This may be simplified because

$$\log(N_\uparrow/N) = \log\tfrac{1}{2}(1 + 2s/N) = -\log 2 + \log(1 + 2s/N)$$
$$\cong -\log 2 + (2s/N) - (2s^2/N^2) \qquad (32)$$

by virtue of the expansion $\log(1 + x) = x - \frac{1}{2}x^2 + \cdots$, valid for $x \ll 1$. Similarly,

$$\log(N_\downarrow/N) = \log\tfrac{1}{2}(1 - 2s/N) \simeq -\log 2 - (2s/N) - (2s^2/N^2). \qquad (33)$$

On substitution in (31) we obtain

$$\log g \cong \tfrac{1}{2}\log(2/\pi N) + N\log 2 - 2s^2/N. \qquad (34)$$

We write this result as

$$\boxed{g(N,s) \cong g(N,0)\exp(-2s^2/N),} \qquad (35)$$

where

$$g(N,0) \simeq (2/\pi N)^{1/2}2^N. \qquad (36)$$

Such a distribution of values of $s$ is called a **Gaussian distribution**. The integral* of (35) over the range $-\infty$ to $+\infty$ for $s$ gives the correct value $2^N$ for the total number of states. Several useful integrals are treated in Appendix A.

The exact value of $g(N,0)$ is given by (15) with $s = 0$:

$$g(N,0) = \frac{N!}{(\frac{1}{2}N)!\,(\frac{1}{2}N)!}. \qquad (37)$$

* The replacement of a sum by an integral, such as $\sum_s(\ldots)$ by $\int(\ldots)ds$, usually does not introduce significant errors. For example, the ratio of

$$\sum_{s=0}^{N} s = \tfrac{1}{2}(N^2 + N) \qquad \text{to} \qquad \int_0^N s\,ds = \tfrac{1}{2}N^2$$

is equal to $1 + (1/N)$, which approaches 1 as $N$ approaches $\infty$.