which is called the **ideal gas law**. In conventional units,

$$pV = Nk_BT. \tag{74}$$

The entropy follows from (49):

$$\sigma = -(\partial F/\partial\tau)_V = N\log[(M\tau/2\pi\hbar^2)^{3/2}V] + \tfrac{3}{2}N - N\log N + N \ , \tag{75}$$

or

$$\boxed{\sigma = N[\log(n_Q/n) + \tfrac{5}{2}] \ ,} \tag{76}$$

with the concentration $n \equiv N/V$. This result is known as the **Sackur-Tetrode equation** for the entropy of a monatomic ideal gas. It agrees with experiment. The result involves $\hbar$ through the term n_Q, so even for the classical ideal gas the entropy involves a quantum concept. We shall derive these results again in Chapter 6 by a direct method that does not explicitly involve the $N!$ or identical particle argument. The energy (69) also follows from $U = F + \tau\sigma$; with use of (71) and (76) we have $U = \frac{3}{2}N\tau$.

Example: Equipartition of energy. The energy $U = \frac{3}{2}N\tau$ from (69) is ascribed to a contribution $\frac{1}{2}\tau$ from each "degree of freedom" of each particle, where the number of degrees of freedom is the number of dimensions of the space in which the atoms move: 3 in this example. In the classical form of statistical mechanics, the partition function contains the kinetic energy of the particles in an integral over the momentum components p_x, p_y, p_z. For one free particle

$$Z_1 \propto \iiint \exp[-({p_x}^2 + {p_y}^2 + {p_z}^2)/2M\tau]\,dp_x\,dp_y\,dp_z \ , \tag{77}$$

a result similar to (61). The limits of integration are $\pm\infty$ for each component. The thermal average energy may be calculated by use of (12) and is equal to $\frac{3}{2}\tau$.

The result is generalized in the classical theory. Whenever the hamiltonian of the system is homogeneous of degree 2 in a canonical momentum component, the classical limit of the thermal average kinetic energy associated with that momentum will be $\frac{1}{2}\tau$. Further, if the hamiltonian is homogeneous of degree 2 in a position coordinate component, the thermal average potential energy associated with that coordinate will also be $\frac{1}{2}\tau$. The result thus