The probability is the ratio of two exponential factors, each of the form $\exp[(N\mu - \varepsilon)/\tau]$. A term of this form is called a **Gibbs factor**. The Gibbs factor is proportional to the probability that the system is in a state s of energy ε_s and number of particles N. The result was first given by J. W. Gibbs, who referred to it as the grand canonical distribution.

The sum of Gibbs factors, taken over all states of the system for all numbers of particles, is the normalizing factor that converts relative probabilities to absolute probabilities:

$$\boxed{\mathcal{Z}(\mu,\tau) = \sum_{N=0}^{\infty} \sum_{s(N)} \exp[(N\mu - \varepsilon_{s(N)})/\tau] = \sum_{\mathrm{ASN}} \exp[(N\mu - \varepsilon_{s(N)})/\tau].} \tag{53}$$

This is called the **Gibbs sum**, or the **grand sum**, or the grand partition function. The sum is to be carried out over all states of the system for all numbers of particles: this defines the abbreviation ASN. We have written ε_s as $\varepsilon_{s(N)}$ to emphasize the dependence of the state on the number of particles N. That is, $\varepsilon_{s(N)}$ is the energy of the state $s(N)$ of the exact N-particle hamiltonian. The term $N = 0$ must be included; if we assign its energy as zero, then the first term in $\mathcal{Z}$ will be 1.

The absolute probability that the system will be found in a state N_1, ε_1 is given by the Gibbs factor divided by the Gibbs sum:

$$P(N_1,\varepsilon_1) = \frac{\exp[(N_1\mu - \varepsilon_1)/\tau]}{\mathcal{Z}}. \tag{54}$$

This applies to a system that is at temperature τ and chemical potential μ. The ratio of any two P's is consistent with our central result (52) for the Gibbs factors. Thus (52) gives the correct relative probabilities for the states N_1, ε_1 and N_2, ε_2. The sum of the probabilities of all states for all numbers of particles of the system is unity:

$$\sum_N \sum_s P(N,\varepsilon_s) = \sum_{\mathrm{ASN}} P(N,\varepsilon_s) = \frac{\sum_{\mathrm{ASN}} \exp[(N\mu - \varepsilon_{s(N)})/\tau]}{\mathcal{Z}} = \frac{\mathcal{Z}}{\mathcal{Z}} = 1\ , \tag{55}$$

by the definition of $\mathcal{Z}$. Thus (54) gives the correct absolute probability.*

---

* Readers interested in probability theory will find Appendix C on the Poisson distribution to be particularly helpful. The method used there to derive the Poisson distribution depends on the Gibbs sum. See also Problem (6.13).