$$Q_N(T,V) = \square \left[\frac{1}{h^d} \int e^{-\frac{h(q,p)}{k_B T}} d^d q \, d^d p \right]^N$$

$$\square \left[\sum_\epsilon g(\epsilon) e^{-\epsilon/k_B T} \right]^N$$

$h = h(q,p)$

$$Q_1(T,V) = \frac{1}{h^d} \int e^{-\frac{h(q,p)}{k_B T}} d^d q \, d^d p$$

$$Q_1(T,V) = \sum_\epsilon g(\epsilon) e^{-\epsilon/k_B T}$$

G1 $d = 3$

$h(q,p) = \frac{p^2}{2m}$

$$\frac{1}{h^3} \int d^3 q \cdot \int e^{-\frac{p^2}{2mk_B T}} d^3 p$$

$$\frac{1}{h^3} V \int_0^\infty e^{-\frac{p^2}{2mk_B T}} 4\pi p^2 dp$$

$$\frac{V}{\left(\frac{L}{\lambda_T}\right)^3}$$

$$\epsilon = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

OH $d = 1$

$h(q,p) = \frac{1}{2} m\omega^2 q^2 + \frac{1}{2m} p^2$

$$\frac{1}{h} \int e^{-\frac{\frac{1}{2}m\omega^2 q^2}{k_B T}} dq \int e^{-\frac{\frac{1}{2}p^2}{2mk_B T}} dp$$

$$\frac{1}{h} \left(\frac{2\pi k_B T}{m\omega^2}\right)^{1/2} \left(2\pi m k_B T\right)^{1/2}$$

$$\frac{k_B T}{\hbar\omega}$$

$$\epsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\sum_{n=0}^{\infty} e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega}{k_B T}}$$

$$\left[2\sinh\left(\frac{1}{2}\frac{\hbar\omega}{k_B T}\right)\right]^{-1}$$

MM $d = 3$

$h(q,p) \equiv h(\theta,\beta) = -\mu H \cos\theta$

$$\frac{1}{h^3} \int e^{+\mu H\cos\theta / k_B T} d^3\theta \int d^3 p$$

$$\vdots$$

$$\int_0^{2\pi} \int_0^{\pi} e^{\frac{\mu H\cos\theta}{k_B T}} \sin\theta \, d\theta \, d\varphi \, 2\pi$$

$$4\pi \sinh\left(\frac{\mu H}{k_B T}\right)$$

$$\frac{\mu H}{k_B T}$$

$$\epsilon(m) = -g\mu_B H m$$

$m = \{-J, \dots, J\}$

$$\sum_{m=-J}^{m=+J} e^{+g\mu_B H m / k_B T}$$

$$\frac{\sinh\left[\left(1+\frac{1}{2J}\right)\frac{g\mu_B H J}{k_B T}\right]}{\sinh\left[\left(\frac{1}{2J}\right)\frac{g\mu_B H J}{k_B T}\right]}$$

2MS