and the other states with one magnet down are formed from (8) by reversing any single magnet. The states (9) and (10) have total moment $M = Nm - 2m$.

Enumeration of States and the Multiplicity Function

aldagai-aldaketa

$N_\uparrow = \frac{1}{2}N + s$

$N_\downarrow = \frac{1}{2}N - s$

$\Delta(\uparrow,\downarrow) = {}^{N}\uparrow - {}^{N}\downarrow = s$

We use the word spin as a shorthand for elementary magnet. It is convenient to assume that N is an even number. We need a mathematical expression for the number of states with $N_\uparrow = \frac{1}{2}N + s$ magnets up and $N_\downarrow = \frac{1}{2}N - s$ magnets down, where s is an integer. When we turn one magnet from the up to the down orientation, $\frac{1}{2}N + s$ goes to $\frac{1}{2}N + s - 1$ and $\frac{1}{2}N - s$ goes to $\frac{1}{2}N - s + 1$. The difference (number up − number down) changes from $2s$ to $2s - 2$. The difference

$$\boxed{N_\uparrow - N_\downarrow = 2s} \tag{11}$$

is called the **spin excess.** The spin excess of the 4 states in Figure 1.5 is 2, 0, 0, −2, from left to right. The factor of 2 in (11) appears to be a nuisance at this stage, but it will prove to be convenient.

The product in (4) may be written symbolically as

$$(\uparrow + \downarrow)^N.$$

↓ "bamaketa"

We may drop the site labels (the subscripts) from (4) when we are interested only in how many of the magnets in a state are up or down, and not in which particular sites have magnets up or down. It we drop the labels and neglect the order in which the arrows appear in a given product, then (5) becomes

$$(\uparrow + \downarrow)^2 = \uparrow\uparrow + 2\uparrow\downarrow + \downarrow\downarrow;$$

further,

$$(\uparrow + \downarrow)^3 = \uparrow\uparrow\uparrow + 3\uparrow\uparrow\downarrow + 3\uparrow\downarrow\downarrow + \downarrow\downarrow\downarrow.$$

We find $(\uparrow + \downarrow)^N$ for arbitrary N by the binomial expansion

$$(x + y)^N = x^N + Nx^{N-1}y + \tfrac{1}{2}N(N-1)x^{N-2}y^2 + \cdots + y^N$$
$$= \sum_{t=0}^{N} \frac{N!}{(N-t)!\,t!}\, x^{N-t}y^t. \tag{12}$$