where the sum over orbitals has been transformed to an integral by the substitution

$$\sum_{n} (\cdot \cdot \cdot) \to \int d\varepsilon \ \mathfrak{D}(\varepsilon)(\cdot \cdot \cdot). \tag{13}$$

Here $\mathfrak{D}(\varepsilon)d\varepsilon$ is the number of orbitals of energy between ε and $\varepsilon + d\varepsilon$. The quantity $\mathfrak{D}(\varepsilon)$ is nearly always called the **density of states**, although it is more accurate to call it the density of orbitals because it refers to the solutions of a one particle problem and not to the states of the N particle system.

Consider an example of the calculation of $\mathfrak{D}(\varepsilon)$. We see from (7) that the number N of free electron orbitals of energy less than or equal to some ε is

$$N(\varepsilon) = (V/3\pi^2)(2m/\hbar^2)^{3/2}\varepsilon^{3/2} , \qquad (14)$$

for volume V. Take the logarithm of both sides:

$$\log N = \frac{3}{2}\log\varepsilon + \text{constant},\tag{15}$$

and take differentials of $\log N$ and $\log \varepsilon$:

$$\frac{dN}{N} = \frac{3}{2} \frac{d\varepsilon}{\varepsilon}.$$
 (16)

The quantity $dN = (3N/2\varepsilon)d\varepsilon$ is the number of orbitals of energy between ε and $\varepsilon + d\varepsilon$, so that

$$\mathfrak{D}(\varepsilon) \equiv dN/d\varepsilon = 3N(\varepsilon)/2\varepsilon \tag{17}$$

is the density of orbitals. The two spin orientations of an electron have been counted throughout this derivation because they were counted in (6). We can write $\mathfrak{D}(\varepsilon)$ as a function of ε alone because

$$N(\varepsilon)/\varepsilon = (V/3\pi^2)(2m/\hbar^2)^{3/2}\varepsilon^{1/2}$$
, (18)

from (14). Then (17) becomes

$$\mathfrak{D}(\varepsilon) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}.$$