for all ε. When this inequality is satisfied we may neglect the term ± 1 in the denominator of (11). Then for either fermions or bosons, the average occupancy of an orbital of energy ε is

$$\boxed{f(\varepsilon) \simeq \exp[(\mu - \varepsilon)/\tau] = \lambda \exp(-\varepsilon/\tau) ,} \tag{13}$$

with $\lambda \equiv \exp(\mu/\tau)$. The limiting result (13) is called the **classical distribution function**. It is the limit of the Fermi-Dirac and Bose-Einstein distribution functions when the average occupancy $f(\varepsilon)$ is very small in comparison with unity. Equation (13), although called classical, is still a result for particles described by quantum mechanics: we shall find that the expression for λ or μ always involves the quantum constant $\hbar$. Any theory which contains $\hbar$ cannot be a classical theory.

We use the classical distribution function $f(\varepsilon) = \lambda \exp(-\varepsilon/\tau)$ to study the thermal properties of the ideal gas. There are many topics of importance: the entropy, chemical potential, heat capacity, the pressure-volume-temperature relation, and the distribution of atomic velocities. To obtain results from the classical distribution function, we need first to find the chemical potential in terms of the concentration of atoms.

Chemical Potential

The chemical potential is found from the condition that the thermal average of the total number of atoms equals the number of atoms known to be present. This number must be the sum over all orbitals of the distribution function $f(\varepsilon_s)$:

$$N = \langle N \rangle = \sum_s f(\varepsilon_s) , \tag{14}$$

where s is the index of an orbital of energy ε_s. We start with a monatomic gas of N identical atoms of zero spin, and later we include spin and molecular modes of motion. The total number of atoms is the sum of the average number of atoms in each orbital. We use (13) in (14) to obtain

$$N = \lambda \sum_s \exp(-\varepsilon_s/\tau). \tag{15}$$

To evaluate this sum, observe that the summation over free particle orbitals is just the partition function Z_1 for a single free atom in volume V, whence $N = \lambda Z_1$.