The largest term in the sum in (18) governs the properties of the total system in thermal equilibrium. For an extremum it is necessary that the differential* of $g(N,U)$ be zero for an infinitesimal exchange of energy:

$$dg = \left(\frac{\partial g_1}{\partial U_1}\right)_{N_1} g_2 dU_1 + g_1\left(\frac{\partial g_2}{\partial U_2}\right)_{N_2} dU_2 = 0; \qquad dU_1 + dU_2 = 0. \quad (19)$$

We divide by g_1g_2 and use the result $dU_2 = -dU_1$ to obtain the thermal equilibrium condition:

$$\frac{1}{g_1}\left(\frac{\partial g_1}{\partial U_1}\right)_{N_1} = \frac{1}{g_2}\left(\frac{\partial g_2}{\partial U_2}\right)_{N_2}, \quad (20a)$$

which we may write as

$$\left(\frac{\partial \log g_1}{\partial U_1}\right)_{N_1} = \left(\frac{\partial \log g_2}{\partial U_2}\right)_{N_2}. \quad (20b)$$

We define the quantity σ, called the **entropy**, by

$$\boxed{\sigma(N,U) \equiv \log g(N,U)\ ,} \quad (21)$$

where σ is the Greek letter sigma. We now write (20) in the final form

$$\boxed{\left(\frac{\partial \sigma_1}{\partial U_1}\right)_{N_1} = \left(\frac{\partial \sigma_2}{\partial U_2}\right)_{N_2}.} \quad (22)$$

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* The notation

$$\left(\frac{\partial g_1}{\partial U_1}\right)_{N_1}$$

means that N_1 is held constant in the differentiation of $g_1(N_1,U_1)$ with respect to U_1. That is, the partial derivative with respect to U_1 is defined as

$$\left(\frac{\partial g_1}{\partial U_1}\right)_{N_1} = \lim_{\Delta U_1 \to 0} \frac{g_1(N_1,U_1 + \Delta U_1) - g_1(N_1,U_1)}{\Delta U_1}.$$

For example, if $g(x,y) = 3x^4y$, then $(\partial g/\partial x)_y = 12x^3y$ and $(\partial g/\partial y)_x = 3x^4$.