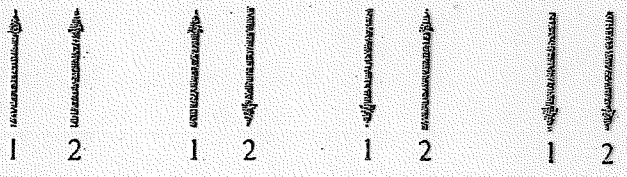

Figure 1.5 The four different states of a system of two elements numbered 1 and 2, where each element can have two conditions. The element is a magnet which can be in condition ↑ or condition ↓.

The sites themselves are assumed to be arranged in a definite order. We may number them in sequence from left to right, as we did in Figure 1.3. According to this convention the state (2) also can be written as

$$\uparrow_1\uparrow_2\downarrow_3\downarrow_4\downarrow_5\uparrow_6\downarrow_7\uparrow_8\uparrow_9\uparrow_{10}\cdots. \tag{3}$$

Both sets of symbols (2) and (3) denote the same state of the system, the state in which the magnetic moment on site 1 is $+m$; on site 2, the moment is $+m$; on site 3, the moment is $-m$; and so forth.

It is not hard to convince yourself that every distinct state of the system is contained in a symbolic product of N factors:

$$(\uparrow_1 + \downarrow_1)(\uparrow_2 + \downarrow_2)(\uparrow_3 + \downarrow_3)\cdots(\uparrow_N + \downarrow_N). \tag{4}$$

The multiplication rule is defined by

$$(\uparrow_1 + \downarrow_1)(\uparrow_2 + \downarrow_2) = \uparrow_1\uparrow_2 + \uparrow_1\downarrow_2 + \downarrow_1\uparrow_2 + \downarrow_1\downarrow_2. \tag{5}$$

The function (4) on multiplication generates a sum of 2^N terms, one for each of the 2^N possible states. Each term is a product of N individual magnetic moment symbols, with one symbol for each elementary magnet on the line. Each term denotes an independent state of the system and is a simple product of the form $\uparrow_1\uparrow_2\downarrow_3\cdots\uparrow_N$, for example.

For a system of two elementary magnets, we multiply $(\uparrow_1 + \downarrow_1)$ by $(\uparrow_2 + \downarrow_2)$ to obtain the four possible states of Figure 1.5:

$$(\uparrow_1 + \downarrow_1)(\uparrow_2 + \downarrow_2) = \uparrow_1\uparrow_2 + \uparrow_1\downarrow_2 + \downarrow_1\uparrow_2 + \downarrow_1\downarrow_2. \tag{6}$$

The sum is not a state but is a way of listing the four possible states of the system. The product on the left-hand side of the equation is called a generating function: it generates the states of the system.