The partition function is the sum over the states (59):

$$Z_1 = \sum_{n_x}\sum_{n_y}\sum_{n_z} \exp[-\hbar^2\pi^2(n_x{}^2 + n_y{}^2 + n_z{}^2)/2ML^2\tau]. \qquad (60)$$

Provided the spacing of adjacent energy values is small in comparison with τ, we may replace the summations by integrations:

$$Z_1 = \int_0^\infty dn_x \int_0^\infty dn_y \int_0^\infty dn_z \exp[-\alpha^2(n_x{}^2 + n_y{}^2 + n_z{}^2)]. \qquad (61)$$

The notation $\alpha^2 \equiv \hbar^2\pi^2/2ML^2\tau$ is introduced for convenience. The exponential may be written as the product of three factors

$$\exp(-\alpha^2 n_x{}^2)\exp(-\alpha^2 n_y{}^2)\exp(-\alpha^2 n_z{}^2) ,$$

so that

$$Z_1 = \left(\int_0^\infty dn_x \exp(-\alpha^2 n_x{}^2)\right)^3 = (1/\alpha)^3\left(\int_0^\infty dx \exp(-x^2)\right)^3 = \pi^{3/2}/8\alpha^3 ,$$

whence

$$Z_1 = \frac{V}{(2\pi\hbar^2/M\tau)^{3/2}} = n_Q V = n_Q/n , \qquad (62)$$

in terms of the concentration $n = 1/V$.

Here

$$\boxed{n_Q \equiv (M\tau/2\pi\hbar^2)^{3/2}} \qquad (63)$$

is called the **quantum concentration**. It is the concentration associated with one atom in a cube of side equal to the thermal average de Broglie wavelength, which is a length roughly equal to $\hbar/M\langle v\rangle \sim \hbar/(M\tau)^{1/2}$. Here $\langle v\rangle$ is a thermal average velocity. This concentration will keep turning up in the thermal physics of gases, in semiconductor theory, and in the theory of chemical reactions.

For helium at atmospheric pressure at room temperature, $n \approx 2.5 \times 10^{19}\ \mathrm{cm}^{-3}$ and $n_Q \approx 0.8 \times 10^{25}\ \mathrm{cm}^{-3}$. Thus, $n/n_Q \approx 3 \times 10^{-6}$, which is very