

Here, $\Delta\sigma$ is the entropy difference:

$$\Delta\sigma \equiv \sigma(N_0 - N_1, U_0 - \varepsilon_1) - \sigma(N_0 - N_2, U_0 - \varepsilon_2). \quad (47)$$

The reservoir is very large in comparison with the system, and $\Delta\sigma$ may be approximated quite accurately by the first order terms in a series expansion in the two quantities N and ε that relate to the system. The entropy of the reservoir becomes

$$\sigma(N_0 - N, U_0 - \varepsilon) = \sigma(N_0, U_0) - N \left(\frac{\partial\sigma}{\partial N_0} \right)_{U_0} - \varepsilon \left(\frac{\partial\sigma}{\partial U_0} \right)_{N_0} + \dots \quad (48)$$

For $\Delta\sigma$ defined by (47) we have, to the first order in $N_1 - N_2$ and in $\varepsilon_1 - \varepsilon_2$,

$$\Delta\sigma = -(N_1 - N_2) \left(\frac{\partial\sigma}{\partial N_0} \right)_{U_0} - (\varepsilon_1 - \varepsilon_2) \left(\frac{\partial\sigma}{\partial U_0} \right)_{N_0}. \quad (49)$$

We know that

$$\frac{1}{\tau} \equiv \left(\frac{\partial\sigma}{\partial U_0} \right)_{N_0}, \quad (50a)$$

by our original definition of the temperature. This is written for the reservoir, but the system will have the same temperature. Also,

$$-\frac{\mu}{\tau} \equiv \left(\frac{\partial\sigma}{\partial N_0} \right)_{U_0}, \quad (50b)$$

by (30).

The entropy difference (49) is

$$\Delta\sigma = \frac{(N_1 - N_2)\mu}{\tau} - \frac{(\varepsilon_1 - \varepsilon_2)}{\tau}. \quad (51)$$

Here $\Delta\sigma$ refers to the reservoir, but $N_1, N_2, \varepsilon_1, \varepsilon_2$ refer to the system. The central result of statistical mechanics is found on combining (46) and (51):

$$\frac{P(N_1, \varepsilon_1)}{P(N_2, \varepsilon_2)} = \frac{\exp[(N_1\mu - \varepsilon_1)/\tau]}{\exp[(N_2\mu - \varepsilon_2)/\tau]}. \quad (52)$$

NORMALIZATION KONSTANTE $\frac{1}{\tau} \propto \exp[-\varepsilon/\tau]$
 $\exp[N\mu]$