(a)

R erradioko eta n-dimentsioko esfera baten bolumena / azalera

0

$$d^n r = \prod_{i=1}^{n} (dx_i) \equiv dV_n$$

$$V_n(R) = \underset{0 \le \sum_{i=1}^{n} x_i^2 \le R^2}{\int \cdots \int} \prod_{i=1}^{n} (dx_i) = \int \cdots \int dV_n$$

$$V_n \propto R^n$$

0'

hauxe da eragina dena

$$L \sim 2\pi R$$
$$S \sim 4\pi R^2$$
$$V \sim \frac{4}{3}\pi R^3$$

eragina, erabil dezagun ereduen [illegible] behar dugunak [illegible]

(1)

$$V_n = C_n R^n$$

[illegible] dezagun [illegible]

deribatuen [illegible]

$$dV_n = S_n(R)\, dR = n C_n R^{n-1} dR$$

eragina de

erabil dezagun hau

(2)

$$* \quad \boxed{\int_{-\infty}^{\infty} \exp(-x^2)\, dx = \pi^{1/2}}$$

dV_n

$$\pi^{n/2} = \left[\int_{x_i=-\infty}^{\infty} \cdots \int \exp\left(-\sum_{i=1}^{n} x_i^2\right)\right] \left[\prod_{i=1}^{n}(dx_i)\right]$$

$$\pi^{n/2} = \left[\int_0^{\infty} \exp(-R^2)\right] n C_n R^{n-1} dR$$

(3)

$$* \quad I_\nu \equiv \int_0^{\infty} e^{-\alpha y^2} y^\nu dy = \frac{1}{2\alpha^{\left(\frac{\nu+1}{2}\right)}} \Gamma\left(\frac{\nu+1}{2}\right), \quad \nu > -1$$

(4)

$$\Gamma(\nu) \equiv (\nu-1)! = \int_0^{\infty} e^{-x} x^{\nu-1} dx, \quad \nu > 0$$

$$\Gamma(\nu) = \frac{1}{\nu} \Gamma(\nu+1)$$

$$\alpha = 1$$

$$I_\nu \equiv \int_0^{\infty} e^{-y^2} y^\nu dy = \frac{1}{2} \Gamma\left(\frac{\nu+1}{2}\right)$$

$$\nu = n-1$$

(5)

$$\Gamma(n)$$

6

$$\pi^{\frac{n}{2}} = n C_n \frac{1}{2} \Gamma\left(\frac{n}{2}\right)$$

$$= C_n \frac{n}{2} \Gamma\left(\frac{n}{2}\right) = C_n \frac{n}{2} \left(\frac{n}{2}-1\right)! = C_n \left(\frac{n}{2}\right)!$$

7

$$\pi^{\frac{n}{2}} = C_n \left(\frac{n}{2}\right)!$$

7

$$\boxed{C_n = \frac{\pi^{\frac{n}{2}}}{\left(\frac{n}{2}\right)!}}$$

8

$$V_n(R) = \frac{\pi^{n/2}}{(n/2)!} R^n$$

$$S_n(R) = \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} R^{n-1}$$