2^N, where $N = 10^{20}$, it is a simplification to look at the logarithm of the number. We take the logarithm of both sides of (15) to obtain

$$\log g(N,s) = \log N! - \log(\tfrac{1}{2}N + s)! - \log(\tfrac{1}{2}N - s)! \ , \tag{22}$$

by virtue of the characteristic property of the logarithm of a product:

$$\log xy = \log x + \log y; \qquad \log(x/y) = \log x - \log y. \tag{23}$$

With the notation

$$N_\uparrow = \tfrac{1}{2}N + s; \qquad N_\downarrow = \tfrac{1}{2}N - s \tag{24}$$

for the number of magnets up and down, (22) appears as

$$\log g(N,s) = \log N! - \log N_\uparrow! - \log N_\downarrow!. \tag{25}$$

We evaluate the logarithm of $N!$ in (25) by use of the **Stirling approximation**, according to which

$$N! \simeq (2\pi N)^{1/2} N^N \exp[-N + 1/(12N) + \cdots] \ , \tag{26}$$

for $N \gg 1$. This result is derived in Appendix A. For sufficiently large N, the terms $1/(12N) + \cdots$ in the argument may be neglected in comparison with N. We take the logarithm of both sides of (26) to obtain

$$\log N! \cong \tfrac{1}{2}\log 2\pi + (N + \tfrac{1}{2})\log N - N. \tag{27}$$

Similarly

$$\log N_\uparrow! \cong \tfrac{1}{2}\log 2\pi + (N_\uparrow + \tfrac{1}{2})\log N_\uparrow - N_\uparrow; \tag{28}$$

$$\log N_\downarrow! \cong \tfrac{1}{2}\log 2\pi + (N_\downarrow + \tfrac{1}{2})\log N_\downarrow - N_\downarrow. \tag{29}$$

After rearrangement of (27),

$$\log N! \cong \tfrac{1}{2}\log(2\pi/N) + (N_\uparrow + \tfrac{1}{2} + N_\downarrow + \tfrac{1}{2})\log N - (N_\uparrow + N_\downarrow) \ , \tag{30}$$

where we have used $N = N_\uparrow + N_\downarrow$. We subtract (28) and (29) from (30) to obtain for (25):

$$\log g \cong \tfrac{1}{2}\log(1/2\pi N) - (N_\uparrow + \tfrac{1}{2})\log(N_\uparrow/N) - (N_\downarrow + \tfrac{1}{2})\log(N_\downarrow/N). \tag{31}$$