

SUMMARY

1. The multiplicity function for a system of N magnets with spin excess $2s = N_\uparrow - N_\downarrow$ is

$$g(N,s) = \frac{N!}{(\frac{1}{2}N + s)!(\frac{1}{2}N - s)!} = \frac{N!}{N_\uparrow! N_\downarrow!}.$$

In the limit $s/N \ll 1$, with $N \gg 1$, we have the Gaussian approximation

$$g(N,s) \simeq (2/\pi N)^{1/2} 2^N \exp(-2s^2/N).$$

2. If all states of the model spin system are equally likely, the average value of s^2 is

$$\langle s^2 \rangle = \int_{-\infty}^{\infty} ds s^2 g(N,s) / \int_{-\infty}^{\infty} ds g(N,s) = \frac{1}{4}N ,$$

in the Gaussian approximation.

3. The fractional fluctuation of s^2 is defined as $\langle s^2 \rangle^{1/2}/N$ and is equal to $1/2N^{1/2}$.
4. The energy of the model spin system in a state of spin excess $2s$ is

$$U(s) = -2smB ,$$

where m is the magnetic moment of one spin and B is the magnetic field.