and is negative, so that the extremum is a maximum. Thus the most probable configuration of the combined system is that for which (12) is satisfied:

$$\frac{s_1}{N_1} = \frac{s - s_1}{N_2} = \frac{s_2}{N_2}. \tag{13}$$

The two systems are in equilibrium with respect to interchange of energy when the fractional spin excess of system 1 is equal to the fractional spin excess of system 2.

We prove that nearly all the accessible states of the combined systems satisfy or very nearly satisfy (13). If $\hat{s}_1$ and $\hat{s}_2$ denote the values of s_1 and s_2 at the maximum, then (13) is written as

$$\frac{\hat{s}_1}{N_1} = \frac{\hat{s}_2}{N_2} = \frac{s}{N}. \tag{14}$$

To find the number of states in the most probable configuration, we insert (14) in (9) to obtain

$$(g_1g_2)_{\max} \equiv g_1(\hat{s}_1)g_2(s - \hat{s}_1) = g_1(0)g_2(0)\exp(-2s^2/N). \tag{15}$$

* To investigate the sharpness of the maximum of g_1g_2 at a given value of s, introduce δ such that

$$s_1 = \hat{s}_1 + \delta; \qquad s_2 = \hat{s}_2 - \delta. \tag{16}$$

Here δ measures the deviation of s_1, s_2 from their values $\hat{s}_1$, $\hat{s}_2$ at the maximum of g_1g_2. Square s_1, s_2 to form

$$s_1{}^2 = \hat{s}_1{}^2 + 2\hat{s}_1\delta + \delta^2; \qquad s_2{}^2 = \hat{s}_2{}^2 - 2\hat{s}_2\delta + \delta^2 ,$$

which we substitute in (9) and (15) to obtain the number of states

$$g_1(N_1,s_1)g_2(N_2,s_2) = (g_1g_2)_{\max}\exp\left(-\frac{4\hat{s}_1\delta}{N_1} - \frac{2\delta^2}{N_1} + \frac{4\hat{s}_2\delta}{N_2} - \frac{2\delta^2}{N_2}\right).$$

We know from (14) that $\hat{s}_1/N_1 = \hat{s}_2/N_2$, so that the number of states in a configuration of deviation δ from equilibrium is

$$g_1(N_1,\hat{s}_1 + \delta)g_2(N_2,\hat{s}_2 - \delta) = (g_1g_2)_{\max}\exp\left(-\frac{2\delta^2}{N_1} - \frac{2\delta^2}{N_2}\right). \tag{17}$$

As a numerical example in which the fractional deviation from equilibrium is very small, let $N_1 = N_2 = 10^{22}$ and $\delta = 10^{12}$; that is, $\delta/N_1 = 10^{-10}$. Then $2\delta^2/N_1 = 200$, and the