30.3 The Bose gas

For the **Bose gas** (a gas composed of bosons), we can use our expressions for N and U in eqns 30.15 and 30.16 to give

$$N = \frac{(2S+1)V}{\lambda_{\rm th}^3} \mathrm{Li}_{3/2}(z) \tag{30.43}$$

and

$$U = \frac{3}{2} N k_{\rm B} T \frac{\mathrm{Li}_{5/2}(z)}{\mathrm{Li}_{3/2}(z)}. \tag{30.44}$$

Example 30.6

Evaluate eqns 30.43 and 30.44 for the case $\mu = 0$.

Solution:

If $\mu = 0$ then $z = 1$. Now $\mathrm{Li}_n(1) = \zeta(n)$ where $\zeta(n)$ is the Riemann zeta function. Therefore

$$N = \frac{(2S+1)V}{\lambda_{\rm th}^3} \zeta\left(\frac{3}{2}\right) \tag{30.45}$$

and

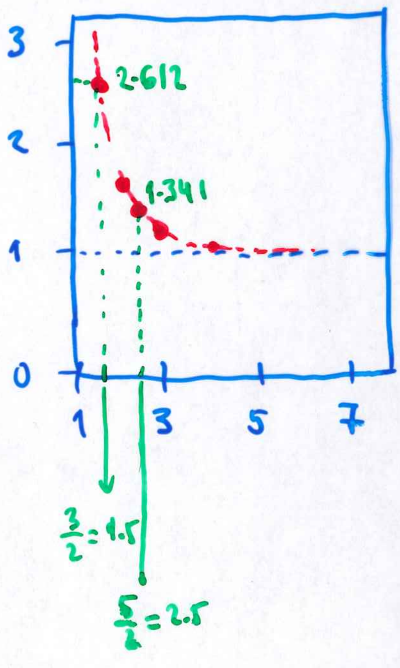
$$U = \frac{3}{2} N k_{\rm B} T \frac{\zeta(\frac{5}{2})}{\zeta(\frac{3}{2})}. \tag{30.46}$$

The numerical values are $\zeta(\frac{3}{2}) = 2.612$, $\zeta(\frac{5}{2}) = 1.341$, and hence we have that $\zeta(\frac{5}{2})/\zeta(\frac{3}{2}) = 0.513$.

$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$

$s > 1$ KONBERGENTEA

$s = 1$ ez!!



Note that these results will not apply to photons because we have assumed at the beginning that $E = \hbar^2 k^2/2m$, whereas for a photon $E = \hbar k c$. This is worked through in the following example.

Example 30.7

Rederive the equation for U for a gas of photons using the formalism of this chapter.

Solution:

The density of states is $g(k)\,\mathrm{d}k = (2S+1)Vk^2\,\mathrm{d}k/(2\pi^2)$. A photon has a spin of 1, but the 0 state is not allowed, so the spin degeneracy factor $(2S+1)$ is in this case only 2. Using $E = \hbar k c$ we arrive at

$$g(E)\,\mathrm{d}E = \frac{V}{\pi^2 \hbar^3 c^3} E^2\,\mathrm{d}E, \tag{30.47}$$

and hence

$$U = \int_0^\infty E\, g(E)\,\mathrm{d}E = \frac{V}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{E^3\,\mathrm{d}E}{z^{-1}\mathrm{e}^{\beta E} - 1}, \tag{30.48}$$