because $\Delta\varepsilon \gg \mu$. Thus the occupation of the first excited orbital at 1 mK is

$$f \cong \frac{1}{1.8 \times 10^{-11}} \cong 5 \times 10^{10} , \tag{64}$$

so that the fraction of the N particles that are in this orbital is $f/N \simeq 5 \times 10^{10}/10^{22} \simeq 5 \times 10^{-12}$, which is very small. We see that the occupancy of the first excited orbital at low temperatures is relatively very much lower than would be expected at first sight from the simple Boltzmann factor (62). The Bose-Einstein distribution is quite strange; it favors a situation in which the greatest part of the population is left in the ground orbital at sufficiently low temperatures. The particles in the ground orbital, as long as their number is $\gg 1$, are called the **Bose-Einstein condensate**. The atoms in the condensate act quite differently from the atoms in excited states.

How do we understand the existence of the condensate? Suppose the atoms were governed by the Planck distribution (Chapter 4), which makes no provision for holding constant the total number of particles; instead, the thermal average number of photons increases with temperature at τ^3, as found in Problem 4.1. If the laws of nature restricted the total number of photons to a value N, we would say that the ground orbital of the photon gas contained the difference $N_0 = N - N(\tau)$ between the number allotted and the number thermally excited. The N_0 nonexcited photons would be described as condensed into the ground orbital, but N_0 becomes essentially zero at a temperature τ_c such that all N photons are excited. There is no actual constraint on the total number of photons; however, there is a constraint on the total number N of material bosons, such as ^{4}He atoms, in a system. This constraint is the origin of the condensation into the ground orbital. The difference between the Planck distribution and the Bose-Einstein distribution is that the latter will conserve the total number of particles, independent of temperature, so that nonexcited atoms are really in the ground state condensate.

Orbital Occupancy Versus Temperature

We saw in (19) that the number of free particle orbitals per unit energy range is

$$\mathfrak{D}(\varepsilon) = \frac{V}{4\pi^2}\left(\frac{2M}{\hbar^2}\right)^{3/2} \varepsilon^{1/2} , \tag{65}$$

for a particle of spin zero. The total number of atoms of helium-4 in the ground and excited orbitals is given by the sum of the occupancies of all orbitals:

$$N = \sum_n f_n = N_0(\tau) + N_e(\tau) = N_0(\tau) + \int_0^\infty d\varepsilon\, \mathfrak{D}(\varepsilon) f(\varepsilon,\tau). \tag{66}$$

We have separated the sum over n into two parts. Here $N_0(\tau)$ has been written for $f(0,\tau)$, the number of atoms in the ground orbital at temperature τ. The integral in (66) gives the number of atoms $N_e(\tau)$ in all excited orbitals, with