The Bose-Einstein distribution function when written for the orbital at $\varepsilon = 0$ is

$$\boxed{N_0(\tau) = \frac{1}{\lambda^{-1} - 1},} \qquad \lambda = \lambda(\tau) \tag{67}$$

as in (54), where λ will depend on the temperature τ. The number of particles in all excited orbitals increases as $\tau^{3/2}$:

$$N_e(\tau) = \frac{V}{4\pi^2}\left(\frac{2M}{\hbar^2}\right)^{3/2} \int_0^\infty d\varepsilon \, \frac{\varepsilon^{1/2}}{\lambda^{-1}\exp(\varepsilon/\tau) - 1}.$$

or, with $x \equiv \varepsilon/\tau$,

$$\boxed{N_e(\tau) = \frac{V}{4\pi^2}\left(\frac{2M}{\hbar^2}\right)^{3/2} \tau^{3/2} \int_0^\infty dx \, \frac{x^{1/2}}{\lambda^{-1}e^x - 1}.} \tag{68}$$

Notice the factor $\tau^{3/2}$ which gives the temperature dependence of N_e.

At sufficiently low temperatures the number of particles in the ground state will be a very large number. Equation (67) tells us that λ must be very close to unity whenever $\boxed{N_0 \text{ is} \gg 1.}$ Then λ is very accurately constant, because a macroscopic value of N_0 forces λ to be close to unity. The condition for the validity of the calculation is that $N_0 \gg 1$, and it is not required that $N_e \ll N$. When $\varepsilon \approx \tau$ in the integrand, the value of the integrand is insensitive to small deviations of λ from 1, so that we can set $\lambda = 1$ in (68), although not in (67).

The value of the integral* in (68) is, when $\lambda = 1$,

$$\int_0^\infty dx \, \frac{x^{1/2}}{e^x - 1} = 1.306\pi^{1/2}. \tag{69}$$

* To evaluate the integral we write

$$\int_0^\infty dx \, \frac{x^{1/2}}{e^x - 1} = \int_0^\infty dx \, \frac{x^{1/2}e^{-x}}{1 - e^{-x}} = \sum_{s=1}^{\infty} \int_0^\infty dx \, x^{1/2}e^{-sx}$$
$$= \left(\sum_{s=1}^{\infty} s^{-3/2}\right) \int_0^\infty dy \, y^{1/2}e^{-y}.$$

The infinite sum is easily evaluated numerically to be 2.612. The integral may be transformed with $y = u^2$ to give

$$2\int_0^\infty du \, u^2 \exp(-u^2) = \tfrac{1}{2}\sqrt{\pi}.$$