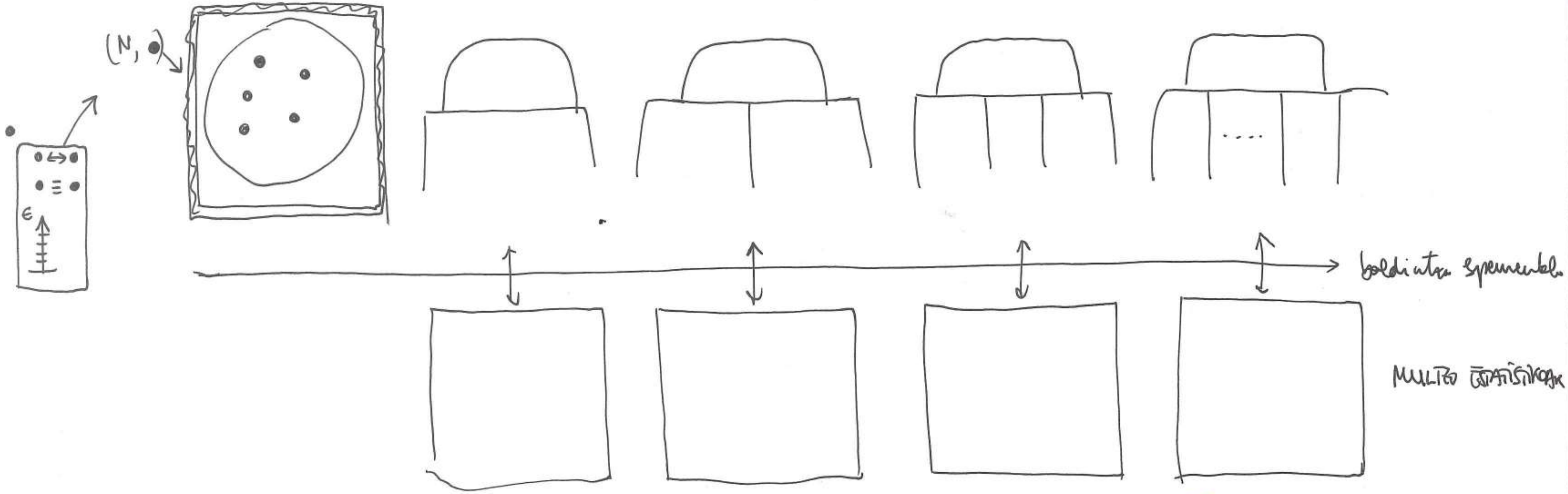
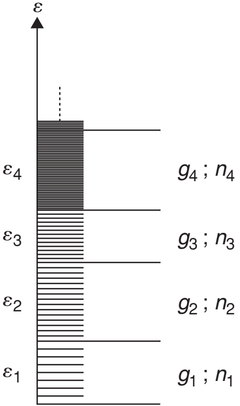


Ω, Σ, Γ $\quad Z, Q$ $\quad \mathcal{Z}\ (z, Z)$ $\quad \mathcal{Z}\ \ldots$ $\quad \mathcal{Z}\ \ldots$

zuzenetan zenbat kasutan ondo zenbatu dugun
kontuan hartuz zenbat gau daitekeen egoera konkretuan



ε
ε_4
g_4 ; n_4
ε_3
g_3 ; n_3
ε_2
g_2 ; n_2
ε_1
g_1 ; n_1

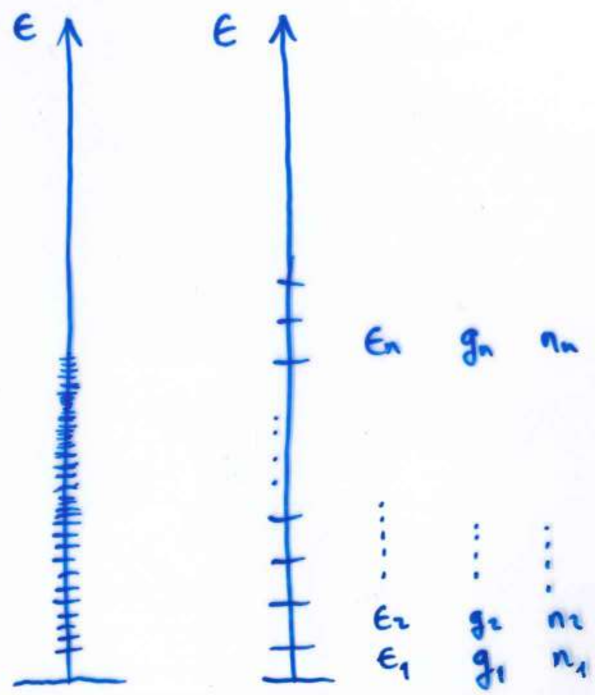
Multzo mikrokanoniko mekaniko-kunatikoan dagoen gas ideala

$$\sum_i n_i = N$$

$$\sum_i n_i \epsilon_i = E$$

$$\Omega(N,V,E) = \sum_{\{n_i\}}{}' W\{n_i\}$$

$$W\{n_i\} = \prod_i w(i)$$



$$w_{\mathrm{BE}}(i) = \frac{(n_i + g_i - 1)!}{n_i!(g_i - 1)!}$$

$$W_{\mathrm{BE}}\{n_i\} = \prod_i \frac{(n_i + g_i - 1)!}{n_i!(g_i - 1)!}$$

$$w_{\mathrm{FD}}(i) = \frac{g_i!}{n_i!(g_i - n_i)!}$$

$$W_{\mathrm{FD}}\{n_i\} = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!}$$

$$\frac{N!}{n_1!n_2!\cdots} \qquad \frac{1}{N!}$$

$$\frac{1}{n_1!n_2!\cdots} = \prod_i \frac{1}{n_i!}$$

$$W_{\mathrm{MB}}\{n_i\} = \prod_i \frac{g_i^{n_i}}{n_i!}$$

M. MIKROKANONIKOA

$$S(N,V,E) = k \ln \Omega(N,V,E) = k \ln \left[\underline{\sum_{\{n_i\}}{}' W\{n_i\}} \right]$$

$$S(N,V,E) \approx k \ln W\{n_i^*\}$$

$\{n_i^*\}$ max

$\Downarrow$

$$\delta \ln W\{n_i\} - \left[\alpha \sum_i \delta n_i + \beta \sum_i \epsilon_i \delta n_i \right]$$

$$\begin{aligned} \ln W\{n_i\} &= \sum_i \ln w_i \quad * \\ &\approx \sum_i \left[n_i \ln\left(\frac{g_i}{n_i} - a\right) - \frac{g_i}{a} \ln\left(1 - a\frac{n_i}{g_i}\right) \right] \end{aligned}$$

$$\sum_i \left[\ln\left(\frac{g_i}{n_i} - a\right) - \alpha - \beta\epsilon_i \right]_{n_i = n_i^*} \delta n_i = 0$$

$$\ln\left(\frac{g_i}{n_i^*} - a\right) - \alpha - \beta\epsilon_i = 0$$

$*$

$$\boxed{a = \begin{cases} -1 & \text{B.E.} \\ 0 & \text{M.B.} \\ 1 & \text{F.D.} \end{cases}}$$

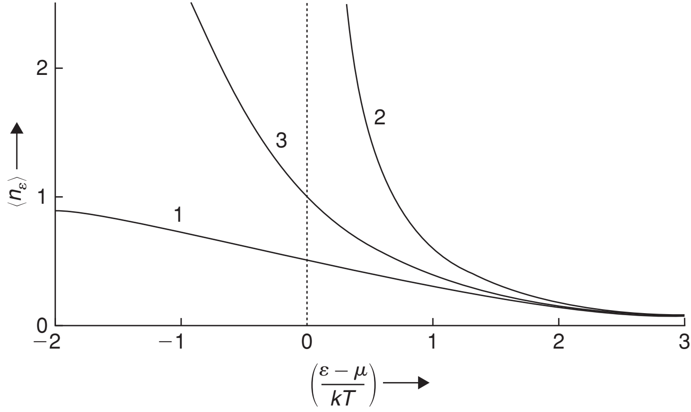
$$n_i^* = \frac{g_i}{e^{\alpha+\beta\epsilon_i} + a}$$

interpretazioa

$$\frac{n_i^*}{g_i} = \frac{1}{e^{\alpha+\beta\epsilon_i} + a}$$

BANAKETA PROBABEEENA, ENERGÍA-MAÍLA BAKARREAN

NON DAGO ENERGÍA-MAÍLEN TALDEKATEEA?


$\langle n_\varepsilon \rangle$
0
1
2
1
2
3
−2
−1
0
1
2
3
$\left(\frac{\varepsilon - \mu}{kT}\right)$

$$\frac{S}{k} \approx \ln W\{n_i^*\} = \left[\sum_i n_i^* \ln\left(\frac{g_i}{n_i} - a\right) - \frac{g_i}{a}\ln\left(1 - a\frac{n_i^*}{g_i}\right)\right]$$
$$= \sum_i \left[n_i^*(\alpha + \beta\epsilon_i) + \frac{g_i}{a}\ln\left\{1 + ae^{-\alpha-\beta\epsilon_i}\right\}\right]$$

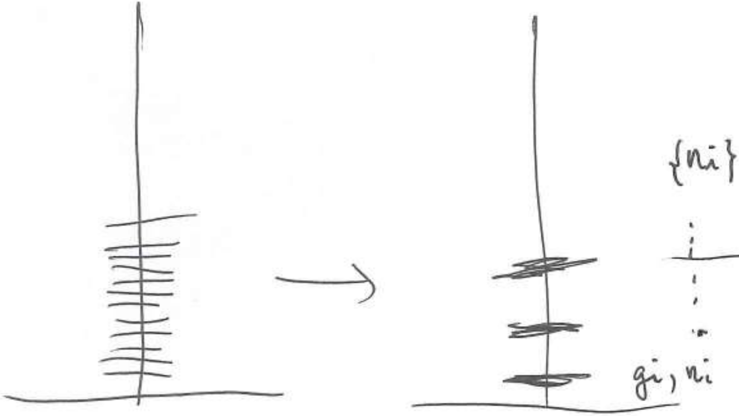
$$\frac{1}{a}\sum_i g_i \ln\left\{1 + ae^{-\alpha-\beta\epsilon_i}\right\} = \frac{S}{k} - \alpha N - \beta E$$

$$\boxed{\alpha = -\frac{\mu}{T} \quad \beta = \frac{1}{kT}},$$

$$\frac{S}{k} + \frac{\mu N}{kT} - \frac{E}{kT} = \frac{G - (E - TS)}{kT} = \frac{PV}{kT}$$

$$PV = \frac{kT}{a}\sum_i \left[g_i \ln\left\{1 + ae^{-\alpha-\beta\epsilon_i}\right\}\right]$$

$$PV = \sum_i g_i e^{-\alpha-\beta\epsilon_i} = kT\sum_i n_i^* = NkT$$



$$\Omega = \Omega(N, V, E)$$

$$E = \sum_i n_i \varepsilon_i$$

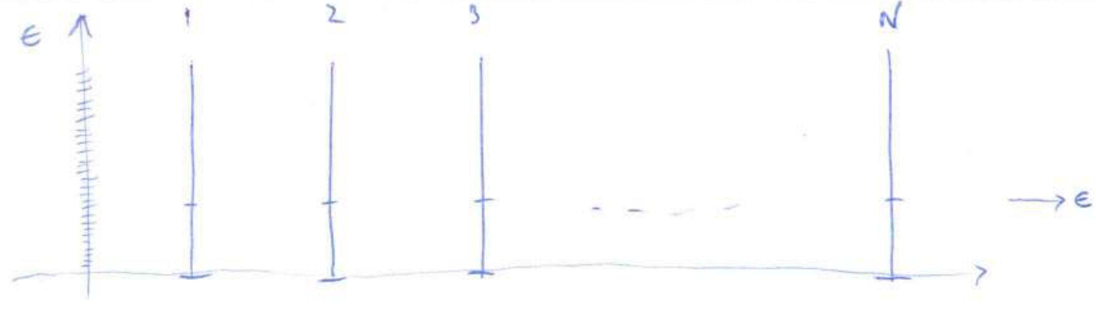
$$N = \sum_i n_i$$

$$\Omega = \sum_{\{n_i\}}{}' W\{n_i\}$$

$$W\{n_i\} = \prod_i W(i)$$

$\{n_i\}$

n_i partikula identikus [illegible] ! $<$ bozonok / fermionok

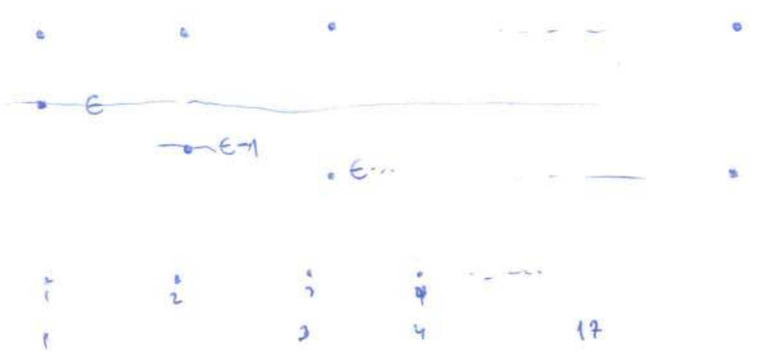


$$Q_N(V,T) = \sum_E e^{-\frac{E}{kT}}$$

$$E = \sum_\epsilon n_\epsilon \epsilon$$

$$n_\epsilon$$

$g\{n_\epsilon\}$ hastapen-faktore estatistikoa



$g\{n_\epsilon\} = 1$

$g\{n_\epsilon\} \begin{cases} 0 \\ 1 \end{cases}$

$g\{n_\epsilon\} = \frac{1}{n_\epsilon!}$
(bereizleen bilketa)

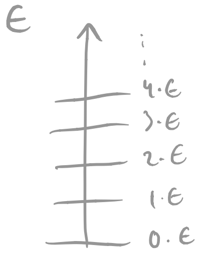
$$w(i) \begin{cases} w_{BE} \\ w_{FD} \\ w_{MB} \end{cases}$$

amaieretan $g_i \to 1$

$$Q_N(V,T) = \sum_{\{n_\epsilon\}}{}' g(\{n_\epsilon\})\, e^{-\frac{1}{kT}\left[\sum_\epsilon n_\epsilon \epsilon\right]}$$

- partikula bakar bati dagokion energia-mailan zenbat partikula dagoen
- ez da endekapena baizik eta zenbat partikula dagoen
- horrela ezaugarritzen baitira sistema osoaren egoerak

eta orain egin behar duena da $g\{n_\epsilon\}$ bakoitza kontuan hartu eta $Q_N(V,T)$ kalkulatzeko "formak" lortu



$\epsilon = n \cdot \epsilon \quad , n = (0, 1, \dots, n, \dots)$

↑ quanta

• partikula bakarraren kasua

identikoak

independenteak

„kuantikoak", izaera < fermioiak / bosoiak - - - - - - - - klasikoak

$$Z_N(T,V) = \sum_E e^{-E/k_BT} \quad \begin{cases} E = \sum_\epsilon n_\epsilon \cdot \epsilon \\ N = \sum_\epsilon n_\epsilon \end{cases} \quad Z_N(T,V) = \sum_{\{n_\epsilon\}}{}' g(\{n_\epsilon\})\, e^{-\frac{1}{k_BT}\sum_\epsilon n_\epsilon \cdot \epsilon}$$

$N = 0, 1, 2, 3, \boxed{-\,-}\,- , \infty$

↓ $N=5$

N finko

posibleak diren multzoaren energia-maila denak [→ aukeratu egingo da ⟹

$g(\{n_\epsilon\})$

egoerak										
1	5	0	0	0	0	0	0	0	1	∅
2	4	1	0	0	0	0	0	0	1	∅
3	3	1	1	0	0	0	0	0	1	∅
4	2	2	1	0	0	0	0	0	1	∅
5	2	1	1	1	0	0	0	0	1	∅
$\boxed{6}$	1	1	1	1	1	0	0	0	1	1
7										
⋮										
n										
⋮										

beste konbinazio asko ere bai

eta horien modukoak baina energia-maila denetara hedatuta

→ fermioien kasua!

MULTZO KANONIKOA

Gas idela, gainerako multzo mekaniko-kuantikoetan

K

$$Q_{\underline{N}}(V,T) = \sum_E e^{-\beta \underline{E}} = \text{MULTZOA}$$

$$E = \sum_\epsilon n_\epsilon \epsilon$$

PARTIKULA BAKARRA

$$\sum_\epsilon n_\epsilon = N$$

batuagarritasuna

$$Q_N(V,T) = \sum_{\{n_\epsilon\}} g\{n_\epsilon\} e^{-\beta \sum_\epsilon n_\epsilon \epsilon},$$

STATISTICAL WEIGHT FACTOR

- partikula motaren (izaeraren) arabera
- zer banaketa hartu behar den kontuan

$$\boxed{\begin{aligned} g_{\mathrm{BE}}\{n_\epsilon\} &= 1 \\ g_{\mathrm{FD}}\{n_\epsilon\} &= \begin{cases} 1 \\ 0 \end{cases} \\ g_{\mathrm{MB}}\{n_\epsilon\} &= \prod_\epsilon \frac{1}{n_\epsilon!} \end{aligned}}$$

$n_\epsilon = 0,1$ direnak
beste edozein kasutan

AURREKO ADIERAZPENETAN $g_i = 1$!!

EZ DITUGU ENERGÍA-MAILAK TALDEKATU

klasikoa

$$
\begin{aligned}
Q_N(V,T) &= \sum_{\{n_\epsilon\}}{}' \left[\left(\prod_\epsilon \tfrac{1}{n_\epsilon!} \prod_\epsilon \left(e^{-\beta\epsilon}\right)^{n_\epsilon}\right)\right] \times \frac{N!}{N!} \\
&= \frac{1}{N!} \sum_{\{n_\epsilon\}}{}' \left[\left(\prod_\epsilon \tfrac{N!}{n_\epsilon!} \prod_\epsilon \left(e^{-\beta\epsilon}\right)^{n_\epsilon}\right)\right]
\end{aligned}
$$

TEOREMA MULTINOMIALA

$$
\begin{aligned}
Q_N(V,T) &= \tfrac{1}{N!} \left[\sum_\epsilon e^{-\beta\epsilon}\right]^N \quad , \\
&= \tfrac{1}{N!} \left[Q_1(V,T)\right]^N
\end{aligned}
$$

hentu espazio batean

$$
\begin{aligned}
Q_1(V,T) &\equiv \sum_\epsilon e^{-\beta\epsilon} \approx \tfrac{2\pi V}{h^3}(2m)^{3/2} \int_0^\infty e^{-\beta\epsilon}\epsilon^{1/2} d\epsilon \\
&= \tfrac{V}{\lambda^3}
\end{aligned}
$$

$$
Q_N(V,T) = \frac{V^N}{N!\lambda^{3N}} \qquad \boxed{\frac{V}{\lambda_T^3}}
$$

Pathria 132. oldal

$$(x_1 + x_2 + \dots x_m)^n = \sum_{k_1 + \dots + k_m = n} \binom{n}{k_1\, k_2\, k_m} \prod_{1 \le t \le m} x_t^{k_t}$$

$$Q_N(V,T) = \sum_{\{n_\epsilon\}}{}' \left[\left(\prod_\epsilon \frac{1}{n_\epsilon!} \right) \prod_\epsilon \left(e^{-\beta\epsilon}\right)^{n_\epsilon} \right]$$

$$\sum_\epsilon n_\epsilon = N \text{ partikula kényszer}$$

$$= \frac{1}{N!} \sum_{\{n_\epsilon\}}{}' \left[\left[\left(\prod_\epsilon \frac{1}{n_\epsilon!} \right) N! \right] \cdot \prod_\epsilon \left(e^{-\beta\epsilon}\right)^{n_\epsilon} \right]$$

$$\underbrace{\left[\frac{N!}{\prod_\epsilon n_\epsilon!} \right] \prod_\epsilon \left(e^{-\beta\epsilon}\right)^{n_\epsilon}}$$

$$\frac{1}{N!} \left[\sum_\epsilon e^{-\beta\epsilon} \right]^N$$

$$\sum_\epsilon e^{-\beta\epsilon} = Q_1(V,T)$$

$$Q_N(V,T) = \frac{1}{N!} \left[Q_1(V,T) \right]^N$$

$$Q_1(V,T) \equiv \left[\frac{V}{\lambda_T^3} \right]$$

+ MK

MK

$$\mathscr{Q}(z,V,T) = \sum_{N=0}^{\infty} z^N Q_N(V,T) = \left[\exp\left(\frac{zV}{\lambda^3}\right)\right]$$

$$Q_N(V,T) = \sum_{\{n_\epsilon\}}{}' \left(e^{-\beta \sum\limits_{\epsilon} n_\epsilon \epsilon} \right)$$

$$\boxed{\begin{aligned} \mathscr{Q}(z,V,T) &= \sum_{N=0}^{\infty} \left[z^N \sum_{\{n_\epsilon\}}{}' e^{-\beta \sum\limits_{\epsilon} n_\epsilon \epsilon} \right], \\ &= \sum_{N=0}^{\infty} \left[\sum_{\{n_\epsilon\}}{}' \left(ze^{-\beta\epsilon}\right)^{n_\epsilon} \right] \end{aligned}}$$

$$\begin{aligned} \mathscr{Q}(z,V,T) &= \sum_{n_0,n_1,\cdots} \left[\left(ze^{-\beta\epsilon_0}\right)^{n_0} \left(ze^{-\beta\epsilon_0}\right)^{n_0} \cdots \right] \\ &= \left[\sum_{n_0} \left(ze^{-\beta\epsilon_0}\right)^{n_0} \right] \left[\sum_{n_0} \left(ze^{-\beta\epsilon_0}\right)^{n_0} \right] \cdots \end{aligned}$$

$$n_\epsilon \begin{cases} \text{B.E.} & 0,1,2,\ldots \\ \text{F.D.} & 0,1 \end{cases}$$

$$\mathscr{Q}(z,V,T) = \prod_{\epsilon} \frac{1}{1 \mp ze^{-\beta\epsilon}}$$

$$\begin{cases} - & \text{B.E.} \quad ze^{-\beta\epsilon} < 1 \\ + & \text{F.D.} \end{cases}$$

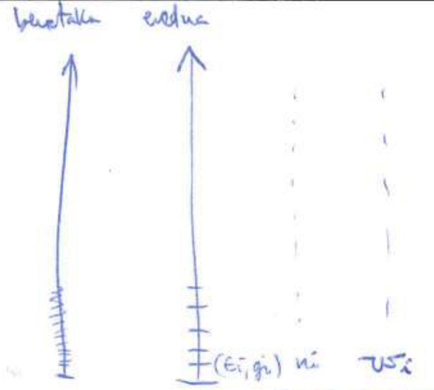
$$\begin{aligned} q(z,V,T) &\equiv \equiv \ln \mathscr{Q}(z,V,T) \\ &= \mp \sum_{\epsilon} \ln\left(1 \mp ze^{-\beta\epsilon}\right) \end{aligned}$$

B.E. $-$

F.E $+$

$$W\{n_i\} = \prod_i w(i)$$

hauxe da ondo zenbatuko duguna
bi partikula mota bereiziz



$\{n_i\}$ $\prod w_i$ banaketa probable bat $W(\{n_i\})$

$$\boxed{\Omega = \sum{}' W(\{n_i\})}$$

$$w_{BE}(i) = \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$$ bereizezin

$$w_{MB}(i) = \frac{(g_i)^{n_i}}{n_i!}$$ bereizgarri
gero bereizezin
Gibbsen zuzenketa

$$w_{FD}(i) = \frac{g_i!}{n_i!\,(g_i - n_i)!}$$ bereizezin

$$\sum{}' \left\{ \begin{aligned} W(\{n_i\}) &= \prod_i w_{BE}(i) = \prod_i \left[\frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!}\right] \\ &= \prod_i w_{FD}(i) = \prod_i \left[\frac{g_i!}{n_i!\,(g_i-n_i)!}\right] \\ &= \prod_i w_{MB}(i) = \prod_i \left[\frac{(g_i)^{n_i}}{n_i!}\right] \end{aligned} \right\} \rightarrow \frac{n_i^*}{g_i} = \frac{1}{e^{\alpha + \frac{\epsilon_i}{kT}} + a} = \frac{1}{e^{\frac{(\epsilon_i - \mu)}{kT}} + a} \qquad a = \begin{cases} -1 \\ +1 \\ 0 \end{cases}$$

maximizatu
denak dira oso handiak

$$\boxed{pV = \frac{kT}{a} \sum_i \left[g_i \ln\left\{1 + a e^{\frac{(\mu - \epsilon_i)}{kT}}\right\}\right]}$$

$$Q_N(V,T) = \sum_{\{n_\epsilon\}}' g(\{n_\epsilon\})\, e^{-\frac{1}{kT}\left(\sum_\epsilon n_\epsilon \epsilon\right)}$$

$$g(\{n_\epsilon\}) = \prod_\epsilon \frac{1}{n_\epsilon!} \Rightarrow \sum_{\{n_\epsilon\}}' \prod_\epsilon \frac{1}{n_\epsilon!} \cdot \prod_\epsilon \left(e^{\left(-\frac{\epsilon}{kT}\right)}\right)^{n_\epsilon}$$

$$\underbrace{\frac{1}{N!} \sum_{\{n_\epsilon\}}' \left[\frac{N!}{\prod_\epsilon n_\epsilon!} \prod_\epsilon \left(e^{-\left(\frac{\epsilon}{kT}\right)}\right)^{n_\epsilon}\right]} \quad + N = \sum_\epsilon n_\epsilon$$

$$\frac{1}{N!} \Bigg[\underbrace{\sum_\epsilon e^{-\frac{\epsilon}{kT}}}_{Q_1(V,T)}\Bigg]^N$$

$$\frac{1}{N!} \left[Q_1(V,T)\right]^N$$

$$g\{n_\epsilon\} = 1 \qquad \sum_{\{n_\epsilon\}}' \left[e^{-\frac{1}{kT}\left(\sum_\epsilon n_\epsilon \epsilon\right)}\right] \rightarrow$$ diferentziak hemen daude

baina hau ez da kalkulatzen
partizio-funtzio makrokanonika baina ez da kalkulatzen.

$$\mathcal{Z}(z,V,T) = \sum_{N=0}^{\infty} \left[z^N \left(\sum_{\{n_\epsilon\}}' e^{-\frac{1}{kT}\left(\sum_\epsilon n_\epsilon \epsilon\right)}\right)\right]$$

$\longrightarrow$ sartu

$$\sum_{N=0}^{\infty} \left[\left(\sum_{\{n_\epsilon\}}' \left(\prod_\epsilon z e^{-\frac{\epsilon}{kT}}\right)^{n_\epsilon}\right)\right]$$

N finkatu eta posibleak diren n_ϵ tan batu, baina gero N tan aldatu
↓ aldatu
n_ϵ posible direnetan batu inolako baldintzarik gabe, balio denetara heldu, independenteki!

$$= \sum_{n_0, n_1} \left[\left(z e^{-\frac{\epsilon_0}{kT}}\right)^{n_0} \left(z e^{-\frac{\epsilon_1}{kT}}\right)^{n_1} \cdots\right] = \left[\sum_{n_0} z e^{-\frac{\epsilon_0}{kT}}\right]^{n_0} \cdots\cdots$$

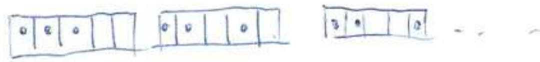
$$\begin{cases} BE & z e^{-\frac{\epsilon}{kT}} < 1 \\ FD & 1 + z e^{-\frac{\epsilon}{kT}} \end{cases}$$ bi balio baino ez daudelako

$$\Omega(N,V,E) = \sum_{\{n_i\}}{}' W\{n_i\} \qquad W\{n_i\} = \prod_i w_i$$

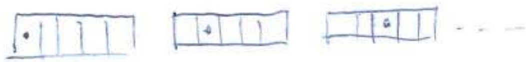
ooo|||| |ooo||| ||oo:|| ||||ooo| ||||ooo
oo|o|||| ......

$$w_{BE} = \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!} \longrightarrow W_{BE} = \prod_i \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!}$$

$$w_{FD} = \frac{g_i!}{n_i!\,(g_i-n_i)!} \longrightarrow W_{FD} = \prod_i \frac{g_i!}{n_i!\,(g_i-n_i)!}$$



$$w_{MB} = \frac{(g_i)^{n_i}}{\prod_i n_i!} \longrightarrow W_{MB} = \prod_i \frac{g_i^{n_i}}{n_i!}$$



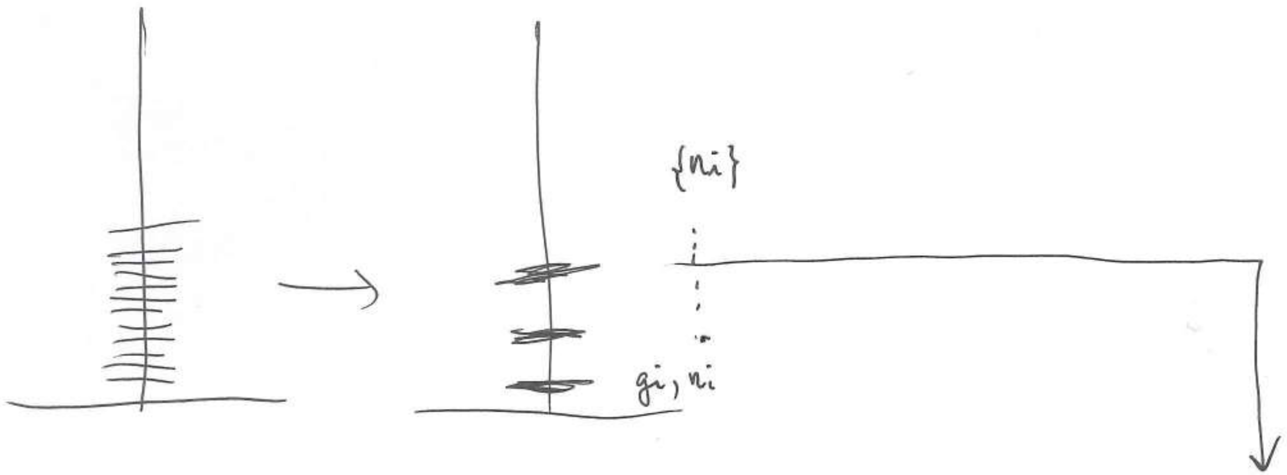
$g_i = 5$

$n_i = 3$

$$S = k_B \ln \left\{ \Omega(N,V,E) = \sum_{\{n_i\}}{}' W\{n_i\} \quad \left\{ \begin{array}{l} \sum_{\{n_i\}}{}' \prod_i \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!} \\ \sum_{\{n_i\}}{}' \prod_i \frac{g_i!}{n_i!\,(g_i-n_i)!} \\ \sum_{\{n_i\}}{}' \prod_i \frac{g_i^{n_i}}{n_i!} \end{array} \right\} \right. \approx S \approx k_B \ln W(n_i^*)$$

maximum taken there

(0)



$$\Omega = \Omega(N, V, E) \qquad \Omega = \sum_{\{n_i\}}{}' W\{n_i\} \qquad W\{n_i\} = \prod_i W(i)$$

$\{n_i\}$

n_i partikula identične nerazlučive ! $<$ bozoni, fermioni

$$E = \sum_i n_i \varepsilon_i$$
$$N = \sum_i n_i$$

$$W\{n_i\} = \prod_i W(i)$$

$$w(i)_{BE} = \frac{n_i + g_i - 1}{n_i!\,(g_i-1)!} \longrightarrow \prod_i \frac{(n_i + g_i - 1)!}{n_i!\,(g_i-1)!}$$

$$w(i)_{FD} = \frac{g_i!}{n_i!\,(g_i-n_i)!} \longrightarrow \prod_i \frac{g_i!}{n_i!\,(g_i-n_i)!}$$

$$w(i)_{MB} = (g_i)^{n_i} \quad \text{bane} \quad \frac{N!}{\prod_i n_i!} \times \left[\frac{1}{N!}\right] \qquad \prod_i \frac{(g_i)^{n_i}}{n_i!}$$

$$S(N, V, E) = k_B \ln \Omega(N, V, E) = k_B \ln \underbrace{\left[\sum_{\{n_i\}}{}' W\{n_i\}\right]}_{\text{maksimalan}}$$

$$k_B \ln W\{n_i^*\} \longrightarrow$$

Lagrange
dana da ovo hvali

(1)

$\ln W\{n_i^*\}$ desgleichen ausrechnen ...

$\ln W\{n_i\}$

$\parallel$

$\prod_i \boxed{W_i}$ BE FD MB

$\sum_i \ln \square$

$\ln\left(\frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!}\right)$

$(n_i+g_i-1)\ln(n_i+g_i-1) - n_i \ln n_i - (g_i-1)\ln(g_i-1)$

$(n_i+g_i)\ln(n_i+g_i) - n_i\ln n_i - g_i \ln g_i$

$n_i \ln\left(\frac{n_i+g_i}{n_i}\right) + g_i \ln\left(\frac{n_i+g_i}{g_i}\right)$

$n_i \ln\left(1+\frac{g_i}{n_i}\right) + g_i \ln\left(1+\frac{n_i}{g_i}\right)$

$\ln\left(\frac{g_i!}{n_i!\,(g_i-n_i)!}\right)$

$g_i \ln g_i - n_i \ln n_i - (g_i-n_i)\ln(g_i-n_i)$

$-g_i \ln\frac{(g_i-n_i)}{g_i} + n_i \ln\left(\frac{g_i-n_i}{n_i}\right)$

$-g_i \ln\left(1-\frac{n_i}{g_i}\right) + n_i \ln\left(1-\frac{g_i}{n_i}\right)$

$$\sum_i \left\{ \ln\left(\frac{g_i}{n_i} - a\right) + \frac{g_i}{a}\ln\left(1 - a\frac{n_i}{g_i}\right)\right\} \quad \begin{cases} a=-1 \\ a=1 \\ a=0 \end{cases}$$

$$\boxed{n_i^* = \frac{g_i}{e^{\alpha+\beta\varepsilon_i}+a}} \longrightarrow \frac{n_i}{g_i} = \frac{1}{e^{\alpha+\beta\varepsilon_i}+a} \quad f_i$$

$$Q_N(V,T) = \sum_E e^{-\frac{E}{k_B T}}$$

$$E = \sum_\epsilon n_\epsilon \epsilon$$

ϵ partikula [illegible] energia-nívók

n_ϵ

$$N = \sum_\epsilon n_\epsilon$$

$$Q_N(V,T) = \sum_{\{n_\epsilon\}}{}' g\{n_\epsilon\} e^{-\left[\frac{1}{k_B T} \sum_\epsilon n_\epsilon \epsilon\right]}$$

$\{n_\epsilon\}$