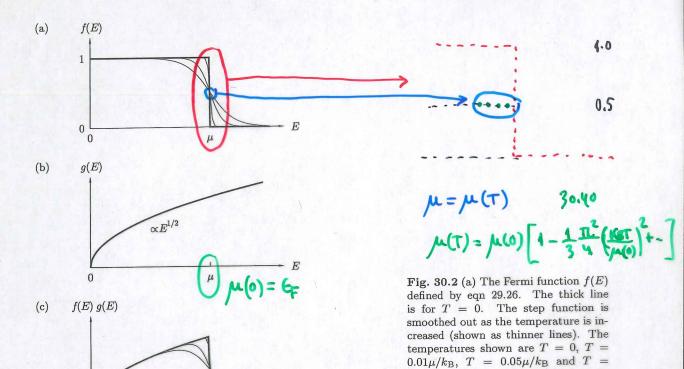
$0.1\mu/k_{\rm B}$. (b) The density of states g(E) for a non-interacting fermion gas in three dimensions is proportional to $E^{1/2}$. (c) f(E)g(E) for the same tem-

M(0) = EF

M(T) = = [1-1(1) (KAT) --]

 $\varepsilon_{F} = 100 (KpT) \Rightarrow \frac{KpT}{\varepsilon} = 10^{-2}$ $\mu(T) = \varepsilon_{F} \Rightarrow \mu \neq \mu(T)$

peratures as in (a).



At T=0, the distribution function f(E) is a Heaviside step function, taking the value 1 for $E<\mu$ and 0 for $E>\mu$. This step is smoothed out as the temperature T increases, as shown in Fig. 30.2(a). The density of states g(E) for a non-interacting fermion gas in three dimensions is proportional to $E^{1/2}$ (as shown in eqn 30.4) and this is plotted in Fig. 30.2(b). The product of f(E)g(E) gives the actual number distribution of fermions, and this is shown in Fig. 30.2(c). The sharp cutoff you would expect at T=0 is smoothed over an energy scale k_BT around the chemical potential μ .

μ

The electrons in a metal can be treated as a non-interacting gas of fermions. Using the number density n of electrons in a metal, one can calculate the Fermi energy using eqn 30.29, and some example results are shown in Table 30.1. The Fermi energies are all several eV; converting each number into a temperature, the so-called Fermi temperature $T_{\rm F} = E_{\rm F}/k_{\rm B}$, yields values of several tens of thousands of Kelvin. Thus the Fermi energy is a large energy scale, and hence for most metals the Fermi function is close to a step function, at pretty much all temperatures below their melting temperature. In this case, the electrons in a metal are said to be in the degenerate limit.

The pressure of these electrons is given (by using eqns 22.49 and 30.17)

$$p = \frac{2U}{3V},\tag{30.30}$$

~f

as