Quantum gases and condensates

30

Exchange symmetry affects the occupation of allowed states in quantum gases. If the density of the gas is very low, such that $n\lambda_{\text{th}}^3 \ll 1$, we can ignore this and forget about exchange symmetry; this is what we do for gases at room temperature. But if the density is high, the effects of exchange symmetry become very important and it really starts to matter whether the particles you are considering are fermions or bosons. In this chapter, we consider quantum gases in detail and explore the possible effects that one can observe.

"gas"

30.1 The non-interacting quantum fluid

HEMENTXE DAGO SISTEMARI BURUZKO INFORMAZIOA, SISTEMA DESKRIBATZEKO INFORMAZIO FISIKOA, ERREALA!!

We first consider a fluid composed of non-interacting particles. To keep things completely general for the moment, we will consider particles with spin S. This means that each allowed momentum state is associated with $2S+1$ possible spin states.[1] If we can ignore interactions between particles, the grand partition function $\mathcal{Z}$ is simply the product of single-particle partition functions, so that

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$$\mathcal{Z} = \prod_k \mathcal{Z}_k^{2S+1}, \tag{30.1}$$

where

$$\mathcal{Z}_k = (1 \pm \mathrm{e}^{-\beta(E_k-\mu)})^{\pm 1} \tag{30.2}$$

is a single particle partition function and where the $\pm$ sign is $+$ for fermions and $-$ for bosons.[2]

[1] If the spin is S, there are $2S+1$ possible states corresponding to the z-component of angular momentum being $-S, -S+1, \ldots S$.

[2] These results follow directly from eqns 29.18 and 29.20.

Example 30.1

Find the grand potential for a three-dimensional gas of non-interacting bosons and fermions with spin S.

Solution:

The grand potential Φ_{G} is obtained from eqn 30.1 as follows:

$$\begin{aligned}
\Phi_{\mathrm{G}} &= -k_{\mathrm{B}}T \ln \mathcal{Z} \\
&= \mp k_{\mathrm{B}}T (2S+1) \sum_k \ln(1 \pm \mathrm{e}^{-\beta(E_k-\mu)}) \\
&= \mp k_{\mathrm{B}}T (2S+1) \int_0^\infty \ln(1 \pm \mathrm{e}^{-\beta(E-\mu)})\, g(E)\, \mathrm{d}E,
\end{aligned} \tag{30.3}$$