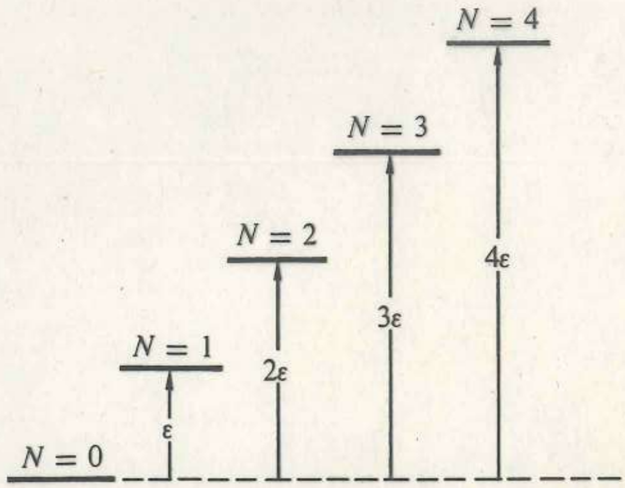

Figure 6.5 Energy-level scheme for noninteracting bosons. Here ε is the energy of an orbital when occupied by one particle; $N\varepsilon$ is the energy of the same orbital when occupied by N particles. Any number of bosons can occupy the same orbital. The lowest level of this orbital contributes a term 1 to the grand sum; the next highest level contributes $\lambda \exp(-\varepsilon/\tau)$; and the subsequent contributions are $\lambda^2 \exp(-2\varepsilon/\tau)$; $\lambda^3 \exp(-3\varepsilon/\tau)$; $\lambda^4 \exp(-4\varepsilon/\tau)$; and so on. The Gibbs sum is $\mathcal{Z} = 1 + \lambda \exp(-\varepsilon/\tau) + \lambda^2 \exp(-2\varepsilon/\tau) + \cdots$.

1 as part of the reservoir. Any arbitrary number of particles may be in the orbital.
2 The Gibbs sum taken for the orbital is

$$\mathcal{Z} = \sum_{N=0}^{\infty} \lambda^N \exp(-N\varepsilon/\tau) = \sum_{N=0}^{\infty} [\lambda \exp(-\varepsilon/\tau)]^N. \tag{7}$$

The upper limit on N should be the total number of particles in the combined system and reservoir. However, the reservoir may be arbitrarily large, so that N may run from zero to infinity. The series (7) may be summed in closed form. Let $x \equiv \lambda \exp(-\varepsilon/\tau)$; then

$$\mathcal{Z} = \sum_{N=0}^{\infty} x^N = \frac{1}{1 - x} = \frac{1}{1 - \lambda \exp(-\varepsilon/\tau)}, \tag{8}$$

provided that $\lambda \exp(-\varepsilon/\tau) < 1$. In all applications, $\lambda \exp(-\varepsilon/\tau)$ will satisfy this inequality; otherwise the number of bosons in the system would not be bounded.

3 The thermal average of the number of particles in the orbital is found from the Gibbs sum by use of (5.62):

$$f(\varepsilon) = \lambda \frac{\partial}{\partial \lambda} \log \mathcal{Z} = -x \frac{d}{dx} \log(1 - x) = \frac{x}{1 - x} = \frac{1}{\lambda^{-1} \exp(\varepsilon/\tau) - 1} \tag{9}$$

or

$$\boxed{f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \mu)/\tau] - 1}.} \tag{10}$$