The total entropy change is

$$\Delta\sigma = \left(\frac{\partial\sigma_1}{\partial U_1}\right)_{N_1}(-\Delta U) + \left(\frac{\partial\sigma_2}{\partial U_2}\right)_{N_2}(\Delta U) = \left(-\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\Delta U. \tag{31}$$

When $\tau_1 > \tau_2$ the quantity in parentheses on the right-hand side is positive, so that the total change of entropy is positive when the direction of energy flow is from the system with the higher temperature to the system with the lower temperature.

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***Example: Entropy increase on heat flow.*** This example makes use of the reader's previous familiarity with heat and specific heat.

(a) Let a 10-g specimen of copper at a temperature of 350 K be placed in thermal contact with an identical specimen at a temperature of 290 K. Let us find the quantity of energy transferred when the two specimens are placed in contact and come to equilibrium at the final temperature $T_f$. The specific heat of metallic copper over the temperature range 15°C to 100°C is approximately 0.389 J g$^{-1}$ K$^{-1}$, according to a standard handbook.

The energy increase of the second specimen is equal to the energy loss of the first; thus the energy increase of the second specimen is, in joules,

$$\Delta U = (3.89\,\mathrm{J\,K^{-1}})(T_f - 290\,\mathrm{K}) = (3.89\,\mathrm{J\,K^{-1}})(350\,\mathrm{K} - T_f)\ ,$$

where the temperatures are in kelvin. The final temperature after contact is

$$T_f = \tfrac{1}{2}(350 + 290)\mathrm{K} = 320\,\mathrm{K}.$$

Thus

$$\Delta U_1 = (3.89\,\mathrm{J\,K^{-1}})(-30\,\mathrm{K}) = -11.7\,\mathrm{J}\ ,$$

and

$$\Delta U_2 = -\Delta U_1 = 11.7\,\mathrm{J}.$$

(b) What is the change of entropy of the two specimens when a transfer of 0.1 J has taken place, almost immediately after initial contact? Notice that this transfer is a small fraction of the final energy transfer as calculated above. Because the energy transfer considered is small, we may suppose the specimens are approximately at their initial temperatures of 350 and 290 K. The entropy of the first body is decreased by

$$\Delta S_1 = \frac{-0.1\,\mathrm{J}}{350\,\mathrm{K}} = -2.86 \times 10^{-4}\,\mathrm{J\,K^{-1}}.$$