

Figure 7.2 Total ground state energy U_0 of one mole of electrons, versus volume.

using (5) and (6). The average kinetic energy per particle is U_0/N and is $\frac{3}{5}$ of the Fermi energy ε_F . At constant N the energy increases as the volume decreases (Figure 7.2), so that the Fermi energy gives a repulsive contribution to the binding of any material; in most metals and in white dwarf and neutron stars it is the most important repulsive interaction. That is, the Fermi energy tends to increase the volume. It is balanced in metals by the Coulomb attraction between electrons and ions and in the stars by gravitational attraction.

Density of States

Thermal averages for independent particle problems have the form

$$\langle X \rangle = \sum_{\mathbf{n}} f(\varepsilon_{\mathbf{n}}, \tau, \mu) X_{\mathbf{n}} ,$$
 (11)

where n denotes the quantum orbital; X_n is the value of the quantity X in the orbital n; and $f(\varepsilon_n, \tau, \mu)$ is the thermal average occupancy, called the distribution function, of the orbital n. We often express $\langle X \rangle$ as an integral over the orbital energy ε . Then (11) becomes

$$\langle X \rangle = \int d\varepsilon \ \mathfrak{D}(\varepsilon) f(\varepsilon, \tau, \mu) X(\varepsilon) \ , \tag{12}$$