and the fractional fluctuation in $2s$ is defined as

$$\mathscr{F} \equiv \frac{\langle (2s)^2 \rangle^{1/2}}{N} = \frac{1}{\sqrt{N}}. \tag{45}$$

The larger N is, the smaller is the fractional fluctuation. This means that the central peak of the distribution function becomes relatively more sharply defined as the size of the system increases, the size being measured by the number of sites N. For 10^{20} particles, $\mathscr{F} = 10^{-10}$, which is very small.

Energy of the Binary Magnetic System

The thermal properties of the model system become physically relevant when the elementary magnets are placed in a magnetic field, for then the energies of the different states are no longer all equal. If the energy of the system is specified, then only the states having this energy may occur. The energy of interaction of a single magnetic moment $\mathbf{m}$ with a fixed external magnetic field $\mathbf{B}$ is

$$U = -\mathbf{m} \cdot \mathbf{B}. \tag{46}$$

This is the potential energy of the magnet $\mathbf{m}$ in the field $\mathbf{B}$.

For the model system of N elementary magnets, each with two allowed orientations in a uniform magnetic field $\mathbf{B}$, the total potential energy U is

$$U = \sum_{i=1}^{N} U_i = -\mathbf{B} \cdot \sum_{i=1}^{N} \mathbf{m}_i = -2smB = -MB\,, \tag{47}$$

using the expression M for the total magnetic moment $2sm$. In this example the spectrum of values of the energy U is discrete. We shall see later that a continuous or quasi-continuous spectrum will create no difficulty. Furthermore, the spacing between adjacent energy levels of this model is constant, as in Figure 1.10. Constant spacing is a special feature of the particular model, but this feature will not restrict the generality of the argument that is developed in the following sections.

The value of the energy for moments that interact only with the external magnetic field is completely determined by the value of s. This functional dependence is indicated by writing $U(s)$. Reversing a single moment lowers $2s$ by -2, lowers the total magnetic moment by $-2m$, and raises the energy by $2mB$. The energy difference between adjacent levels is denoted by $\Delta\varepsilon$, where

$$\Delta\varepsilon = U(s) - U(s+1) = 2mB. \tag{48}$$