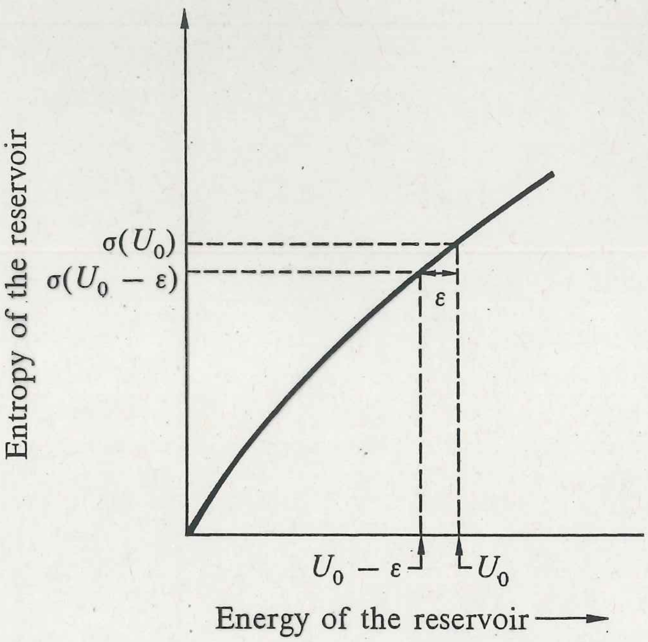

Figure 3.2 The change of entropy when the reservoir transfers energy ε to the system. The fractional effect of the transfer on the reservoir is small when the reservoir is large, because a large reservoir will have a high entropy.

accessible to $\mathcal{R} + \mathcal{S}$ is

$$g_{\mathcal{R}} \times 1 = g_{\mathcal{R}} \ , \tag{1}$$

because for our present purposes we have specified the state of $\mathcal{S}$.

If the system energy is ε_s, the reservoir energy is $U_0 - \varepsilon_s$. The number of states accessible to the reservoir in this condition is $g_{\mathcal{R}}(U_0 - \varepsilon_s)$, as in Figure 3.2. The ratio of the probability that the system is in quantum state 1 at energy ε_1 to the probability that the system is in quantum state 2 at energy ε_2 is the ratio of the two multiplicities:

$$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = \frac{\text{Multiplicity of } \mathcal{R} \text{ at energy } U_0 - \varepsilon_1}{\text{Multiplicity of } \mathcal{R} \text{ at energy } U_0 - \varepsilon_2} = \frac{g_{\mathcal{R}}(U_0 - \varepsilon_1)}{g_{\mathcal{R}}(U_0 - \varepsilon_2)} . \tag{2}$$

This result is a direct consequence of what we have called the fundamental assumption. The two situations are shown in Figure 3.3. Although questions about the system depend on the constitution of the reservoir, we shall see that the dependence is only on the temperature of the reservoir.

$\sigma = k_B \ln g$

$\frac{\sigma}{k_B} = \log g$

$e^{\frac{\sigma}{k_B}} = g$

If the reservoirs are very large, the multiplicities are very, very large numbers. We write (2) in terms of the entropy of the reservoir:

$$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = \frac{\exp[\sigma_{\mathcal{R}}(U_0 - \varepsilon_1)]}{\exp[\sigma_{\mathcal{R}}(U_0 - \varepsilon_2)]} = \exp[\sigma_{\mathcal{R}}(U_0 - \varepsilon_1) - \sigma_{\mathcal{R}}(U_0 - \varepsilon_2)] . \tag{3}$$