Fluktuazioak multzo makrokanonikoan: (i) dentsitatea

$$\overline{N} = \frac{\sum\limits_{r,s} N_r e^{\alpha N_r - \beta E_s}}{\sum\limits_{r,s} e^{\alpha N_r - \beta E_s}}$$

$$\left(\frac{\partial \overline{N}}{\partial \alpha}\right)_{\beta,E_s} = -\overline{N^2} + \overline{N}^2$$

$$\overline{(\Delta N)^2} = \overline{N^2} - \overline{N}^2 = \left(\frac{\partial \overline{N}}{\partial \alpha}\right)_{T,V} = k_{\rm B}T \left(\frac{\partial \overline{N}}{\partial \mu}\right)_{T,V}$$

$$\frac{\overline{(\Delta n)^2}}{\overline{n}^2} = \frac{\overline{(\Delta N)^2}}{\overline{N}^2} = \frac{k_{\rm B}T}{\overline{N}^2} \left(\frac{\partial \overline{N}}{\partial \mu}\right)_{T,V}$$

$$\overline{\frac{(\Delta n)^2}{\overline{n}^2}} = \frac{k_{\rm B}T v^2}{V^2} \left(\frac{\partial (V/v)}{\partial \mu}\right)_{T,V} = -\frac{k_{\rm B}T}{V} \left(\frac{\partial v}{\partial \mu}\right)_{T}$$

$$d\mu = vdP - sdT$$

$$\overline{\frac{(\Delta n)^2}{\overline{n}^2}} = -\frac{k_{\rm B}T}{V} \frac{1}{V} \left(\frac{\partial v}{\partial P}\right)_{T} = \frac{k_{\rm B}T}{V} \kappa_{T}$$