can be distributed among the oscillators. That is, we want the multiplicity function $g(N,n)$ for the N oscillators. The oscillator multiplicity function is not the same as the spin multiplicity function found earlier.

We begin the analysis by going back to the multiplicity function for a single oscillator, for which $g(1,n) = 1$ for all values of the quantum number s, here identical to n. To solve the problem of (53) below, we need a function to represent or generate the series

$$\sum_{n=0}^{\infty} g(1,n)t^n = \sum_{n=0}^{\infty} t^n. \tag{51}$$

All $\sum$ run from 0 to ∞. Here t is just a temporary tool that will help us find the result (53), but t does not appear in the final result. The answer is

$$\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n , \tag{52}$$

provided we assume $|t| < 1$. For the problem of N oscillators, the generating function is

$$\left(\frac{1}{1-t}\right)^N = \left(\sum_{s=0}^{\infty} t^s\right)^N = \sum_{n=0}^{\infty} g(N,n)t^n , \tag{53}$$

because the number of ways a term t^n can appear in the N-fold product is precisely the number of ordered ways in which the integer n can be formed as the sum of N non-negative integers.

We observe that

$$\begin{aligned} g(N,n) &= \lim_{t\to 0} \frac{1}{n!}\left(\frac{d}{dt}\right)^n \sum_{s=0}^{\infty} g(N,s)t^s \\ &= \lim_{t\to 0} \frac{1}{n!}\left(\frac{d}{dt}\right)^n (1-t)^{-N} \\ &= \frac{1}{n!} N(N+1)(N+2)\cdots(N+n-1). \end{aligned} \tag{54}$$

Thus for the system of oscillators,

$$g(N,n) = \frac{(N+n-1)!}{n!\,(N-1)!}. \tag{55}$$

This result will be needed in solving a problem in the next chapter.