 sec to be questionable. Any tones with an attack time which is greater than this makes the tone no longer piano-like.

The attack time for G tones had similar limits which were 0 to 0.05 sec, 0.05 to 0.12 sec, and greater than 0.12 sec. For the higher pitched tones, the attack time was about 25% smaller than those given above.

A determination of decay time was made by giving a piano-like attack to the synthesized G″, G, and G_2 tones, and then varying the decay time. The following limits produced undampened piano tones: 5–9 sec for G″, 2–5.5 sec for G, 1–4 sec for G_2. Decay times slower than these sounded like sustained-tone instruments. Faster decay times produced an unnatural decay. To sound like a damped piano tone, the decay time can be between 0.8 and 7 sec, and must be damped by the damping button (a control on the attack and decay amplifier which produces a change in the decay rate during actual decay. This change can be made large, thus adjusting the decay to a very fast rate, and for simulating the damping pedal on the piano).

EFFECT OF HARMONIC CONTENT ON THE PIANO QUALITY OF MUSICAL TONES

Since there are an infinite number of arrangements for the harmonics of a tone of given frequency, it is difficult to circumscribe those that are piano-like. An attempt to do this was made in the following way. The partials were set up on the synthesizer so that the level of each successive partial was a constant number of decibels less than that of the partial just below it in frequency. As an example of this treatment applied to G″ (when the difference is 2 db), this harmonic content was produced.

Partial number	1	2	3	4	5	6	7	etc.
Relative level, db	0	−2	−4	−6	−8	−10	−12	etc.

Judgment tests, made with such tones, indicated the following. Tests were for the most part centered around G″, G, and G_2 with attack and decay approximating those of the piano.

Tone G″

(1) The limits for a piano-like quality extended from 2.5 to 1.5 db per partial.
(2) When the level difference between components was large, that is when essentially only the fundamental was present, the piano-like quality was gone and the tone tended toward that of a kettledrum.

(3) From this point until the difference approached 2.5 db, the quality approached that of a piano but would be referred to by musicians as dead, hollow, or having no edge.
(4) A difference of 1 db per partial produced a tone that had too much edge. This quality of tone approached that of a harpsichord rather than a piano.
(5) Differences less than this produced tones that were entirely too edgy.
(6) The first five or six components could be changed around in any position and the tone was still piano-like provided the remaining components conformed to the limits given above.

Tone G

(1) Limits for a piano-like quality were from 13.0 to 5.0 db per partial.
(2) When a single partial, namely the fundamental, was used, the tone could not be called piano-like.
(3) Differences greater than 13.0 db resulted in dead or dull tones.
(4) Differences less than 5.0 db produced tones with too much edge.
(5) The first 2 or 3 components could be changed and the tone was still piano-like provided requirement (1) was fulfilled.

Tone G_2

(1) Piano-like quality had limits from 40 to 7.5 db per partial.
(2) The fundamental alone has a piano-like quality. However, adding the next partial in the above limits, improves the tone quality.
(3) Less than 7.5 db per partial produces a tone which is too edgy.

The midpoints on the limits of G'', G, and G_2 are, respectively, 2, 8, and 32 db per partial, each being four times that of the preceding tone. A single partial above the fifth or sixth could be eliminated without producing any noticeable effect. However, if it were raised 4 or 5 db from its position in the series, it was distinctly noticeable and the resulting tone was less pleasing.

These conclusions above were based on synthetic tones whose components were harmonic. When the components had inharmonic frequencies equal to those in the piano, results obtained were approximately the same as those stated above with one very important exception. The lack of being harmonic gives rise to the peculiar quality known as piano quality, namely, the live-ness or warmth. This is very important for the first three octaves on the piano.