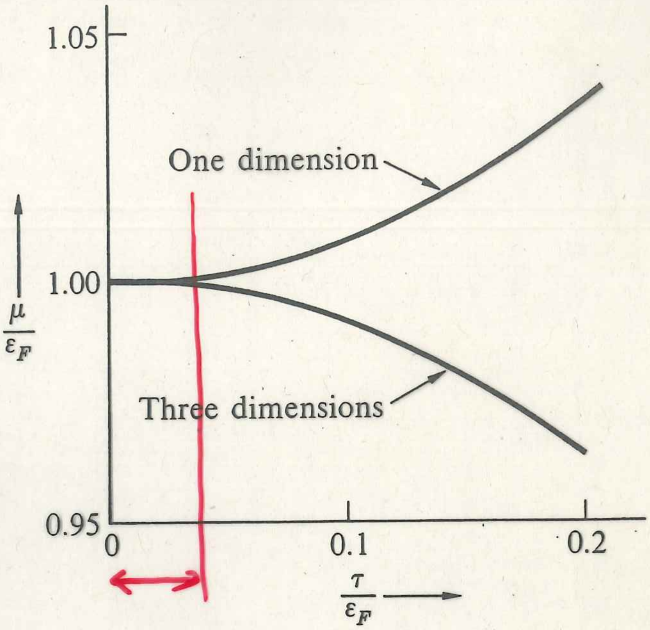
gure 7.7 Variation with temperature of the ıemical potential μ, for free electron Fermi ıses in one and three dimensions. In common etals $\tau/\varepsilon_F \approx 0.01$ at room temperature, so .at μ is closely equal to ε_F. These curves were ılculated from series expansions of the integral ır the number of particles in the system.



We set

$$x \equiv (\varepsilon - \varepsilon_F)/\tau \ , \tag{31}$$

and it follows from (29) and (30) that

$$\boxed{C_{el} = \tau \mathfrak{D}(\varepsilon_F) \int_{-\varepsilon_F/\tau}^{\infty} dx\, x^2 \frac{e^x}{(e^x + 1)^2}.} \tag{32}$$

We may safely replace the lower limit by $-\infty$ because the factor e^x in the integrand is already negligible at $x = -\varepsilon_F/\tau$ if we are concerned with low temperatures such that $\varepsilon_F/\tau \sim 100$ or more. The integral* becomes

$$\boxed{\int_{-\infty}^{\infty} dx\, x^2 \frac{e^x}{(e^x + 1)^2} = \frac{\pi^2}{3} \ ,} \tag{33}$$

* The integral is not elementary, but may be evaluated from the more familiar result

$$\int_0^{\infty} dx \frac{x}{e^{ax} + 1} = \frac{\pi^2}{12a^2}$$

on differentiation of both sides with respect to the parameter a.