The entropy of the second body is increased by

$$\Delta S_2 = \frac{0.1\,\mathrm{J}}{290\,\mathrm{K}} = 3.45 \times 10^{-4}\,\mathrm{J\,K^{-1}}.$$

The total entropy increases by

$$\Delta S_1 + \Delta S_2 = (-2.86 + 3.45) \times 10^{-4}\,\mathrm{J\,K^{-1}} = 0.59 \times 10^{-4}\,\mathrm{J\,K^{-1}}.$$

In fundamental units the increase of entropy is

$$\Delta\sigma = \frac{0.59 \times 10^{-4}}{k_B} = \frac{0.59 \times 10^{-4}\,\mathrm{J\,K^{-1}}}{1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}} = 0.43 \times 10^{19}\ , \tag{32}$$

where k_B is the Boltzmann constant. This result means that the number of accessible states of the two systems increases by the factor $\exp(\Delta\sigma) = \exp(0.43 \times 10^{19})$.

Law of Increase of Entropy

We can show that the total entropy always increases when two systems are brought into thermal contact. We have just demonstrated this in a special case. If the total energy $U = U_1 + U_2$ is constant, the total multiplicity after the systems are in thermal contact is

$$g(U) = \sum_{U_1} g_1(U_1)g_2(U - U_1)\ , \tag{33}$$

by (18). This expression contains the term $g_1(U_{10})g_2(U - U_{10})$ for the initial multiplicity before contact and many other terms besides. Here U_{10} is the initial energy of system 1 and $U - U_{10}$ is the initial energy of system 2. Because all terms in (33) are positive numbers, the multiplicity is always increased by establishment of thermal contact between two systems. This is a proof of the law of increase of entropy for a well-defined operation.

The significant effect of contact, the effect that stands out even after taking the logarithm of the multiplicity, is not just that the number of terms in the summation is large, but that the largest single term in the summation may be very, very much larger than the initial multiplicity. That is,

$$(g_1 g_2)_{\max} \equiv g_1(\hat{U}_1)g_2(U - \hat{U}_1) \tag{34}$$