which implies that

$$\mu(T) = \mu(0)\left[1 - \frac{\pi^2}{12}\left(\frac{k_{\mathrm{B}}T}{\mu(0)}\right)^2 + \ldots\right]. \tag{30.40}$$

In fact, equating E_{F} and μ is good to 0.01% for typical metals even at room temperature, although it is worthwhile keeping in the back of one's mind that the two quantities are not the same.

We can also compute the heat capacity of electrons in a metal by a similar technique, as shown in the following example.

Example 30.5

Compute the heat capacity of non-interacting free electrons in a three-dimensional metal.

Solution:

$$\begin{aligned} U &= \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty E^{3/2} f(E)\,\mathrm{d}E \\ &= \frac{V}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu(T)^{5/2}\left[1 + \frac{5\pi^2}{8}\left(\frac{k_{\mathrm{B}}T}{\mu(0)}\right)^2 + \ldots\right] \\ &= \frac{3}{5}N\mu(T)\left[1 + \frac{\pi^2}{2}\left(\frac{k_{\mathrm{B}}T}{\mu(0)}\right)^2 + \ldots\right] \\ &= \frac{3}{5}N\mu(0)\left[1 + \frac{5\pi^2}{12}\left(\frac{k_{\mathrm{B}}T}{\mu(0)}\right)^2 + \ldots\right] \end{aligned} \tag{30.41}$$

and hence

$$C_V = \frac{3}{2}Nk_{\mathrm{B}}\left(\frac{\pi^2}{3}\frac{k_{\mathrm{B}}T}{\mu(0)}\right) + O(T^3). \tag{30.42}$$

Thus the contribution to the heat capacity from electrons is linear in temperature (recall from Chapter 24 that the heat capacity from lattice vibrations (phonons) is proportional to T^3 at low temperature) and will therefore dominate the heat capacity of a metal at very low temperatures.

The **Fermi surface** is the set of points in k-space whose energy is equal to the chemical potential. If the chemical potential lies in a gap[7] between energy bands, then the material is a semiconductor or an insulator and there will be no Fermi surface. Thus a metal is a material with a Fermi surface.

[7] The periodic potential which exists in crystalline metals can lead to the formation of energy gaps, i.e. intervals in energy in which there are no allowed states.