$$\lambda_T \equiv \frac{h}{\sqrt{2\pi m (K_B T)}} \to \lambda_T^3 = \left[\frac{h^2}{2\pi m (K_B T)}\right]^{3/2}$$

$$n_Q \equiv \frac{1}{\lambda_T^3} \Rightarrow \quad n_Q = \left(\frac{2\pi m (K_B T)}{h^2}\right)^{3/2} \to \left(\frac{\cancel{2\pi} m (K_B T)}{\hbar^2 (2\pi)^{\cancel{2}}}\right)^{3/2}$$

$$\left(\frac{m\tau}{2\pi\hbar^2}\right)^{3/2}$$

It is a fundamental result of quantum theory that all particles, including atoms and molecules, are either fermions or bosons. They behave alike in the classical regime in which the concentration is small in comparison with the quantum concentration,

$$\boxed{n \ll n_Q \equiv (M\tau/2\pi\hbar^2)^{3/2}.} \tag{1}$$

Whenever $\boxed{n \geq n_Q}$ the gas is said to be in the quantum regime and is called a **quantum gas**. The difference in physical properties between a quantum gas of fermions and one of bosons is dramatic, and both are unlike a gas in the classical regime. A Fermi gas or liquid has a high kinetic energy, low heat capacity, low magnetic susceptibility, low interparticle collision rate, and exerts a high pressure on the container, even at absolute zero. A Bose gas or liquid has a high concentration of particles in the ground orbital, and these particles—called the Bose condensate—may act as a superfluid, with practically zero viscosity.

For many systems the concentration n is fixed, and the temperature is the important variable. The quantum regime obtains when the temperature is below

$$\boxed{\tau_0 \equiv (2\pi\hbar^2/M)n^{2/3},} \tag{2}$$

defined by the condition $n = n_Q$. A gas in the quantum regime with $\tau \ll \tau_0$ is often said to be a **degenerate gas***.

It was realized by Nernst that the entropy of a classical gas diverges as $\log\tau$ as $\tau \to 0$. Quantum theory removes the difficulty: both fermion and boson gases approach a unique ground state as $\tau \to 0$, so that the entropy goes to zero. We say that the entropy is squeezed out on cooling a quantum gas (see Problems 3 and 8).

In the classical regime (Chapter 6) the thermal average number of particles in an orbital of energy ε is given by

$$f(\varepsilon) \simeq \exp[(\mu - \varepsilon)/\tau]. \tag{3}$$

* Here we have the second distinct usage of the word "degenerate" in statistical physics. The first usage was introduced in Chapter 1, where we called an energy level degenerate if more than one state has the same energy.

$$\mu = -K_B T \ln \frac{V}{N\lambda_T^3} \to \frac{\mu}{K_B T} = -\ln \frac{V}{N}\cdot\frac{1}{\lambda_T^3}$$

$$\Rightarrow \frac{\mu}{K_B T} = \ln \underbrace{\frac{N}{V}}_{\equiv\, n}\, \underbrace{\lambda_T^3}_{\equiv\, \frac{1}{n_Q}}$$

$$\frac{\mu}{\tau} = \ln\left(\frac{n}{n_Q}\right)$$