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ASKATASUN-GRADUA: transle

$\epsilon = \frac{p^2}{2m}$

$\vec{p} = \hbar \vec{k}$

$\vec{k} = \frac{\pi}{L}(n_x, n_y, n_z)$

where $g(E)$ is the density of states, which can be derived as follows. States in k-space are uniformly distributed, and so

$$\boxed{g(k)\,\mathrm{d}k = \frac{4\pi k^2\,\mathrm{d}k}{(2\pi/L)^3} \times (2S+1) = \frac{(2S+1)Vk^2\,\mathrm{d}k}{2\pi^2}}, \tag{30.4}$$

where $(2S+1)$ is the spin degeneracy factor and $V = L^3$ is the volume. Using $E = \hbar^2 k^2/2m$ we can transform this into

$$\boxed{g(E)\,\mathrm{d}E = \frac{(2S+1)VE^{1/2}\,\mathrm{d}E}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}}, \tag{30.5}$$

and hence

$$\Phi_{\mathrm{G}} = \mp k_{\mathrm{B}}T\frac{(2S+1)V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \ln(1 \pm \mathrm{e}^{-\beta(E-\mu)})\,E^{1/2}\,\mathrm{d}E, \tag{30.6}$$

which after integrating by parts yields

$$\boxed{\Phi_{\mathrm{G}} = -\frac{2}{3}\frac{(2S+1)V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{3/2}\,\mathrm{d}E}{\mathrm{e}^{-\beta(E-\mu)} \pm 1}}. \tag{30.7}$$

The grand potential evaluated in the previous example can be used to derive various thermodynamic functions for fermions and bosons.[3] Another way to get to the same result is to evaluate the mean occupation $n_{\boldsymbol{k}}$ of a state with wave vector $\boldsymbol{k}$, which is given by

$$n_{\boldsymbol{k}} = k_{\mathrm{B}}T\frac{\partial}{\partial \mu}\mathcal{Z}_{\boldsymbol{k}} = \frac{1}{\mathrm{e}^{\beta(E_{\boldsymbol{k}}-\mu)} \pm 1}, \tag{30.8}$$

$f(\epsilon)$ KITTEL

and then use this expression to derive directly quantities such as

$$N = \sum_{\boldsymbol{k}} n_{\boldsymbol{k}} = \int_0^\infty \frac{g(E)\,\mathrm{d}E}{\mathrm{e}^{\beta(E_{\boldsymbol{k}}-\mu)} \pm 1}, \tag{30.9}$$

and

$$U = \sum_{\boldsymbol{k}} n_{\boldsymbol{k}} E_{\boldsymbol{k}} = \int_0^\infty \frac{E\,g(E)\,\mathrm{d}E}{\mathrm{e}^{\beta(E_{\boldsymbol{k}}-\mu)} \pm 1}. \tag{30.10}$$

For reasons which will become more clear below, we will write $\mathrm{e}^{\beta\mu}$ as the **fugacity** z, i.e.

$$\boxed{z = \mathrm{e}^{\beta\mu}}. \tag{30.11}$$

These give expressions for N and U as follows:

$$N = \underbrace{\left[\frac{(2S+1)V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\right]}\underbrace{\int_0^\infty \frac{E^{1/2}\,\mathrm{d}E}{z^{-1}\mathrm{e}^{\beta E} \pm 1}} \tag{30.12}$$

and

$$U = \left[\frac{(2S+1)V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\right]\int_0^\infty \frac{E^{3/2}\,\mathrm{d}E}{z^{-1}\mathrm{e}^{\beta E} \pm 1}. \tag{30.13}$$

$\int_0^\infty \frac{E^n}{\frac{1}{z}e^{\beta E} \pm 1}\,dE \quad n\begin{cases} 1/2 \to 3/2 \\ 3/2 \to 5/2 \end{cases}$

[3] Note that in the derived expressions, the $\pm$ sign means $+$ for fermions and $-$ for bosons.