

is in thermal contact with a large reservoir at temperature τ . The number of particles in the system is assumed constant.

2. The partition function is

$$Z \equiv \sum_s \exp(-\varepsilon_s/\tau).$$

3. The pressure is given by

$$p = -(\partial U/\partial V)_\sigma = \tau(\partial \sigma/\partial V)_U.$$

4. The Helmholtz free energy is defined as $F \equiv U - \tau\sigma$. It is a minimum in equilibrium for a system held at constant τ , V .

5. $\sigma = -(\partial F/\partial \tau)_V$; $p = -(\partial F/\partial V)_\tau$.

6. $F = -\tau \log Z$. This result is very useful in calculations of F and of quantities such as p and σ derived from F .

7. For an ideal monatomic gas of N atoms of spin zero,

$$Z_N = (n_Q V)^N / N!,$$

if $n = N/V \ll n_Q$. The quantum concentration $n_Q \equiv (M\tau/2\pi\hbar^2)^{3/2}$. Further,

$$pV = N\tau; \quad \sigma = N[\log(n_Q/n) + \frac{5}{2}]; \quad C_V = \frac{3}{2}N.$$

8. A process is reversible if the system remains infinitesimally close to the equilibrium state at all times during the process.

PROBLEMS

1. *Free energy of a two state system.* (a) Find an expression for the free energy as a function of τ of a system with two states, one at energy 0 and one at energy ε . (b) From the free energy, find expressions for the energy and entropy of the system. The entropy is plotted in Figure 3.11.

2. *Magnetic susceptibility.* (a) Use the partition function to find an exact expression for the magnetization M and the susceptibility $\chi \equiv dM/dB$ as a function of temperature and magnetic field for the model system of magnetic moments in a magnetic field. The result for the magnetization is $M = nm \tanh(mB/\tau)$, as derived in (46) by another method. Here n is the particle