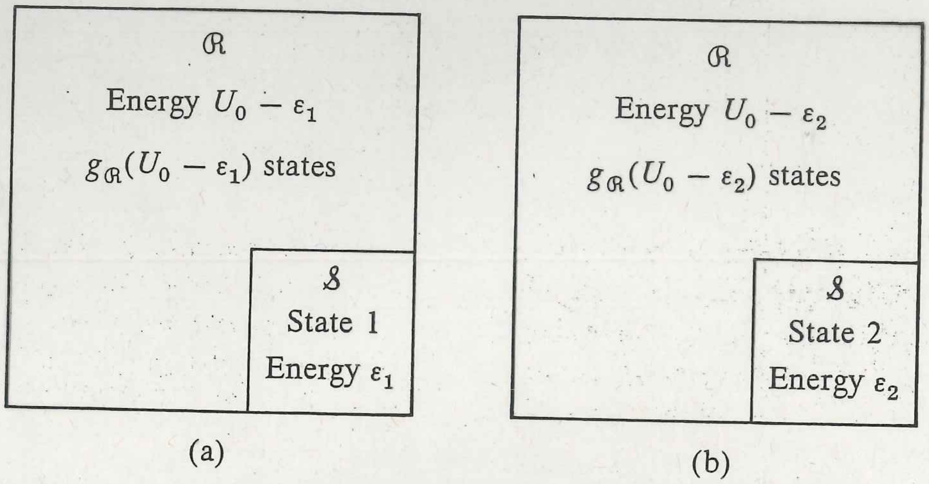

Figure 3.3 The system in (a), (b) is in quantum state 1, 2. The reservoir has $g_{\mathcal{R}}(U_0 - \varepsilon_1)$, $g_{\mathcal{R}}(U_0 - \varepsilon_2)$ accessible quantum states, in (a) and (b) respectively.

With

$$\Delta\sigma_{\mathcal{R}} \equiv \sigma_{\mathcal{R}}(U_0 - \varepsilon_1) - \sigma_{\mathcal{R}}(U_0 - \varepsilon_2) \ , \tag{4}$$

the probability ratio for the two states 1, 2 of the system is simply

$$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = \exp(\Delta\sigma_{\mathcal{R}}). \tag{5}$$

Let us expand the entropies in (4) in a Taylor series expansion about $\sigma_{\mathcal{R}}(U_0)$. The Taylor series expansion of $f(x)$ about $f(x_0)$ is

$$f(x_0 + a) = f(x_0) + a\left(\frac{df}{dx}\right)_{x=x_0} + \frac{1}{2!}a^2\left(\frac{d^2f}{dx^2}\right)_{x=x_0} + \cdots. \tag{6}$$

Thus

$$\begin{aligned} \sigma\ (U_0 - \varepsilon) &= \sigma_{\mathcal{R}}(U_0) - \varepsilon(\partial\sigma_{\mathcal{R}}/\partial U)_{V,N} + \cdots \\ &= \sigma_{\mathcal{R}}(U_0) - \varepsilon/\tau + \cdots \ , \end{aligned} \tag{7}$$

where $1/\tau \equiv (\partial\sigma_{\mathcal{R}}/\partial U)_{V,N}$ gives the temperature. The partial derivative is taken