

1 OCCUPANCY

2 GIBBS SUM

3 DISTRIBUTION FUNCTION $f(\epsilon, \mu, \tau)$

$$\sum_{N=0}^{\infty} \lambda^N e^{-\epsilon N / \tau} = 1 + \lambda e^{-\epsilon / \tau} + \lambda^2 e^{-2\epsilon / \tau} + \dots$$

The two different occupancy rules give rise to two different Gibbs sums for each orbital: there is a boson sum over all integral values of the orbital occupancy N , and there is a fermion sum in which $N = 0$ or $N = 1$ only. Different Gibbs sums lead to different quantum distribution functions $f(\epsilon, \tau, \mu)$ for the thermal average occupancy. If conditions are such that $f \ll 1$, it will not matter whether the occupancies $N = 2, 3, \dots$ are excluded or are allowed. Thus when $f \ll 1$ the fermion and boson distribution functions must be similar. This limit in which the orbital occupancy is small in comparison with unity is called the classical regime.

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We now treat the Fermi-Dirac distribution function for the thermal average occupancy of an orbital by fermions and the Bose-Einstein distribution function for the thermal average occupancy of an orbital by bosons. We show the equivalence of the two functions in the limit of low occupancy, and we go on to treat the properties of a gas in this limit. In Chapter 7 we treat the properties of fermion and boson gases in the opposite limit, where the nature of the particles is absolutely crucial for the properties of the gas.

FERMI-DIRAC DISTRIBUTION FUNCTION

We consider a system composed of a single orbital that may be occupied by a fermion. The system is placed in thermal and diffusive contact with a reservoir, as in Figures 6.1 and 6.2. A real system may consist of a large number N_0 of fermions, but it is very helpful to focus on one orbital and call it the system. All other orbitals of the real system are thought of as the reservoir. Our problem is to find the thermal average occupancy of the orbital thus singled out. An orbital can be occupied by zero or by one fermion. No other occupancy is allowed by the Pauli exclusion principle. The energy of the system will be taken to be zero if the orbital is unoccupied. The energy is ϵ if the orbital is occupied by one fermion.

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2 The Gibbs sum now is simple: from the definition in Chapter 5 we have

$$\mathcal{Z} = 1 + \lambda \exp(-\epsilon/\tau). \quad (1)$$

The term 1 comes from the configuration with occupancy $N = 0$ and energy $\epsilon = 0$. The term $\lambda \exp(-\epsilon/\tau)$ comes when the orbital is occupied by one fermion, so that $N = 1$ and the energy is ϵ . The thermal average value of the occupancy of the orbital is the ratio of the term in the Gibbs sum with $N = 1$ to the entire Gibbs sum:

$$\langle N(\epsilon) \rangle = \frac{\lambda \exp(-\epsilon/\tau)}{1 + \lambda \exp(-\epsilon/\tau)} = \frac{1}{\lambda^{-1} \exp(\epsilon/\tau) + 1}. \quad (2)$$

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