$$P(z,V,T) = \frac{k_B T}{V} q(z,V,T) \equiv \frac{k_B T}{V} \ln \mathcal{Q}(z,V,T)$$

$$N(z,V,T) = z \left[\frac{\partial}{\partial z} q(z,V,T)\right]_{V,T} = k_B T \left[\frac{\partial}{\partial \mu} q(\mu,V,T)\right]_{V,T}$$

$$U(z,V,T) = -\left[\frac{\partial}{\partial \beta} q(z,V,T)\right]_{z,V} = k_B T^2 \left[\frac{\partial}{\partial T} q(z,V,T)\right]_{z,V}$$

$$A = N\mu - PV = N k_B T \ln z - k_B T \ln \mathcal{Q}(z,V,T)$$

$$F = -k_B T \ln \frac{\mathcal{Q}(z,V,T)}{z^N}$$

$F[\mu] \rightarrow F[\mu](\mu, V, T)$

$$S = \frac{U - A}{T} = k_B T \left(\frac{\partial q}{\partial T}\right)_{z,V} - Nk \ln z + kq$$