	n ($10^{28}\,\mathrm{m}^{-3}$)	E_F (eV)	$\frac{2}{3}nE_\mathrm{F}$ ($10^9\mathrm{N\,m}^{-2}$)	B ($10^9\mathrm{N\,m}^{-2}$)
Li	4.70	4.74	23.8	11.1
Na	2.65	3.24	9.2	6.3
K	1.40	2.12	3.2	3.1
Cu	8.47	7.00	63.3	137.8
Ag	5.86	5.49	34.3	103.6

Table 30.1 Properties of selected metals

as is appropriate for non-relativistic electrons (see Table 25.1). The mean energy of the electrons at $T = 0$ is given by

$$\langle E\rangle = \frac{\int_0^{E_\mathrm{F}} E g(E)\,\mathrm{d}E}{\int_0^{E_\mathrm{F}} g(E)\,\mathrm{d}E}, \tag{30.31}$$

which with $g(E) \propto E^{1/2}$ gives $\langle E\rangle = \frac{3}{5}E_\mathrm{F}$. Writing $U = n\langle E\rangle$, we have that the bulk modulus B is

$$B = -V\frac{\partial p}{\partial V} = \frac{10U}{9V} = \frac{2}{3}nE_\mathrm{F}. \tag{30.32}$$

This expression is evaluated in Table 30.1 and gives results which are of the same order of magnitude as experimental values.

The next example computes an integral which is useful for considering analytically the effect of finite temperature.

Example 30.4

Evaluate the integral $\boxed{I = \int_0^\infty \phi(E)f(E)\,\mathrm{d}E}$ as a power series in temperature.

Solution:

Consider the function $\psi(E) = \int_0^E \phi(E')\,\mathrm{d}E'$, which is defined so that $\phi(E) = \mathrm{d}\psi/\mathrm{d}E$ and therefore

$$\begin{aligned} I = \int_0^\infty \frac{\mathrm{d}\psi}{\mathrm{d}E}f(E)\,\mathrm{d}E &= [f(E)\phi(E)]_0^\infty - \int_0^\infty \psi(E)\frac{\mathrm{d}f}{\mathrm{d}E}\,\mathrm{d}E \\ &= -\int_0^\infty \psi(E)\frac{\mathrm{d}f}{\mathrm{d}E}\,\mathrm{d}E. \end{aligned} \tag{30.33}$$

Now put $x = (E-\mu)/k_\mathrm{B}T$ and hence

$$\frac{\mathrm{d}f}{\mathrm{d}E} = -\frac{1}{k_\mathrm{B}T}\frac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}. \tag{30.34}$$

$dx = \frac{1}{k_BT} \cdot dE \Rightarrow dE = k_BT\, dx$ $\qquad \frac{1}{dx} = k_BT \frac{1}{dE}$

$E = 0 \Rightarrow x = \frac{-\mu}{k_BT} = \frac{-E_F}{k_BT}$