4'

$$\frac{\partial^2}{\partial E^2}\left[\ln\left(g(E)\cdot e^{-\beta E}\right)\right] = \frac{\partial^2}{\partial E^2}\left[\ln g(E) - \beta E\right] = \frac{\partial^2}{\partial E^2}\left(\ln g(E)\right) - \beta\underbrace{\frac{\partial^2}{\partial E^2}E}_{=0}$$

$$= \frac{\partial}{\partial E}\left[\left(\frac{1}{g(E)}\right)\left(\frac{\partial g(E)}{\partial E}\right)\right]$$

$$= \frac{\frac{\partial^2 g(E)}{\partial E^2}\cdot g(E) - \frac{\partial g(E)}{\partial E}\,\frac{\partial g(E)}{\partial E}}{g^2(E)}$$

$$= \frac{\left(\frac{\partial^2 g(E)}{\partial E^2}\right)}{g(E)} - \frac{\left(\frac{\partial g(E)}{\partial E}\right)^2}{g^2(E)}$$

$$= \frac{\partial^2}{\partial E^2}\left(\ln g(E)\right)$$

$$\frac{S}{k_B}$$

$$= \frac{\partial}{\partial E}\left(\frac{\partial S/k_B}{\partial E}\right) = \frac{\partial}{\partial E}\left(\frac{1}{k_B}\frac{1}{T}\right)$$

$$= \frac{1}{k_B}\frac{\partial}{\partial E}\left(\frac{1}{T}\right) \qquad = \frac{1}{k_B}\left(-\frac{1}{T^2}\right)\underbrace{\left(\frac{\partial T}{\partial E}\right)}_{C_V}$$

$$\partial\left(\frac{1}{T}\right) = \left(-\frac{1}{T^2}\right)\partial T$$