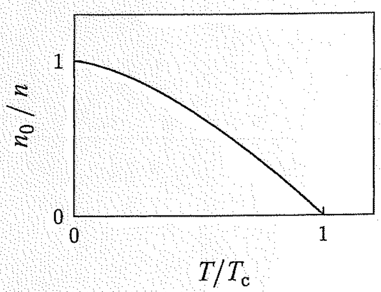

Fig. 30.4 The number of particles in the ground state as a function of temperature, after eqn 30.60.

Hence

$$\frac{n_0}{n} = \frac{n - n_1}{n} = 1 - \left(\frac{T}{T_c}\right)^{3/2}. \tag{30.60}$$

This function is plotted in Fig. 30.4 and shows how the number of particles in the ground state grows as the temperature is cooled below T_c. This macroscopic occupation of the ground state is known as **Bose–Einstein condensation**.[8] Note that this transition is not driven by interactions between particles (as we had for the liquid-gas transition); we have so far only considered non-interacting particles; the transition is driven purely by the requirements of exchange symmetry on the quantum statistics of the bosons.

[8] This is often abbreviated to BEC.

The term 'condensation' often implies a condensation in space, as when liquid water condenses on a cold window in a steamy bathroom. However, for Bose–Einstein condensation it is a condensation in k-space, with a macroscopic occupation of the lowest energy state occurring below T_c.

Example 30.8

Find the internal energy $U(T)$ at temperature T for the Bose gas.

Solution:

The internal energy of the system only depends on the excited states, since the macroscopically occupied ground state has zero energy. Since $z = 1$ for $T \leq T_c$, we have that

$$\begin{aligned} U &= \frac{3}{2} N_1 k_B T \frac{\zeta(\frac{5}{2})}{\zeta(\frac{3}{2})} \\ &= \frac{3}{2} N k_B T \frac{\zeta(\frac{5}{2})}{\zeta(\frac{3}{2})} \left(\frac{T}{T_c}\right)^{3/2} \\ &= 0.77 N k_B T_c \left(\frac{T}{T_c}\right)^{5/2}. \end{aligned} \tag{30.61}$$

For $T > T_c$ we have (from eqn 30.46)

$$U = \frac{3}{2} N k_B T \frac{\mathrm{Li}_{5/2}(z)}{\mathrm{Li}_{3/2}(z)}. \tag{30.62}$$

This example gives the high–temperature results as a function of the fugacity, but z is temperature-dependent. For a system with a fixed number N of bosons, we can extract z via $N/V = (2S+1)\mathrm{Li}_{3/2}(z)/\lambda_{\mathrm{th}}^3$ and equating this with eqn 30.59 yields

$$\frac{T}{T_c} = \left[\frac{\zeta(\frac{3}{2})}{\mathrm{Li}_{3/2}(z)}\right], \tag{30.63}$$