In general, for fermions, the requirement that $\hat{P}_{12}|\psi\rangle = -|\psi\rangle$ means that if $|\psi\rangle$ is a two-particle state consisting of two particles in the *same* quantum state, i.e. if $\psi = |\varphi\rangle|\varphi\rangle$, then

$$\hat{P}_{12}|\varphi\rangle|\varphi\rangle = |\varphi\rangle|\varphi\rangle = -|\varphi\rangle|\varphi\rangle, \tag{29.13}$$

so that

$$|\varphi\rangle|\varphi\rangle = 0, \tag{29.14}$$

i.e. the doubly-occupied state cannot exist. This, again, illustrates the Pauli exclusion principle, namely that two identical fermions cannot coexist in the same quantum state.

29.3 The statistics of identical particles

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In the last section, we have demonstrated that exchange symmetry has an important effect on the statistics of two identical particles. Now we want to do the same for cases in which we have many more than two identical particles. Our derivation will be easiest if we do this by finding the grand partition function $\mathcal{Z}$ (see Section 22.3) for a system comprised either of fermions or bosons. In this approach, the the total number of particles is not fixed, and this is an easy constraint to apply as we shall see. If one is treating a system in which the number of particles *is* fixed, we can always fix it at the end of our calculation. Our method will be to use the expression $\mathcal{Z} = \sum_\alpha e^{\beta(\mu N_\alpha - E_\alpha)}$ (from eqn 22.20). Here, α denotes a particular state of the system. We assume that there are certain possible quantum states in which to place our particles, and that the energy cost of putting a particle into the i^{th} state is given by E_i. We will put n_i particles into the i^{th} state; here n_i is called the **occupation number** of the i^{th} state. A particular configuration of the system is then described by the product

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$$\left[e^{\beta(\mu-E_1)}\right]^{n_1} \times \left[e^{\beta(\mu-E_2)}\right]^{n_2} \times \cdots = \prod_i e^{n_i\beta(\mu-E_i)}. \tag{29.15}$$

The grand partition function is the sum of such products for all sets of occupation numbers which are allowed by the symmetry of the particles. Hence

$$\boxed{\mathcal{Z} = \sum_{\{n_i\}} \prod_i e^{n_i\beta(\mu-E_i)},} \tag{29.16}$$

where the symbol $\{n_i\}$ denotes a set of occupation numbers allowed by the symmetry of the particles.

Fortunately, the total number of particles $\sum_i n_i$ does not have to be fixed,[3] because that would have been a fiddly constraint to apply to this expression. In fact, we will only be considering two cases: fermions, for which $\{n_i\} = \{0, 1\}$ (independent of i), and bosons, for which $\{n_i\} = \{0, 1, 2, 3, \ldots\}$ (independent of i). This allows us to factor out the terms in the product for each state i and hence write

[3] The total number of particles is not fixed in the grand canonical ensemble, which is the one we are using here.

$$\boxed{\mathcal{Z} = \prod_i \sum_{\{n_i\}} e^{n_i\beta(\mu-E_i)}} \tag{29.17}$$

$$\underbrace{\sum_{\{n_i\}} \prod_i}_{} \longrightarrow \prod_i \sum_{\{n_i\}}$$

BATU DAITEKE

PARTIKULA KOPURUA $\neq$ kte $\Rightarrow$