



Figure 7.6 Plot of the chemical potential μ versus temperature τ for a gas of noninteracting fermions in three dimensions. For convenience in plotting, the units of μ and τ are $0.763\epsilon_F$.

$$\Delta U = \int_{\epsilon_F}^{\infty} d\epsilon (\epsilon - \epsilon_F) f(\epsilon) D(\epsilon) + \int_0^{\epsilon_F} d\epsilon (\epsilon_F - \epsilon) (1 - f(\epsilon)) D(\epsilon)$$

can group terms to obtain

$$\Delta U \rightarrow \int_{\epsilon_F}^{\infty} d\epsilon (\epsilon - \epsilon_F) f(\epsilon) D(\epsilon) + \int_0^{\epsilon_F} d\epsilon (\epsilon - \epsilon_F) f(\epsilon) D(\epsilon)$$

$$C_{el} = \frac{dU}{d\tau} = \int_0^{\infty} d\epsilon (\epsilon - \epsilon_F) \frac{df}{d\tau} D(\epsilon). \quad (28)$$

At the temperatures of interest in metals $\tau/\epsilon_F < 0.01$, and we see from Figure 7.5 (1) that the derivative $df/d\tau$ is large only at energies near ϵ_F . It is a good approximation to evaluate the density of orbitals $D(\epsilon)$ at ϵ_F and take it outside of the integral: $D(\epsilon) \rightarrow D(\epsilon_F)$

$$C_{el} \cong D(\epsilon_F) \int_0^{\infty} d\epsilon (\epsilon - \epsilon_F) \frac{df}{d\tau}. \quad (29)$$

Examination of the graphs in Figures 7.6 and 7.7 of the variation of μ with τ suggests that when $\tau \ll \epsilon_F$ we ignore the temperature dependence of the chemical potential μ in the Fermi-Dirac distribution function and replace μ by the constant ϵ_F . We have then:

$$\frac{df}{d\tau} = \frac{\epsilon - \epsilon_F}{\tau^2} \cdot \frac{\exp[(\epsilon - \epsilon_F)/\tau]}{\{\exp[(\epsilon - \epsilon_F)/\tau] + 1\}^2}. \quad (30)$$

$$\frac{x}{z} \quad \frac{e^x}{(e^x + 1)^2}$$

$$\mu \rightarrow \epsilon_F$$

$$\mu(\tau) \neq \epsilon_F(\tau)$$