Every distinct state of a binary alloy system on N sites is contained in the symbolic product of N factors:

$$(A_1 + B_1)(A_2 + B_2)(A_3 + B_3) \cdots (A_N + B_N) , \tag{19}$$

in analogy to (4). The average composition of a binary alloy is specified conventionally by the chemical formula $A_{1-x}B_x$, which means that out of a total of N atoms, the number of A atoms is $N_A = (1-x)N$ and the number of B atoms is $N_B = xN$. Here x lies between 0 and 1.

The symbolic expression

$$(A + B)^N = \sum_{t=0}^{N} \frac{N!}{(N-t)!\,t!} A^{N-t} B^t , \tag{20}$$

is analogous to the result (12). The coefficient of the term in $A^{N-t}B^t$ gives the number $g(N,t)$ of possible arrangements or states of $N - t$ atoms A and t atoms B on N sites:

$$g(N,t) = \frac{N!}{(N-t)!\,t!} = \frac{N!}{N_A!\,N_B!} , \tag{21}$$

which is identical to the result (15) for the spin model system, except for notation.

Sharpness of the Multiplicity Function

T FiNKo
$\Downarrow$
E FiNKo
$E = \langle E \rangle$ ...

We know from common experience that systems held at constant temperature usually have well-defined properties; this stability of physical properties is a major prediction of thermal physics. The stability follows as a consequence of the exceedingly sharp peak in the multiplicity function and of the steep variation of that function away from the peak. We can show explicitly that for a very large system, the function $g(N,s)$ defined by (15) is peaked very sharply about the value $s = 0$. We look for an approximation that allows us to examine the form of $g(N,s)$ versus s when $N \gg 1$ and $|s| \ll N$. We cannot look up these values in tables: common tables of factorials do not go above $N = 100$, and we may be interested in $N \approx 10^{20}$, of the order of the number of atoms in a solid specimen big enough to be seen and felt. An approximation is clearly needed, and a good one is available.

It is convenient to work with $\log g$. Except where otherwise specified, all logarithms are understood to be log base e, written here as log. The international standard usage is ln for log base e, but it is clearer to write log when there is no ambiguity whatever. When you confront a very, very large number such as