small compared to unity, so that helium is very dilute under normal conditions. Whenever $n/n_Q \ll 1$ we say that the gas is in the **classical regime**. An **ideal gas is defined as a gas of noninteracting atoms in the classical regime**.

The thermal average energy of the atom in the box is, as in (12),

$$U = \frac{\sum_n \varepsilon_n \exp(-\varepsilon_n/\tau)}{Z_1} = \tau^2(\partial \log Z_1/\partial\tau) \ , \tag{64}$$

because $Z_1{}^{-1}\exp(-\varepsilon_n/\tau)$ is the probability the system is in the state n. From (62),

$$\log Z_1 = -\tfrac{3}{2}\log(1/\tau) + \text{terms independent of } \tau \ ,$$

so that for an ideal gas of one atom

$$\boxed{U = \tfrac{3}{2}\tau.} \tag{65}$$

If $\tau = k_B T$, where k_B is the Boltzmann constant, then $U = \frac{3}{2}k_B T$, the well-known result for the energy per atom of an ideal gas.

The thermal average occupancy of a free particle orbital satisfies the inequality

$$Z_1{}^{-1}\exp(-\varepsilon_n/\tau) < Z_1{}^{-1} = n/n_Q \ ,$$

which sets an upper limit of 4×10^{-6} for the occupancy of an orbital by a helium atom at standard concentration and temperature. For the classical regime to apply, this occupancy must be $\ll 1$. We note that ε_n as defined by (59) is always positive for a free atom.

Example: N atoms in a box. There follows now a tricky argument that we will use temporarily until we develop in Chapter 6 a powerful method to deal with the problem of many noninteracting identical atoms in a box. We first treat an ideal gas of N atoms in a box, all atoms of different species or different isotopes. This is a simple extension of the one atom result. We then discuss the major correction factor that arises when all atoms are identical, of the same isotope of the same species.