$$g(E) = \frac{1}{2\pi i} \int_{\beta' - i\infty}^{\beta' + i\infty} e^{\beta E} Q(\beta)\, d\beta \qquad \beta' > 0$$

$$Q_N(\beta) = \left(\frac{k_B T}{\hbar\omega}\right)^N = \left(\frac{1}{\beta\hbar\omega}\right)^N$$

$$g(E) = \frac{1}{2\pi i} \int_{\beta' - i\infty}^{\beta' + i\infty} e^{\beta E} \left(\frac{1}{\beta\hbar\omega}\right)^N d\beta \qquad \beta' > 0$$

$$\left(\frac{1}{\hbar\omega}\right)^N \underbrace{\boxed{\frac{1}{2\pi i} \int_{\beta' - i\infty}^{\beta' + i\infty} e^{\beta E} \frac{1}{\beta^N}\, d\beta \qquad \beta' > 0}}_{\equiv\ \frac{E^{N-1}}{(N-1)!}} \qquad E > 0$$

$$g(E) = \left(\frac{1}{\hbar\omega}\right)^N \frac{E^{N-1}}{(N-1)!}$$

$$S = k_B \ln\left[g(E)\right]$$

$$S = k_B \ln\left[\left(\frac{1}{\hbar\omega}\right)^N \frac{E^{N-1}}{(N-1)!}\right] + \text{S.H.} \quad (N \gg\gg 1)$$

$$= k_B \left\{ -N \ln(\hbar\omega) + (N-1)\ln E - (N-1)\ln(N-1) + (N-1) \right\}$$

$$N \simeq N-1$$

$$= k_B \left\{ -N \ln(\hbar\omega) + N \ln E - N \ln N + N \right\}$$

$$\boxed{S \cong k_B N \left\{ \ln\left(\frac{E}{\hbar\omega \cdot N}\right) + 1 \right\}} \longrightarrow \frac{1}{T} \equiv \left(\frac{\partial S}{\partial E}\right)_N$$

$$\frac{1}{T} = k_B N \frac{1}{(E/\cancel{\hbar\omega N})} \cdot \frac{1}{\cancel{N\hbar\omega}} \longrightarrow T = \frac{E}{N k_B} \Rightarrow \boxed{\frac{E}{N} = k_B T} \longrightarrow S \approx k_B N \left\{ \ln\left(\frac{k_B T}{\hbar\omega}\right) + 1 \right\}$$

$$Q_N(\beta) = \left[Q_1(\beta)\right]^N$$

$$Q_1(\beta) = \left(2\sinh\left(\frac{1}{2}\frac{\hbar\omega}{k_B T}\right)\right)^{-1}$$

$$Q_N(\beta) = \left[2\sinh\left(\frac{1}{2}\frac{\hbar\omega}{k_B T}\right)\right]^{-N}$$

$$\boxed{Q_N(\beta) = e^{-\frac{N}{2}\frac{\hbar\omega}{k_B T}}\left[1 - e^{-\frac{\hbar\omega}{k_B T}}\right]^{-N}}$$

binomialnu zapisati

$$(1+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k \qquad \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$(1-x)^n = \sum_{k=0}^{n}\binom{n}{k}(-x)^k \qquad \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

važno $n < 0$!!

$$\left(\frac{1}{1+x}\right)^n = \frac{1}{(1+x)^n} = 1 - nx + \frac{(-n)(-n-1)}{2!}x^2 + \frac{(-n)(-n-1)(-n-2)}{3!}x^3 + \dots$$

$$= \sum_{k=0}^{\infty}\binom{n+k-1}{k}(-1)^k x^k \qquad -(n)[-(n+1)][-(n+2)]$$

$$\left(\frac{1}{1-x}\right)^n = \sum_{k=0}^{\infty}\binom{n+k-1}{k}\underbrace{(-1)^k(-1)^k}_{(-1)^{2k} = 1}x^k = \sum_{k=0}^{\infty}\binom{n+k-1}{k}x^k$$

$$x \equiv e^{-\frac{\hbar\omega}{k_B T}}$$

https://brilliant.org/wiki/negative-binomial-theorem

$$Q_N(\beta) = \sum_{R=0}^{\infty} \binom{N+R-1}{R} e^{-\frac{1}{k_B T}\left(\frac{1}{2}N\hbar\omega + R\hbar\omega\right)}$$

$$Q_N(\beta) = \int_0^{\infty} g(E)\, e^{-\frac{E}{k_B T}}\, dE$$

definizio(a) [illegible]

alderaketa

$$g(E) = \underbrace{\sum_{R=0}^{\infty} \binom{N+R-1}{R} \delta\left(E - \left\{R + \frac{1}{2}N\right\}\hbar\omega\right)}_{\text{interpretazioa}}$$

irudia

$$\binom{N+R-1}{R} = \frac{(R+N-1)!}{R!\,(N-1)!} \equiv \Omega(R)$$

Mikrokanonikoa

$$S = k_B \ln \Omega(R) \Rightarrow S = k_B \ln \frac{(R+N-1)!}{R!\,(N-1)!}$$

$$N \ggg 1 \Rightarrow N \simeq N-1$$

+ SH.

$$\boxed{S \cong k_B \left\{ (N+R)\ln(N+R) - R\ln R - N\ln N \right\}}$$

+ fisika

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N \Rightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial R}\right)_N \left(\frac{\partial R}{\partial E}\right)_N$$

$$E = \frac{1}{2}N\hbar\omega + R\hbar\omega \Rightarrow R = \left(E - \frac{1}{2}N\hbar\omega\right)/\hbar\omega \Rightarrow \left(\frac{\partial R}{\partial E}\right) = \frac{1}{\hbar\omega}$$

$$\left(\frac{\partial S}{\partial R}\right) = k_B \left\{ \ln(N+R) + \cancel{(N+R)\frac{1}{N+R}} - \ln R - \cancel{R \cdot \frac{1}{R}} \right\}$$

$$= k_B \ln\left(\frac{N+R}{R}\right)$$

$$\frac{1}{T} = \frac{1}{\hbar\omega} \cdot k_B \ln\left(\frac{N + \left(E - \frac{1}{2}N\hbar\omega\right)\frac{1}{\hbar\omega}}{E - \frac{1}{2}N\hbar\omega}\right)$$

$$\boxed{\frac{1}{T} = \frac{1}{\hbar\omega} \cdot k_B \ln\left(\frac{E + \frac{1}{2}N\hbar\omega}{E - \frac{1}{2}N\hbar\omega}\right)}$$

$$\Downarrow$$

$$e^{\frac{\hbar\omega}{k_BT}} = \frac{E + \frac{1}{2}N\hbar\omega}{E - \frac{1}{2}N\hbar\omega} \rightarrow \left(E - \frac{1}{2}\hbar\omega N\right)e^{\frac{\hbar\omega}{k_BT}} = E + \frac{1}{2}\hbar\omega N$$

$$E\left(e^{\frac{\hbar\omega}{k_BT}} - 1\right) = \frac{1}{2}\hbar\omega N\left(1 + e^{\frac{\hbar\omega}{k_BT}}\right)$$

$$E = \frac{1}{2}\hbar\omega N \frac{\left(1 + e^{\frac{\hbar\omega}{k_BT}}\right)}{\left(e^{\frac{\hbar\omega}{k_BT}} - 1\right)}$$

$$\boxed{\frac{E}{N} = \frac{1}{2}\hbar\omega \frac{\left(e^{\frac{\hbar\omega}{k_BT}} + 1\right)}{\left(e^{\frac{\hbar\omega}{k_BT}} - 1\right)}}$$

Kanonikoki lortutako emaitza

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limite klasikoa!

$$\underbrace{\frac{E}{N}}_{} \ggg \hbar\omega$$

osziladoreko batezbesteko energia

$$R = \left(E - \frac{1}{2}N\hbar\omega\right)\frac{1}{\hbar\omega} \quad \text{definizioa}$$

$$R = \frac{1}{\hbar\omega} E\left(1 - \frac{\hbar\omega}{(E/N)}\right) \approx R \simeq \frac{E}{\hbar\omega}$$

$$\Downarrow$$

$$R \gg N$$

$$+ \quad \frac{(R+N-1)!}{R!\,(N-1)!} = \frac{\underbrace{(R+N-1)}_{\sim R}\underbrace{(R+N-2)}_{\sim R} \cdots \underbrace{(R+1)}_{\sim R}}{(N-1)!} \approx \frac{R^{N-1}}{(N-1)!} \quad \text{eta } R = \frac{E}{\hbar\omega}$$

$$S = k_B \ln\frac{R^{N-1}}{(N-1)!} \Rightarrow S = k_B \ln\frac{(E/\hbar\omega)^{N-1}}{(N-1)!} \Rightarrow S = k_B \ln\left[\frac{(E/\hbar\omega)^N}{N!}\right]$$

$$N - 1 \simeq N$$

$$+ SH$$

$$S = k_B \left\{ N \ln \frac{E}{\hbar\omega} - N \ln N + N \right\}$$

$$k_B \left\{ N \ln \frac{E}{N\hbar\omega} + N \right\}$$

$$\boxed{S = k_B N \left\{ \ln \frac{E}{N\hbar\omega} + 1 \right\}} \longrightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N$$