We may write the exponents of x and y in a slightly different, but equivalent, form by replacing t with $\frac{1}{2}N - s$:

$$(x + y)^N = \sum_{s=-\frac{1}{2}N}^{\frac{1}{2}N} \frac{N!}{(\frac{1}{2}N + s)!\,(\frac{1}{2}N - s)!}\, x^{\frac{1}{2}N+s} y^{\frac{1}{2}N-s}. \tag{13}$$

With this result the symbolic expression $(\uparrow + \downarrow)^N$ becomes

$$\boxed{(\uparrow + \downarrow)^N = \sum_s \frac{N!}{(\frac{1}{2}N + s)!\,(\frac{1}{2}N - s)!} \uparrow^{\frac{1}{2}N+s} \downarrow^{\frac{1}{2}N-s}.} \tag{14}$$

The coefficient of the term in $\uparrow^{\frac{1}{2}N+s} \downarrow^{\frac{1}{2}N-s}$ is the number of states having $N_\uparrow = \frac{1}{2}N + s$ magnets up and $N_\downarrow = \frac{1}{2}N - s$ magnets down. This class of states has spin excess $\boxed{N_\uparrow - N_\downarrow = 2s}$ and net $\boxed{\text{magnetic moment } 2sm}$. Let us denote the number of states in this class by $g(N,s)$, for a system of N magnets: M

$$\boxed{g(N,s) = \frac{N!}{(\frac{1}{2}N + s)!\,(\frac{1}{2}N - s)!} = \frac{N!}{N_\uparrow!\,N_\downarrow!}.} \tag{15}$$

Thus (14) is written as

$$(\uparrow + \downarrow)^N = \sum_{s=-\frac{1}{2}N}^{\frac{1}{2}N} g(N,s) \uparrow^{\frac{1}{2}N+s} \downarrow^{\frac{1}{2}N-s}. \tag{16}$$

We shall call $g(N,s)$ the **multiplicity function**; it is the number of states having the same value of s. The reason for our definition emerges when a magnetic field is applied to the spin system: in a magnetic field, states of different values of s have different values of the energy, so that our g is equal to the multiplicity of an energy level in a magnetic field. Until we introduce a magnetic field, all states of the model system have the same energy, which may be taken as zero. Note from (16) that the total number of states is given by

$$\sum_{s=-\frac{1}{2}N}^{s=\frac{1}{2}N} g(N,s) = (1 + 1)^N = 2^N. \tag{17}$$

Examples related to $g(N,s)$ for $N = 10$ are given in Figures 1.6 and 1.7. For a coin, "heads" could stand for "magnet up" and "tails" could stand for "magnet down."