(3) – MATEMATIKOKI : $\boxed{S_{max} \Leftrightarrow U_{min}}$

$S_{max} \Rightarrow \left\{ \left(\frac{\partial S}{\partial X}\right)_u = 0 \;\; ; \;\; \left(\frac{\partial^2 S}{\partial X^2}\right)_u < 0 \right\}$ ①

$U_{min} \Rightarrow \left\{ \left(\frac{\partial U}{\partial X}\right)_s = 0 \;\; ; \;\; \left(\frac{\partial^2 U}{\partial X^2}\right)_s > 0 \right\}$ ②

(i) – ① ⇒ ②

- $\left(\frac{\partial U}{\partial X}\right)_s = -\frac{\left(\frac{\partial S}{\partial X}\right)_u}{\left(\frac{\partial S}{\partial U}\right)_X} = -\left(\frac{\partial U}{\partial S}\right)_X \left(\frac{\partial S}{\partial X}\right)_u = -T\left(\frac{\partial S}{\partial X}\right)_u = 0$, with $\left(\frac{\partial S}{\partial X}\right)_u = 0$

- $\left(\frac{\partial^2 U}{\partial X^2}\right)_s = \frac{\partial}{\partial X}\left[\left(\frac{\partial U}{\partial X}\right)_s\right] = \frac{\partial}{\partial X}\left[-\left(\frac{\partial U}{\partial S}\right)_X \left(\frac{\partial S}{\partial X}\right)_u\right] = -\left\{\left(\frac{\partial^2 U}{\partial X \partial S}\right)\left(\frac{\partial S}{\partial X}\right)_u + \left(\frac{\partial U}{\partial S}\right)_X \left(\frac{\partial^2 S}{\partial X^2}\right)_u\right\}$

$= -\left\{\left(\frac{\partial^2 U}{\partial X \partial S}\right)\left(\frac{\partial S}{\partial X}\right)_u + T\left(\frac{\partial^2 S}{\partial X^2}\right)_u\right\}$, with $\left(\frac{\partial S}{\partial X}\right)_u = 0$

$\Rightarrow \left(\frac{\partial^2 U}{\partial X^2}\right)_s = -T\left(\frac{\partial^2 S}{\partial X^2}\right)_u \Rightarrow \left(\frac{\partial^2 U}{\partial X^2}\right)_s > 0$

(ii) – ② ⇒ ①

- $\left(\frac{\partial S}{\partial X}\right)_u = -\frac{\left(\frac{\partial U}{\partial X}\right)_s}{\left(\frac{\partial U}{\partial S}\right)_X} = -\left(\frac{\partial U}{\partial X}\right)_s \left(\frac{\partial S}{\partial U}\right)_X = -\frac{1}{T}\left(\frac{\partial U}{\partial X}\right)_s = 0$, with $\left(\frac{\partial U}{\partial X}\right)_s = 0$

- $\left(\frac{\partial^2 S}{\partial X^2}\right)_u = \frac{\partial}{\partial X}\left[\left(\frac{\partial S}{\partial X}\right)_u\right] = \frac{\partial}{\partial X}\left[-\left(\frac{\partial S}{\partial U}\right)_X \left(\frac{\partial U}{\partial X}\right)_s\right] = -\left\{\left(\frac{\partial^2 S}{\partial X \partial U}\right)\left(\frac{\partial U}{\partial X}\right)_s + \left(\frac{\partial S}{\partial U}\right)_X \left(\frac{\partial^2 U}{\partial X^2}\right)_s\right\}$

$= -\left\{\left(\frac{\partial^2 S}{\partial X \partial U}\right)\left(\frac{\partial U}{\partial X}\right)_s + \frac{1}{T}\left(\frac{\partial^2 U}{\partial X^2}\right)_s\right\}$, with $\left(\frac{\partial U}{\partial X}\right)_s = 0$

$\Rightarrow \left(\frac{\partial^2 S}{\partial X^2}\right)_u = -\frac{1}{T}\left(\frac{\partial^2 U}{\partial X^2}\right)_s \Rightarrow \left(\frac{\partial^2 S}{\partial X^2}\right)_u < 0$