s	$U(s)/mB$	$g(s)$	$\log g(s)$
-5	$+10$	1	0
-4	$+8$	10	2.30
-3	$+6$	45	3.81
-2	$+4$	120	4.79
-1	$+2$	210	5.35
0	0	252	5.53
$+1$	-2	210	5.35
$+2$	-4	120	4.79
$+3$	-6	45	3.81
$+4$	-8	10	2.30
$+5$	-10	1	0

Figure 1.10 Energy levels of the model system of 10 magnetic moments m in a magnetic field B. The levels are labeled by their s values, where $2s$ is the spin excess and $\frac{1}{2}N + s = 5 + s$ is the number of up spins. The energies $U(s)$ and multiplicities $g(s)$ are shown. For this problem the energy levels are spaced equally, with separation $\Delta\varepsilon = 2mB$ between adjacent levels.

Example: Multiplicity function for harmonic oscillators. The problem of the binary model system is the simplest problem for which an exact solution for the multiplicity function is known. Another exactly solvable problem is the harmonic oscillator, for which the solution was originally given by Max Planck. The original derivation is often felt to be not entirely simple. The beginning student need not worry about this derivation. The modern way to do the problem is given in Chapter 4 and is simple.

The quantum states of a harmonic oscillator have the energy eigenvalues

$$\varepsilon_s = s\hbar\omega \; , \tag{49}$$

where the quantum number s is a positive integer or zero, and ω is the angular frequency of the oscillator. The number of states is infinite, and the multiplicity of each is one. Now consider a system of N such oscillators, all of the same frequency. We want to find the number of ways in which a given total excitation energy

$$\varepsilon = \sum_{i=1}^{N} s_i\hbar\omega = n\hbar\omega \tag{50}$$