cancel. The overall entropy change $d\sigma$ will be zero. If we denote these interdependent values of dU and dV by $(\delta U)_\sigma$ and $(\delta V)_\sigma$, the entropy change will be zero:

$$0 = \left(\frac{\partial\sigma}{\partial U}\right)_V (\delta U)_\sigma + \left(\frac{\partial\sigma}{\partial V}\right)_U (\delta V)_\sigma. \tag{28}$$

After division by $(\delta V)_\sigma$,

$$0 = \left(\frac{\partial\sigma}{\partial U}\right)_V \frac{(\delta U)_\sigma}{(\delta V)_\sigma} + \left(\frac{\partial\sigma}{\partial V}\right)_U. \tag{29}$$

But the ratio $(\delta U)_\sigma/(\delta V)_\sigma$ is the partial derivative of U with respect to V at constant σ:

$$(\delta U)_\sigma/(\delta V)_\sigma \equiv (\partial U/\partial V)_\sigma. \tag{30}$$

With this and the definition $1/\tau \equiv (\partial\sigma/\partial U)_V$, Eq. (29) becomes

$$\left(\frac{\partial U}{\partial V}\right)_\sigma = -\tau\left(\frac{\partial\sigma}{\partial V}\right)_U. \tag{31}$$

By (26) the left-hand side of (31) is equal to $-p$, whence

$$\boxed{p = \tau\left(\frac{\partial\sigma}{\partial V}\right)_U.} \tag{32}$$

Thermodynamic Identity

Consider again the differential (27) of the entropy; substitute the new result for the pressure and the definition of τ to obtain

$$d\sigma = \frac{1}{\tau}dU + \frac{p}{\tau}dV, \tag{33}$$

or

$$\boxed{\tau d\sigma = dU + pdV.} \tag{34a}$$