Thus the number of atoms in excited states is

$$\boxed{N_e = \frac{1.306V}{4}\left(\frac{2M\tau}{\pi\hbar^2}\right)^{3/2} = 2.612 n_Q V\,,} \tag{70}$$

$n_Q \equiv \frac{1}{\lambda_T^3}$

where $n_Q \equiv (M\tau/2\pi\hbar^2)^{3/2}$ is again the quantum concentration. We divide N_e by N to obtain the fraction of atoms in excited orbitals:

$$N_e/N \simeq 2.612 n_Q V/N = 2.612 n_Q/n. \tag{71}$$

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The value $\lambda \simeq 1$ or $1 - 1/N$ which led to (71) is valid as long as a large number of atoms are in the ground state. All particles have to be in some orbital, either in an excited orbital or in the ground orbital. The number in excited orbitals is relatively insensitive to small changes in λ, but the rest of the particles have to be in the ground orbital. To assure this we must take λ very close to 1 as long as N_0 is a large number. Even 10^3 is a large number for the occupancy of an orbital. Yet within $\Delta\tau/\tau_E = 10^{-6}$ of the transition, where τ_E is defined by (72) below, the occupancy of the ground orbital is $>10^{15}$ atoms cm^{-3} at the concentration of liquid ^{4}He. Thus our argument is highly accurate at $\Delta\tau/\tau_E = 10^{-6}$.

Einstein Condensation Temperature

We define the **Einstein condensation temperature*** τ_E as the temperature for which the number of atoms in excited states is equal to the total number of atoms. That is, $N_e(\tau_E) = N$. Above τ_E the occupancy of the ground orbital is not a macroscopic number; below τ_E the occupancy is macroscopic. From (70) with N for N_e we find for the condensation temperature

$$\boxed{\tau_E \equiv \frac{2\pi\hbar^2}{M}\left(\frac{N}{2.612V}\right)^{2/3}.} \tag{72}$$

Now (71) may be written as

$$N_e/N \simeq (\tau/\tau_E)^{3/2}\,, \tag{73}$$

where N is the total number of atoms. The number of atoms in excited orbitals varies as $\tau^{3/2}$ at temperatures below τ_E, as shown in Figure 7.11. The calculated value of T_E for atoms of ^{4}He is $\approx 3\,\mathrm{K}$.

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* A. Einstein, Akademie der Wissenschaften, Berlin, Sitzungsberichte **1924**, 261; **1925**, 3.