Schedulability analysis of limited-preemptive moldable gang tasks

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TU Delft

Daily Supervisor Geoffrey Nelissen

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3rd of June, 2020

Real-time systems

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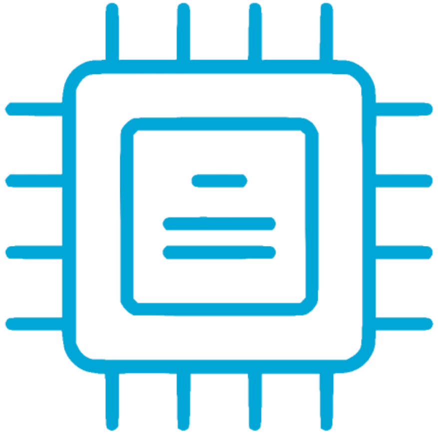
- Multicore systems



Definitions

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- Task
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- Task
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  - Instance of a task
- Schedule
  - A particular assignment of jobs to the processors and time intervals
- Scheduling policy
  - Algorithm that produces a schedule
  - FIFO, Round-Robin, JLFP, EDF

What is gang?

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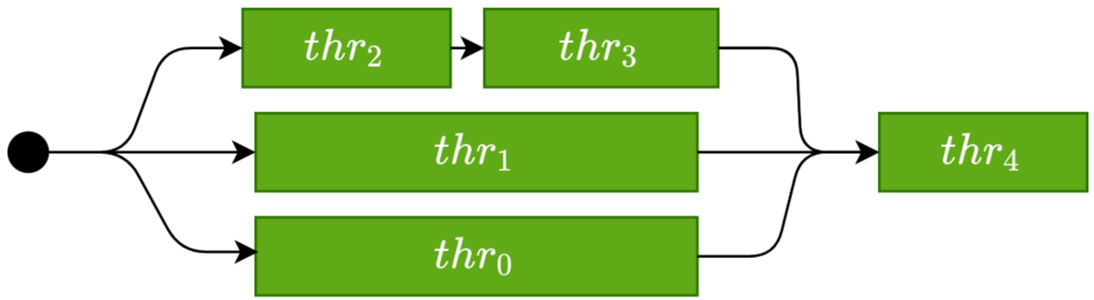

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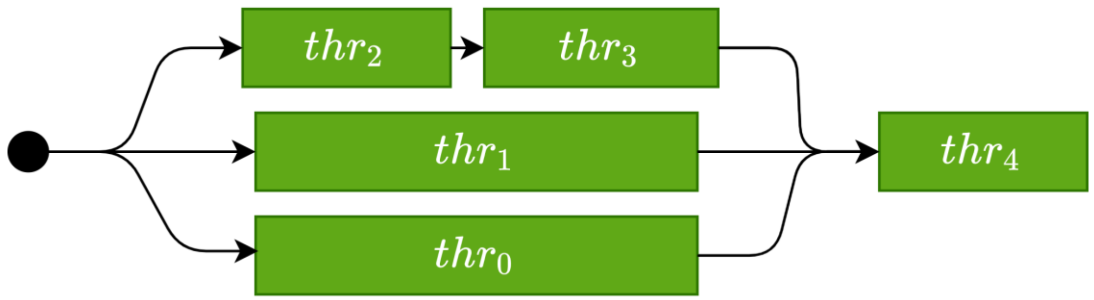
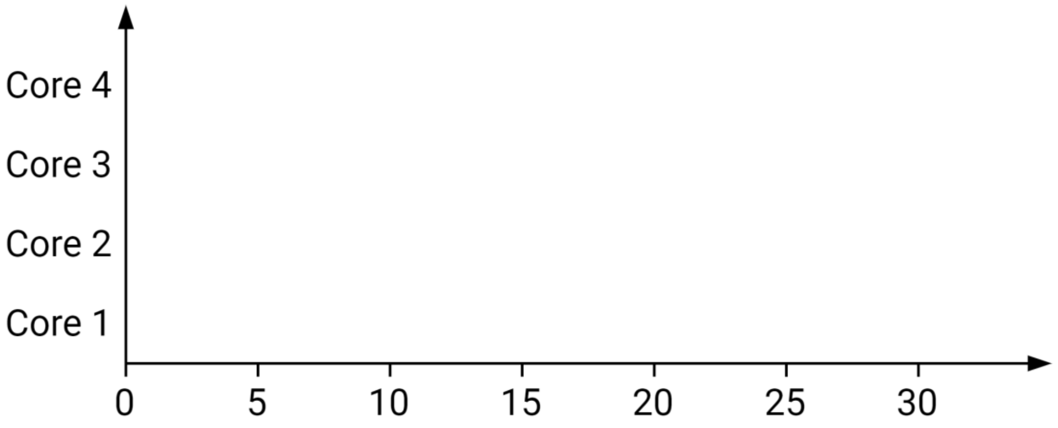


Global scheduling



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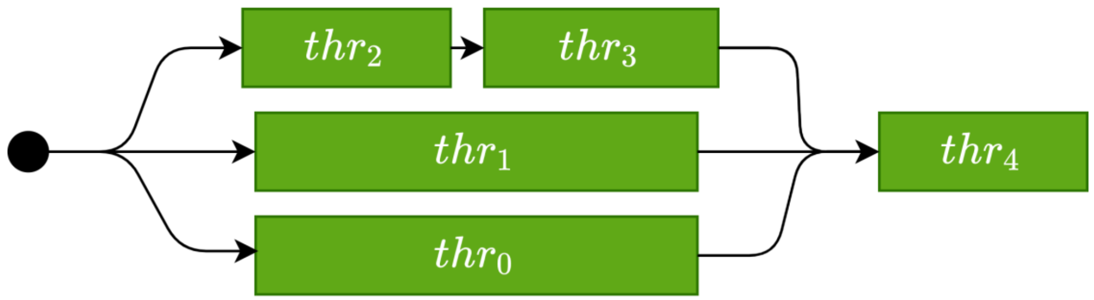

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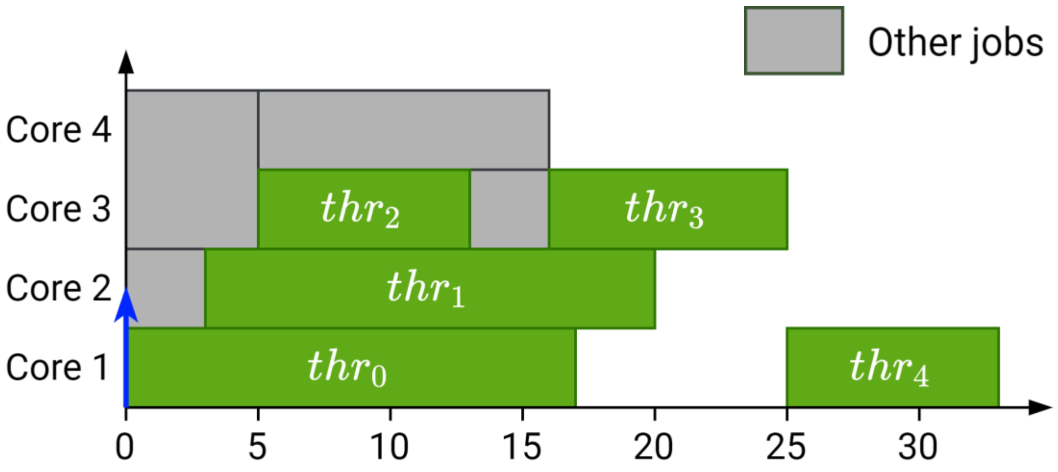

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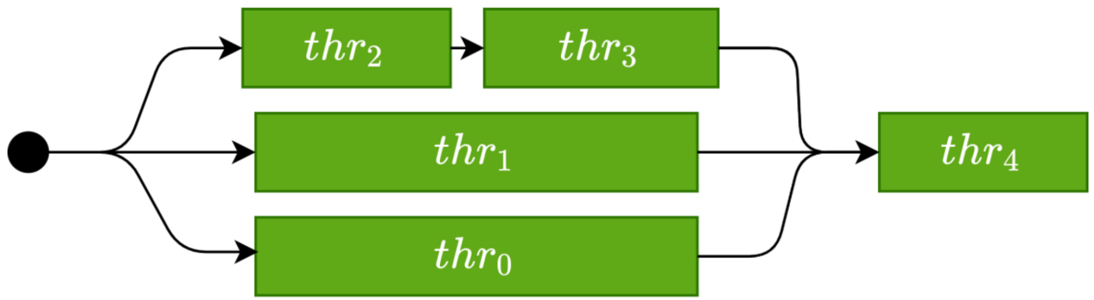
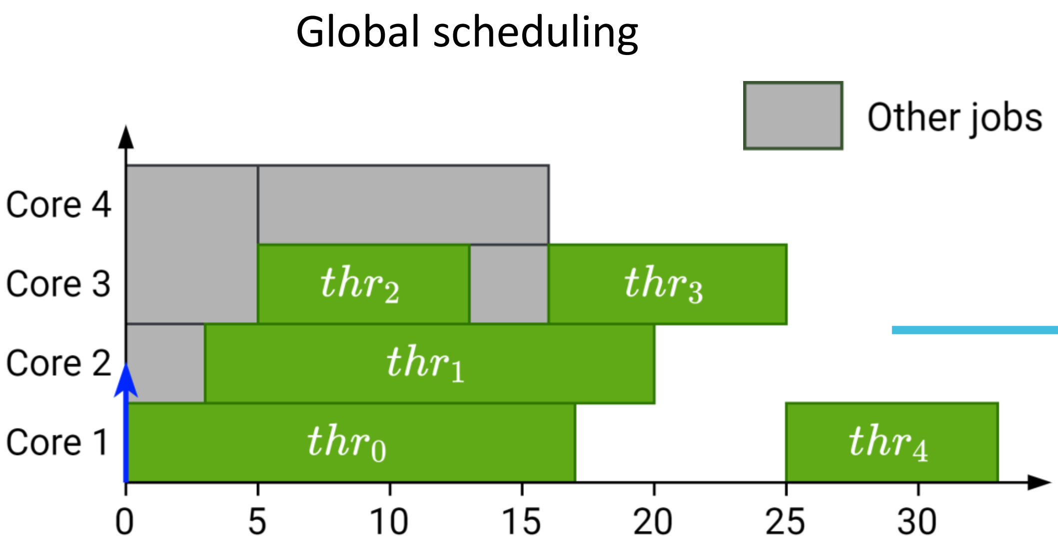
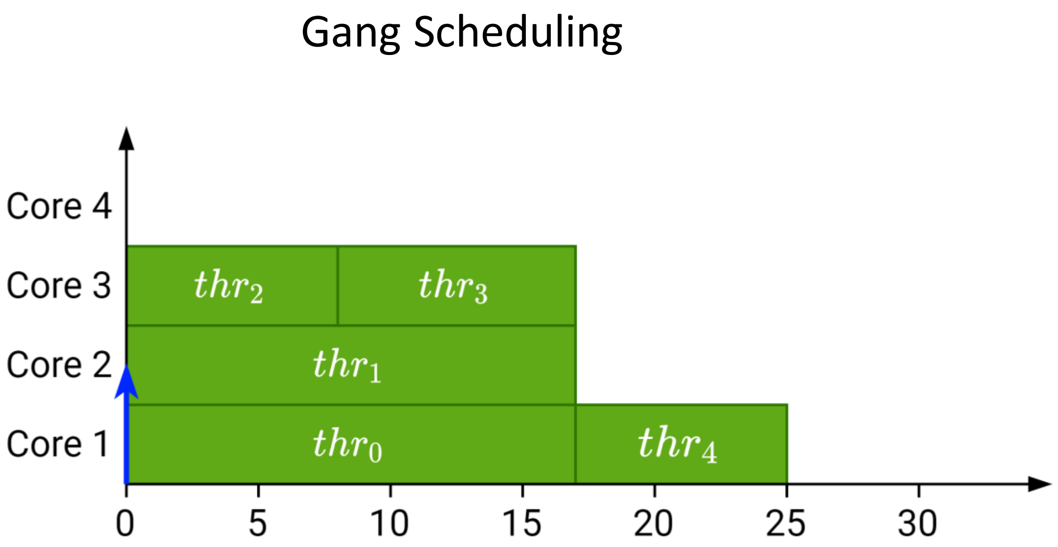


Parallel threads together as a “gang”





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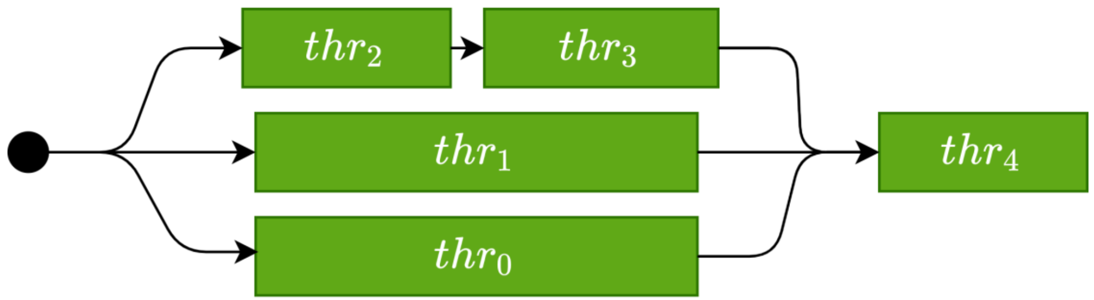

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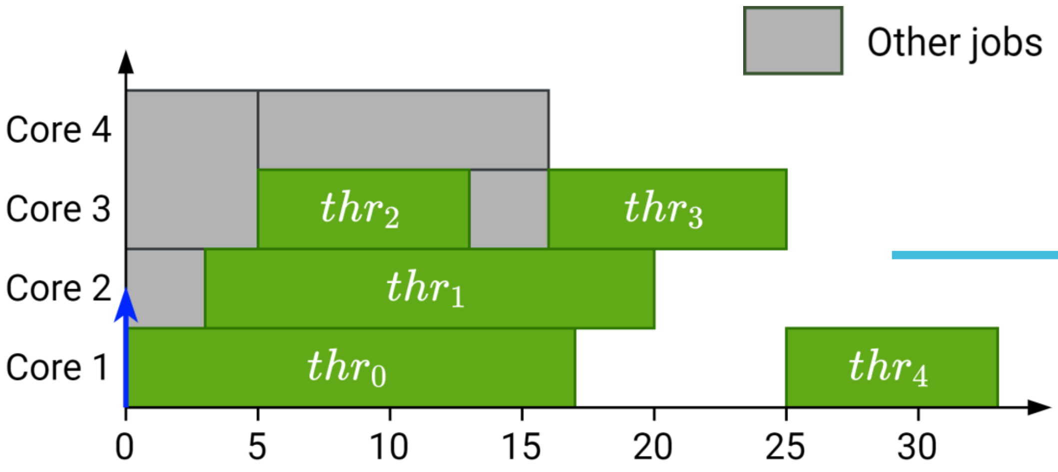

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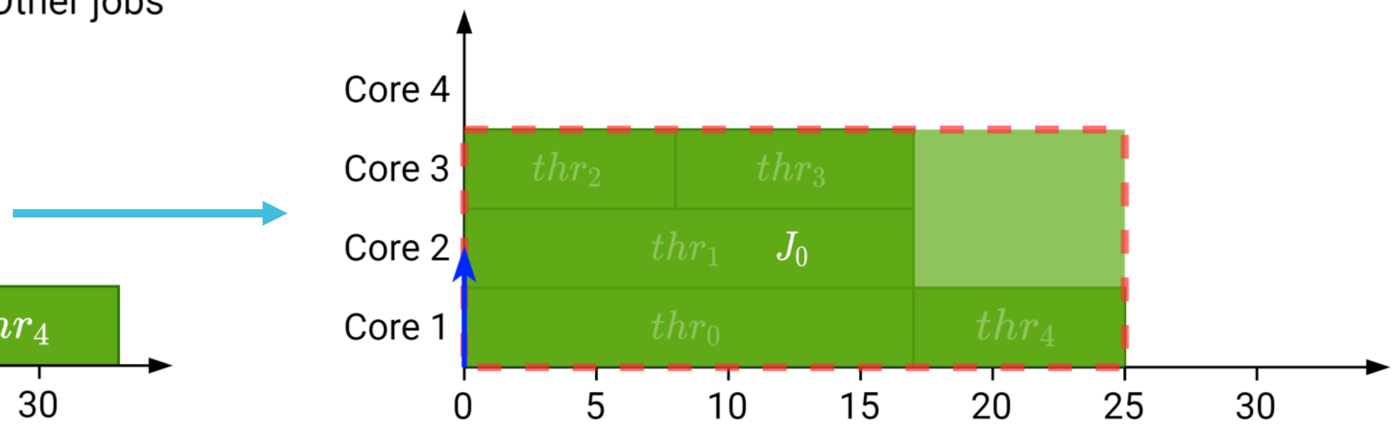

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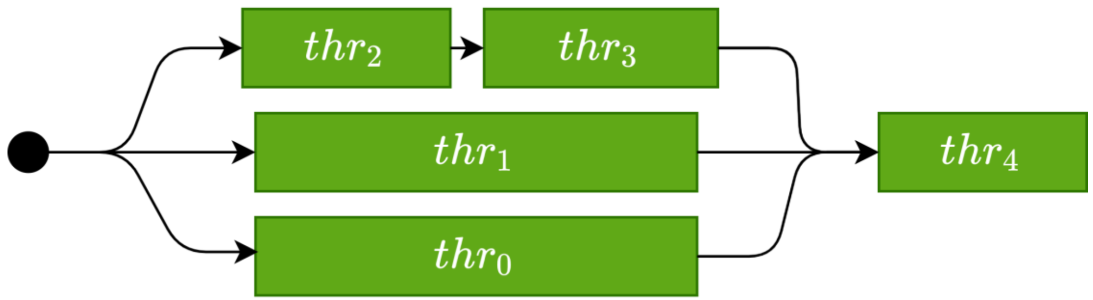
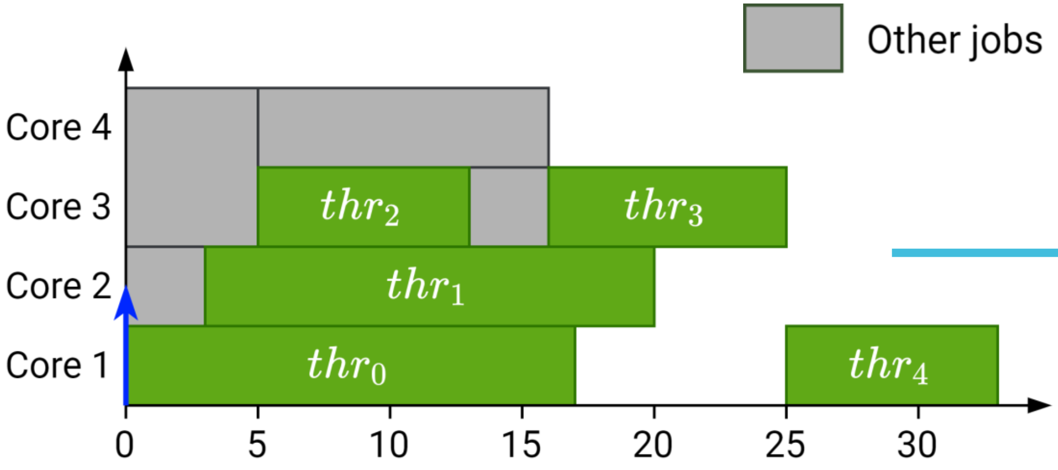


Parallel threads together as a "gang"

Execution does not start until there are enough cores

Global scheduling



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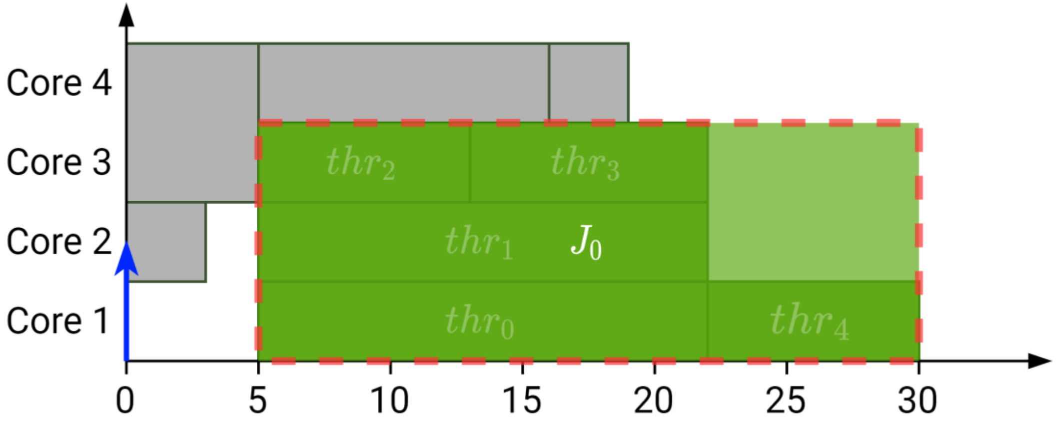

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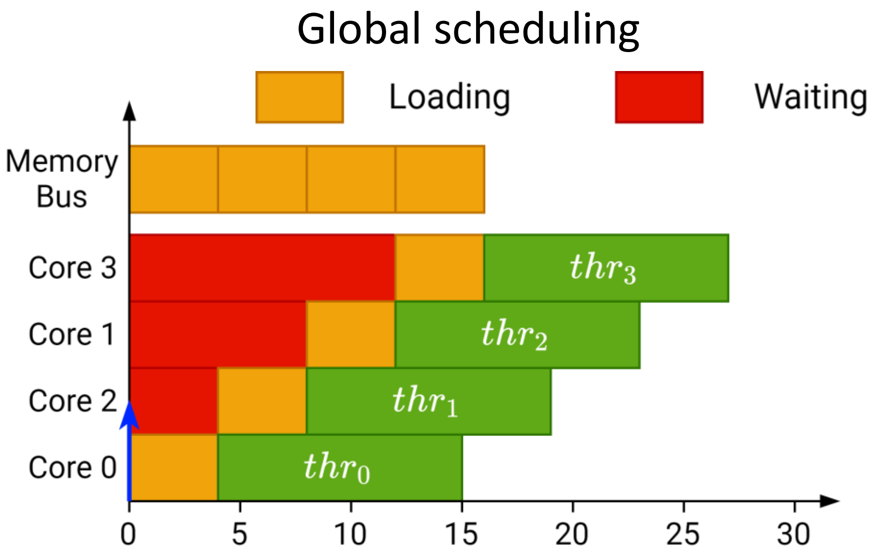

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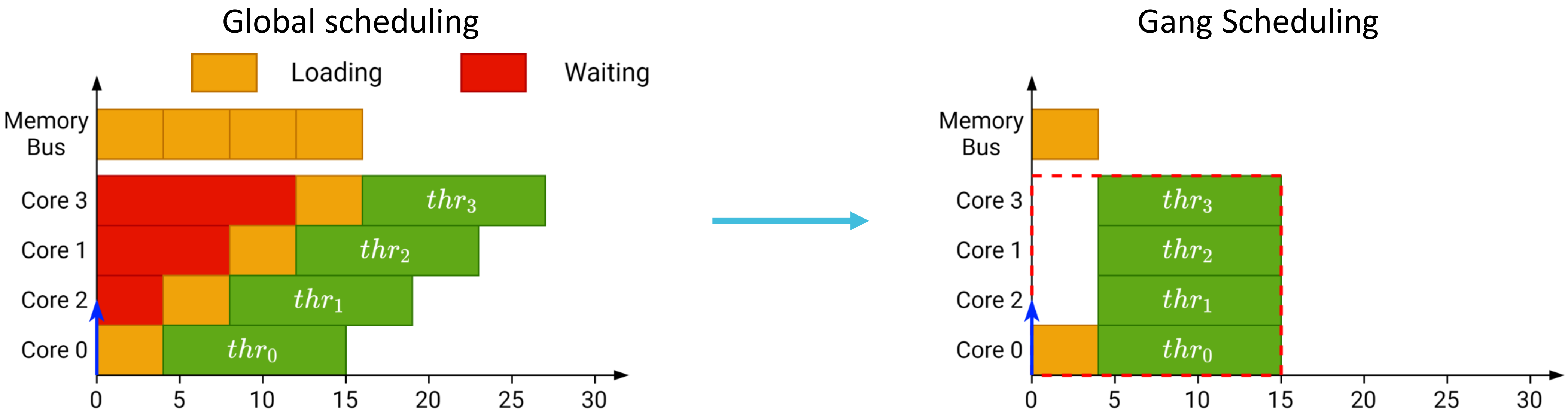
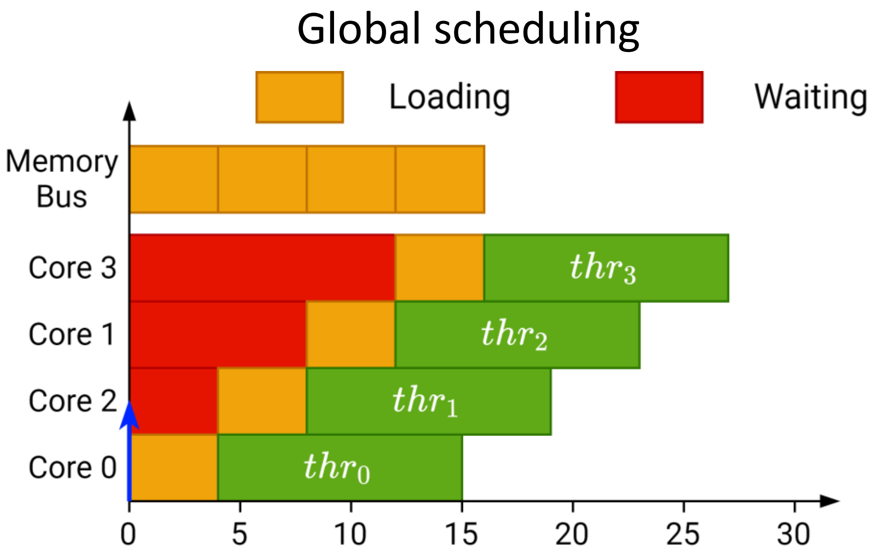
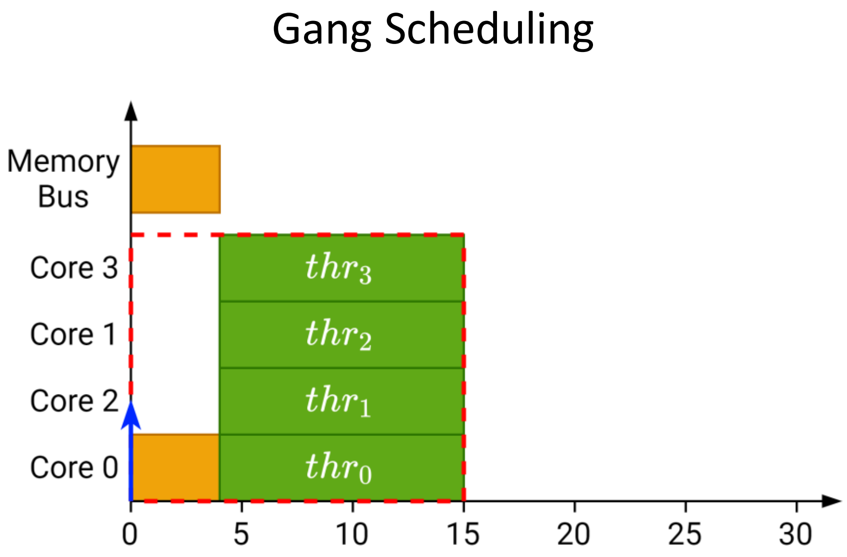


Why gang?

- More efficient synchronization
- Reduces variability in the execution
- Avoids overhead when loading initial data
- Shows its full potential when executed non-preemptively





Types of gang

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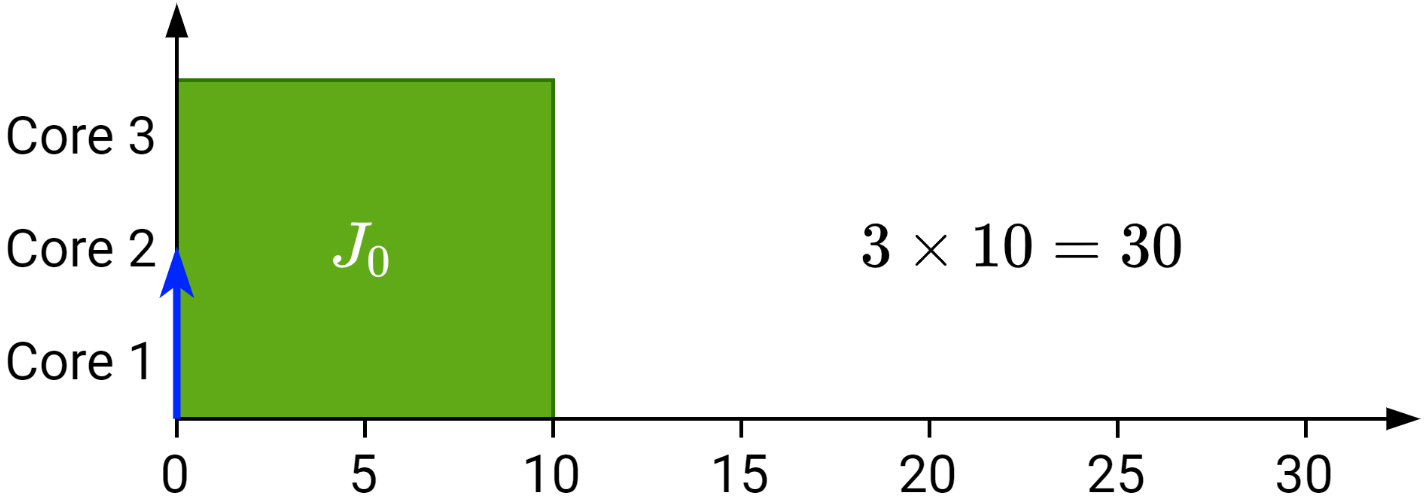

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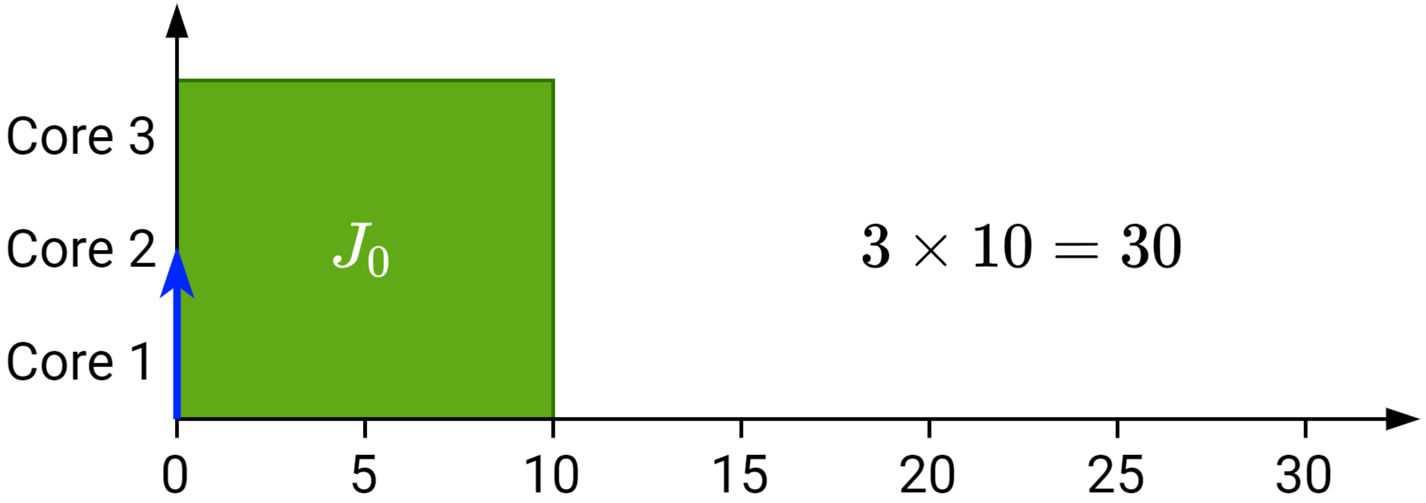

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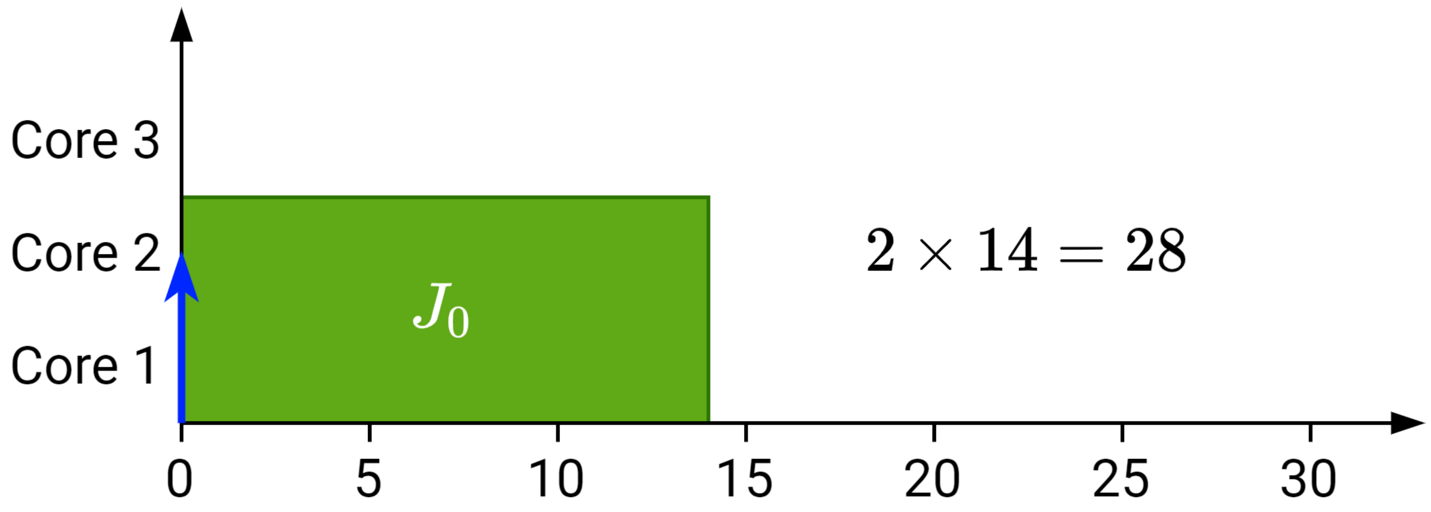

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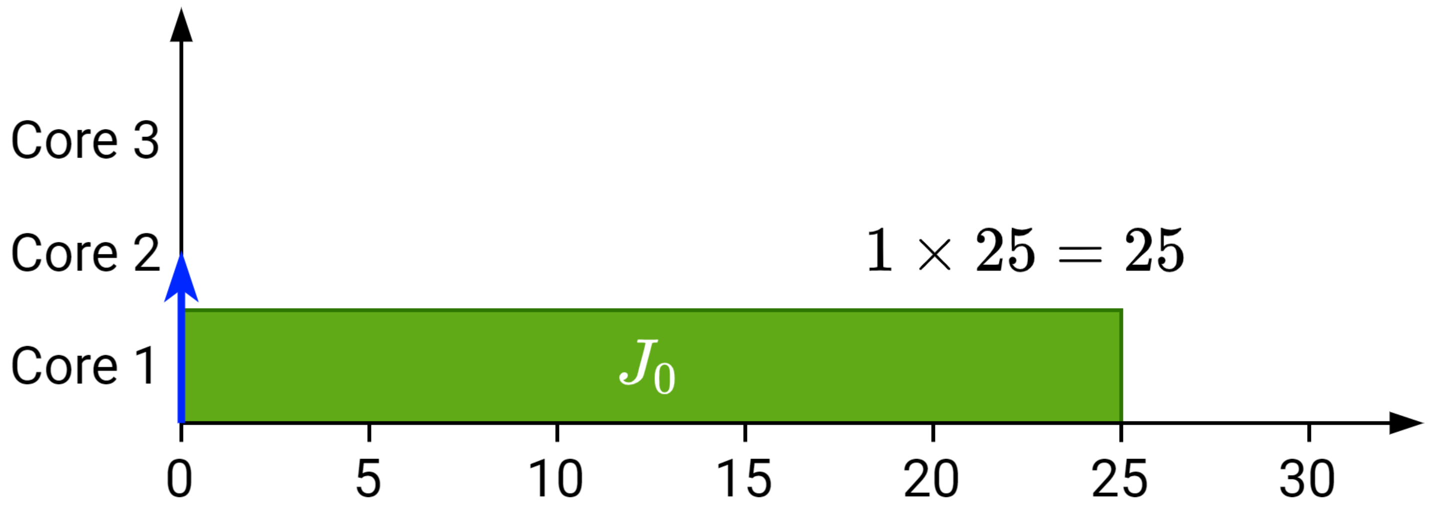

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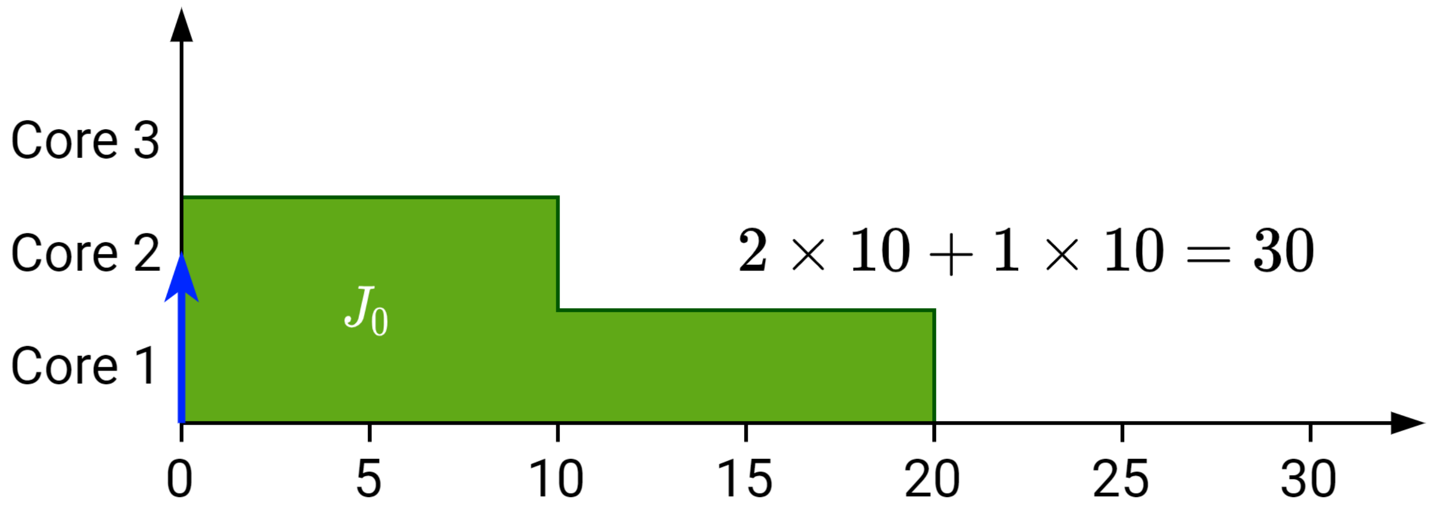

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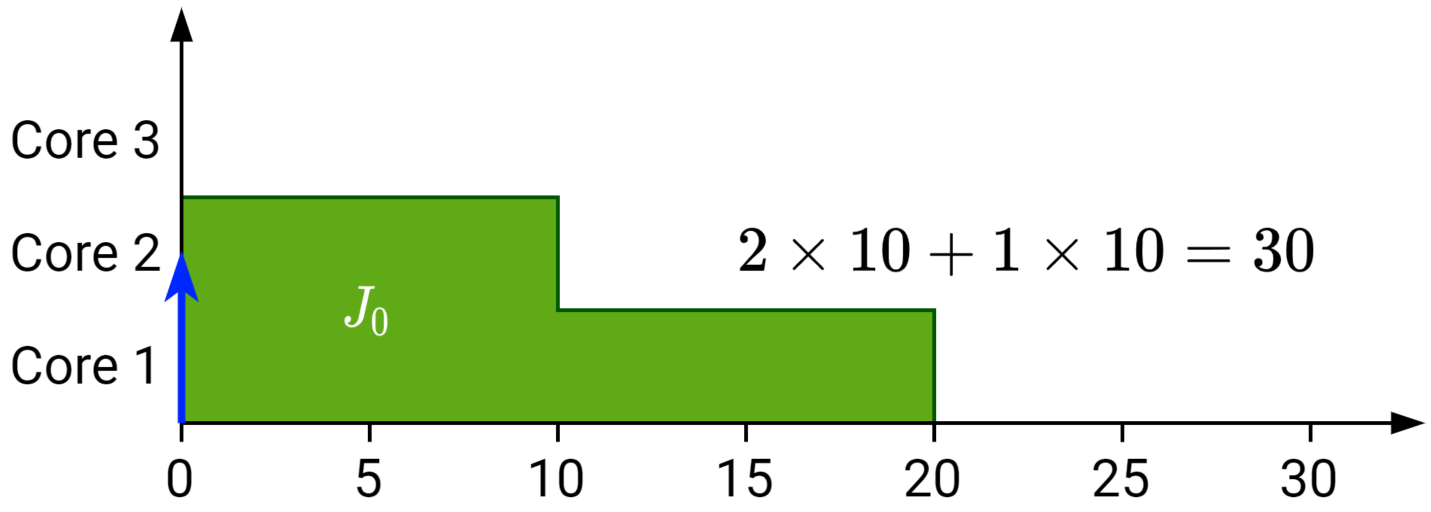

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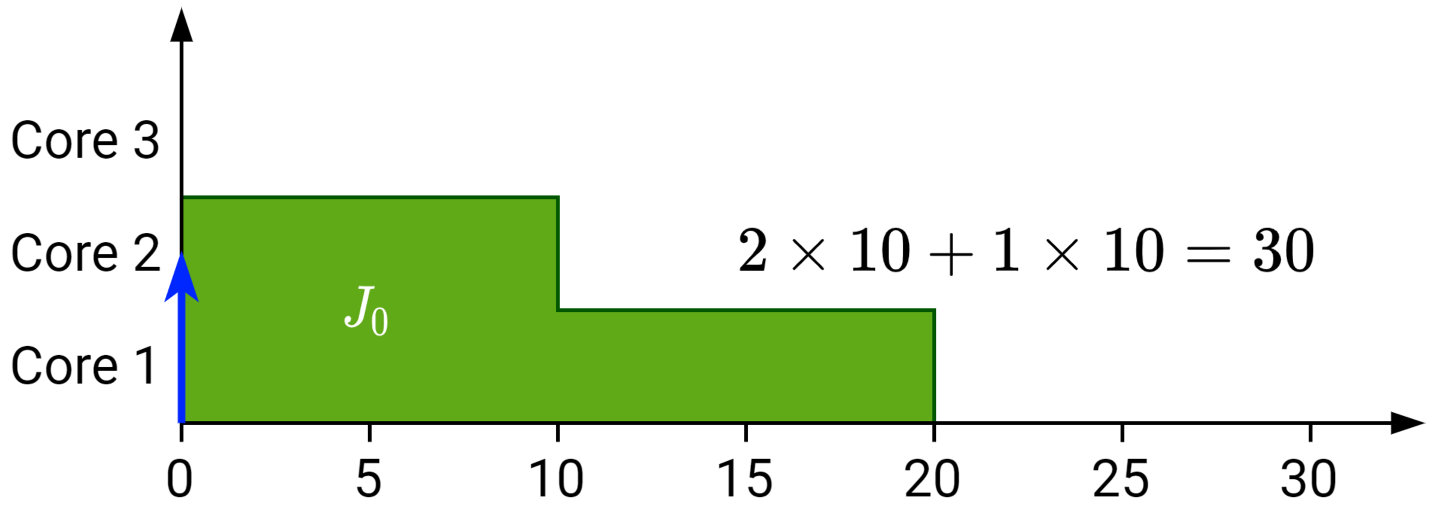

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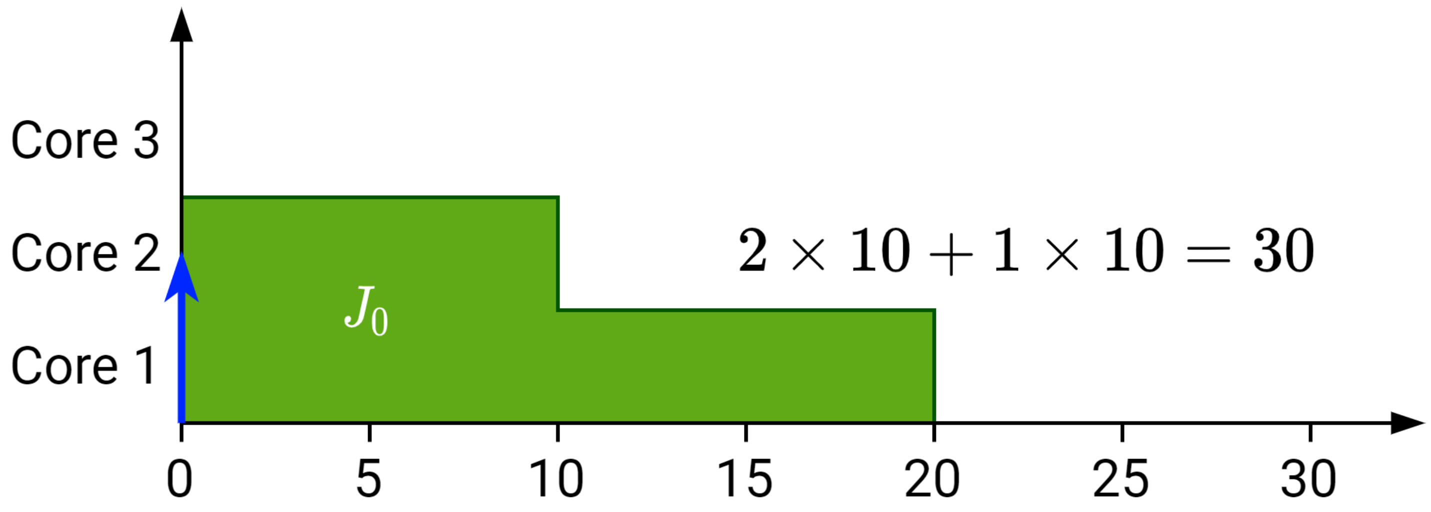

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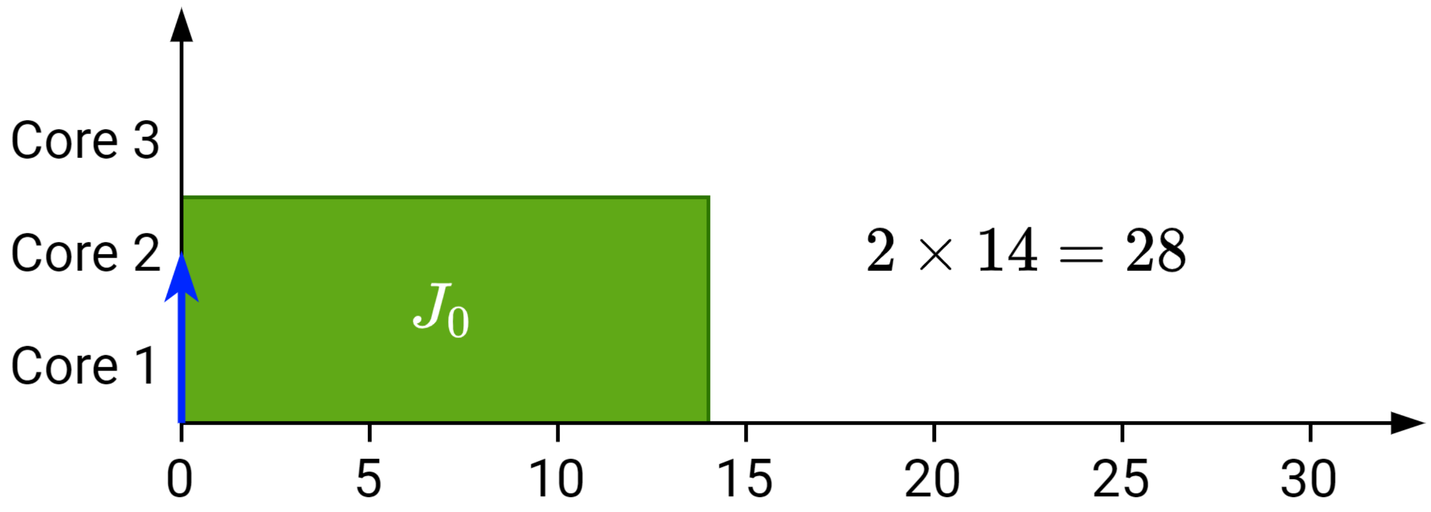
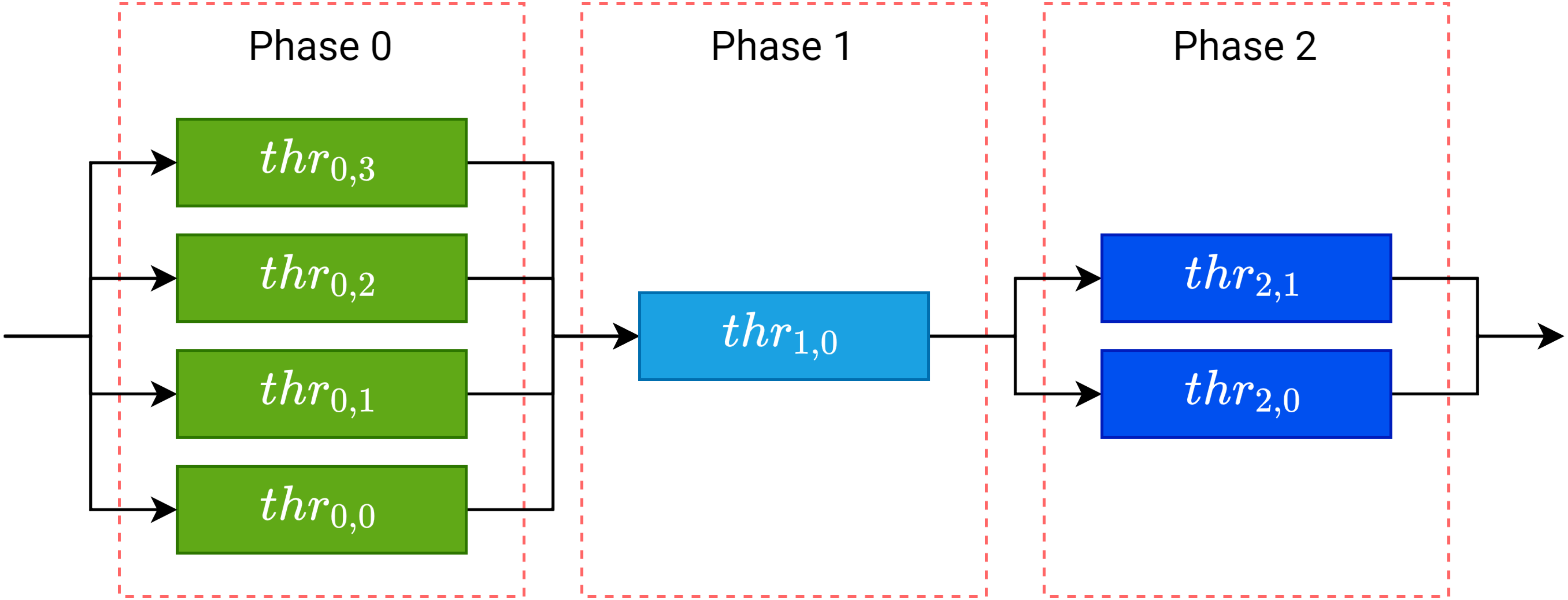
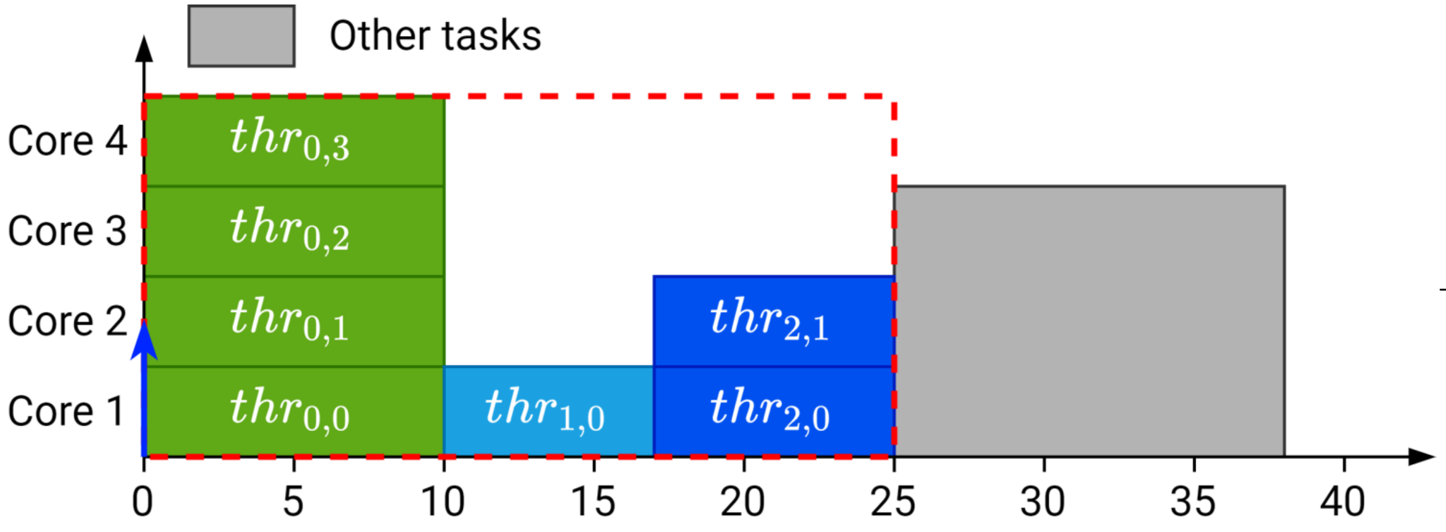


Bundled scheduling[1] vs limited-preemptive

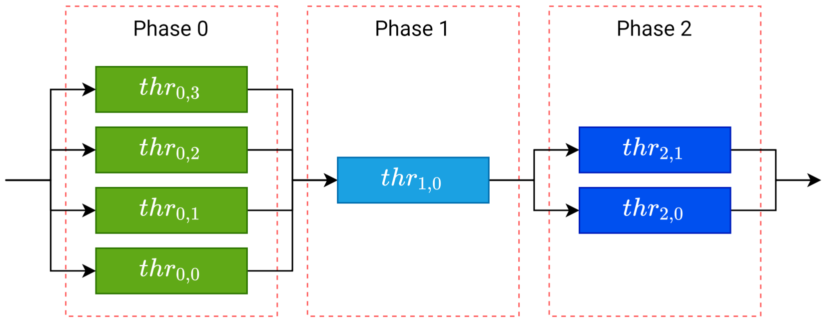


Bundled scheduling[1] vs limited-preemptive

- Rigid gang reserves the whole block



Rigid gang scheduling



[1]Wasly et al., 2017

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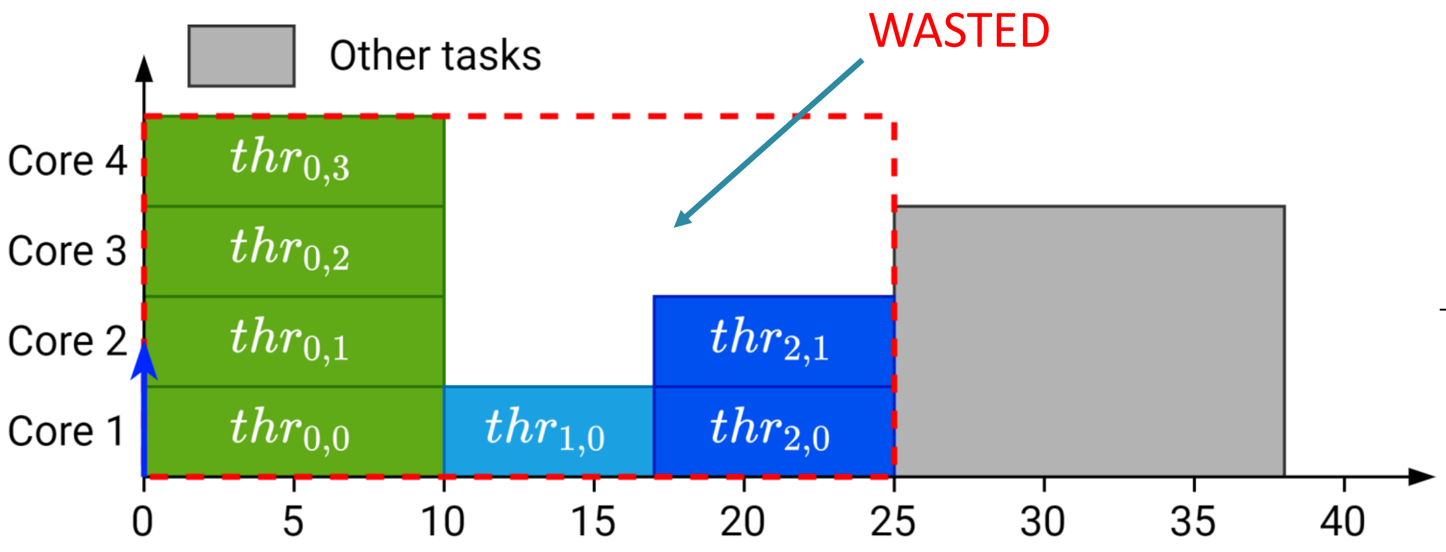

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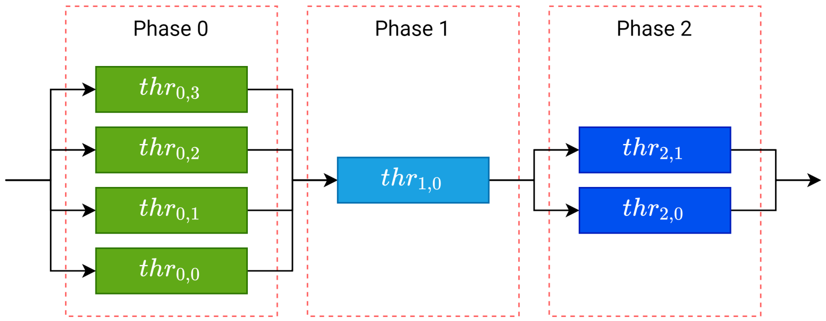

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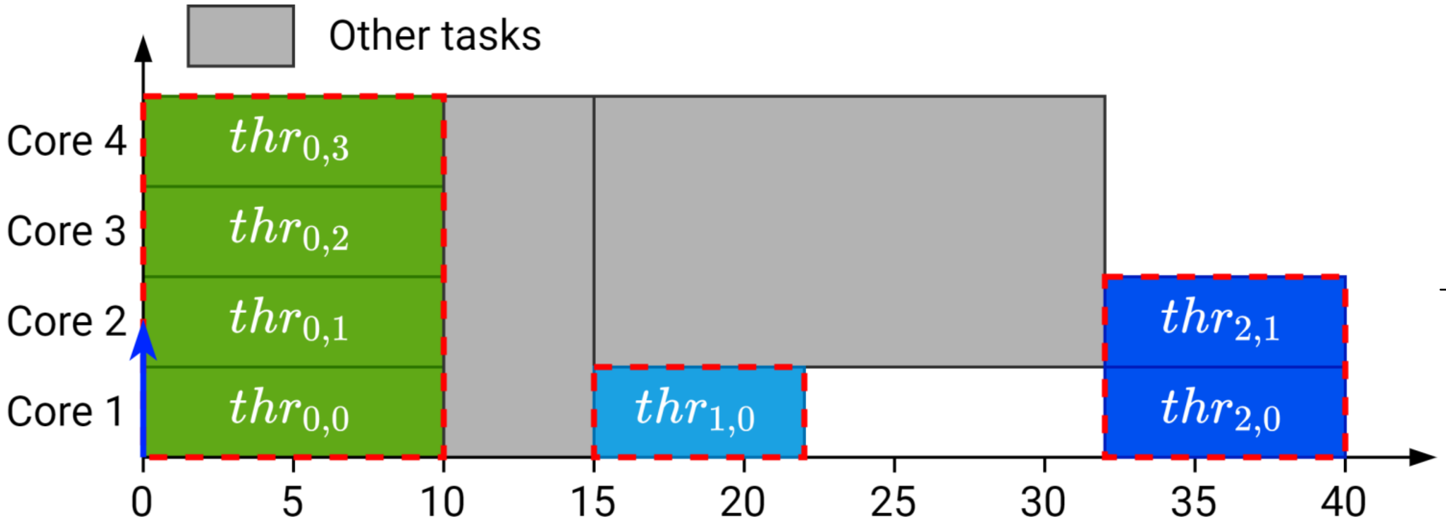
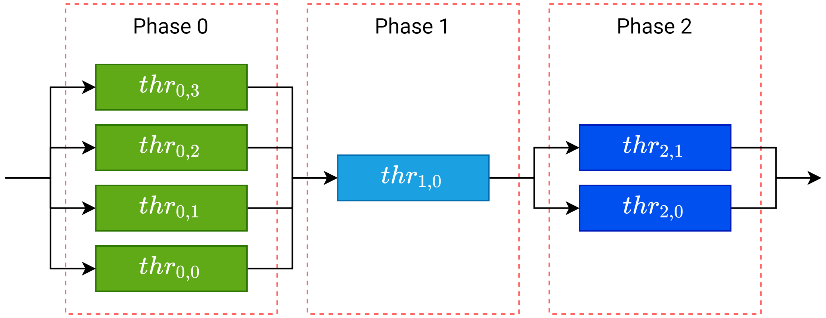
Bundled scheduling[1] vs limited-preemptive

- Rigid gang reserves the whole block
- Bundled creates rigid blocks with dependencies



Bundled model scheduling



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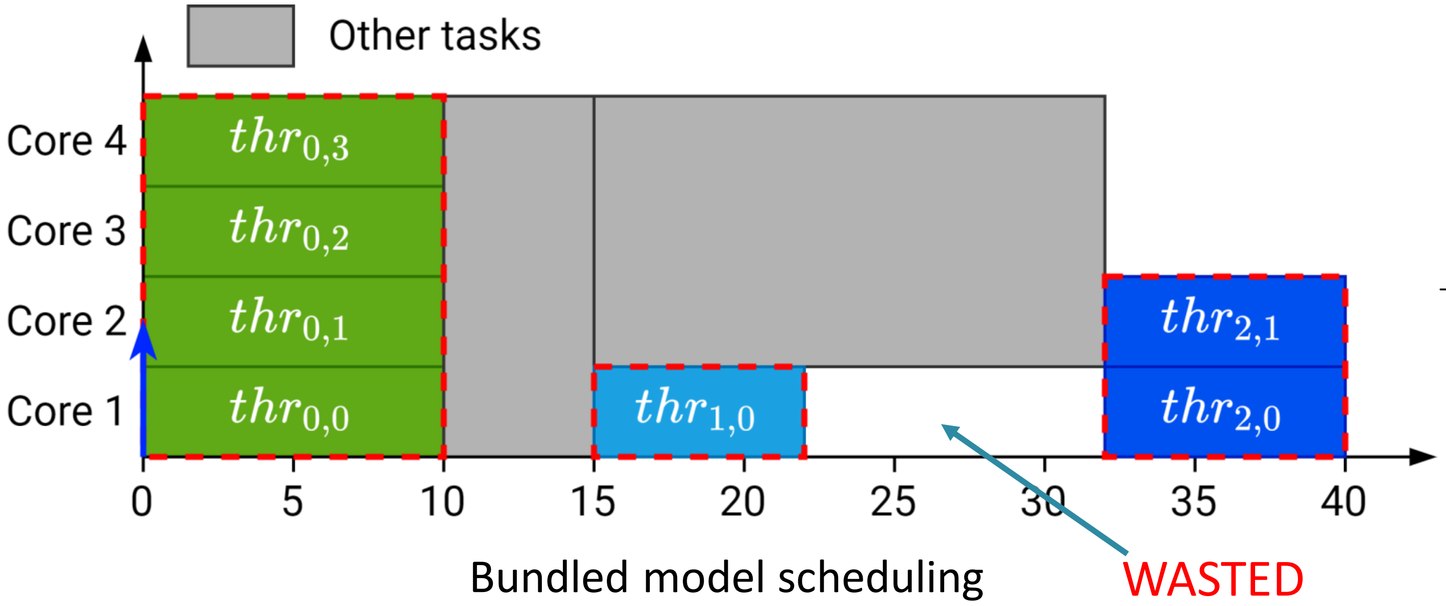

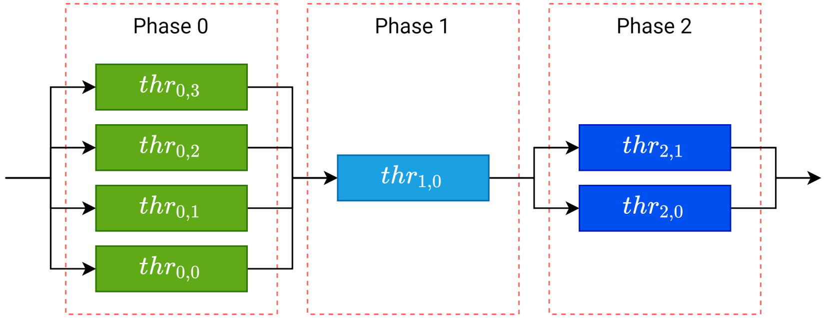

Bundled scheduling[1] vs limited-preemptive

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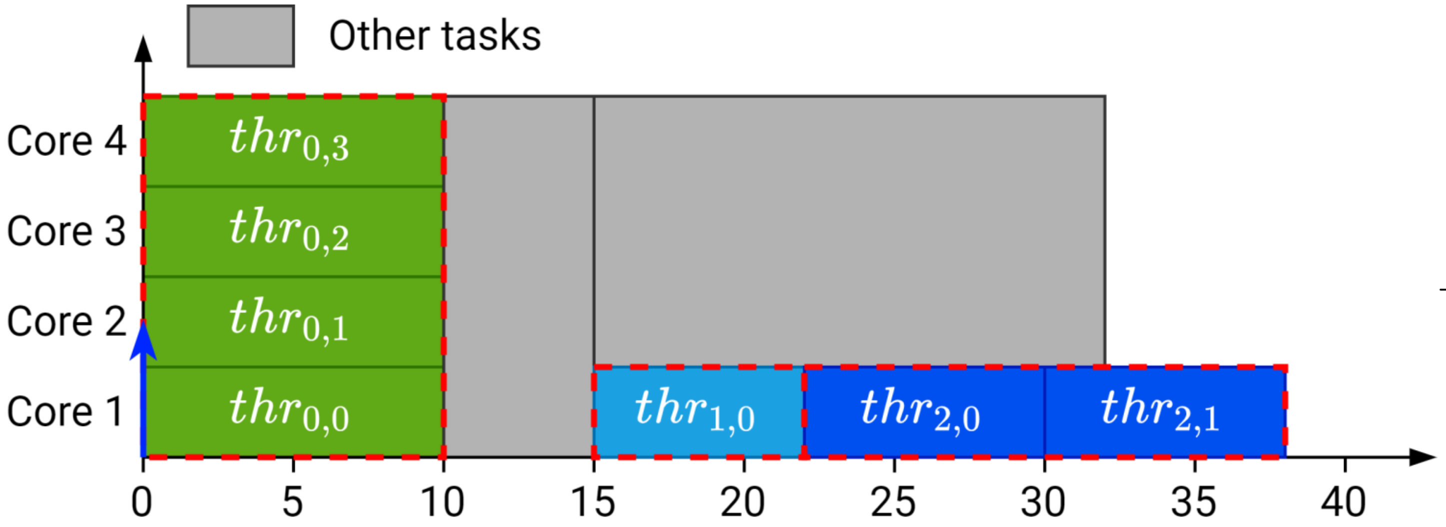
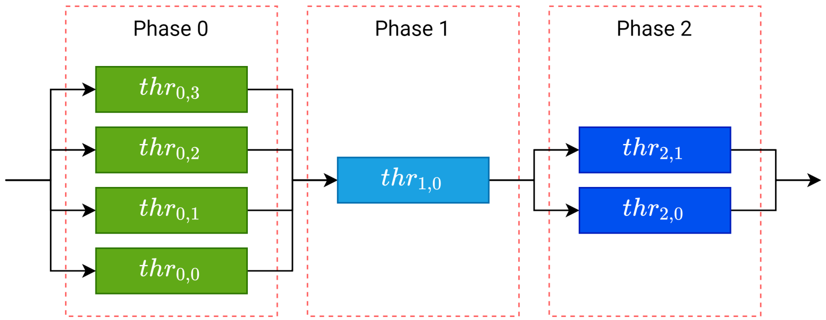
- Limited-Preemptive creates moldable blocks with dependencies



Limited-Preemptive scheduling



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Job-level fixed-priority scheduling (JLFP) for gang

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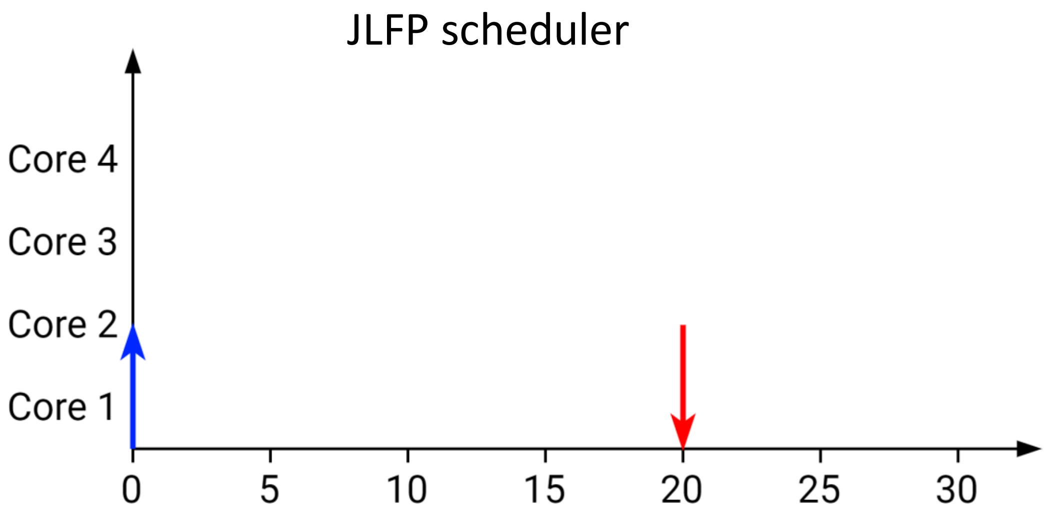

	Priority	Min cores	Max cores	Deadline	Execution time
J_0	High	2	3	∞	15, 10
J_1	Low	2	2	20	15

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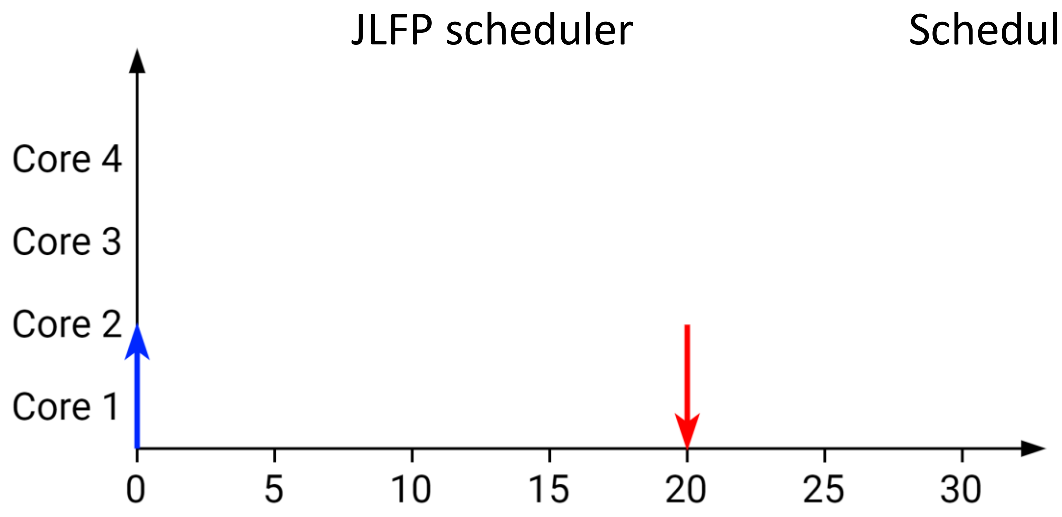

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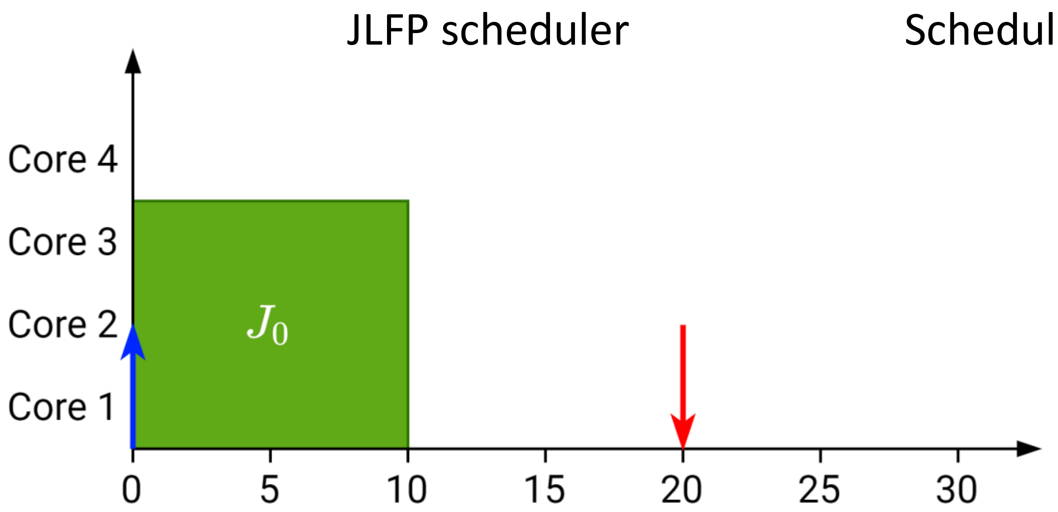

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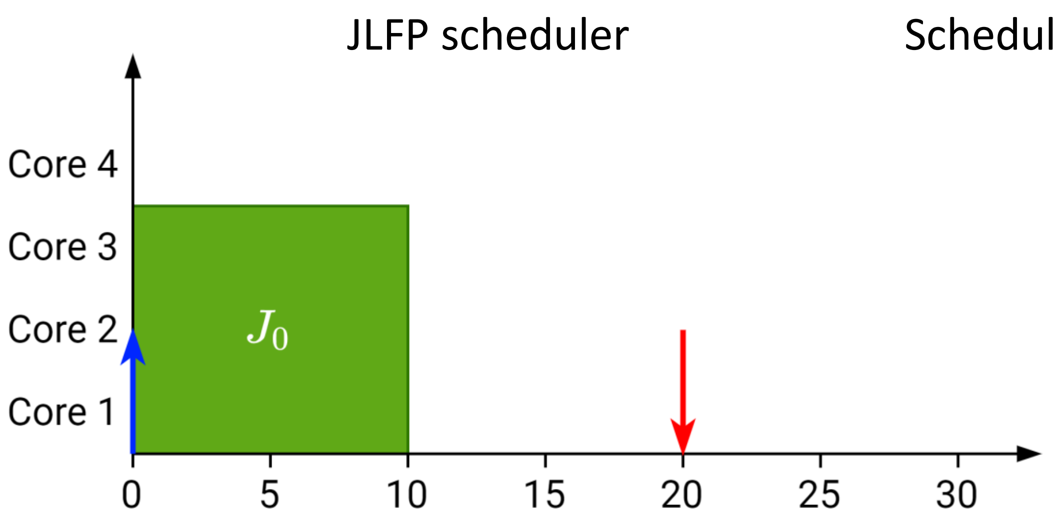

Scheduling: J_1

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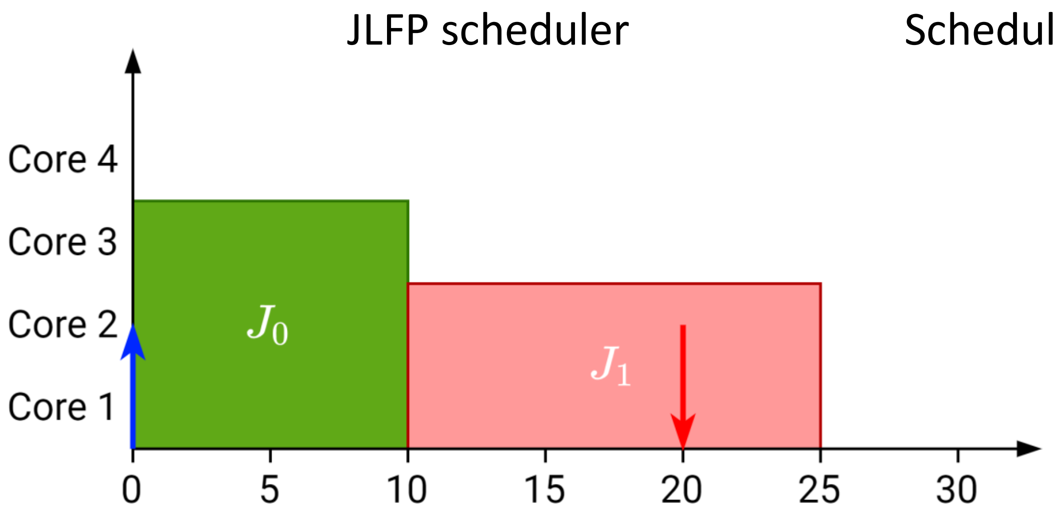

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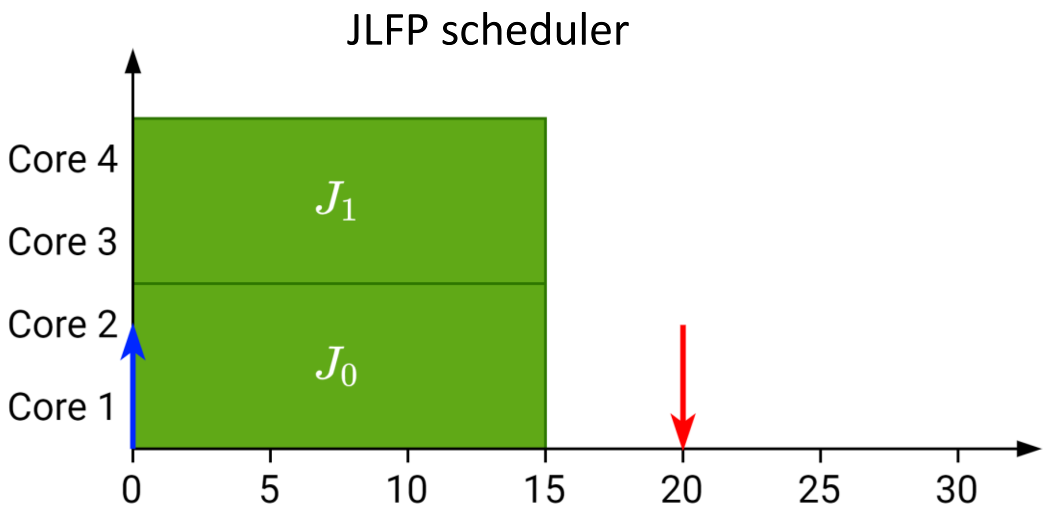

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Preemptive solutions

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Schedulability tests

- Job-level fixed-priority[2]
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Schedulers

- Optimal for rigid gang (DP-Fair)[4]

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Non-preemptive solutions

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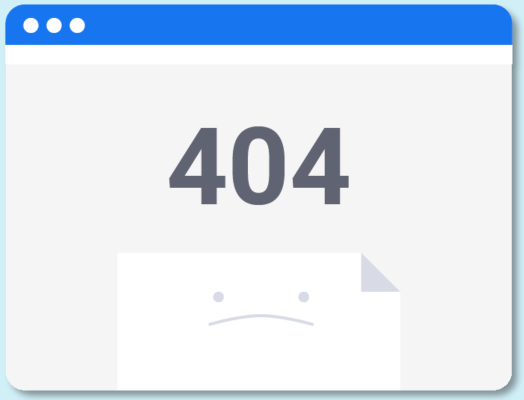

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Our work

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2. Propose a new scheduling algorithm to improve the schedulability of limited-preemptive moldable gang tasks

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2. Propose a new scheduling algorithm to improve the schedulability of limited-preemptive moldable gang tasks
   - Extend analysis to support this new algorithm

Agenda

- Gang schedulability analysis
- New scheduling policy

Schedule abstraction graph

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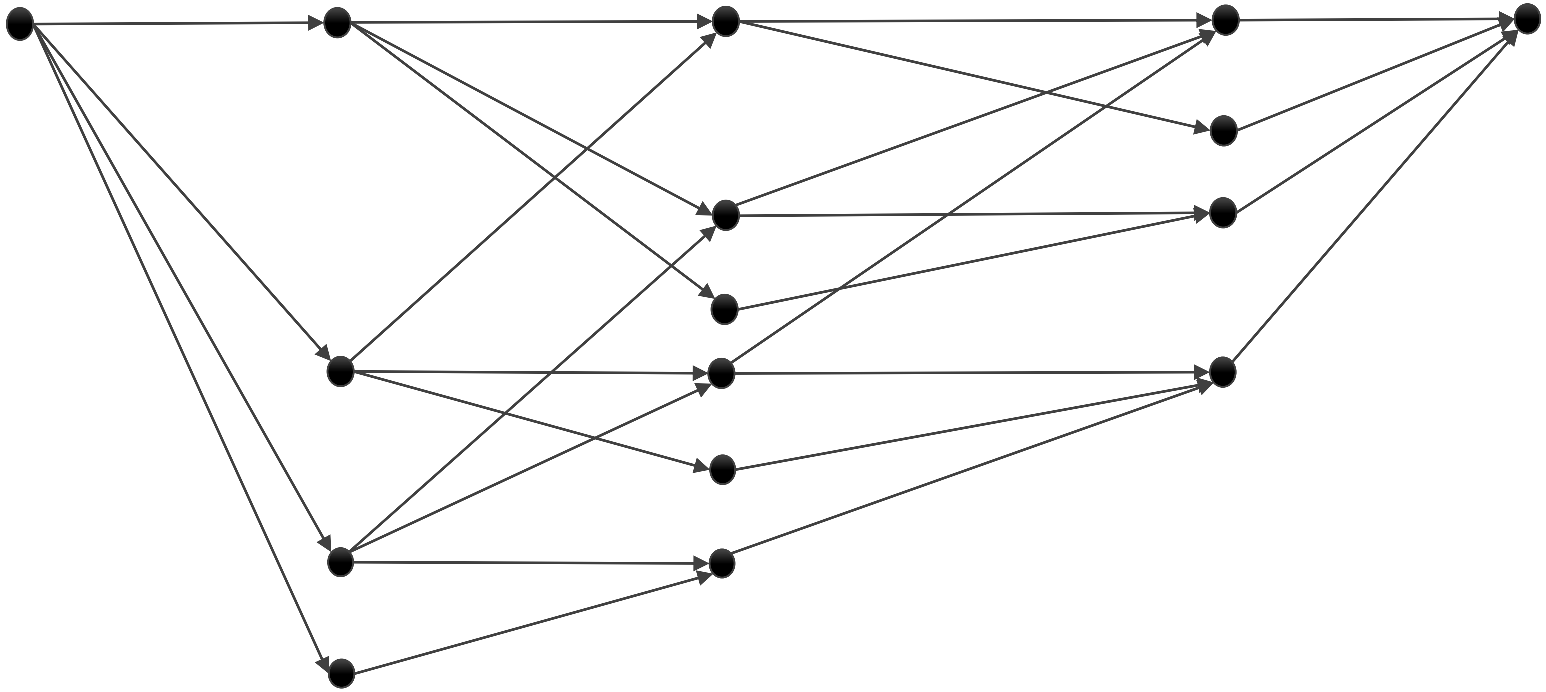

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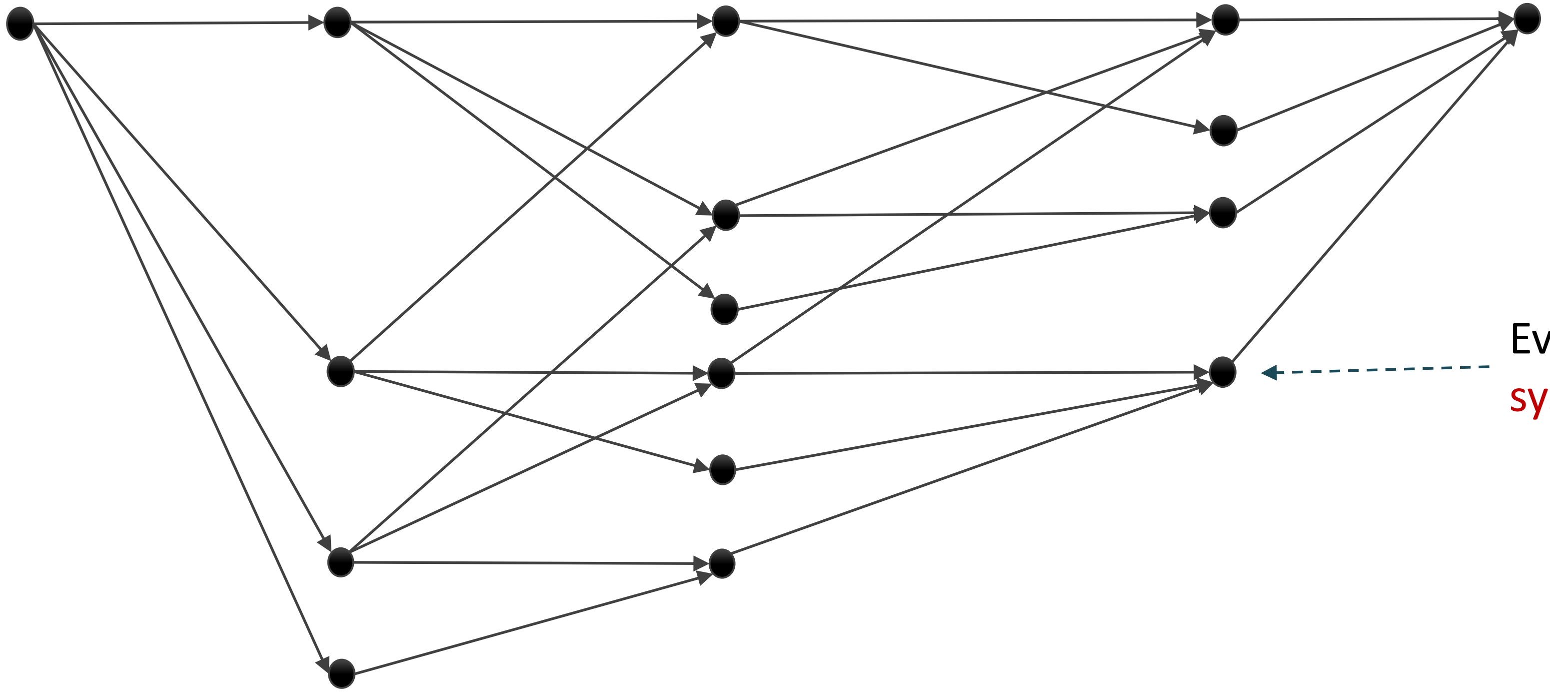
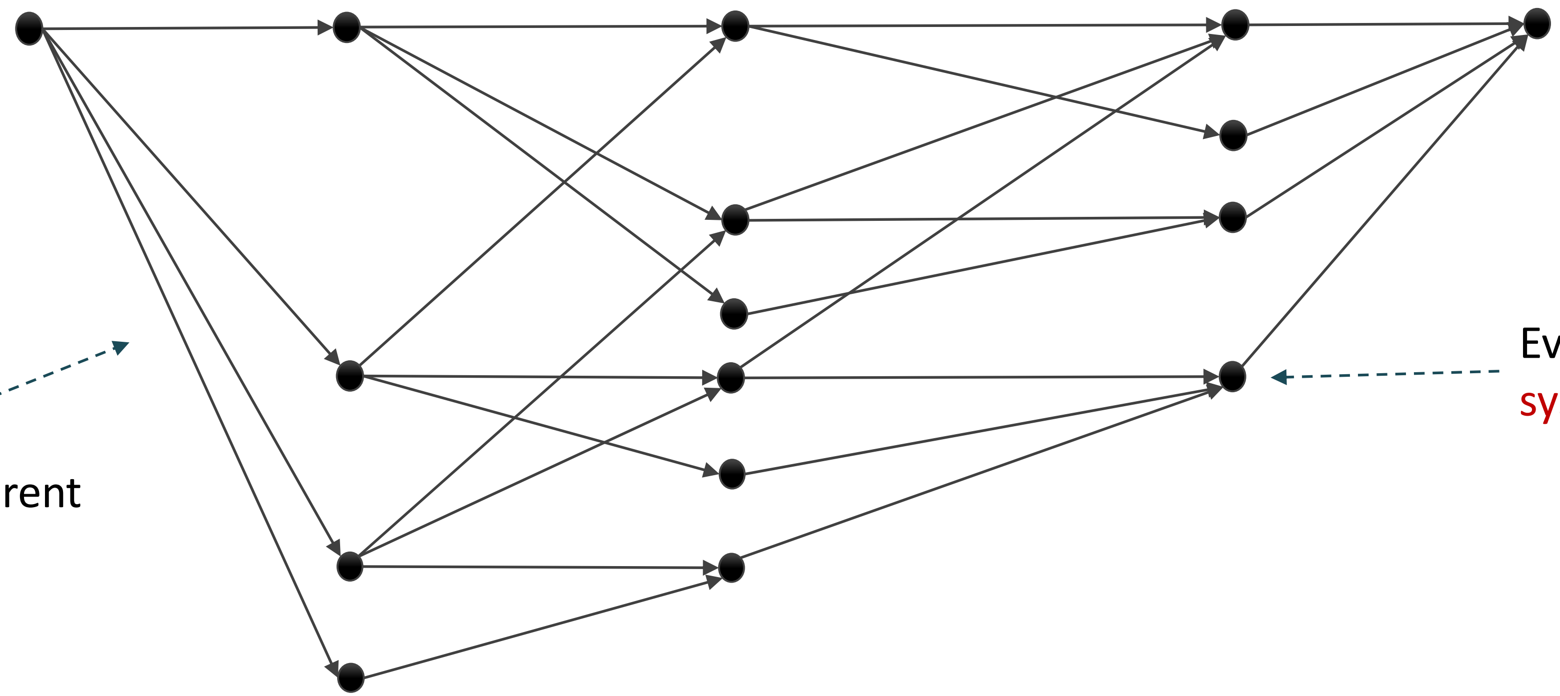


Every node is a different system state

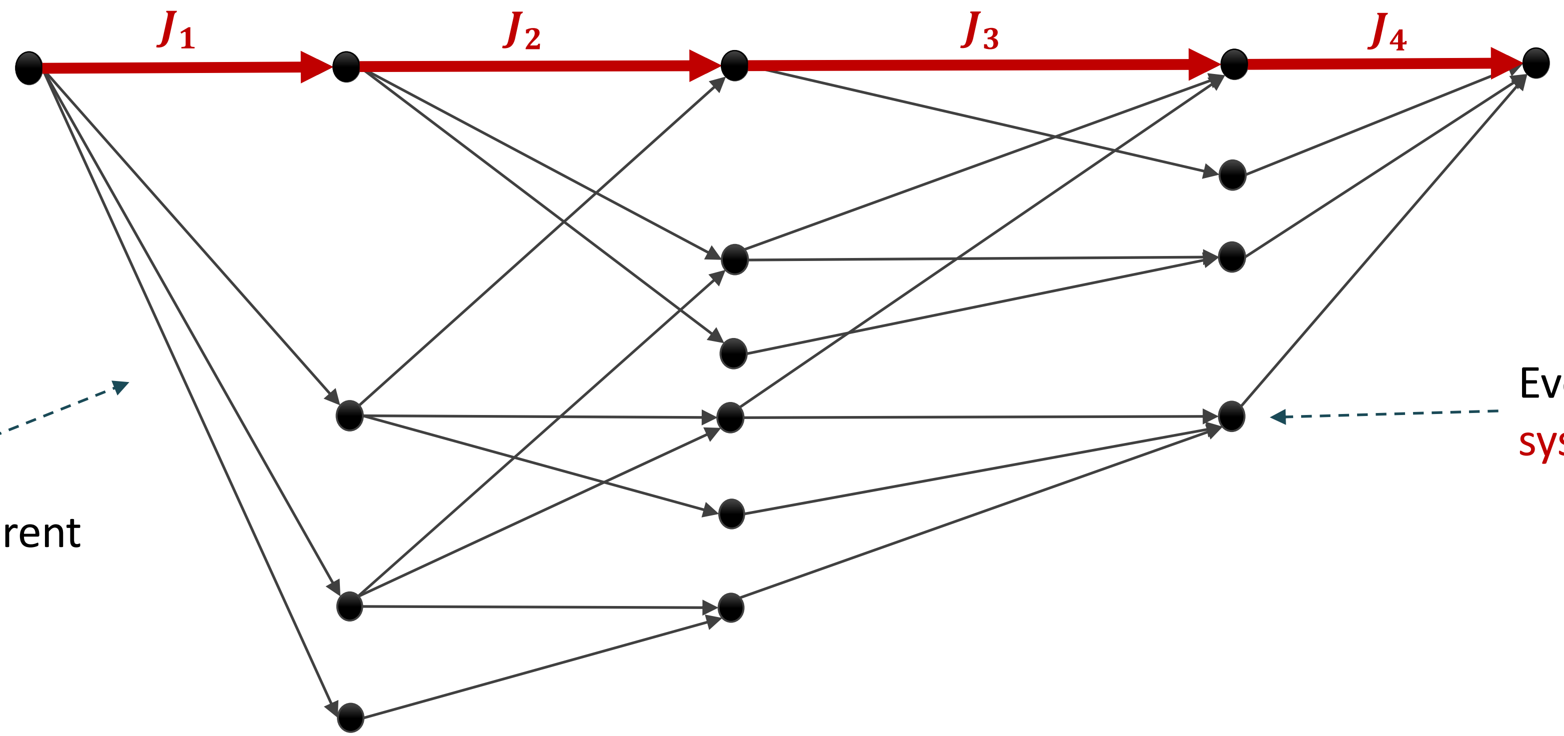
Schedule abstraction graph



Every edge is a different scheduler decision

Every node is a different system state

Schedule abstraction graph



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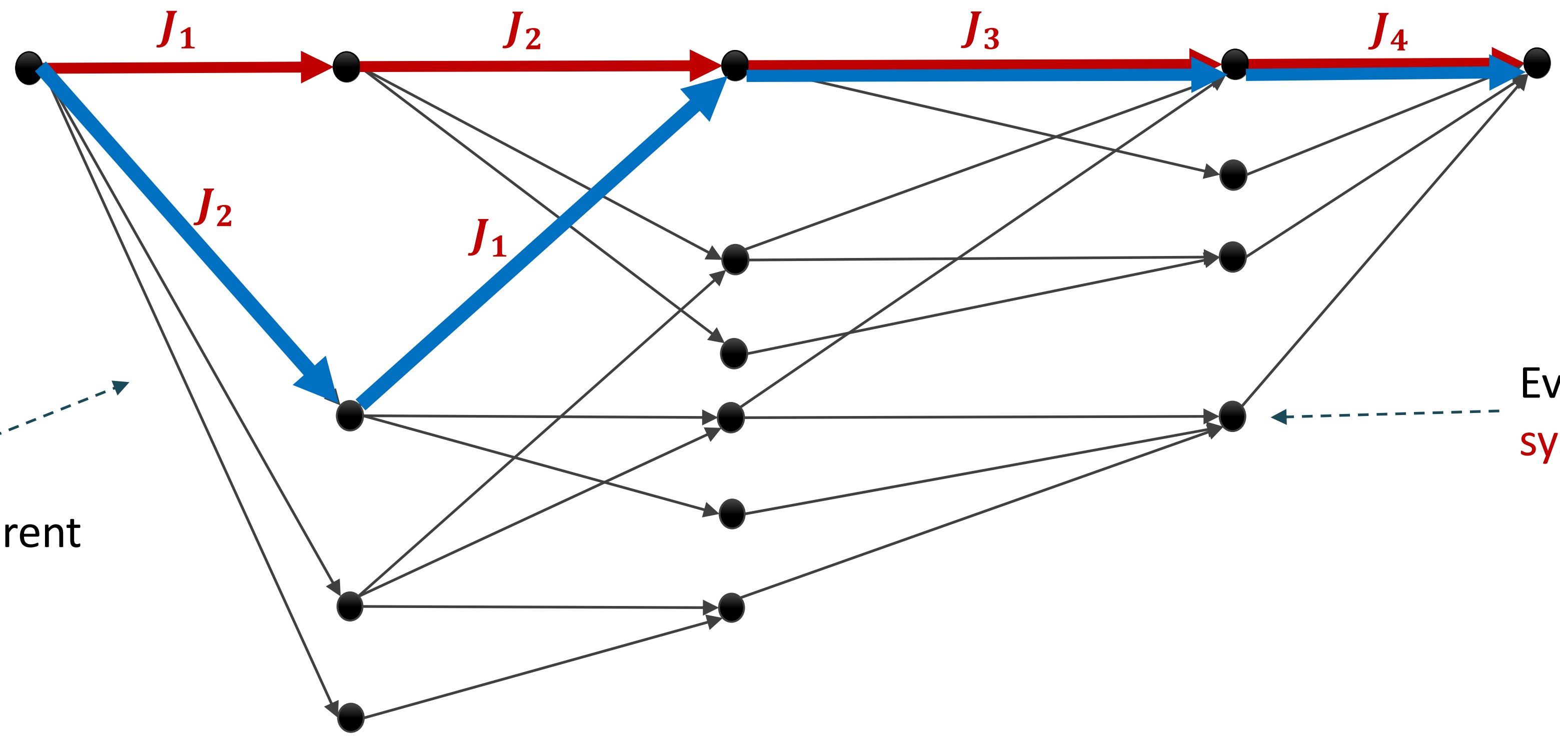
Schedule abstraction graph

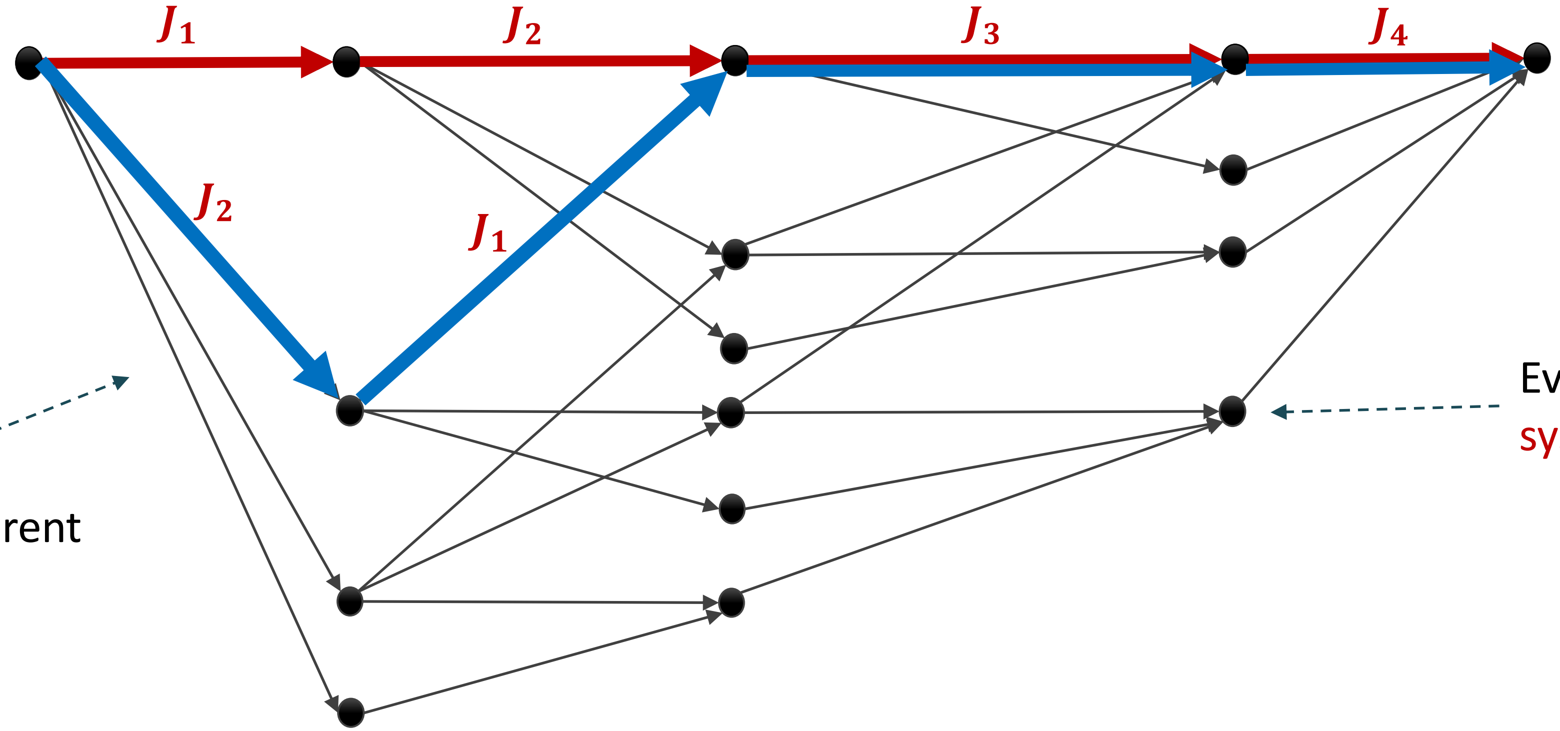


Every edge is a different scheduler decision

Every node is a different system state

Schedule abstraction graph

- It is a technique that allows:



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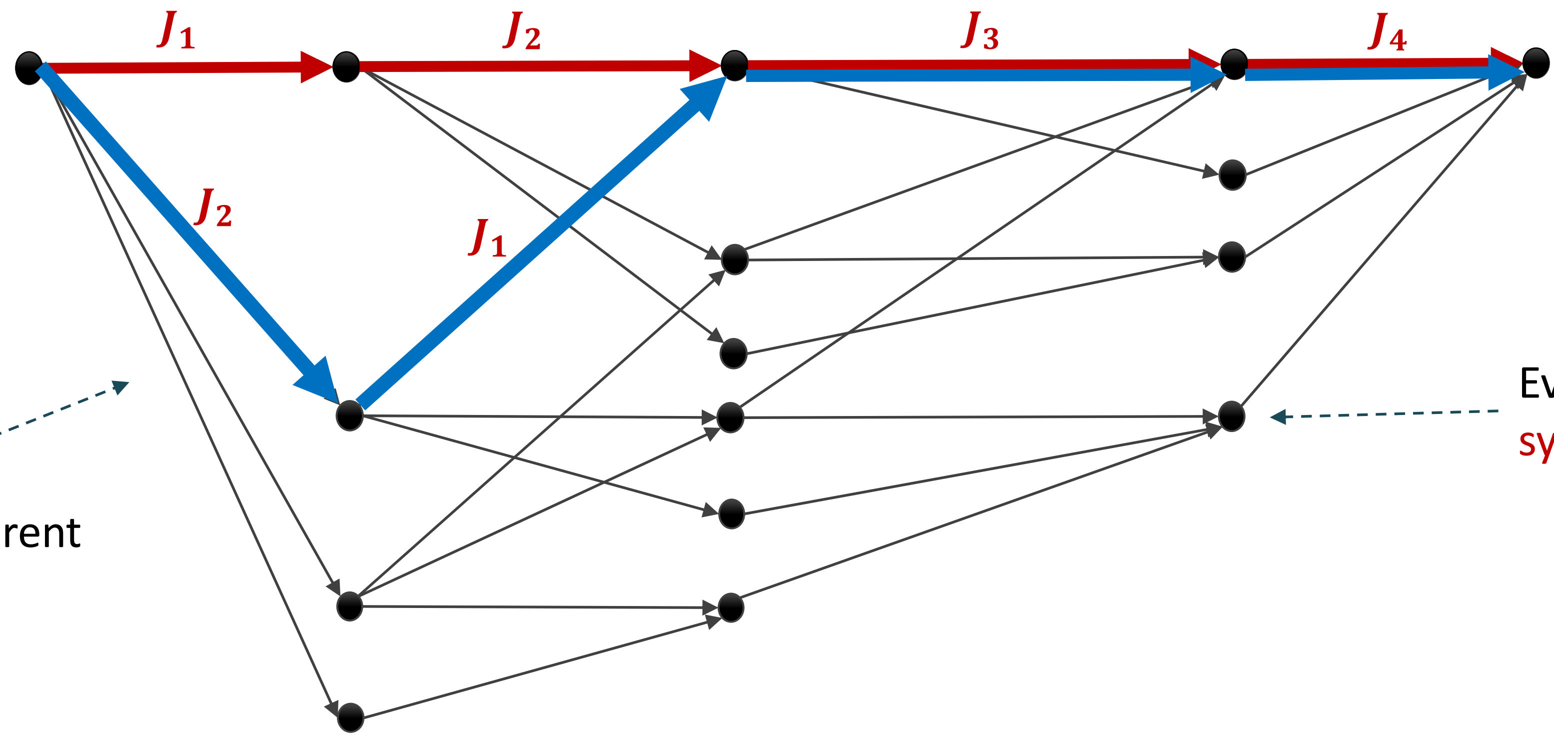
Schedule abstraction graph

- It is a technique that allows:
  - Search for all possible schedules



Every edge is a different scheduler decision

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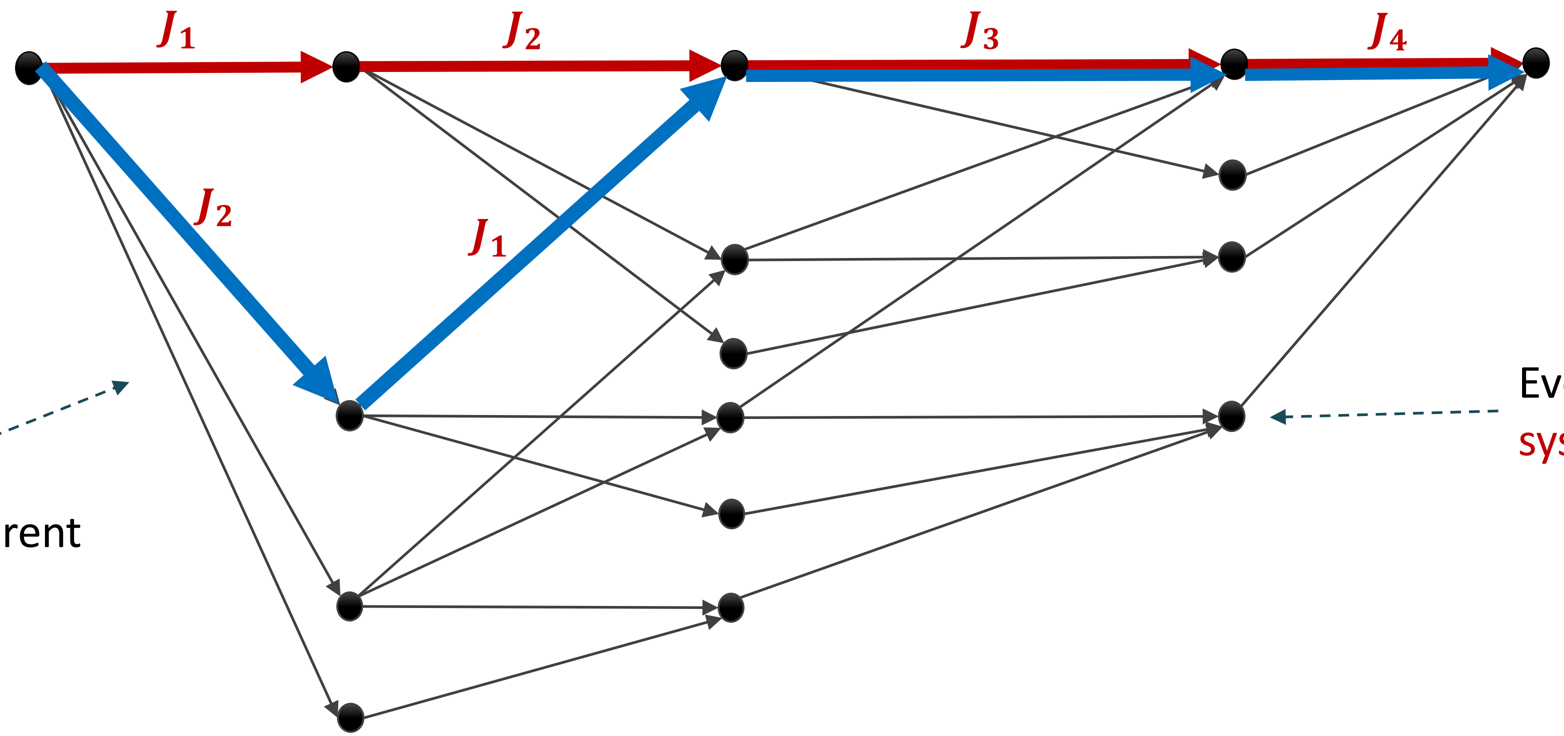
Schedule abstraction graph

- It is a technique that allows:
  - Search for all possible schedules
  - Aggregate “similar” schedules



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SAG analysis changes for gang

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- Previously a state was created for every schedulable job

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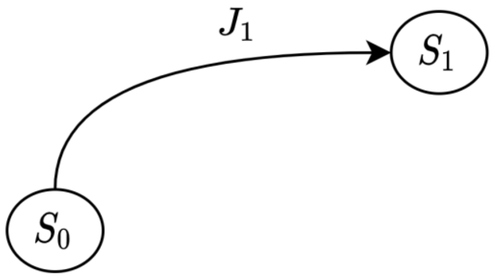

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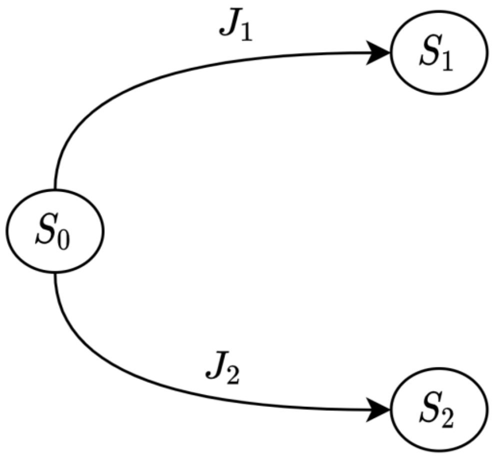
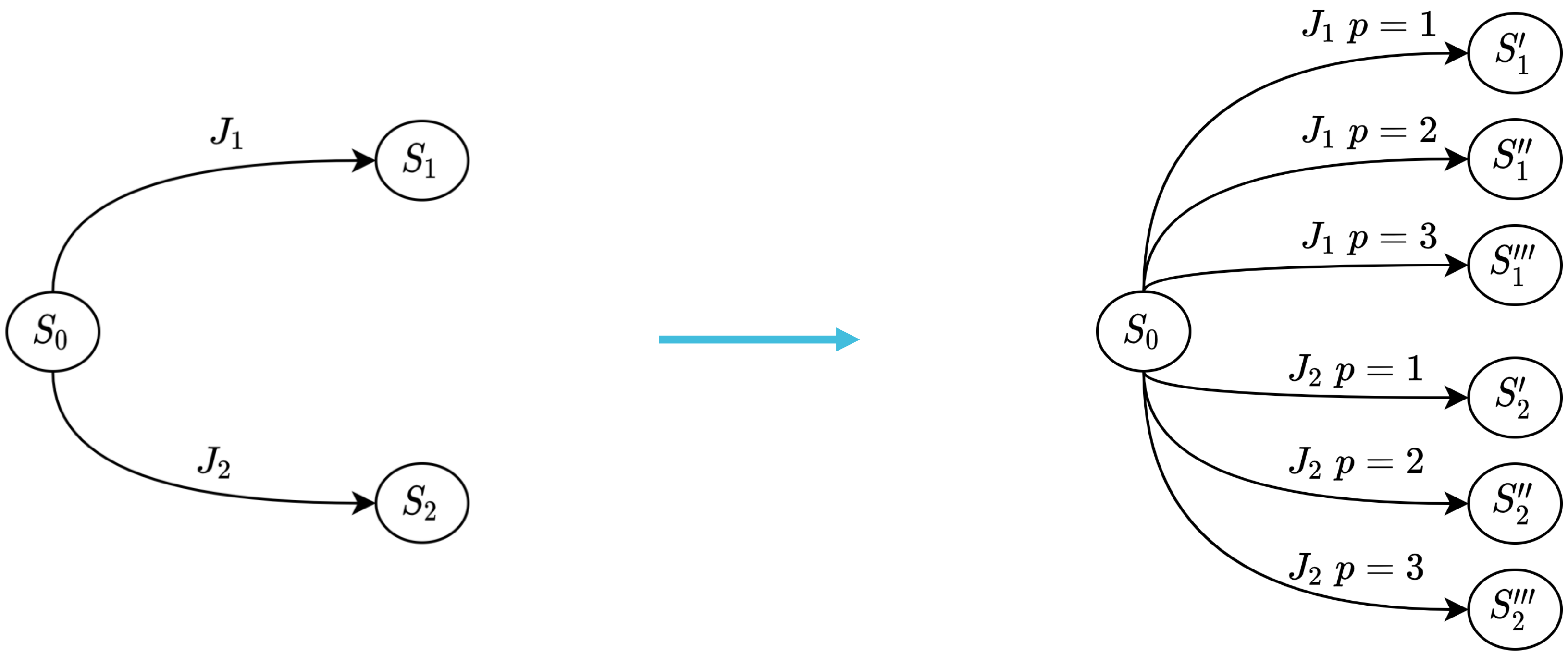


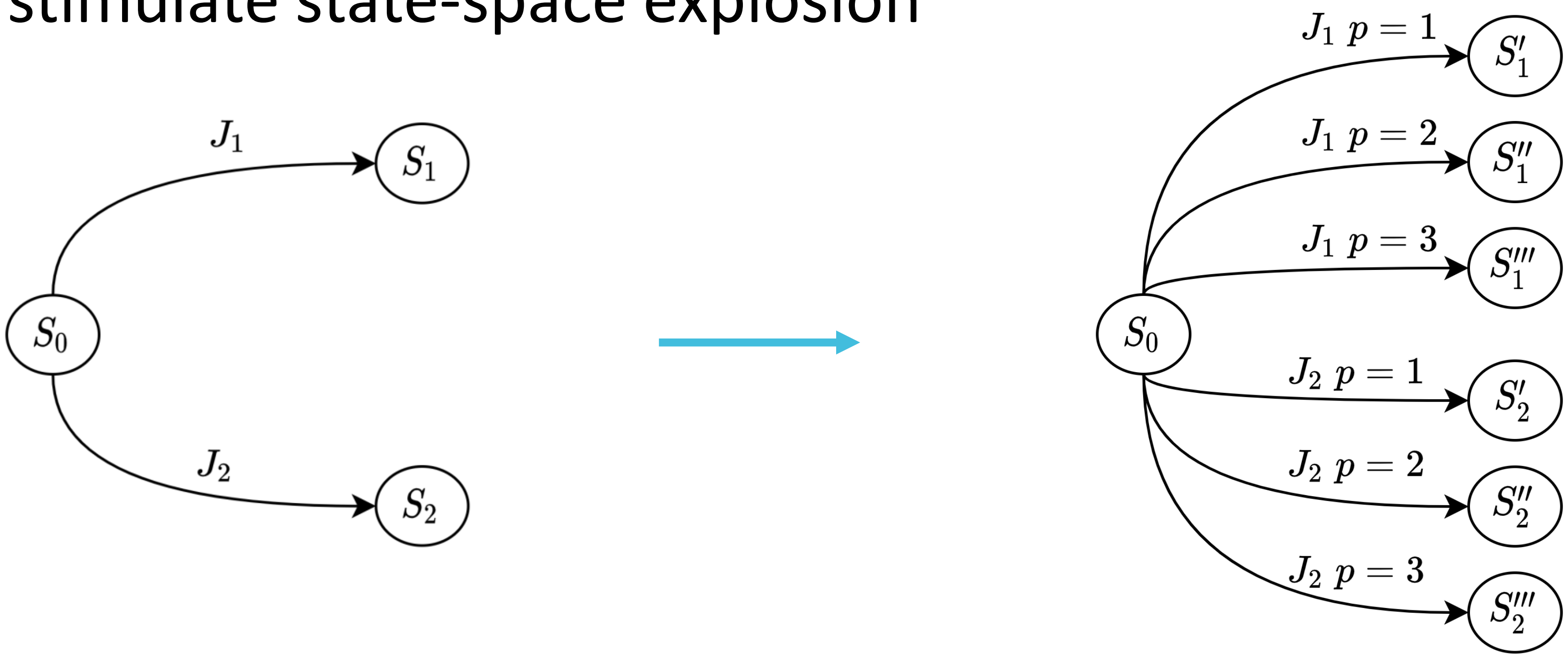
SAG analysis changes for gang

- Previously a state was created for every schedulable job
- Now a state is created for every job and possible number of cores



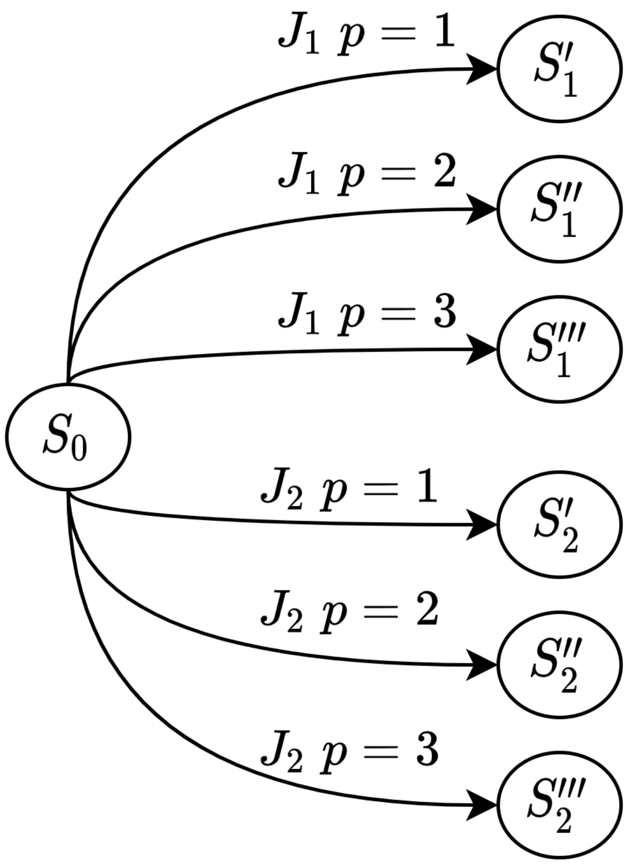
SAG analysis changes for gang

- Previously a state was created for every schedulable job
- Now a state is created for every job and possible number of cores
- Can stimulate state-space explosion



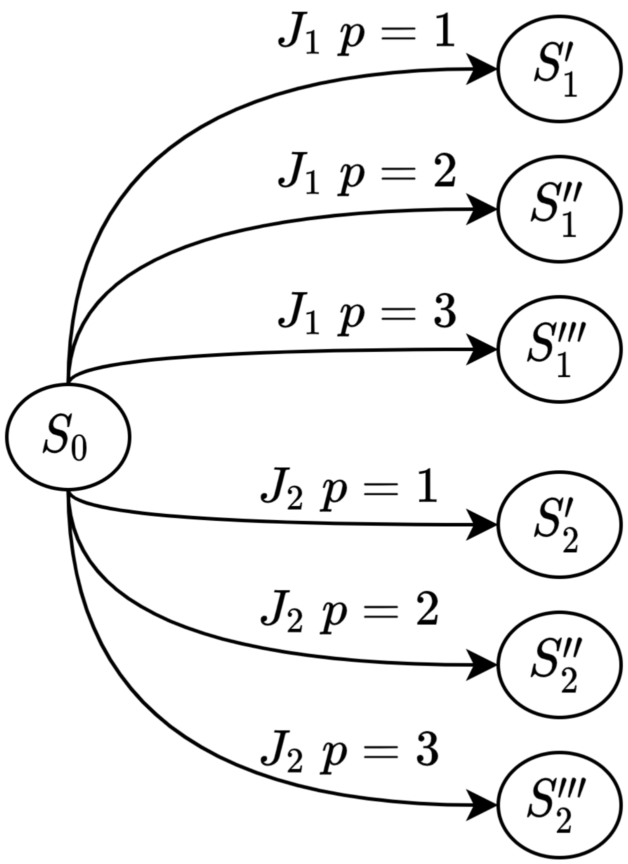
SAG analysis changes for gang

- cores available



SAG analysis changes for gang

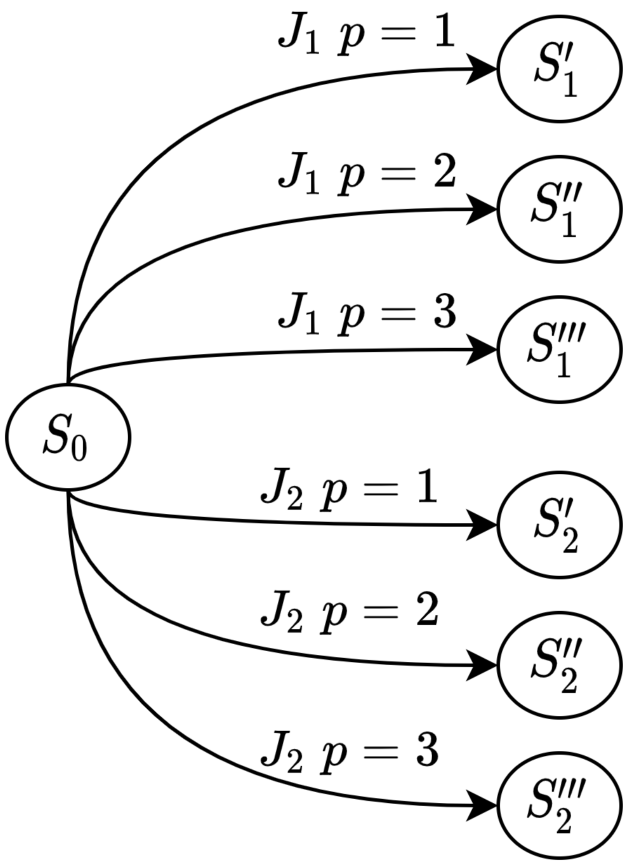
- Exploring more states
  - cores available



SAG analysis changes for gang

- Exploring more states
  - Is safe, does not make analysis invalid

  - cores available



SAG analysis changes for gang

- Exploring more states
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  - 👎 Slower and more pessimistic

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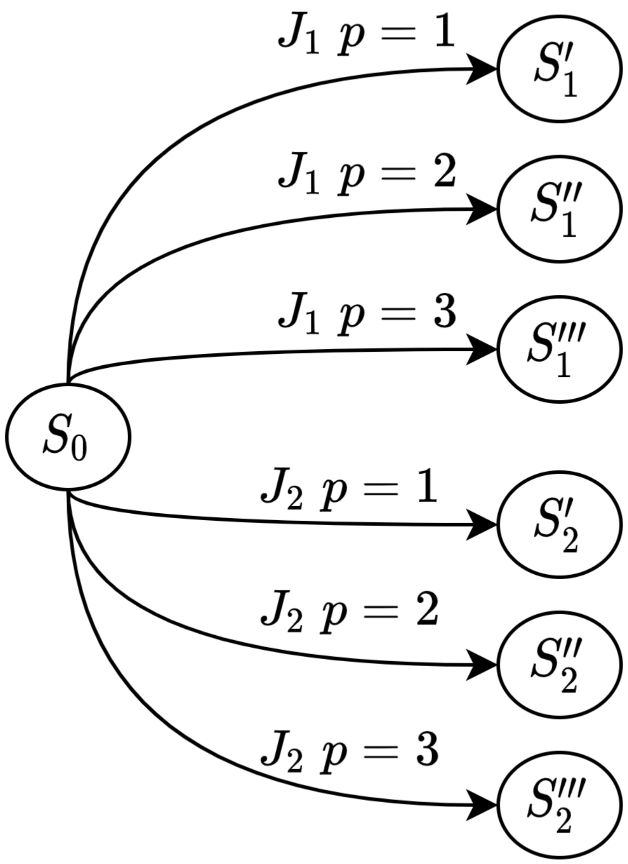

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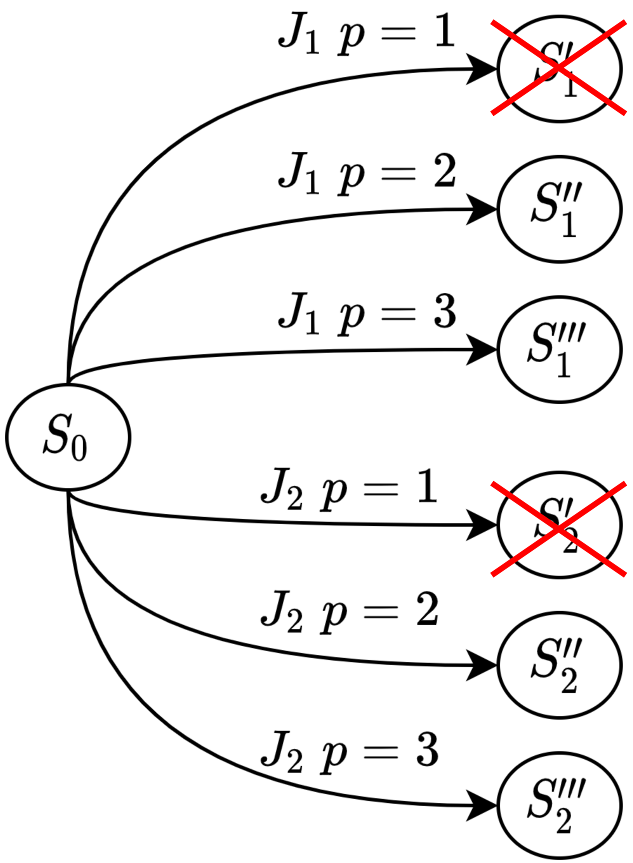

SAG analysis changes for gang

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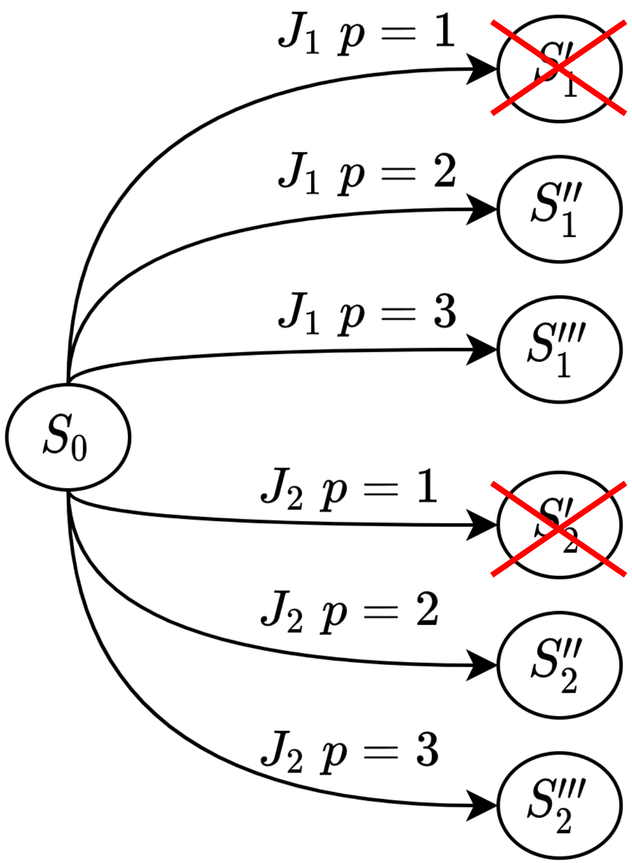

SAG analysis changes for gang

- Exploring more states
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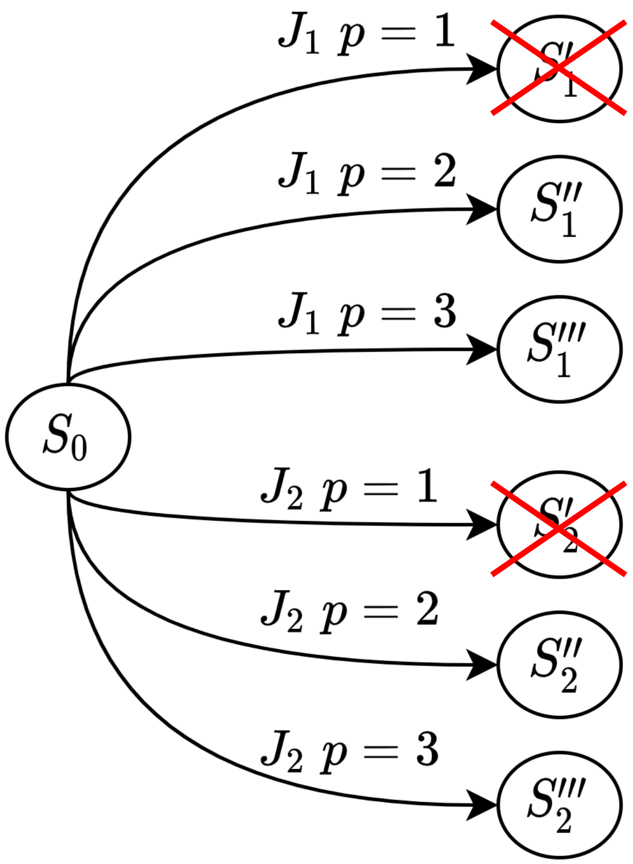

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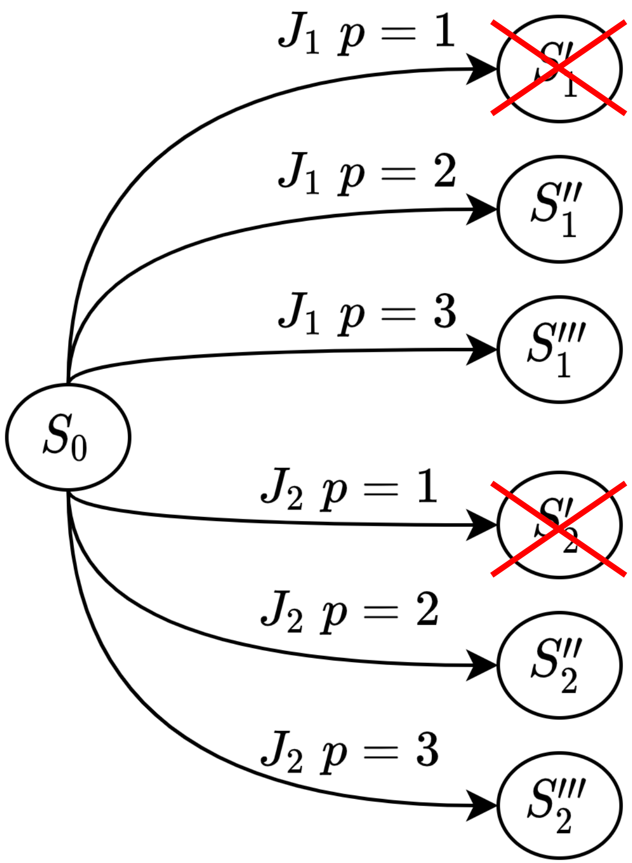

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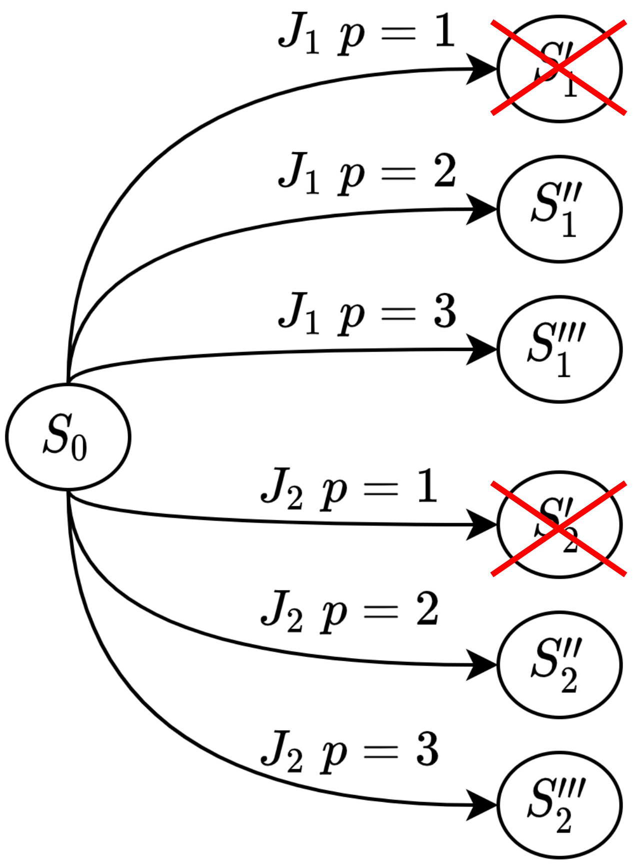

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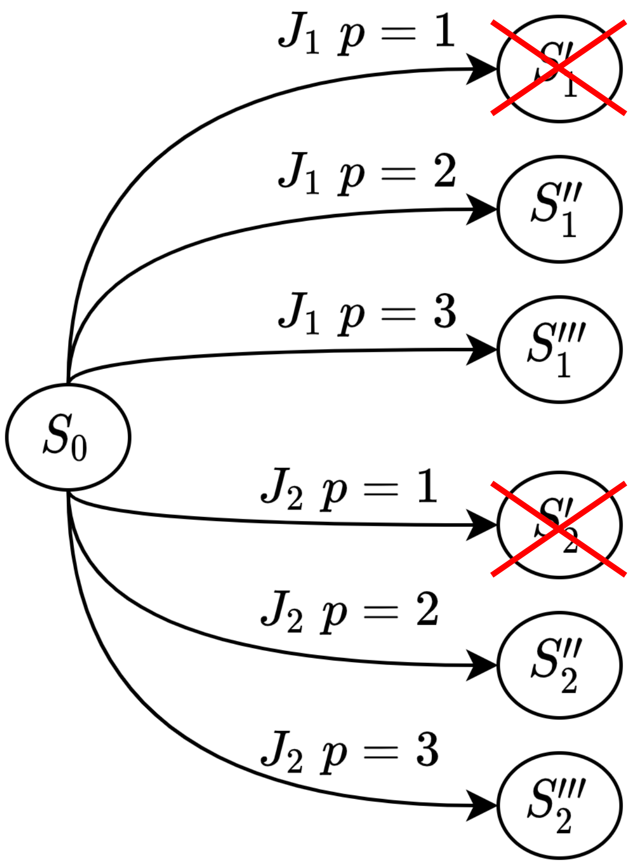
- Proofs



Our analysis’ results

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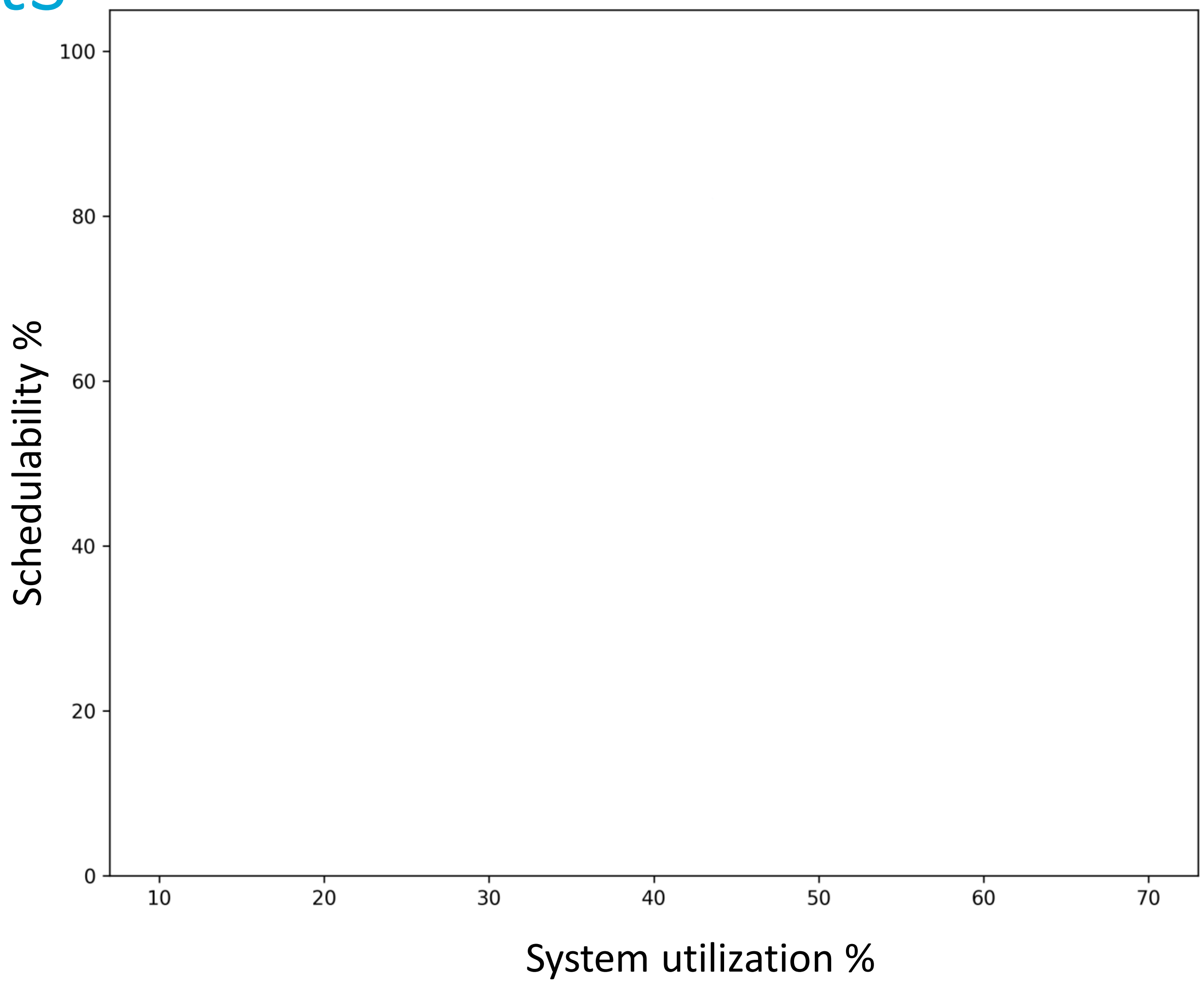
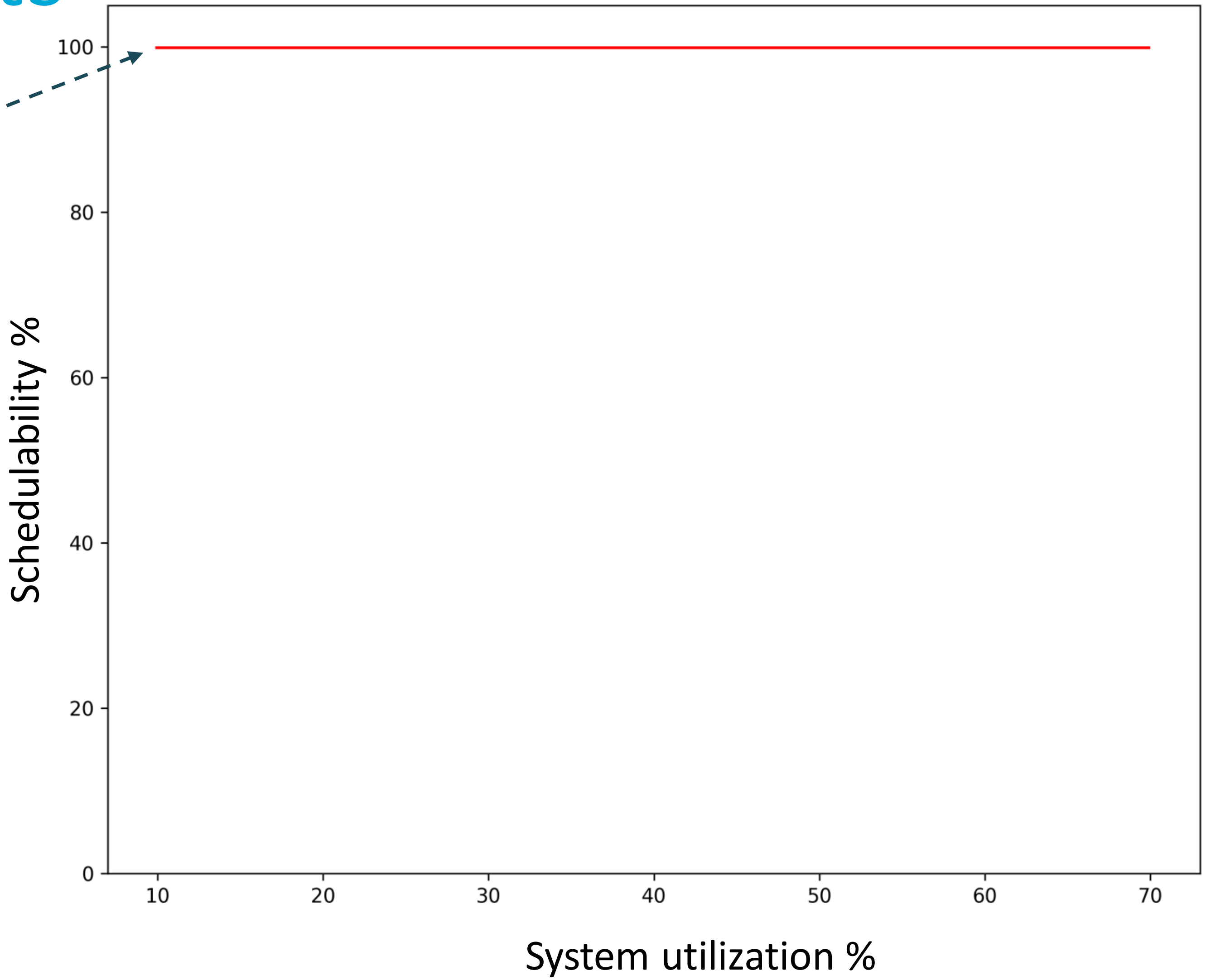


Our analysis' results

All task sets pass necessary test

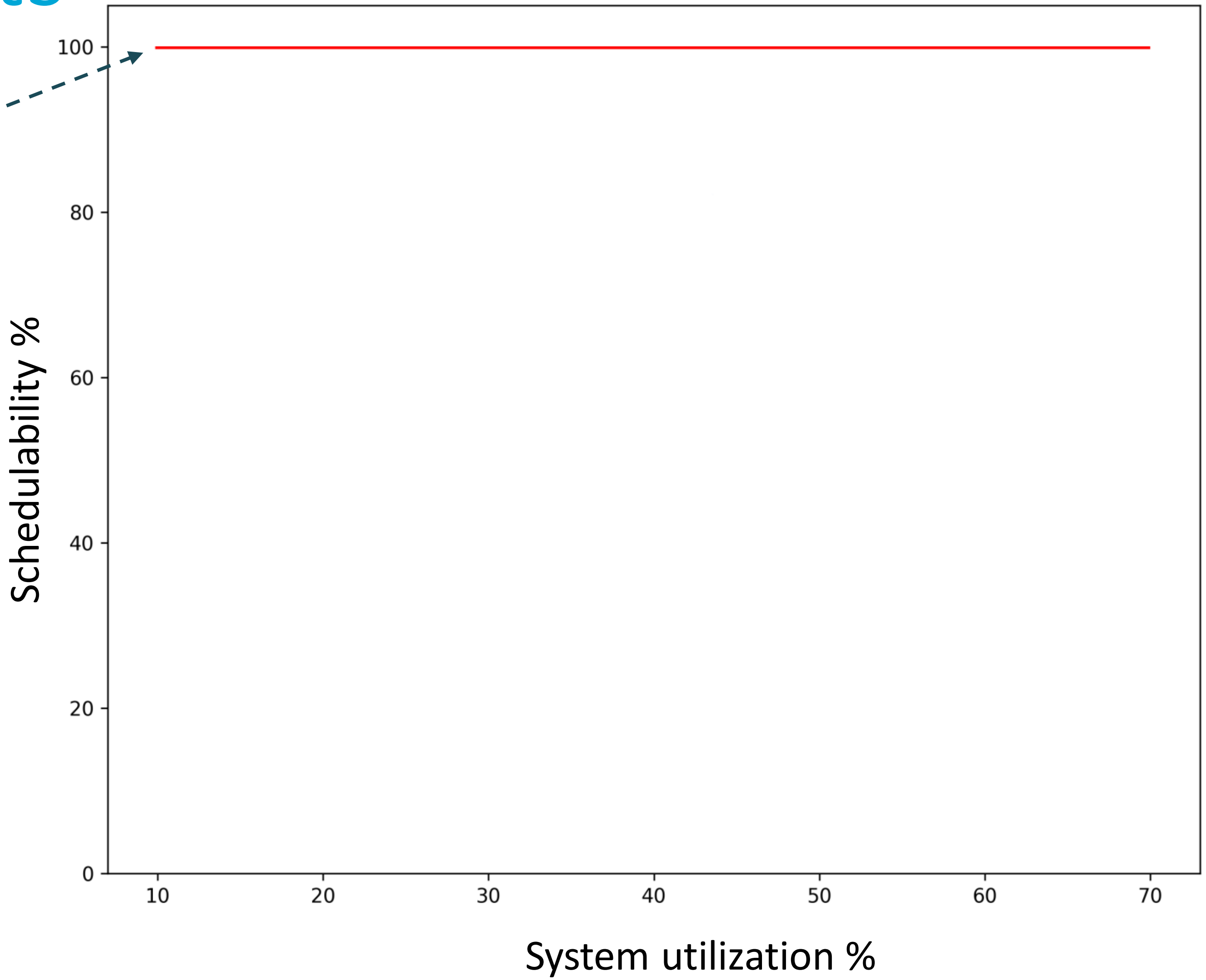


Our analysis' results

All task sets pass necessary test

- System processors: 8
- System tasks: 4
- Execution-time variation: 25%
- Segments per task: 1

Randomly generated task sets



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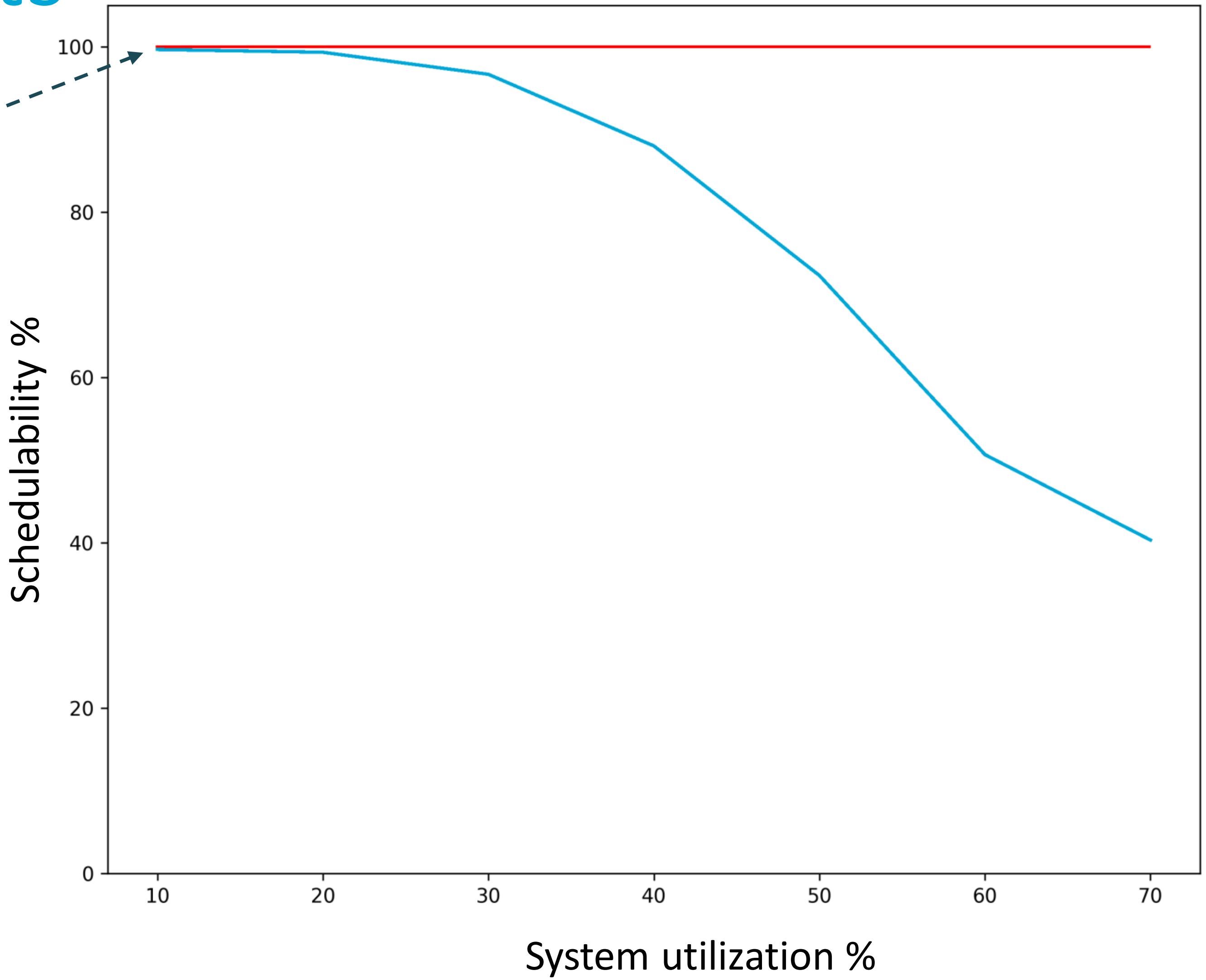

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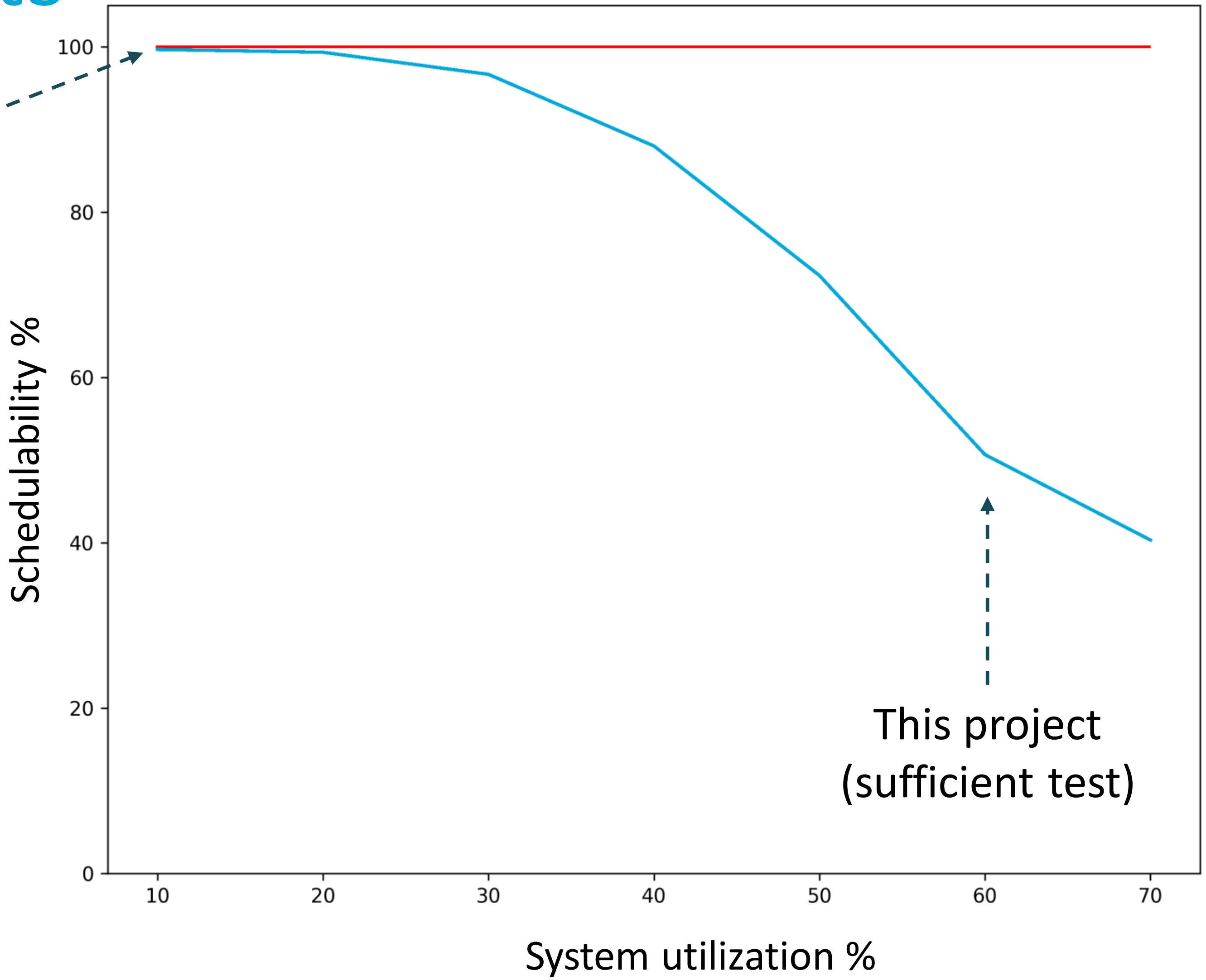

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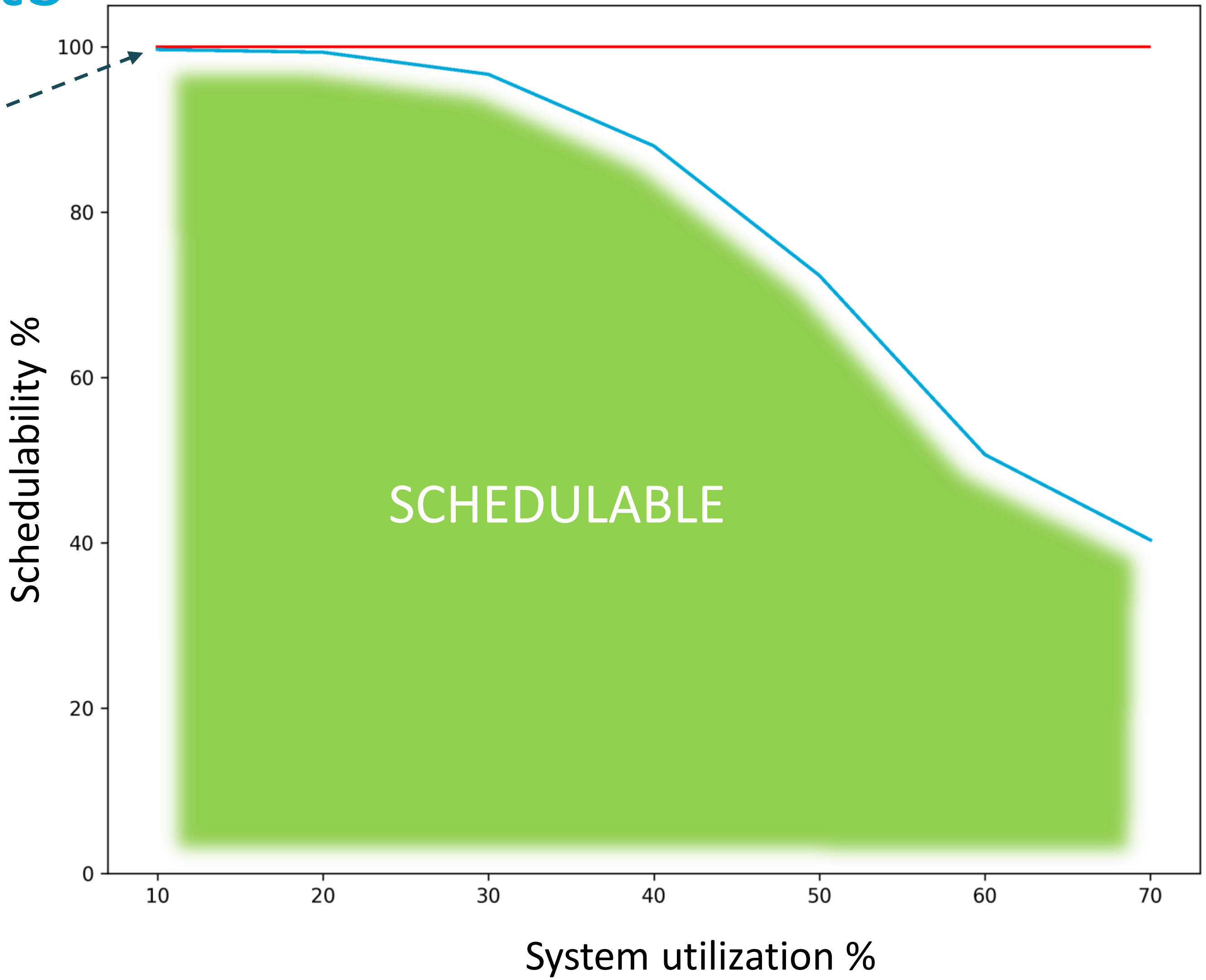

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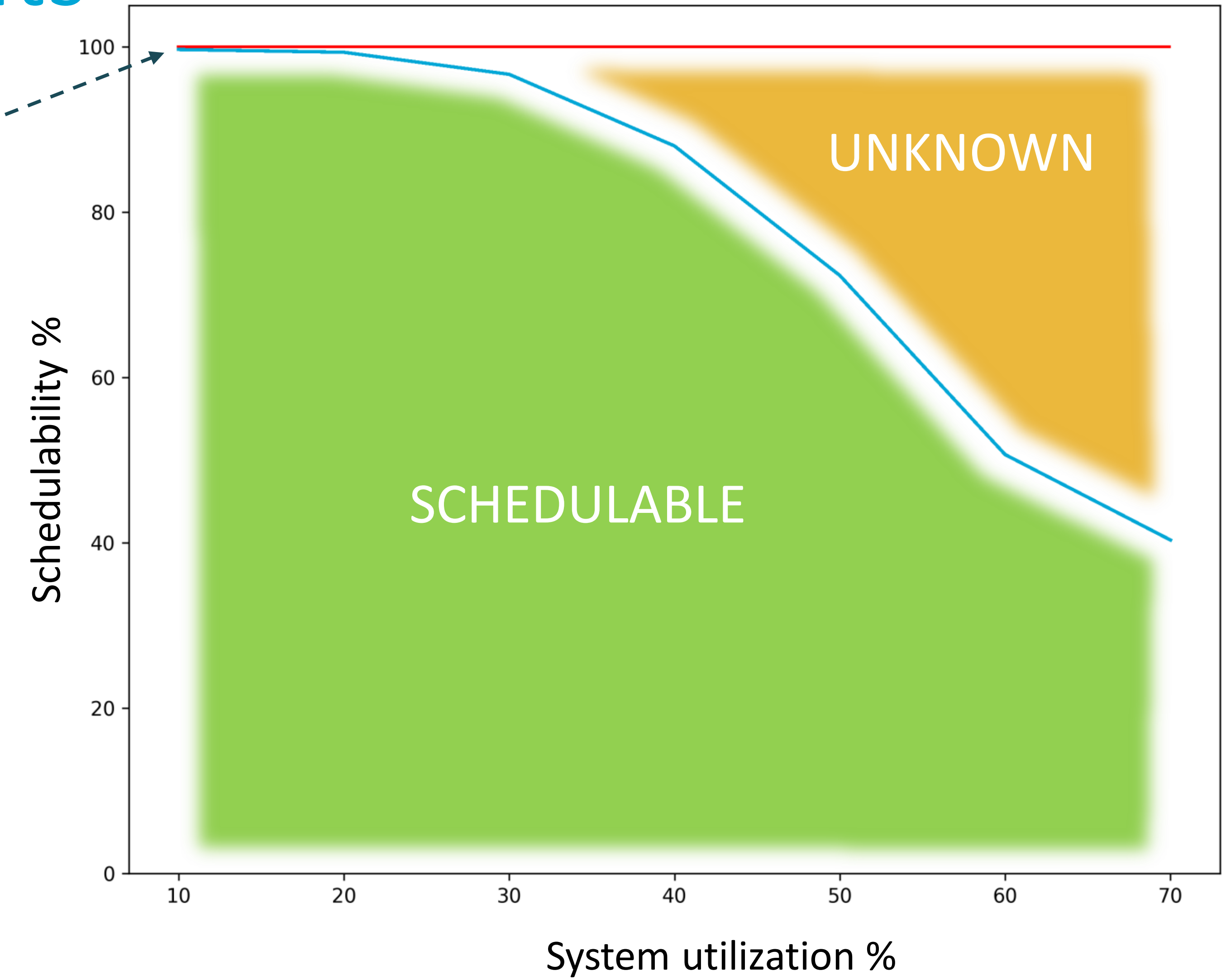

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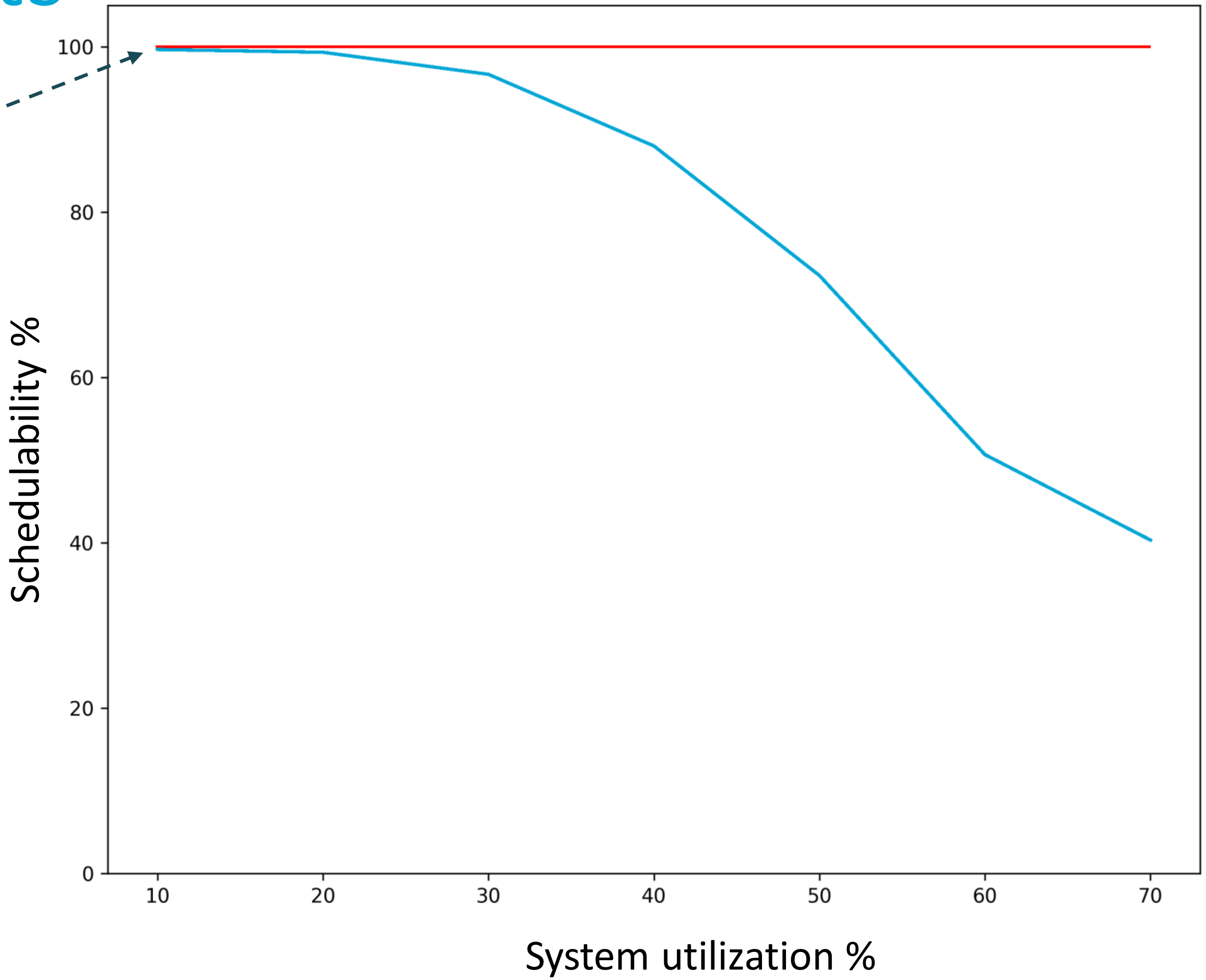
All task sets pass necessary test



- System processors: 8
- System tasks: 4
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- Segments per task: 1

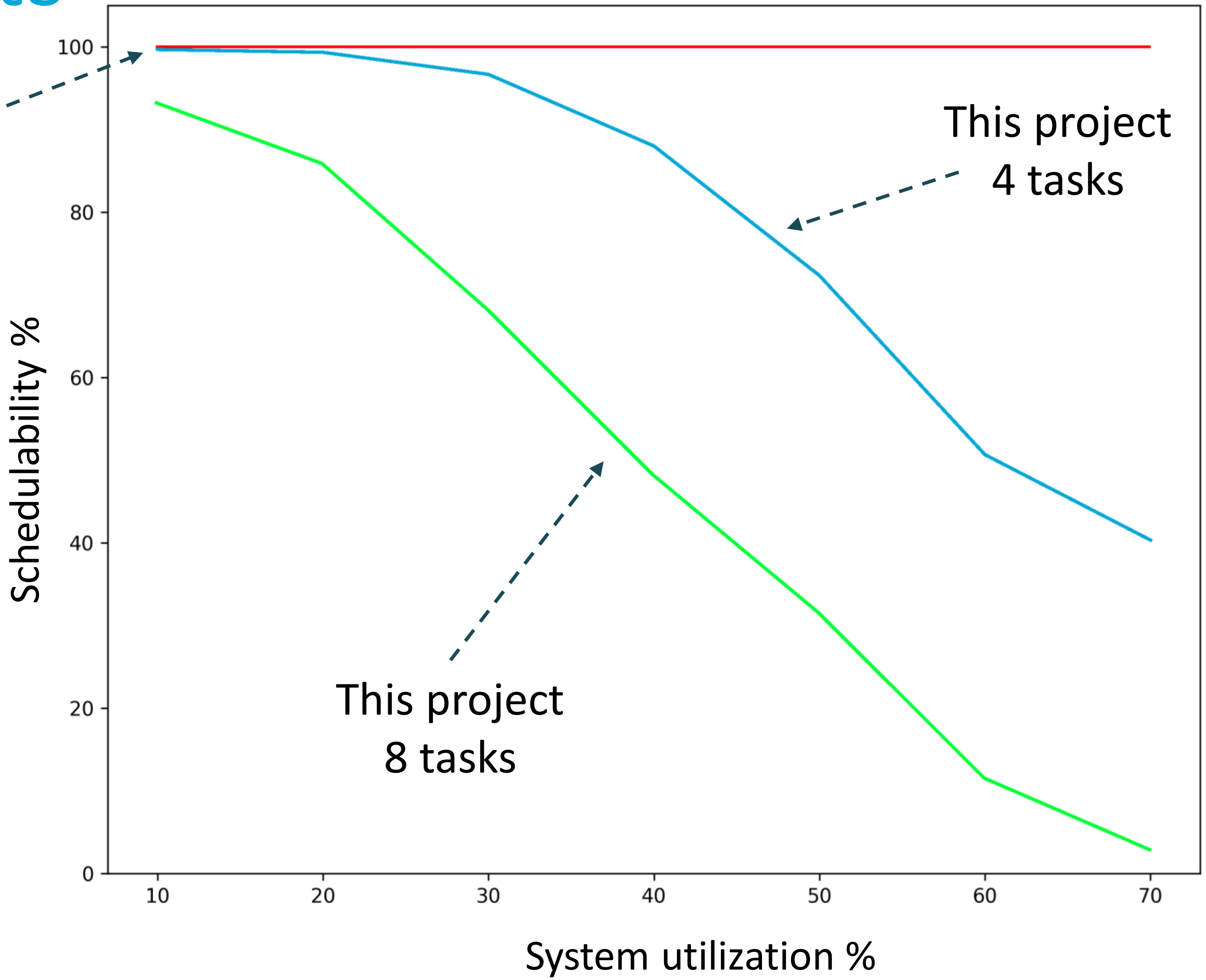
Randomly generated task sets

Our analysis' results

All task sets pass necessary test

- System processors: 8
- System tasks: 4 and 8
- Execution-time variation: 25%
- Segments per task: 1

Randomly generated task sets

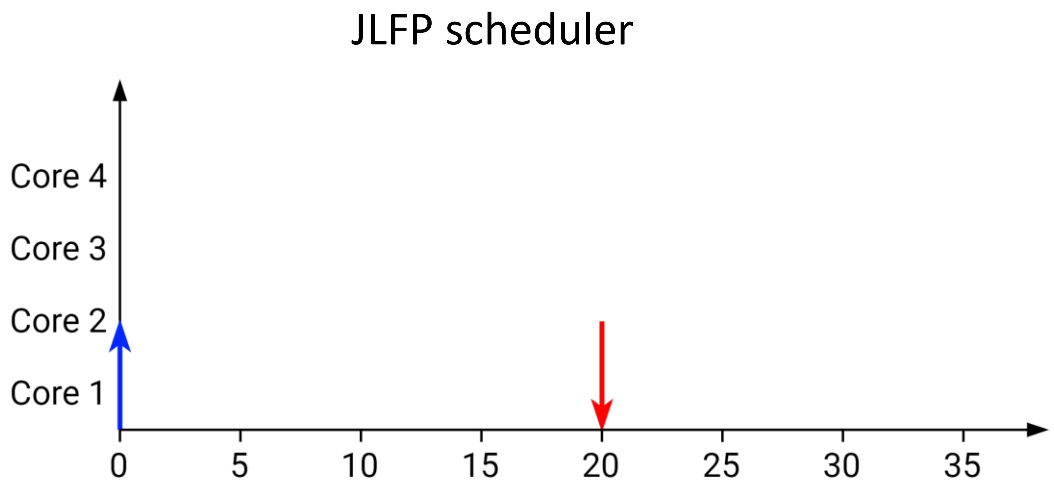


Agenda

- ~~Gang schedulability analysis~~
- New scheduling policy

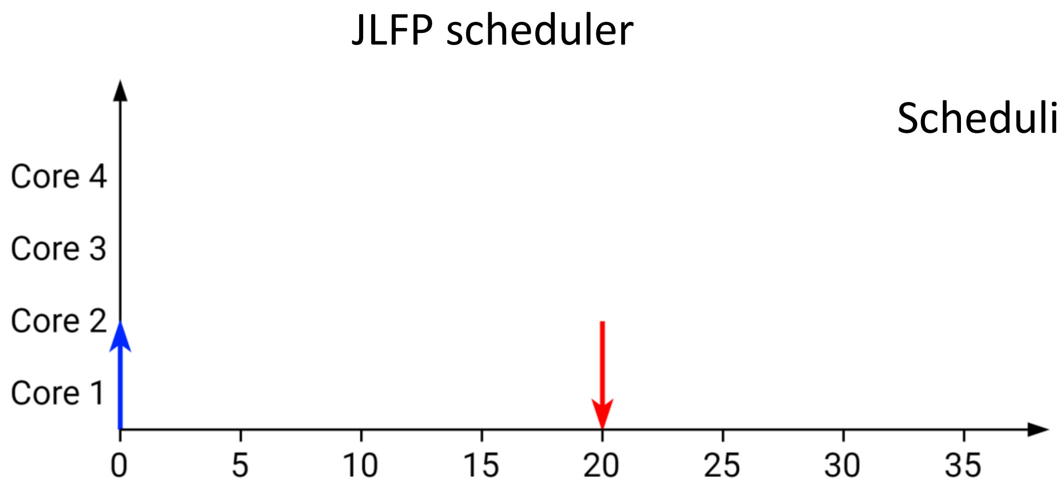
JLFP limitations with moldable gang

JLFP limitations with moldable gang



	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

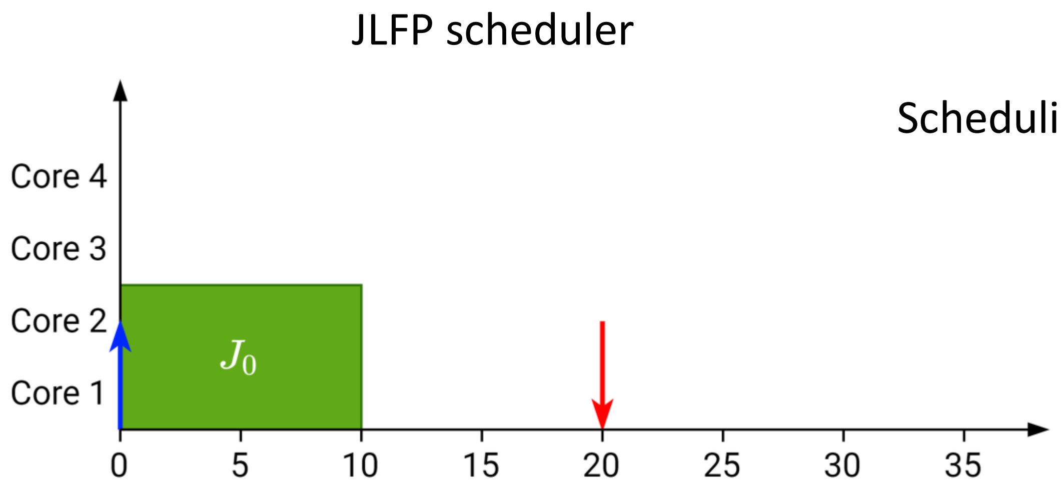
JLFP limitations with moldable gang



Scheduling: J_0

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

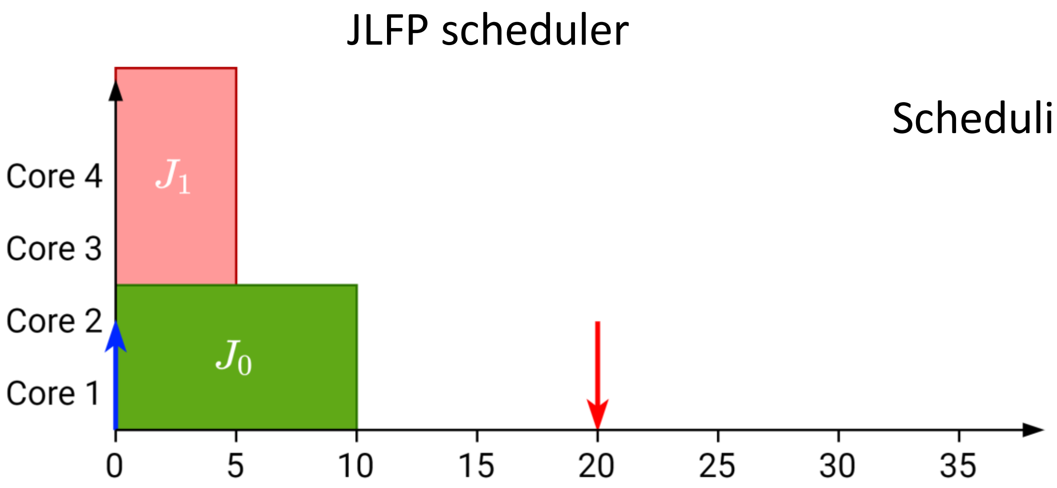
JLFP limitations with moldable gang



Scheduling: J_1

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

JLFP limitations with moldable gang



Scheduling: J_1

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
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J_2	Mid-low	1	∞	20
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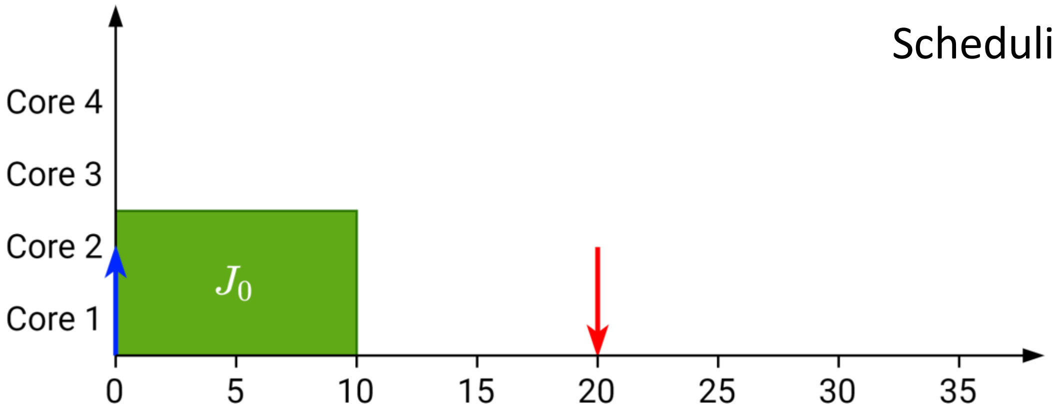
JLFP limitations with moldable gang

JLFP scheduler

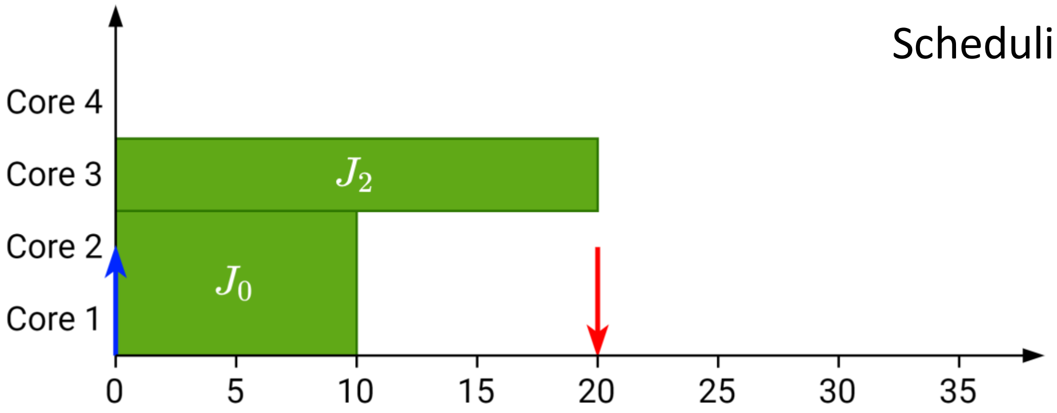


Scheduling: J_2

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

JLFP limitations with moldable gang

JLFP scheduler

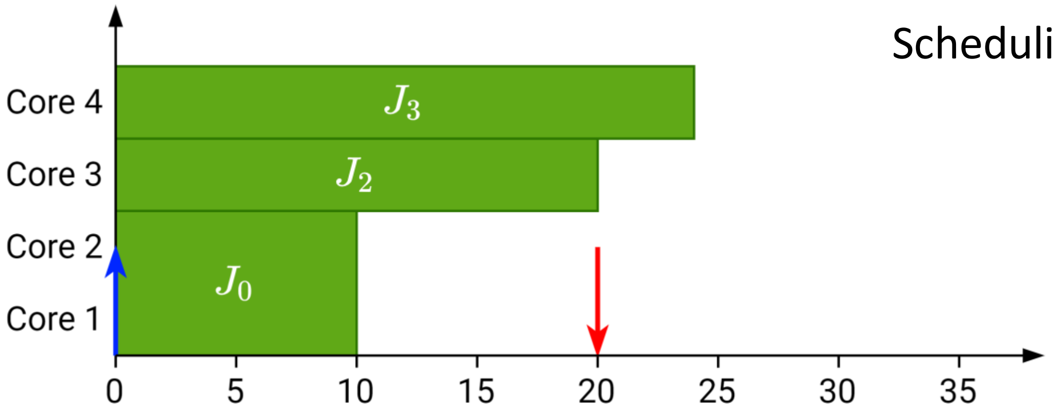


Scheduling: J_3

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

JLFP limitations with moldable gang

JLFP scheduler



Scheduling: J_1

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
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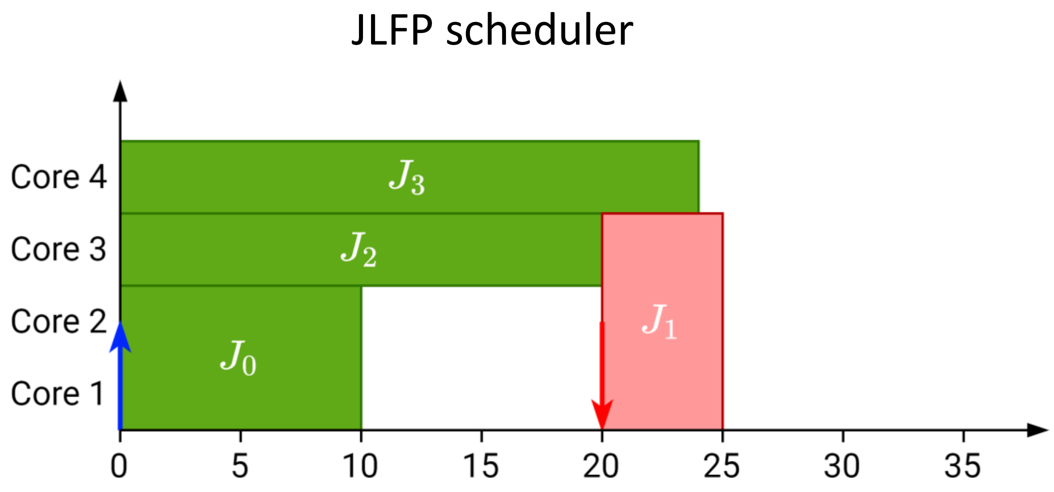
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

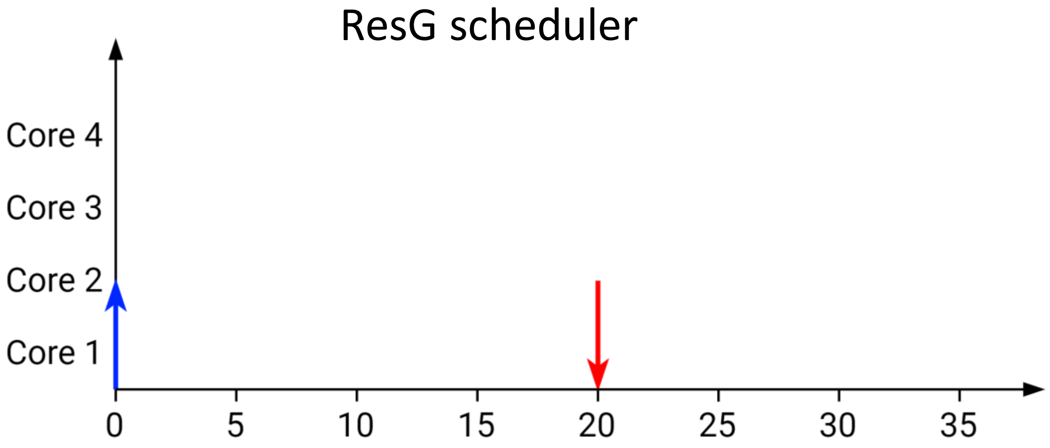
JLFP limitations with moldable gang



	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

Reservation-based gang scheduler

- Reservation-based



	Priority	Cores	Deadline	Execution time
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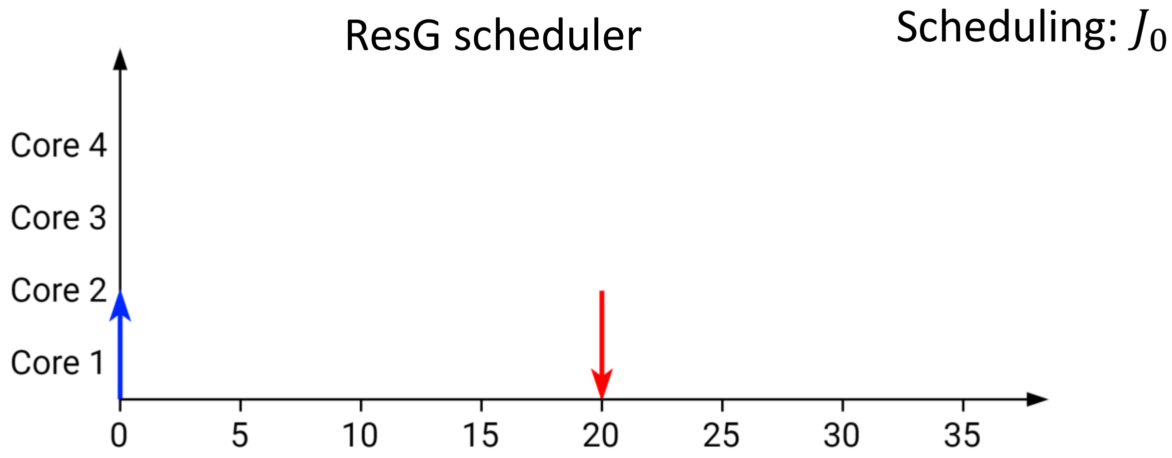

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
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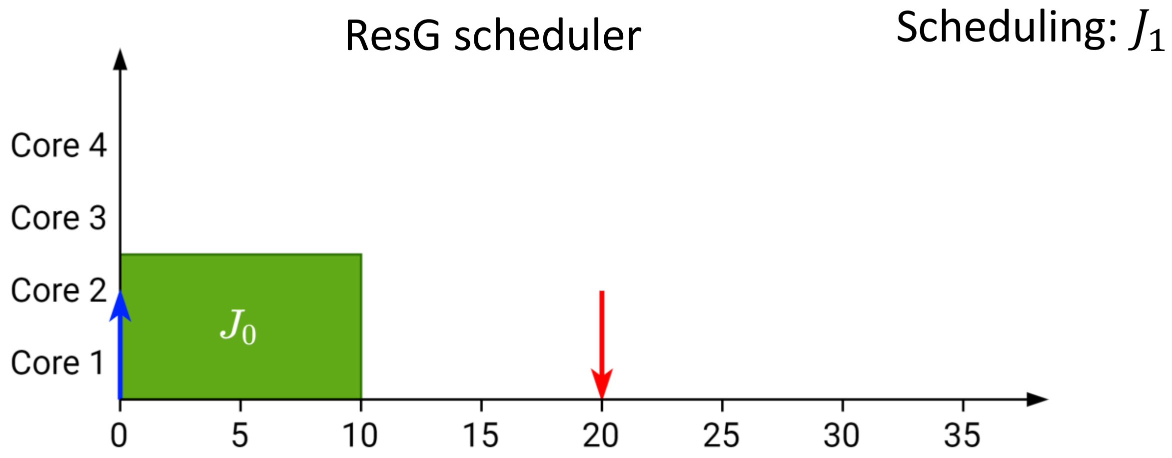

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J_0	High	2	∞	10
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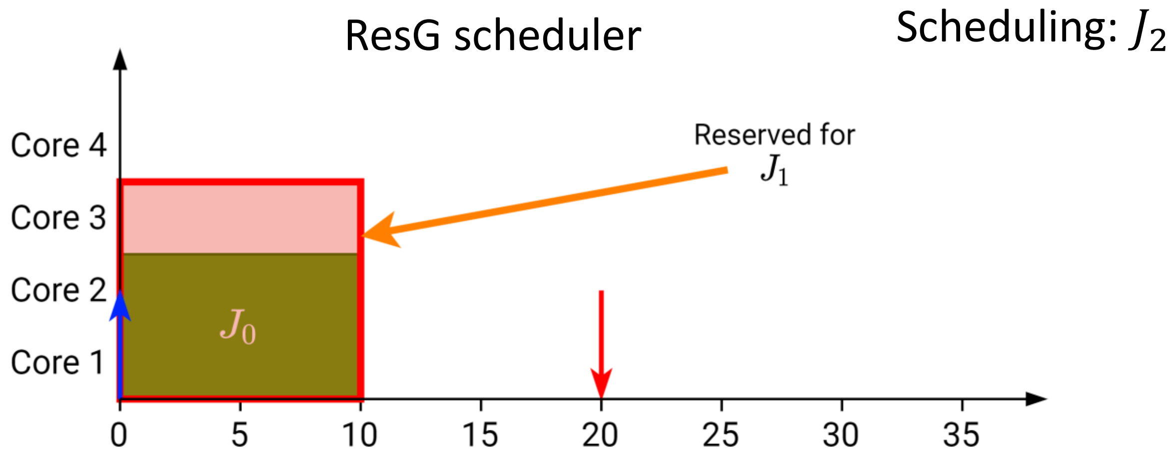
Reservation-based gang scheduler

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
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Reservation-based gang scheduler

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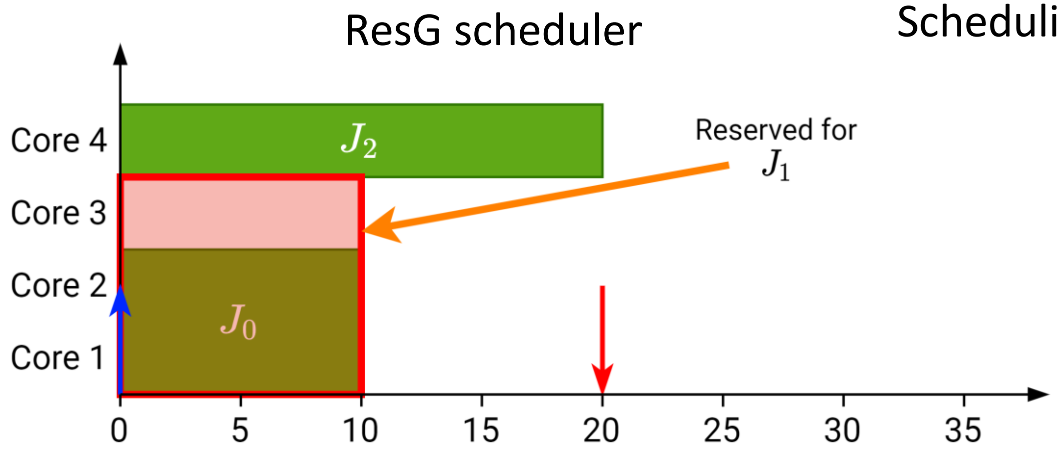
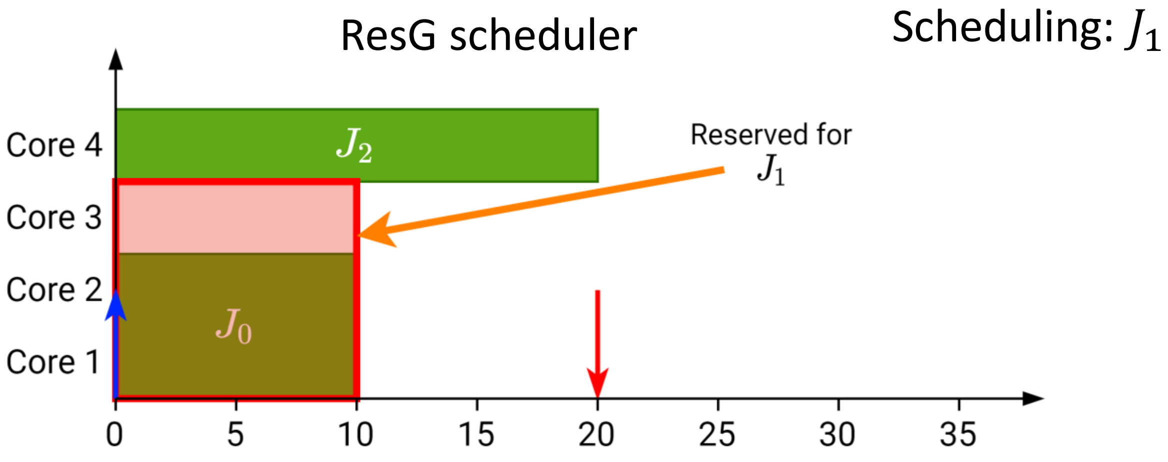


Scheduling: J_3

	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

Reservation-based gang scheduler

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
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Reservation-based gang scheduler

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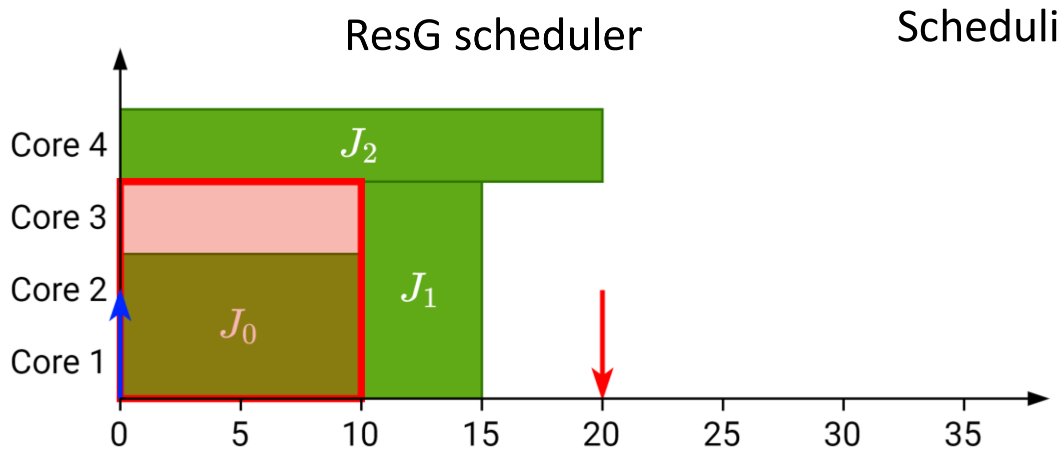

	Priority	Cores	Deadline	Execution time
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J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
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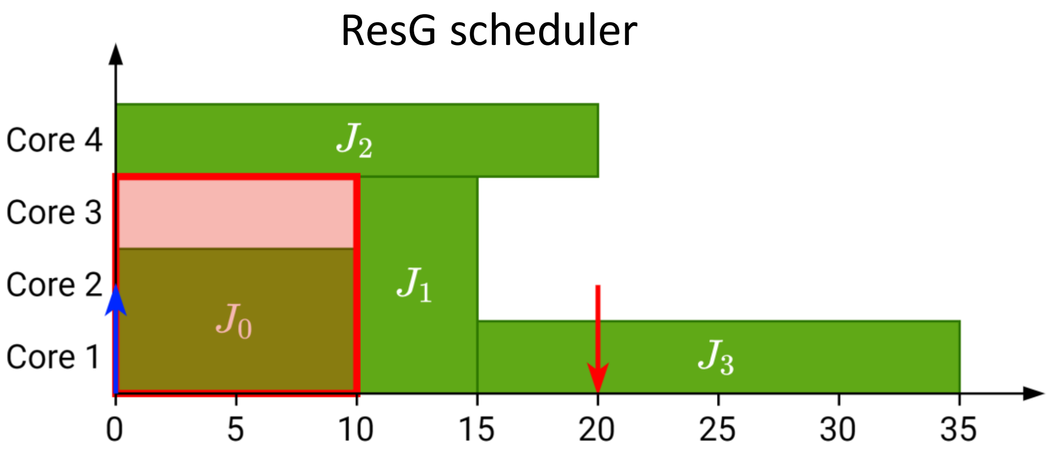
Reservation-based gang scheduler

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



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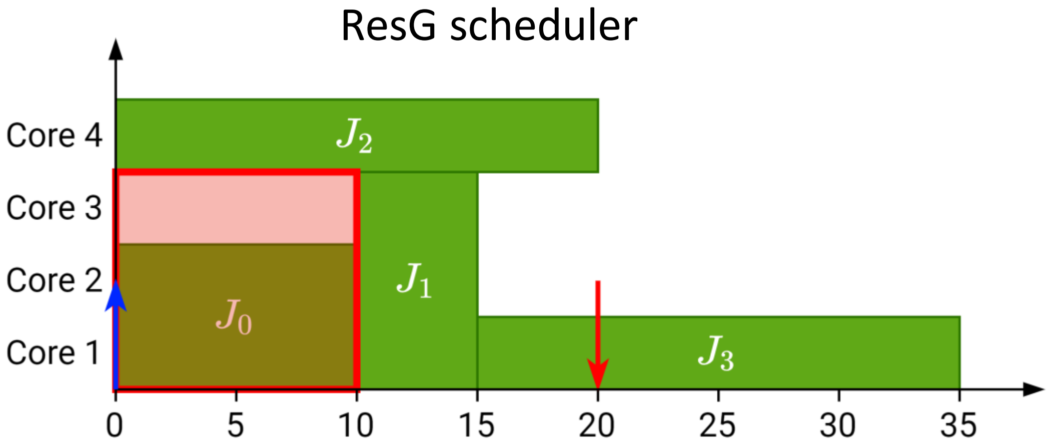
Reservation-based gang scheduler

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks
- Non-work conserving scheduler



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Results JLFP vs ResG in simulator

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- Evaluated in simulator

Results JLFP vs ResG in simulator

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- Randomly generated task sets

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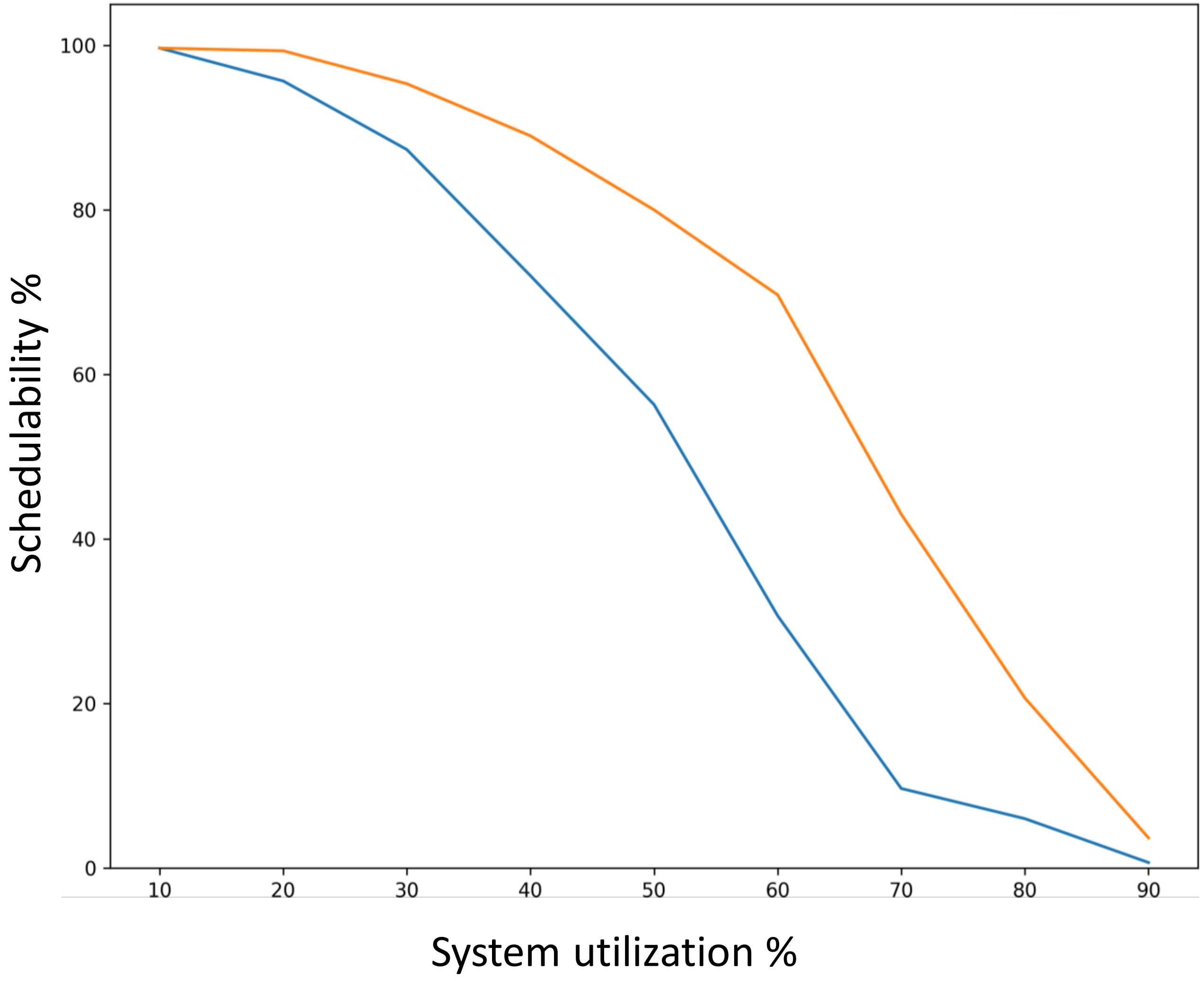

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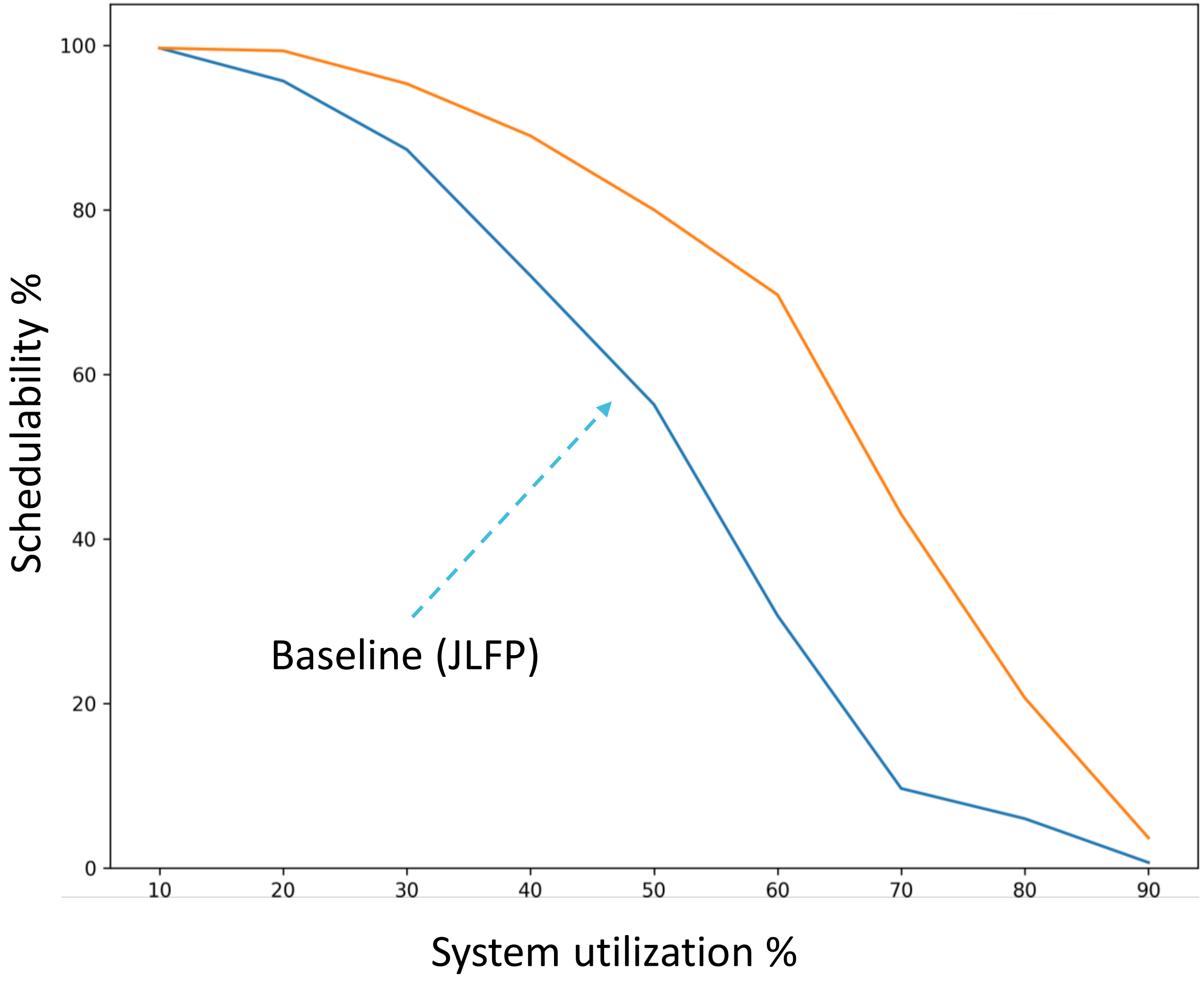
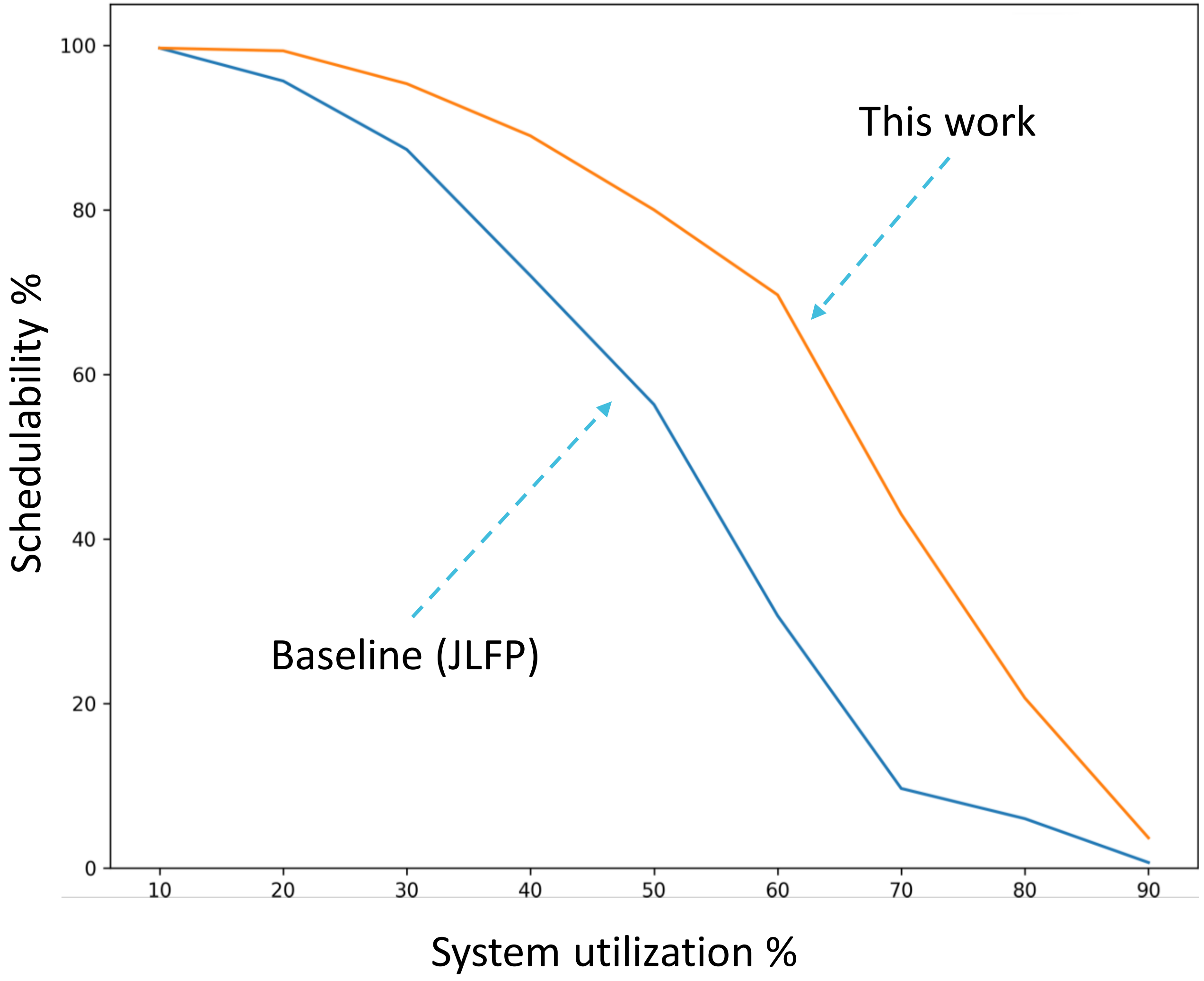


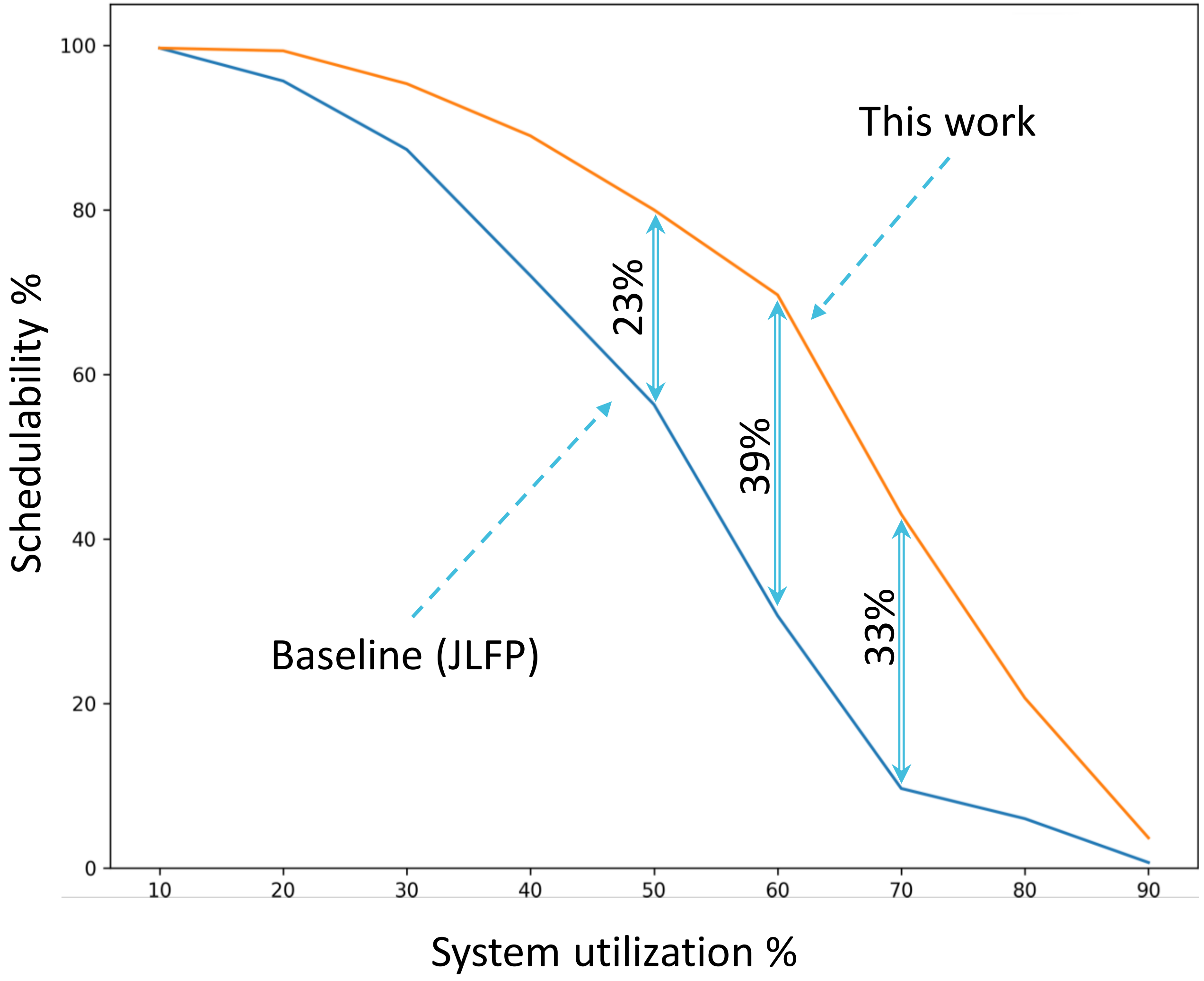
Results JLFP vs ResG in simulator

- Evaluated in simulator
- Randomly generated task sets



Results JLFP vs ResG in simulator

- Evaluated in simulator
- Randomly generated task sets



Conclusions

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- With a better scheduling policy one can improve the schedulability of moldable gang tasks

Summary

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- A new analysis for gang tasks using SAG has been defined

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- A new scheduling policy that uses gang moldable properties has been created

Next steps

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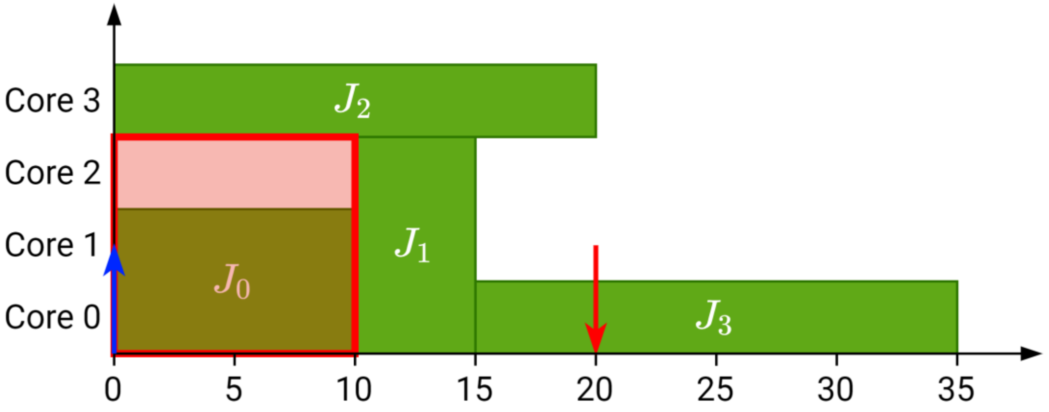
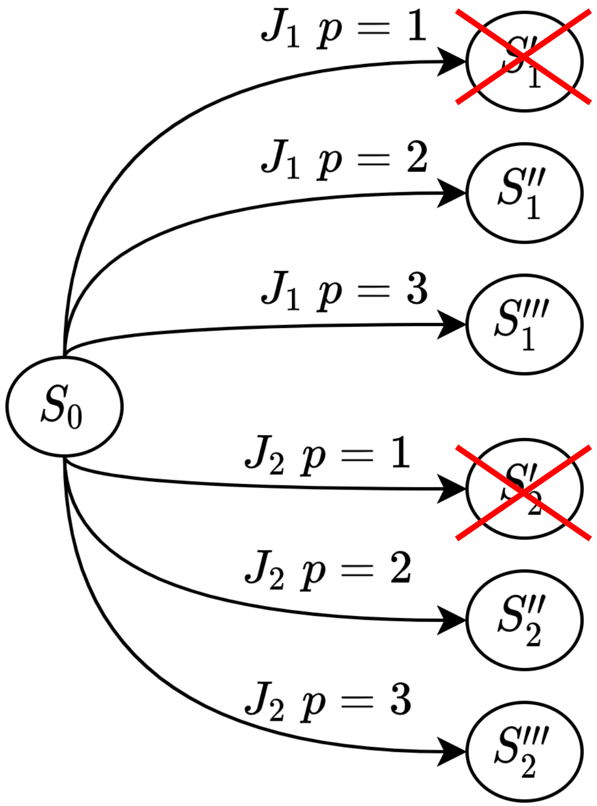
- Provide analysis for ResG scheduler and respective proofs
- Thorough evaluation of results using SURFSara cluster

Questions?



Core 3
Core 2
Core 1
Core 0
J_2
J_1
J_0
J_3
0
5
10
15
20
25
30
35



S_0
J_1 $p = 1$
S_1'
J_1 $p = 2$
S_1''
J_1 $p = 3$
S_1'''
J_2 $p = 1$
S_2'
J_2 $p = 2$
S_2''
J_2 $p = 3$
S_2'''

SAG analysis changes for gang

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$$t_h\left(J_i, J_j\right) = \begin{cases} r_j^{\max} & \text{if } m_j^{\min} \le p \\ \max\left\{r_j^{\max}, A_{m_j^{\min}}^{\max}\right\} & \text{otherwise} \end{cases}$$

$$t_{gang} = \begin{cases} A_p^{\min} & \text{if } p = m_i^{\max} \\ A_p^{exact} & \text{otherwise} \end{cases} \qquad t_{avail} = \begin{cases} A_{p+1}^{\max} - 1 & \text{if } p < m_i^{\max} \\ +\infty & \text{otherwise} \end{cases}$$

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Check if execution with p cores is possible

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SAG analysis changes for gang

p cores available

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$p + 1$ cores **not** available

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How does SAG work?

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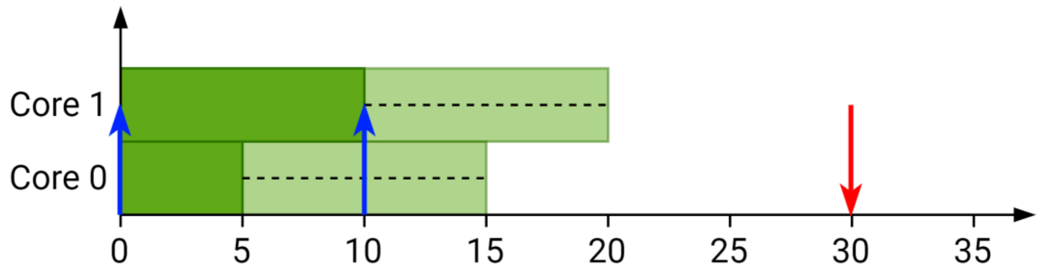
J_i	C_i^{min}	C_i^{max}	r_i	d_i	P_i
J_1	10	15	10	30	1
J_2	5	5	0	100	2



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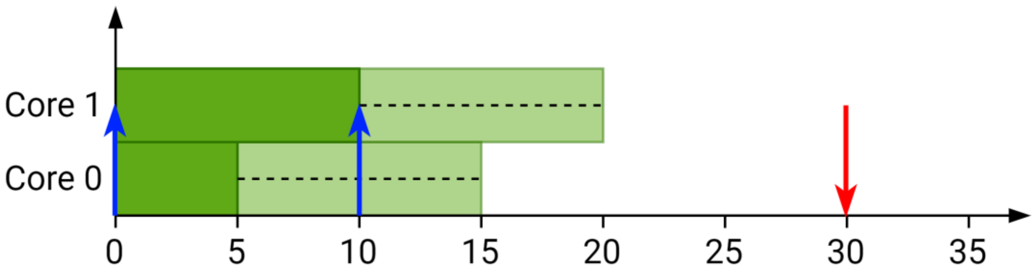

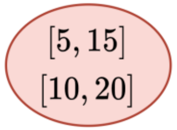

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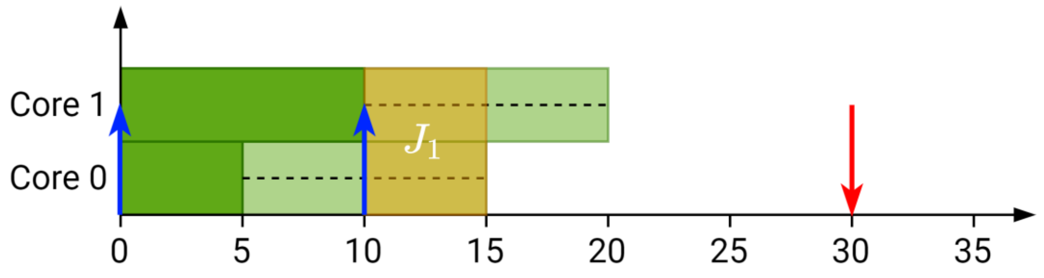

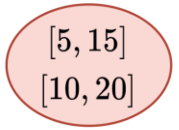

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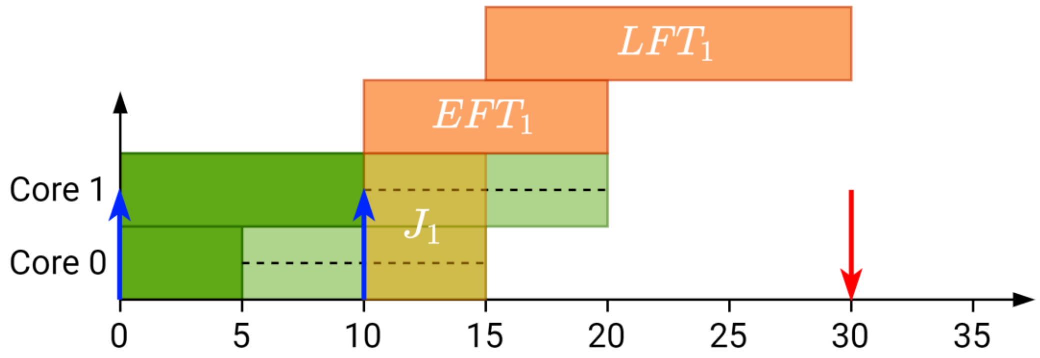

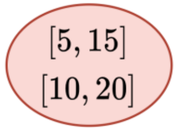

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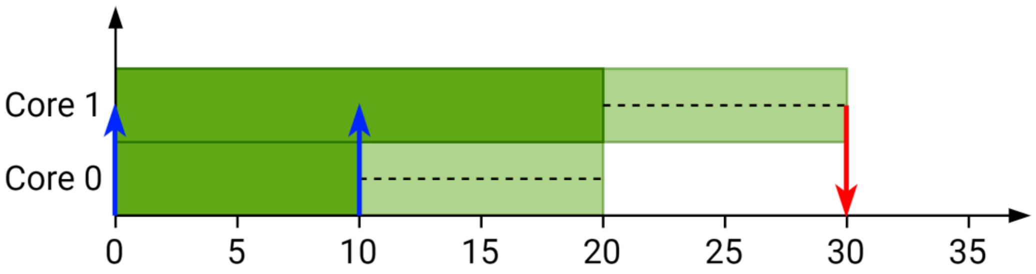

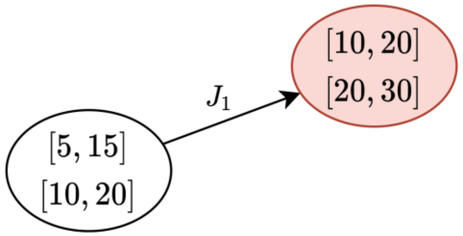

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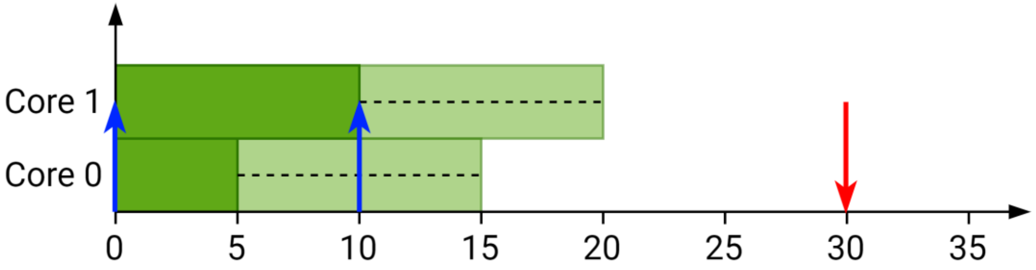

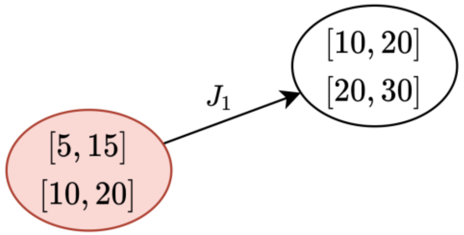

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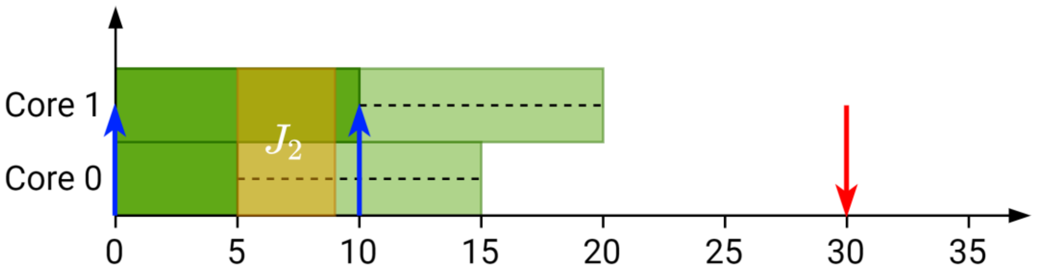

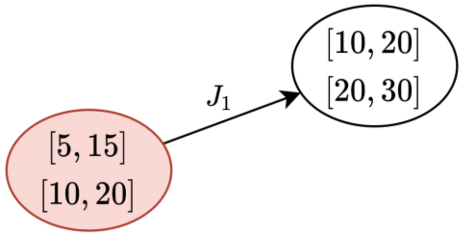

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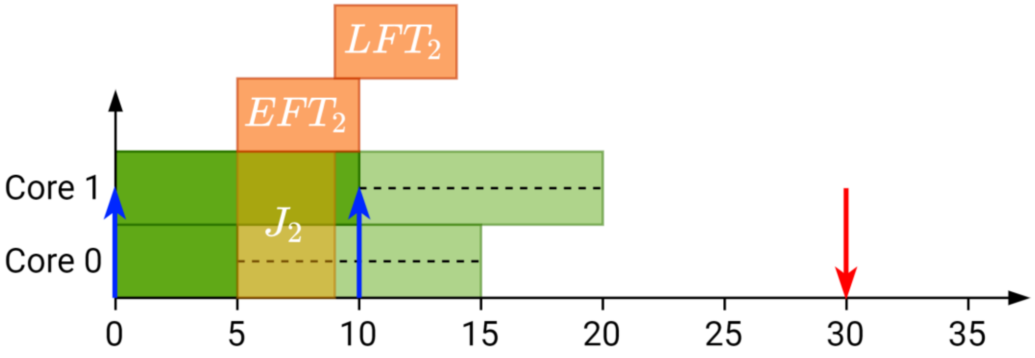

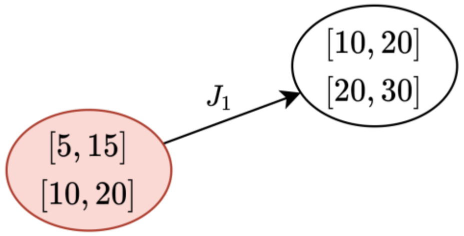

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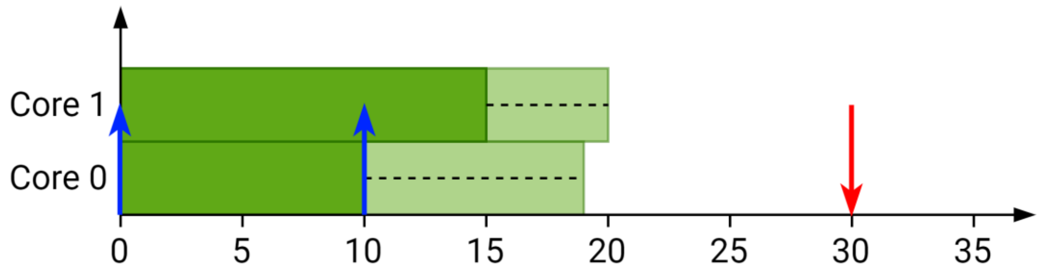

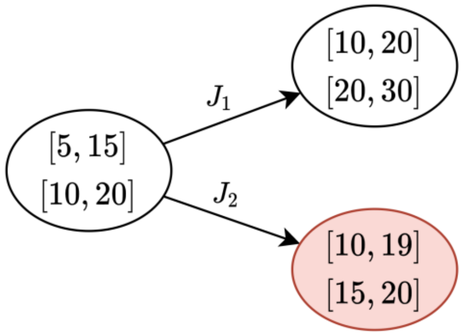

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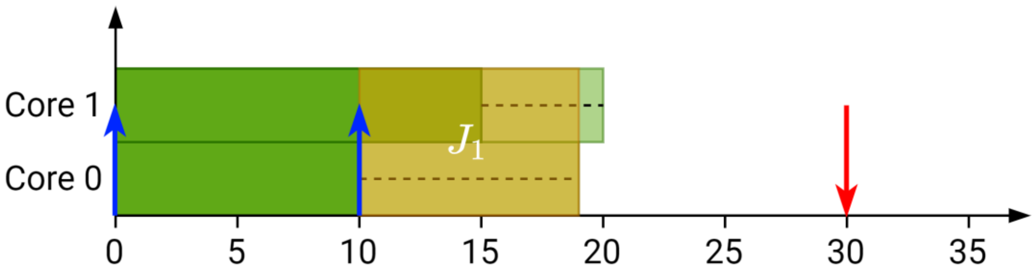

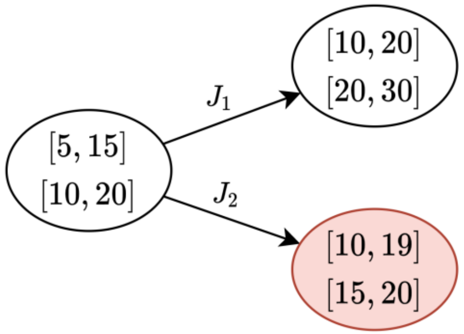

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