$$Q_N(T,V) = \boxed{} \left[\frac{1}{h^d} \int e^{-\frac{h(q,p)}{k_B T}} d^d q\, d^d p \right]^N \qquad \boxed{} \left[\sum_{\epsilon} g(\epsilon) e^{-\epsilon/k_B T} \right]^N$$

$h = h(q,p)$ $\qquad Q_1(T,V) = \frac{1}{h^d} \int e^{-\frac{h(q,p)}{k_B T}} d^d q\, d^d p$ $\qquad Q_1(T,V) = \sum_{\epsilon} g(\epsilon) e^{-\epsilon/k_B T}$

GI $\quad d=3$

$h(q,p) = \frac{p^2}{2m}$

$$\frac{1}{h^3} \int d^3 q \cdot \int e^{-\frac{p^2}{2mk_B T}} d^3 p$$

$$\frac{1}{h^3} V \int_0^\infty e^{-\frac{p^2}{2mk_B T}} 4\pi p^2 dp$$

$$\left(\frac{L}{\lambda_T}\right)^3$$

$$\epsilon = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2)$$

OH $\quad d=1$

$h(q,p) = \frac{1}{2} m \omega^2 q^2 + \frac{1}{2m} p^2$

$$\frac{1}{h} \int e^{-\frac{\frac{1}{2} m \omega^2 q^2}{k_B T}} dq \int e^{-\frac{1}{2m}\frac{p^2}{k_B T}} dp$$

$$\frac{1}{h} \left(\frac{2\pi k_B T}{m\omega^2}\right)^{1/2} (2\pi m k_B T)^{1/2}$$

$$\frac{k_B T}{\hbar \omega}$$

$$\epsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\sum_{n=0}^{\infty} e^{-\frac{(n+\frac{1}{2})\hbar\omega}{k_B T}}$$

$$\left[2\sinh\left(\frac{1}{2}\frac{\hbar\omega}{k_B T}\right)\right]^{-1}$$

MM $\quad d = 3$

$h(q,p) \equiv h(\theta, p_\theta) = -\mu H \cos\theta$

$$\frac{1}{h^3} \int e^{+\frac{\mu H \cos\theta}{k_B T}} d^3\theta \int d^3 p$$

$$\cancel{\frac{1}{h^3}} \int_0^{2\pi} \int_0^{\pi} e^{\frac{\mu H \cos\theta}{k_B T}} \sin\theta\, d\theta\, d\varphi\, \cancel{}$$

$$4\pi \frac{\sinh\left(\frac{\mu H}{k_B T}\right)}{\frac{\mu H}{k_B T}}$$

$$\epsilon(m) = -g \mu_B H m$$

$$m = \{-J, \dots, J\}$$

$$\sum_{m=-J}^{+J} e^{+\frac{g \mu_B H m}{k_B T}}$$

$$\frac{\sinh\left(\left[1 + \frac{1}{2J}\right]\frac{g\mu_B H J}{k_B T}\right)}{\sinh\left(\left[\frac{1}{2J}\right]\frac{g\mu_B H J}{k_B T}\right)}$$

2MS