



Fig. 30.2 (a) The Fermi function  $f(E)$  defined by eqn 29.26. The thick line is for  $T = 0$ . The step function is smoothed out as the temperature is increased (shown as thinner lines). The temperatures shown are  $T = 0$ ,  $T = 0.01\mu/k_B$ ,  $T = 0.05\mu/k_B$  and  $T = 0.1\mu/k_B$ . (b) The density of states  $g(E)$  for a non-interacting fermion gas in three dimensions is proportional to  $E^{1/2}$ . (c)  $f(E)g(E)$  for the same temperatures as in (a).

At  $T = 0$ , the distribution function  $f(E)$  is a Heaviside step function, taking the value 1 for  $E < \mu$  and 0 for  $E > \mu$ . This step is smoothed out as the temperature  $T$  increases, as shown in Fig. 30.2(a). The density of states  $g(E)$  for a non-interacting fermion gas in three dimensions is proportional to  $E^{1/2}$  (as shown in eqn 30.4) and this is plotted in Fig. 30.2(b). The product of  $f(E)g(E)$  gives the actual number distribution of fermions, and this is shown in Fig. 30.2(c). The sharp cutoff you would expect at  $T = 0$  is smoothed over an energy scale  $k_B T$  around the chemical potential  $\mu$ .

The electrons in a metal can be treated as a non-interacting gas of fermions. Using the number density  $n$  of electrons in a metal, one can calculate the Fermi energy using eqn 30.29, and some example results are shown in Table 30.1. The Fermi energies are all several eV; converting each number into a temperature, the so-called **Fermi temperature**  $T_F = E_F/k_B$ , yields values of several tens of thousands of Kelvin. Thus the Fermi energy is a large energy scale, and hence for most metals the Fermi function is close to a step function, at pretty much all temperatures below their melting temperature. In this case, the electrons in a metal are said to be in the **degenerate limit**.

The pressure of these electrons is given (by using eqns 22.49 and 30.17) as

$$p = \frac{2U}{3V}, \quad (30.30)$$

$$\mu = \mu(T) \quad 30.40$$

$$\mu(T) = \mu(0) \left[ 1 - \frac{1}{3} \frac{\pi^2}{4} \left( \frac{k_B T}{\mu(0)} \right)^2 + \dots \right]$$

$$\mu(0) \equiv E_F$$

$$\mu(T) = E_F \left[ 1 - \frac{1}{3} \frac{\pi^2}{4} \left( \frac{k_B T}{E_F} \right)^2 + \dots \right]$$

$$E_F = 100 (k_B T) \Rightarrow \frac{k_B T}{E_F} = 10^{-2}$$



$$\mu(T) = E_F \Rightarrow \mu \neq \mu(T)$$