

The probability that \mathcal{S} is ionized ($N = 0$) is

$$P(\text{ionized}) = P(0,0) = \frac{1}{\mathcal{Z}} = \frac{1}{1 + 2 \exp[(\mu + I)/\tau]}. \quad (73)$$

The probability that \mathcal{S} is neutral (un-ionized) is

$$P(\text{neutral}) = P(1\uparrow, -I) + P(1\downarrow, -I), \quad (74)$$

which is just $1 - P(0,0)$.

SUMMARY

1. The chemical potential is defined as $\mu(\tau, V, N) \equiv (\partial F / \partial N)_{\tau, V}$ and may also be found from $\mu = (\partial U / \partial N)_{\sigma, V} = -\tau (\partial \sigma / \partial N)_{U, V}$. Two systems are in diffusive equilibrium if $\mu_1 = \mu_2$.
2. The chemical potential is made up of two parts, external and internal. The external part is the potential energy of a particle in an external field of force. The internal part is of thermal origin; for an ideal monatomic gas $\mu(\text{int}) = \tau \log(n/n_Q)$, where n is the concentration and $n_Q \equiv (M\tau/2\pi\hbar^2)^{3/2}$ is the quantum concentration.
3. The Gibbs factor

$$P(N, \varepsilon_s) = \exp[(N\mu - \varepsilon_s)/\tau] / \mathcal{Z}$$

gives the probability that a system at chemical potential μ and temperature τ will have N particles and be in a quantum state s of energy ε_s .

4. The Gibbs sum

$$\mathcal{Z} \equiv \sum_{\text{ASN}} \exp[(N\mu - \varepsilon_{s(N)})/\tau]$$

is taken over all states for all numbers of particles.

5. The absolute activity λ is defined by $\lambda \equiv \exp(\mu/\tau)$.
6. The thermal average number of particles is

$$\langle N \rangle = \lambda \frac{\partial}{\partial \lambda} \log \mathcal{Z}.$$