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Example: Two spin systems in thermal contact. We investigate for the model spin system the sharpness of the product (7) near the maximum (8) as follows. We form the product of the multiplicity functions for $g_1(N_1,s_1)$ and $g_2(N_2,s_2)$, both of the form of (1.35):

$$g_1(N_1,s_1)g_2(N_2,s_2) = g_1(0)g_2(0)\exp\left(-\frac{2s_1^{\,2}}{N_1} - \frac{2s_2^{\,2}}{N_2}\right), \tag{9}$$

where $g_1(0)$ denotes $g_1(N_1,0)$ and $g_2(0)$ denotes $g_2(N_2,0)$. We replace s_2 by $s - s_1$:

$$g_1(N_1,s_1)g_2(N_2,s - s_1) = g_1(0)g_2(0)\exp\left(-\frac{2s_1^{\,2}}{N_1} - \frac{2(s - s_1)^2}{N_2}\right). \tag{10}$$

This product* gives the number of states accessible to the combined system when the spin excess of the combined system is $2s$, and the spin excess of the first system is $2s_1$.

We find the maximum value of (10) as a function of s_1 when the total spin excess $2s$ is held constant; that is, when the energy of the combined systems is constant. It is convenient to use the property that the maximum of $\log y(x)$ occurs at the same value of x as the maximum of $y(x)$. The calculation can be done either way. From (10),

$$\log g_1(N_1,s_1)g_2(N_2,s - s_1) = \log g_1(0)g_2(0) - \frac{2s_1^{\,2}}{N_1} - \frac{2(s - s_1)^2}{N_2}. \tag{11}$$

This quantity is an extremum when the first derivative with respect to s_1 is zero. An extremum may be a maximum, a minimum, or a point of inflection. The extremum is a maximum if the second derivative of the function is negative, so that the curve bends downward.

At the extremum the first derivative is

$$\frac{\partial}{\partial s_1}\{\log g_1(N_1,s_1)g_2(N_2,s - s_1)\} = -\frac{4s_1}{N_1} + \frac{4(s - s_1)}{N_2} = 0\ , \tag{12}$$

where N_1, N_2, and s are held constant as s_1 is varied. The second derivative $\partial^2/\partial s_1^{\,2}$ of Equation (11) is

$$-4\left(\frac{1}{N_1} + \frac{1}{N_2}\right)$$

* The product function of two Gaussian functions is always a Gaussian.