$$\mathcal{Z} = 1 + \lambda e^{-\epsilon/\tau} + \lambda^2 e^{-2\epsilon/\tau} + \lambda^3 e^{-3\epsilon/\tau} + \lambda^4 e^{-4\epsilon/\tau} + \cdots$$

$$f = \frac{\Box}{\mathcal{Z}} \approx \frac{[\cdots]}{1 + [\cdots]} \approx [\cdots]$$
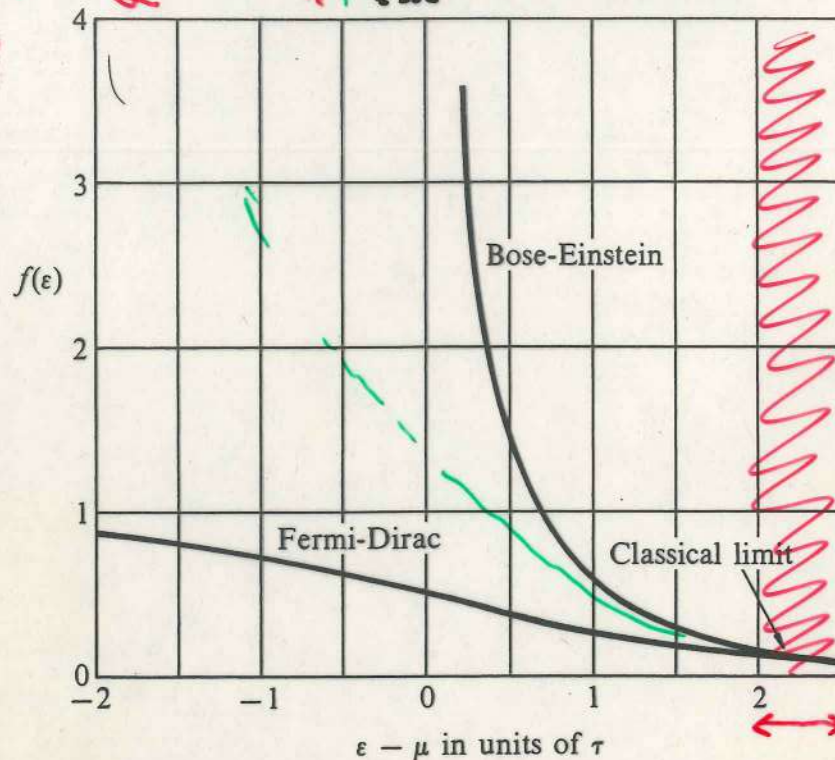
Figure 6.6 Comparison of Bose-Einstein and Fermi-Dirac distribution functions. The classical regime is attained for $(\epsilon - \mu) \gg \tau$, where the two distributions become nearly identical. We shall see in Chapter 7 that in the degenerate regime at low temperature the chemical potential μ for a FD distribution is positive, and changes to negative at high temperature.

This defines the **Bose-Einstein distribution function**. It differs mathematically from the Fermi-Dirac distribution function only by having -1 instead of $+1$ in the denominator. The change can have very significant physical consequences, as we shall see in Chapter 7. The two distribution functions are compared in Figure 6.6. The ideal gas represents the limit $\epsilon - \mu \gg \tau$ in which the two distribution functions are approximately equal, as discussed below. The choice of the zero of the energy ϵ is always arbitrary. The particular choice made in any problem will affect the value of the chemical potential μ, but the value of the difference $\epsilon - \mu$ has to be independent of the choice of the zero of ϵ. This point is discussed further in (20) below.

A gas is in the **classical regime** when the average number of atoms in each orbital is much less than one. The average orbital occupancy for a gas at room temperature and atmospheric pressure is of the order of only 10^{-6}, safely in the classical regime. Differences between fermions (half-integral spin) and bosons