Schedulability analysis of limited-preemptive moldable gang tasks

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TU Delft

Daily Supervisor Geoffrey Nelissen

Co-supervisor Mitra Nasri

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Real-time systems

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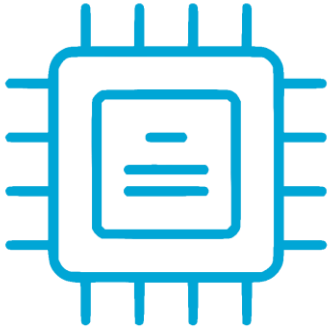
- Multicore systems





Definitions

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- Task
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- Job
  - Instance of a task
- Schedule
  - A particular assignment of jobs to the processors and time intervals
- Scheduling policy
  - Algorithm that produces a schedule
  - FIFO, Round-Robin, JLFP, EDF

What is gang?

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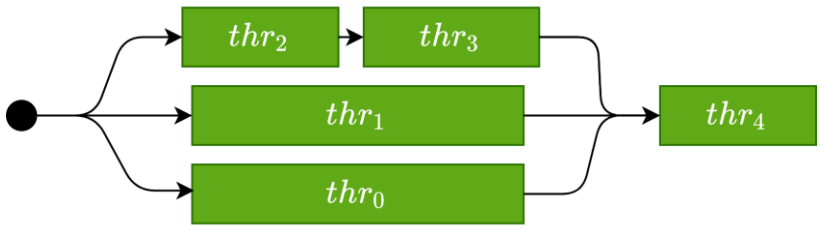

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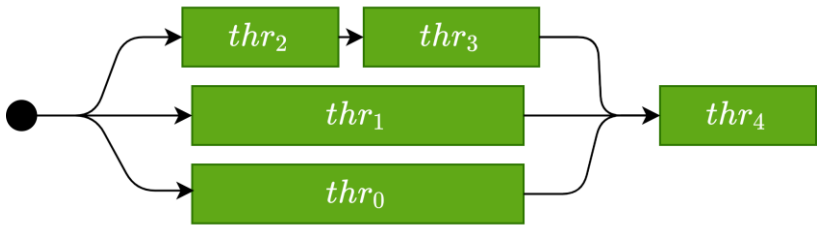
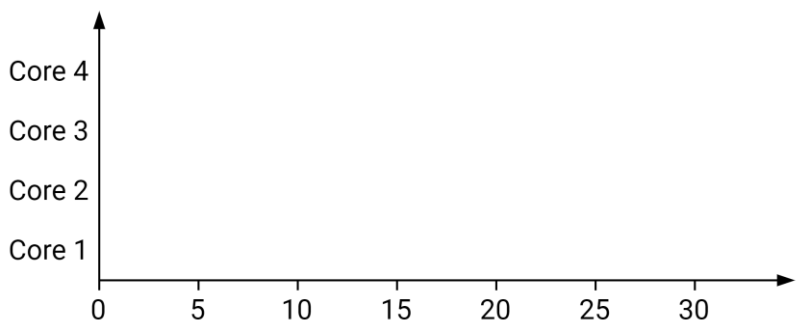


Global scheduling



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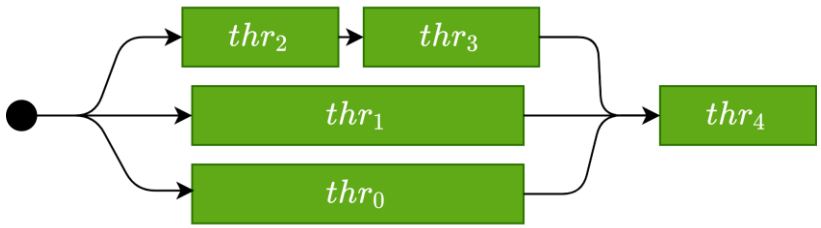
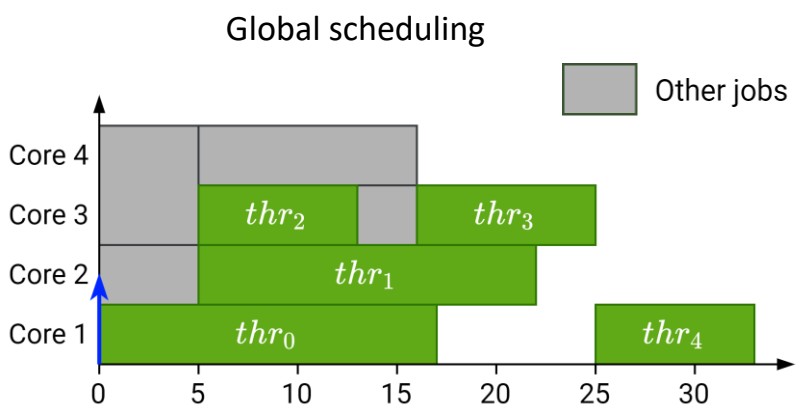
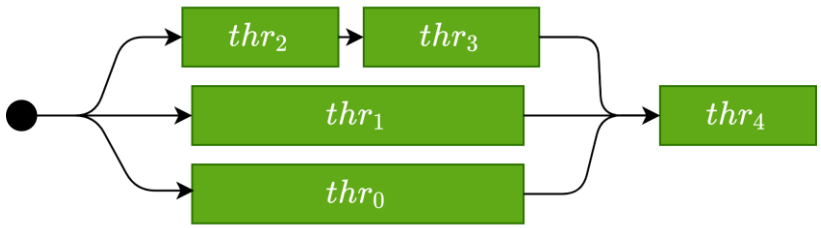
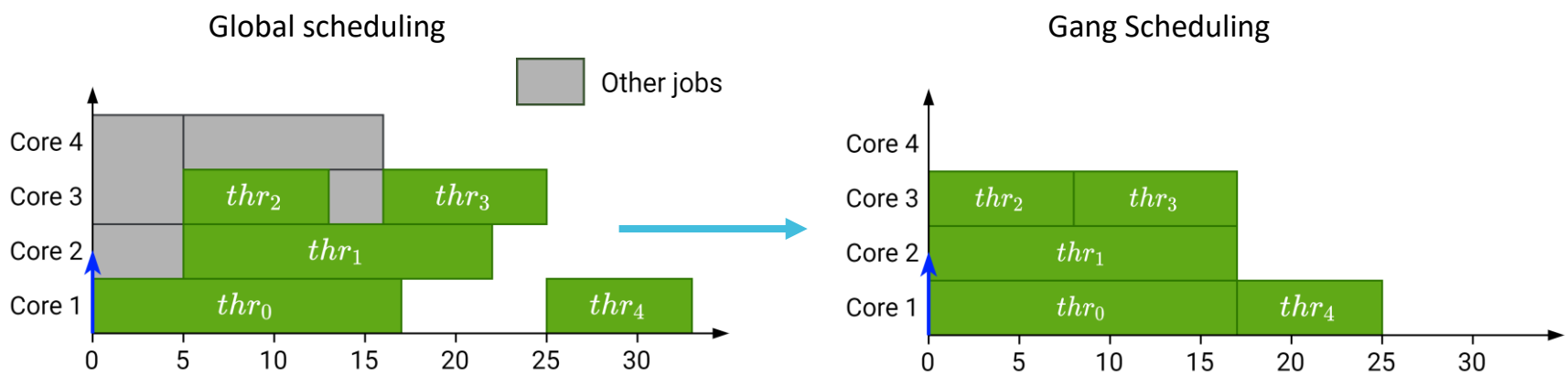

thr_2
thr_3
thr_1
thr_4
thr_0


Global scheduling
Other jobs
Core 4
Core 3
Core 2
Core 1
thr_2
thr_3
thr_1
thr_0
thr_4
0
5
10
15
20
25
30

What is gang?



Parallel threads together as a “gang”



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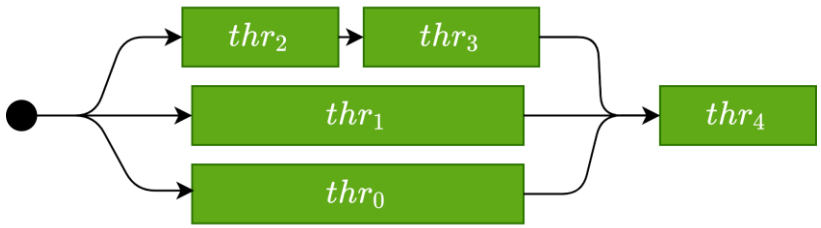

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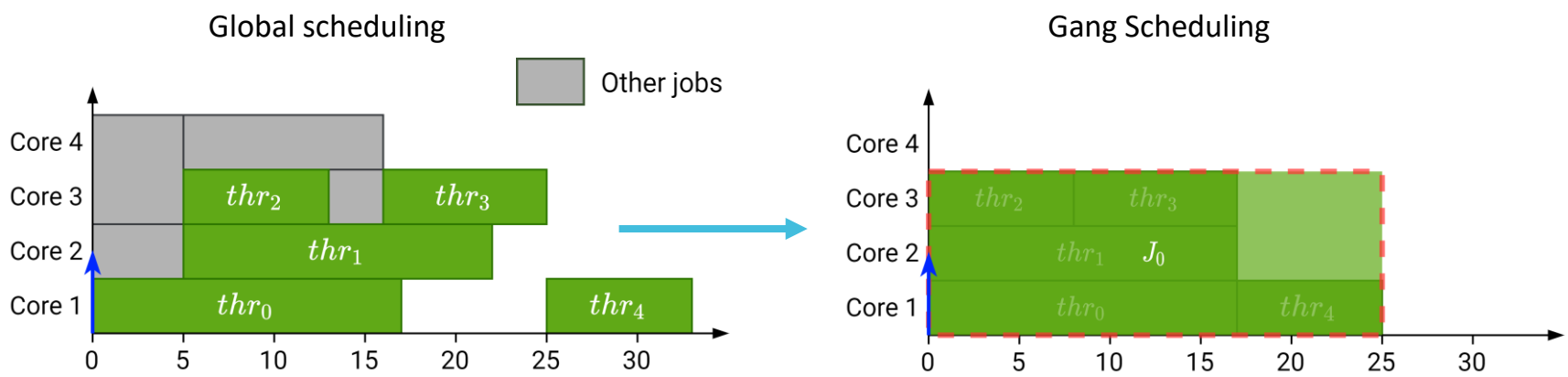

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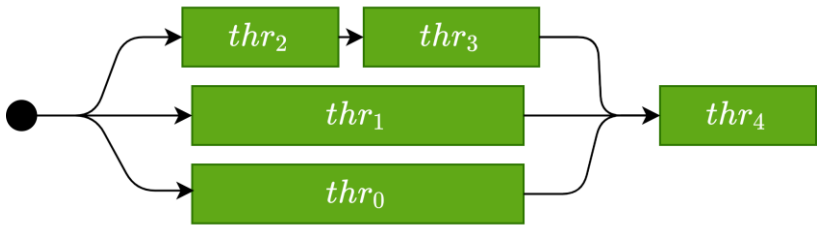

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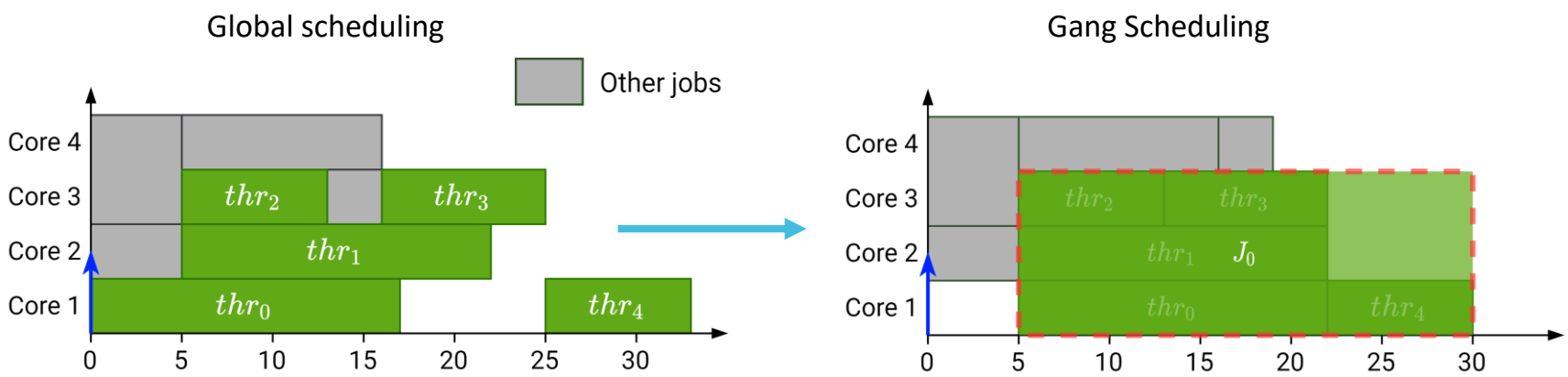

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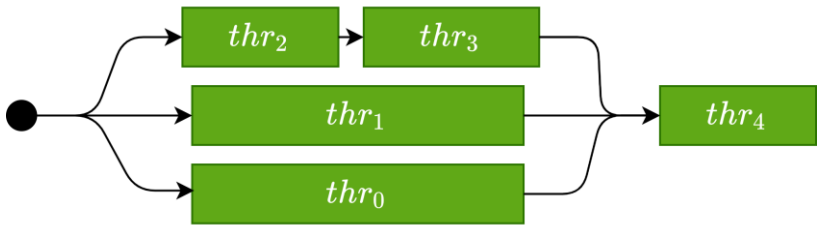

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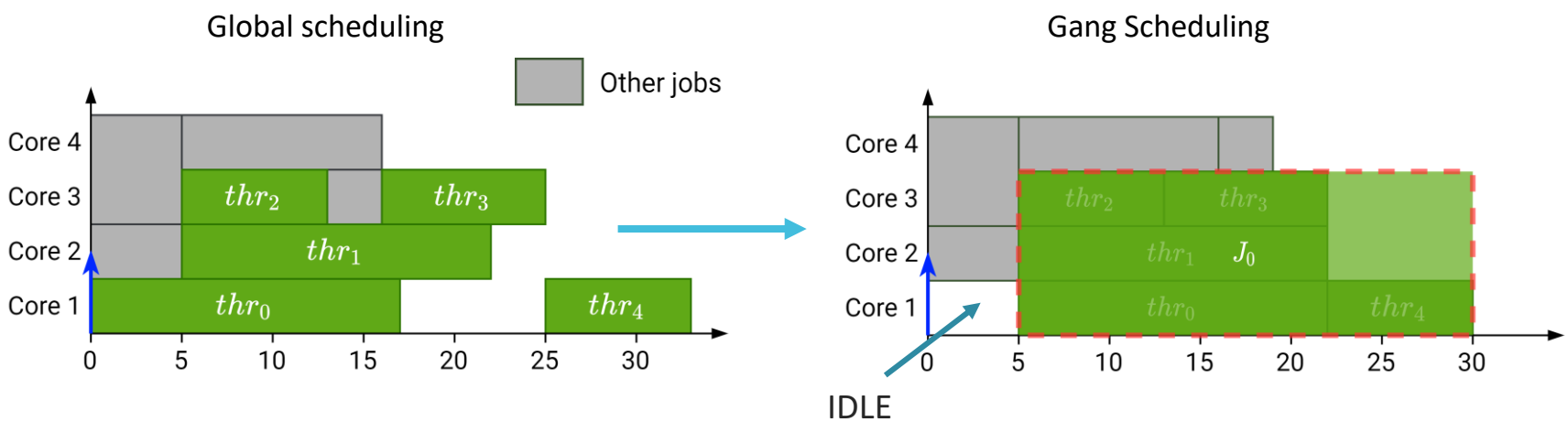

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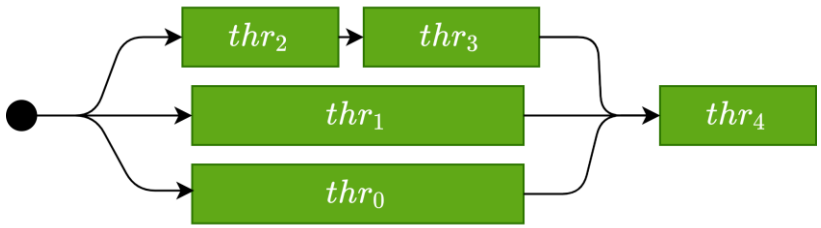

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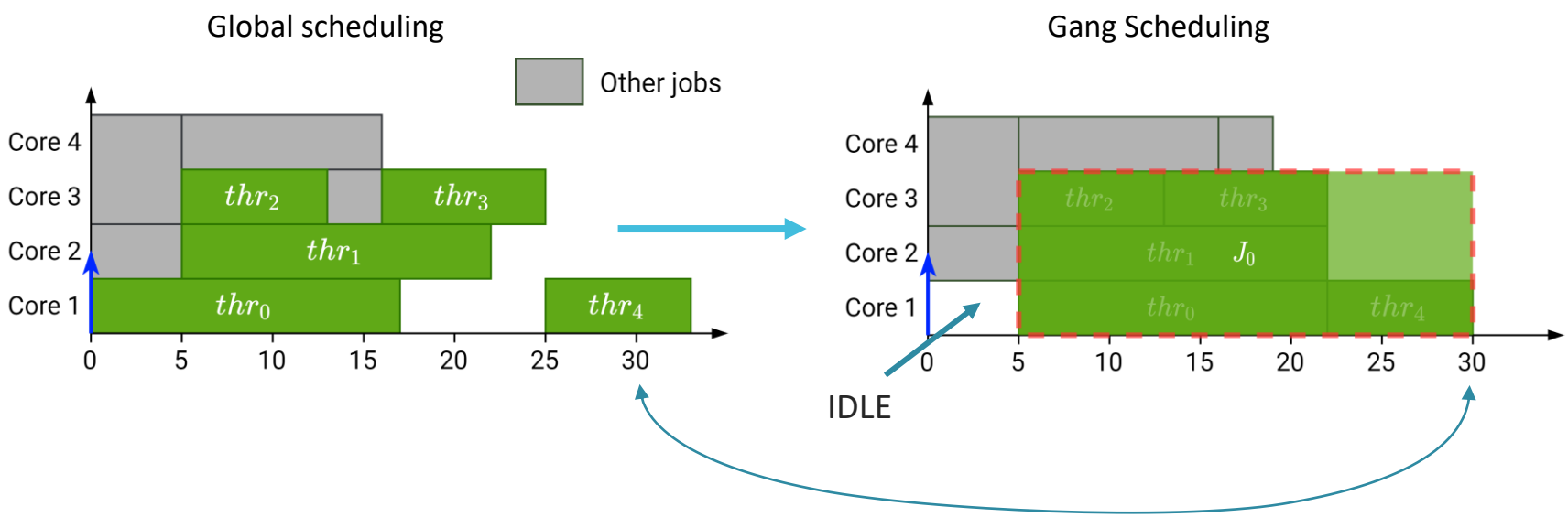

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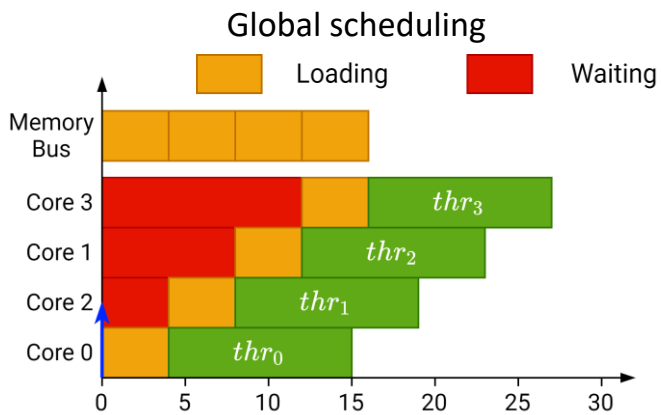

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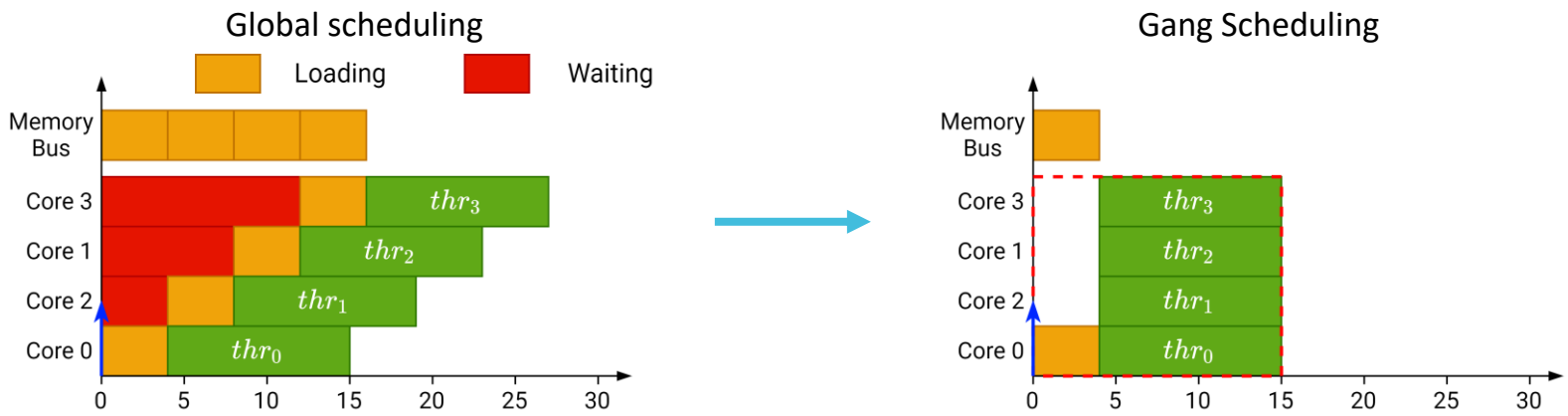
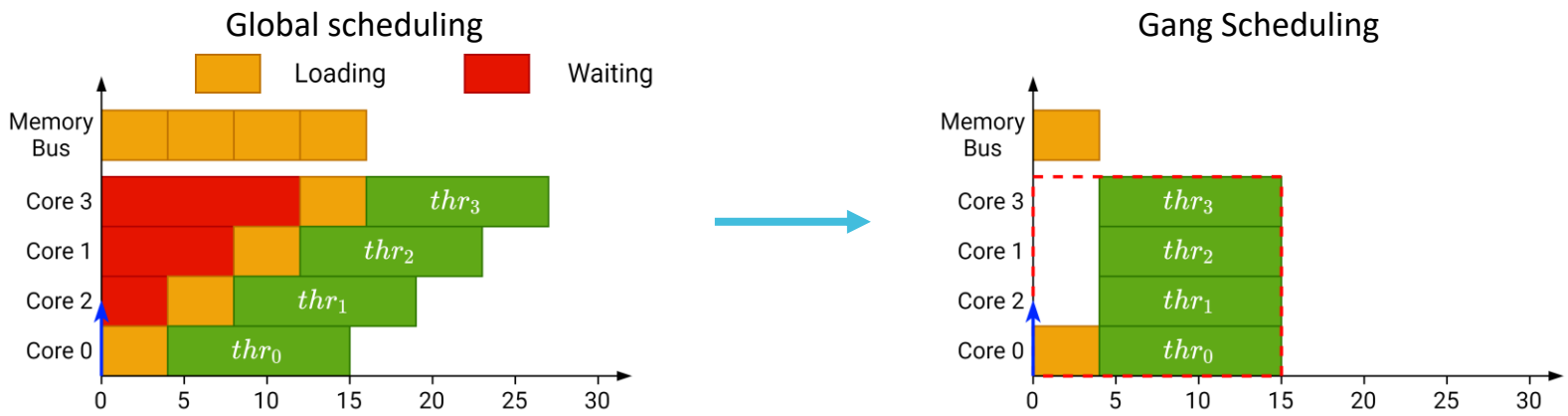


Why gang?

- More efficient synchronization
- Reduces variability in the execution
- Avoids overhead when loading initial data
- Shows its full potential when executed non-preemptively



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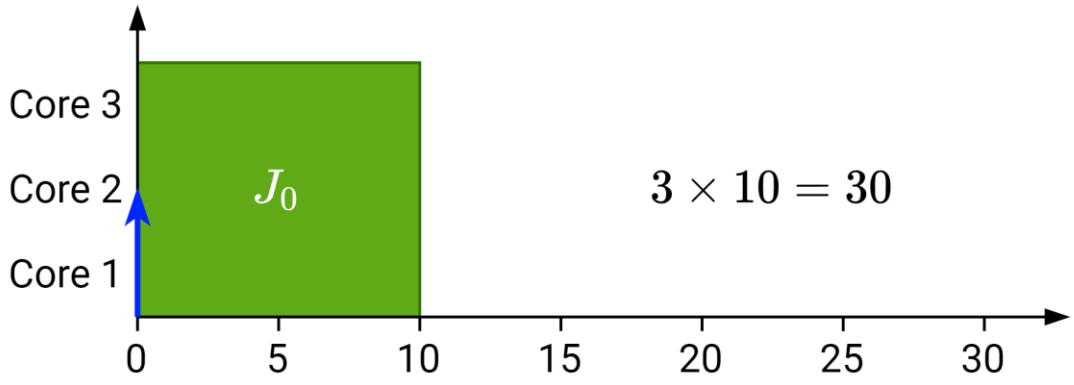

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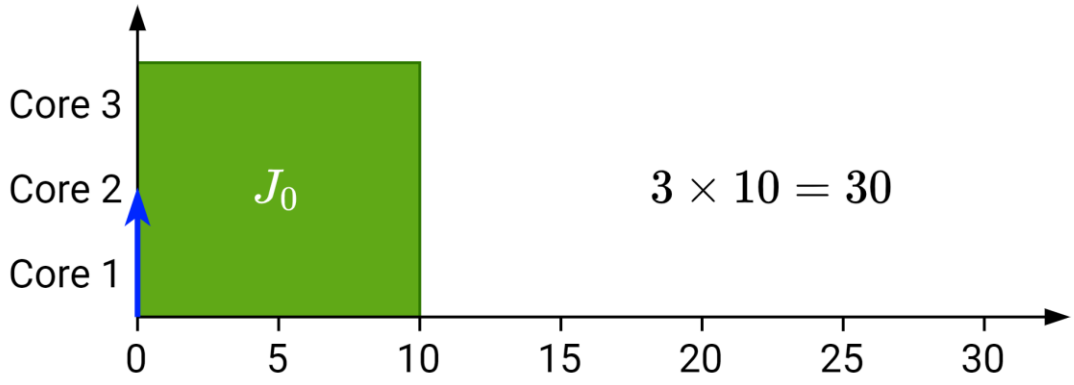

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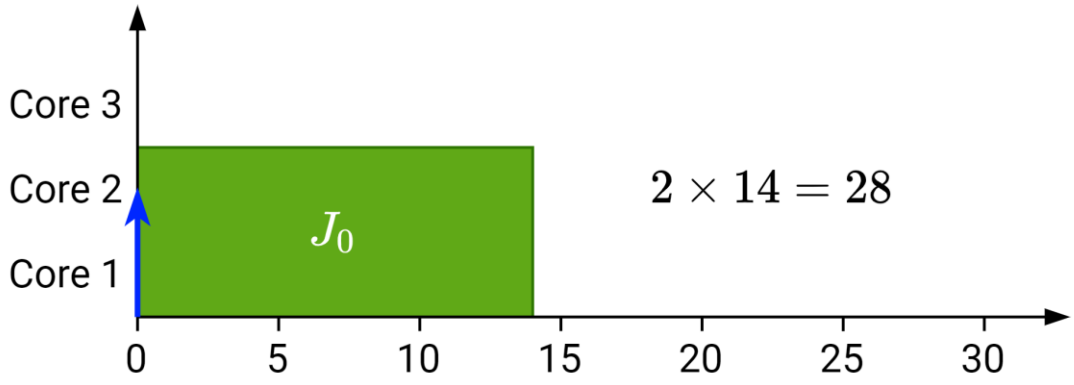

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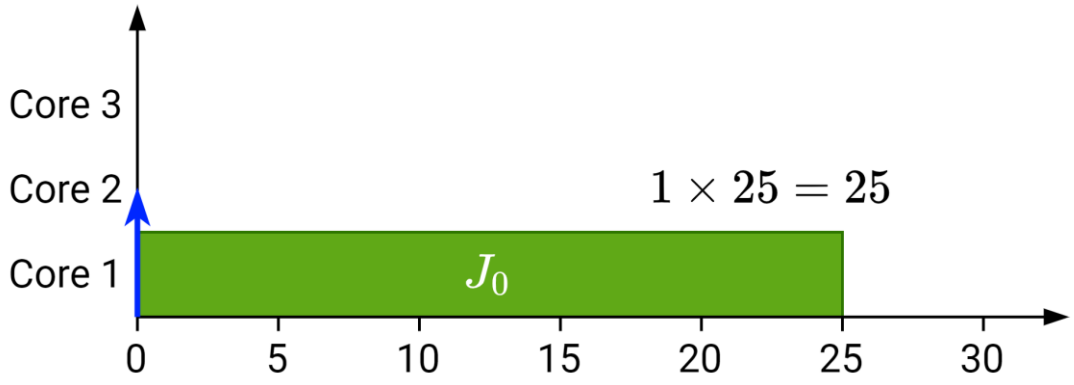

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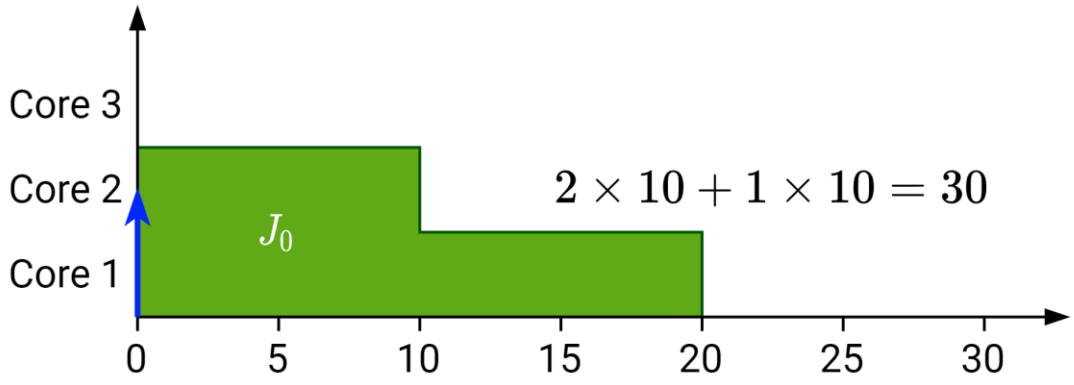

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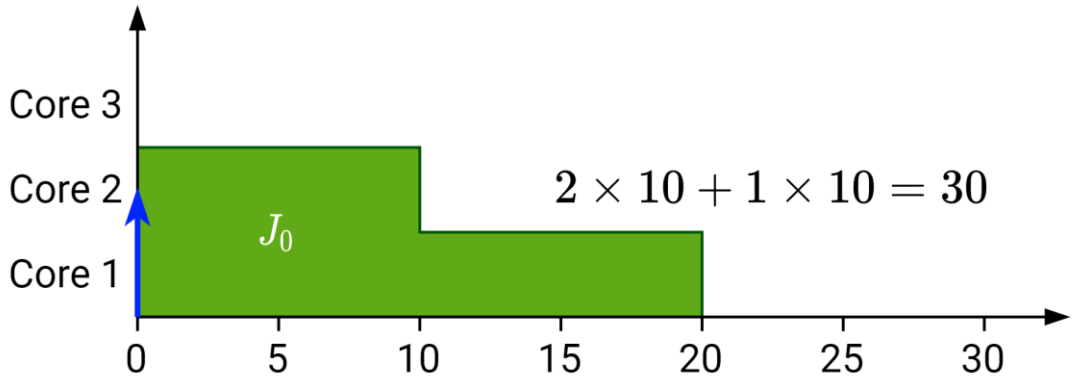

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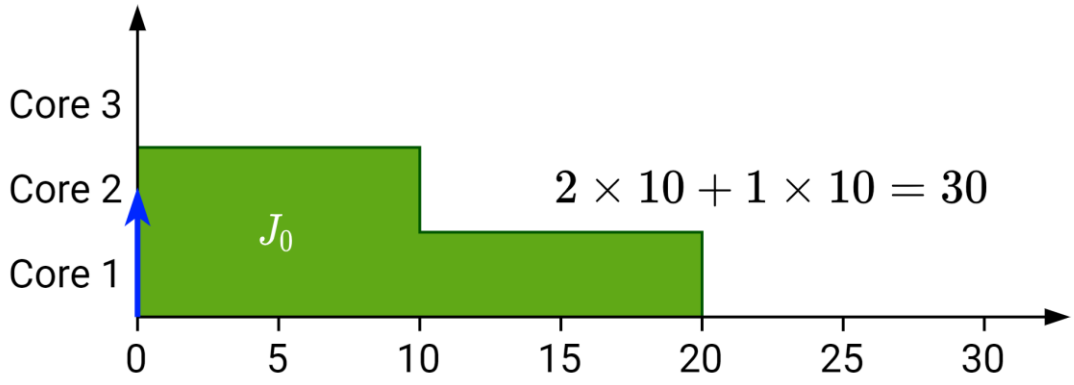

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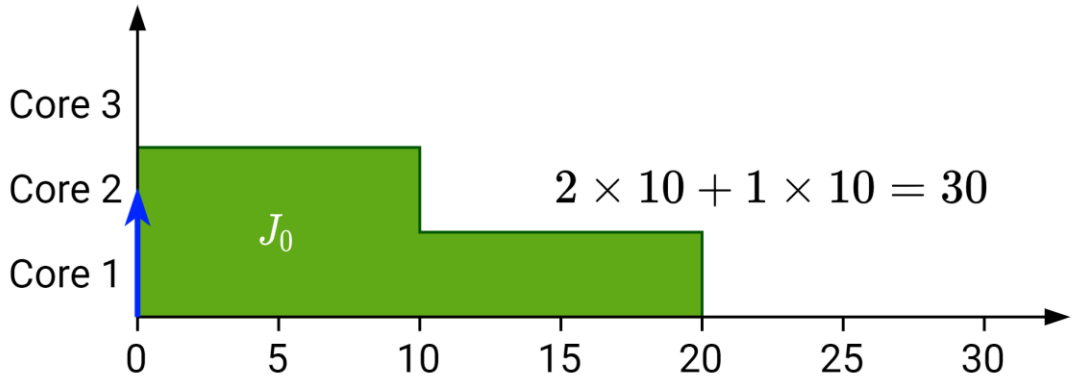

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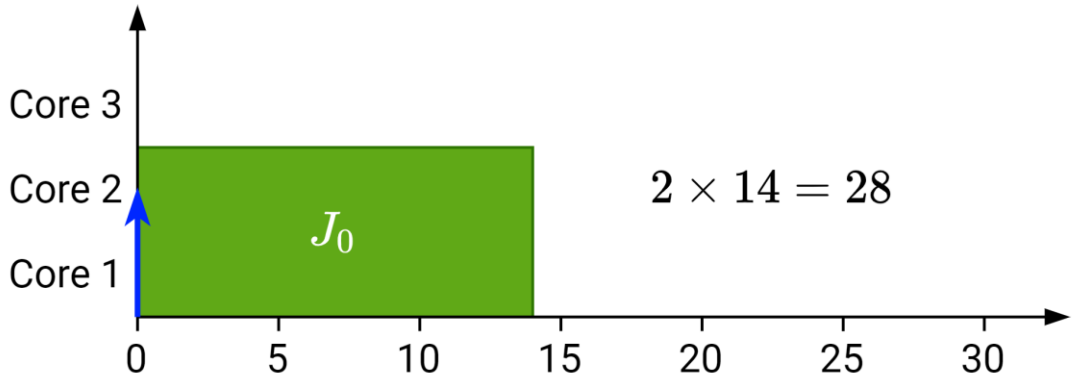
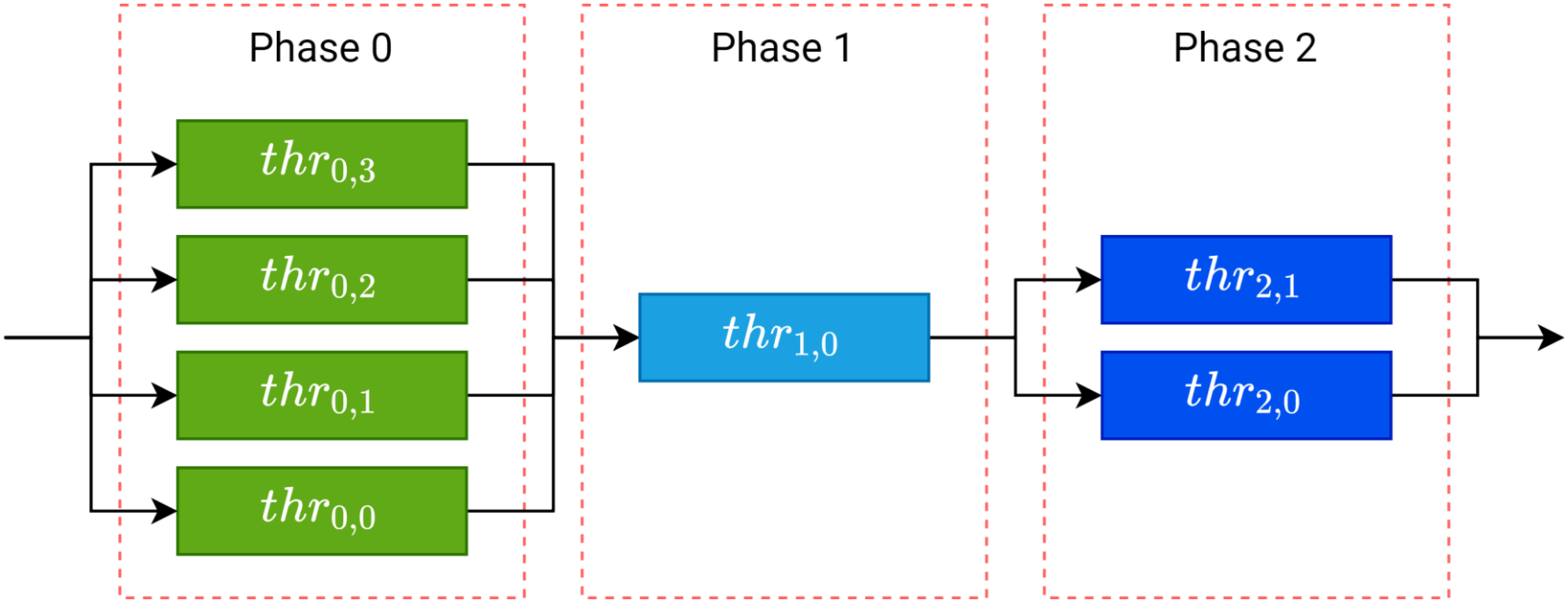


Bundled scheduling[1] vs limited-preemptive



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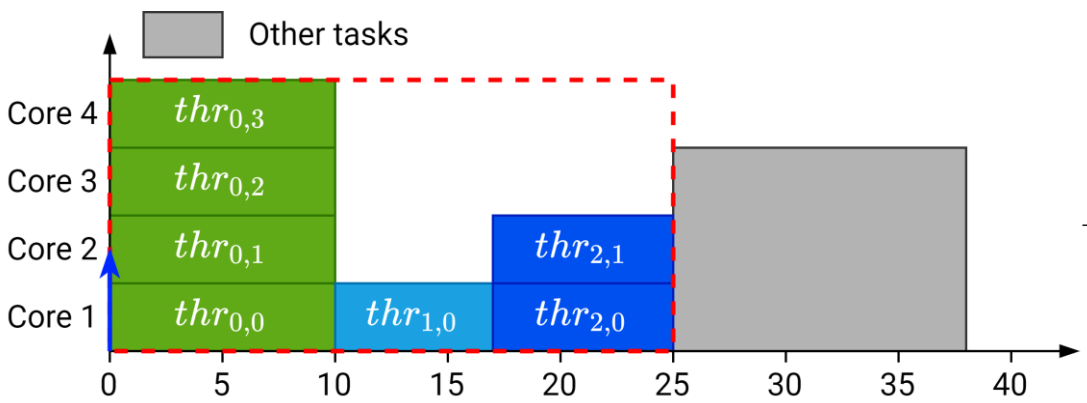
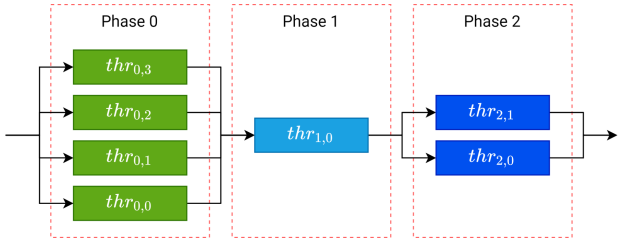


Rigid gang scheduling



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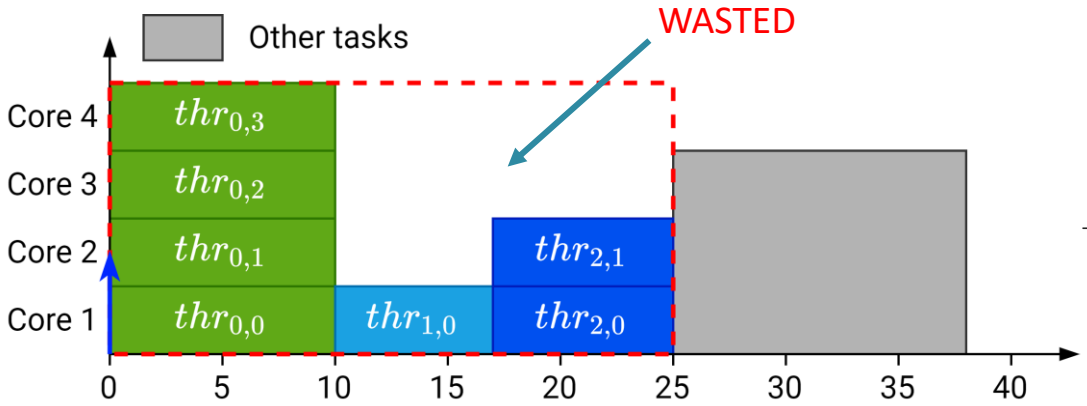

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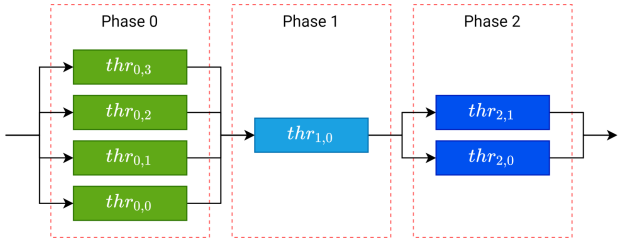

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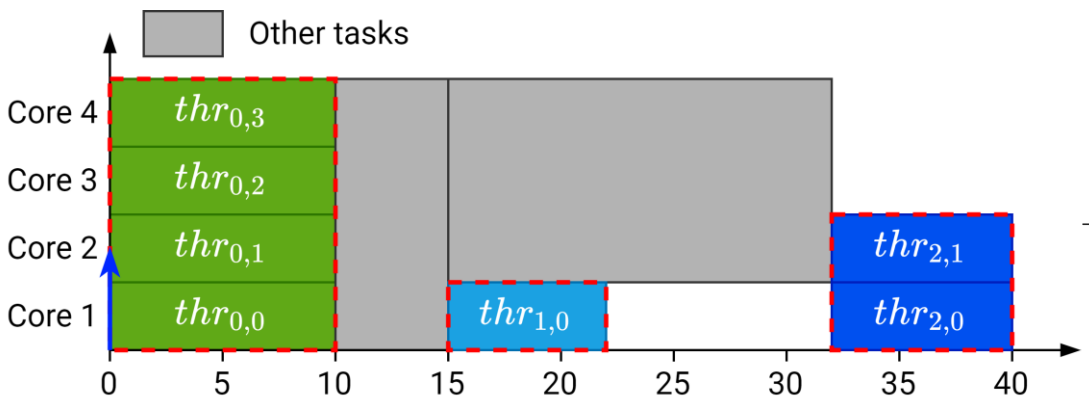
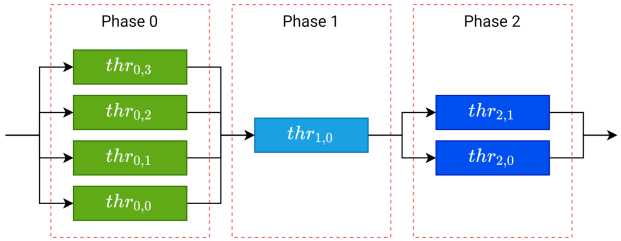
- Rigid gang reserves the whole block
- Bundled creates rigid blocks with dependencies



Bundled model scheduling



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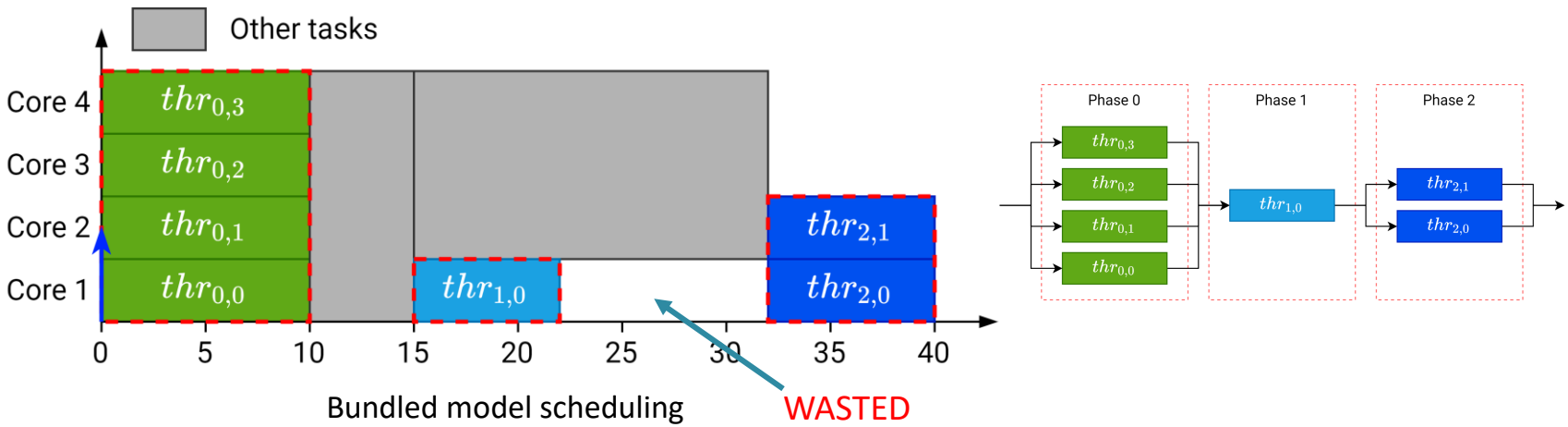
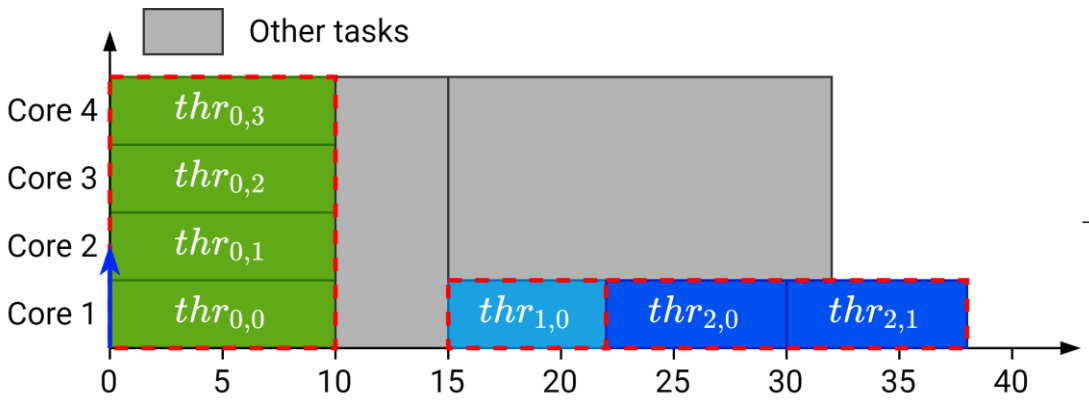
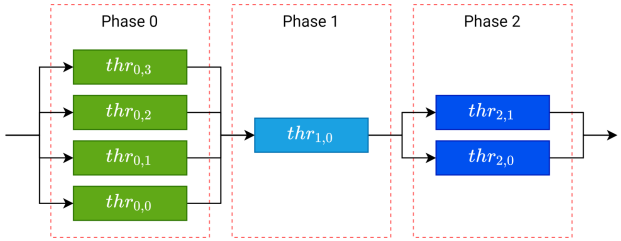


Bundled scheduling[1] vs limited-preemptive

- Rigid gang reserves the whole block
- Bundled creates rigid blocks with dependencies
- Limited-Preemptive creates moldable blocks with dependencies



Limited-Preemptive scheduling



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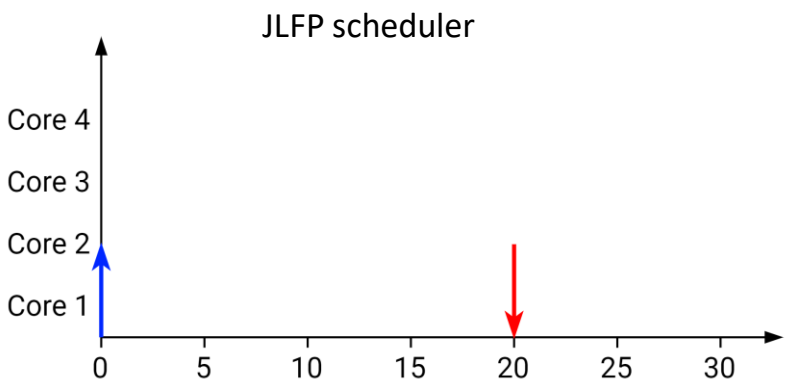

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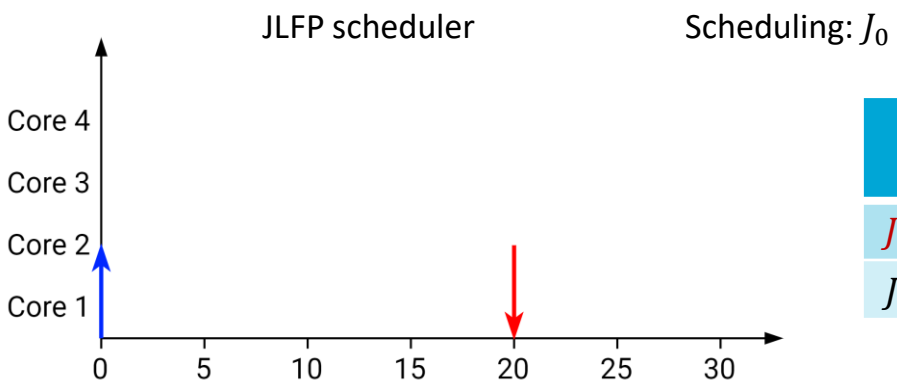

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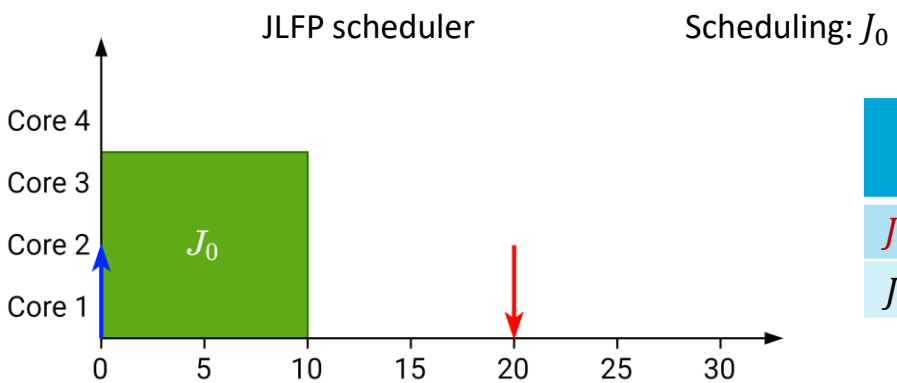

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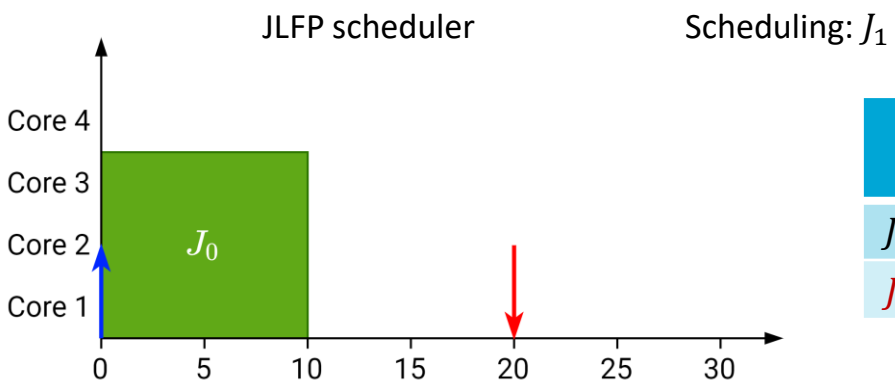

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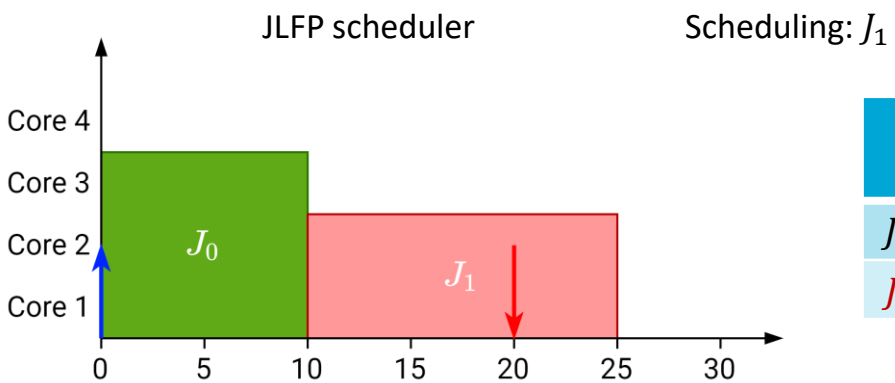

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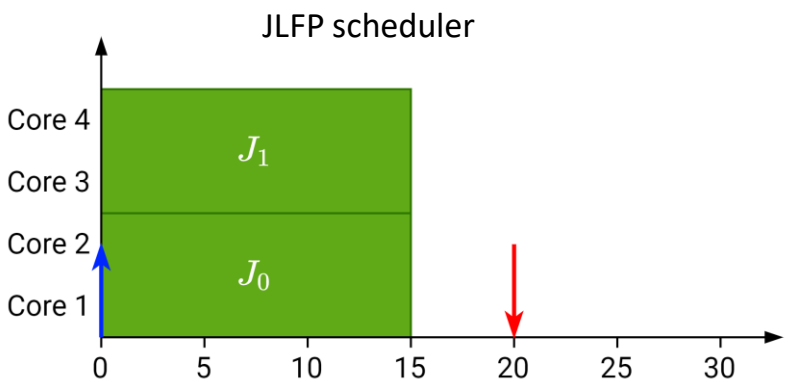

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Preemptive solutions

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- Job-level fixed-priority[2]

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Preemptive solutions

Schedulability tests

- Job-level fixed-priority[2]
- Earliest deadline first[3]

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Schedulers

- Optimal for rigid gang (DP-Fair)[4]

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Preemptive solutions

Schedulability tests

- Job-level fixed-priority[2]
- Earliest deadline first[3]

Schedulers

- Optimal for rigid gang (DP-Fair)[4]
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Our work

Project goals

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1. Design an accurate schedulability analysis for limited-preemptive moldable gang tasks

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Project goals

1. Design an accurate schedulability analysis for limited-preemptive moldable gang tasks

2. Evaluate the impact of the level of parallelism assigned to the jobs

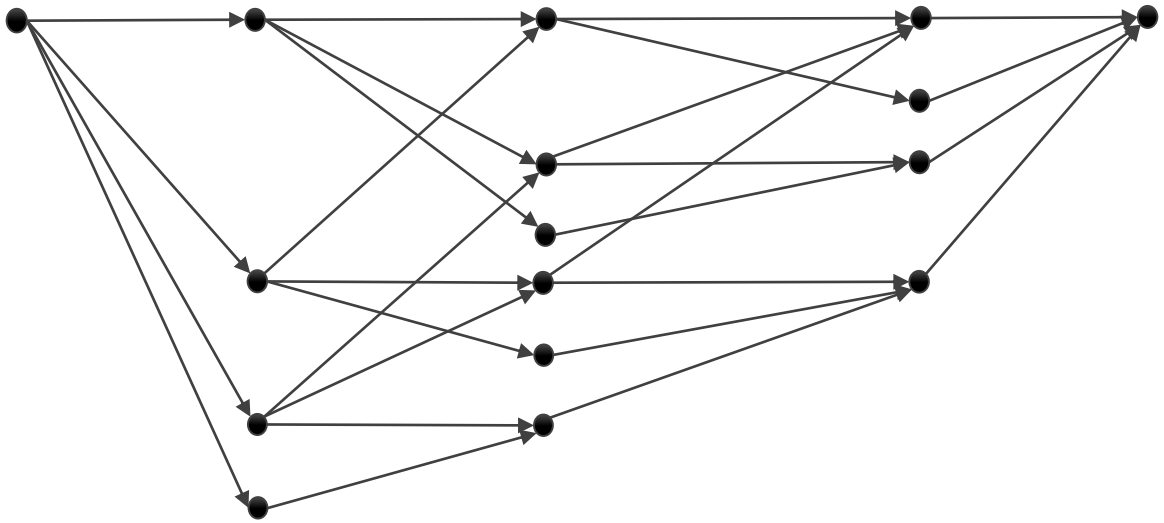
3. Propose a new scheduling algorithm to improve the schedulability of limited-preemptive moldable gang tasks

Agenda

- Gang schedulability analysis
- New scheduling policy

Schedule abstraction graph

Schedule abstraction graph



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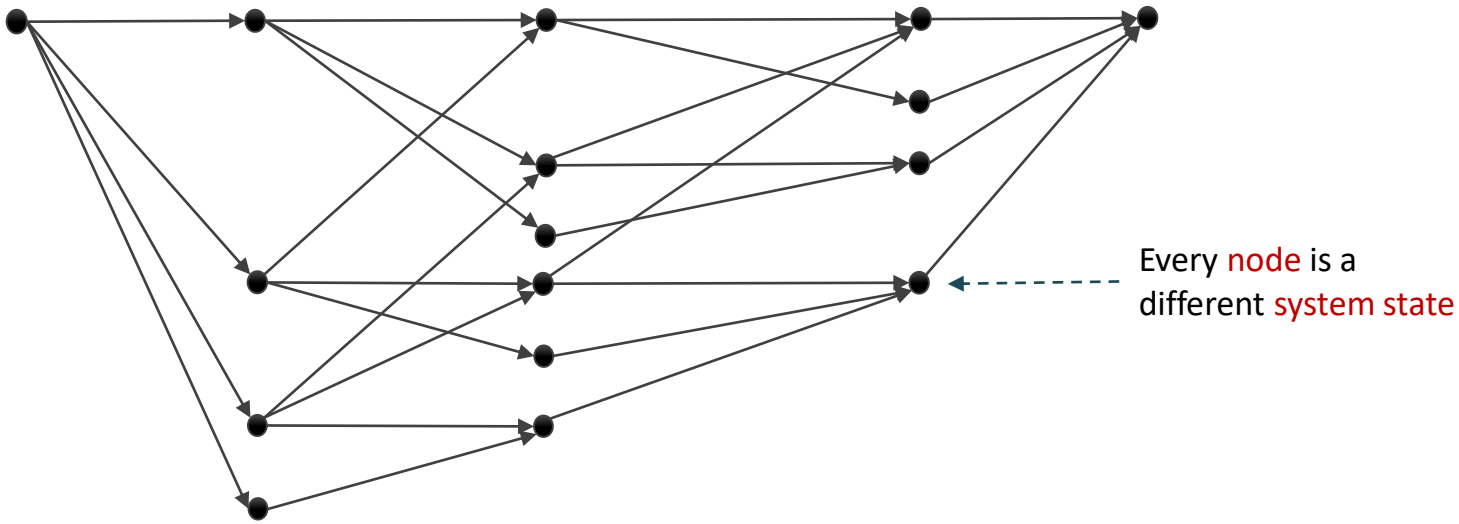

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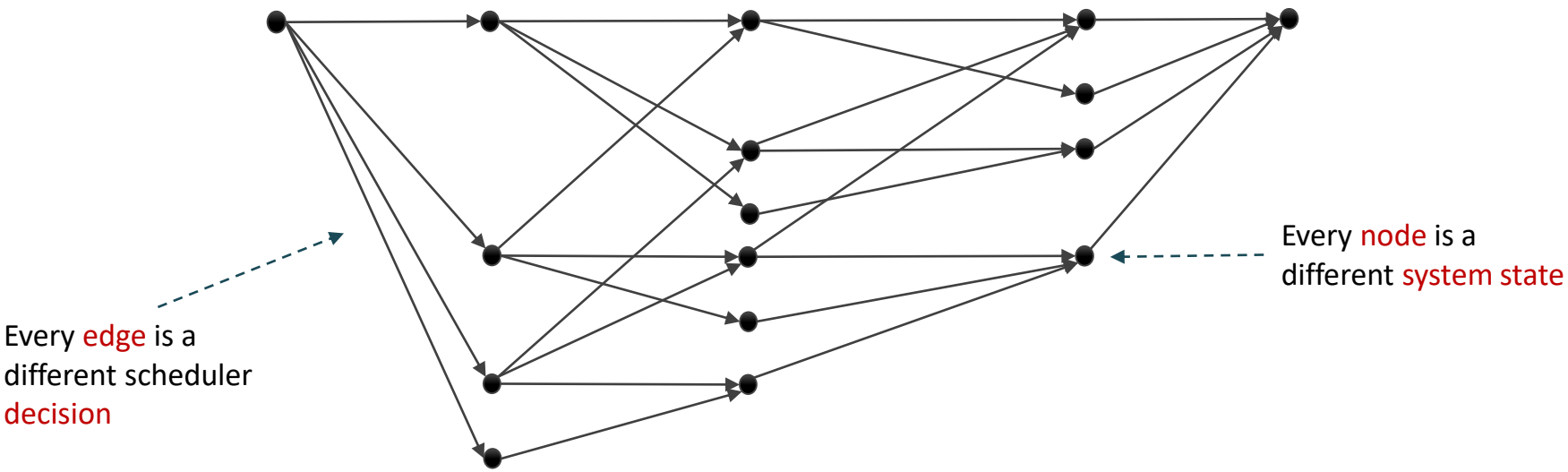

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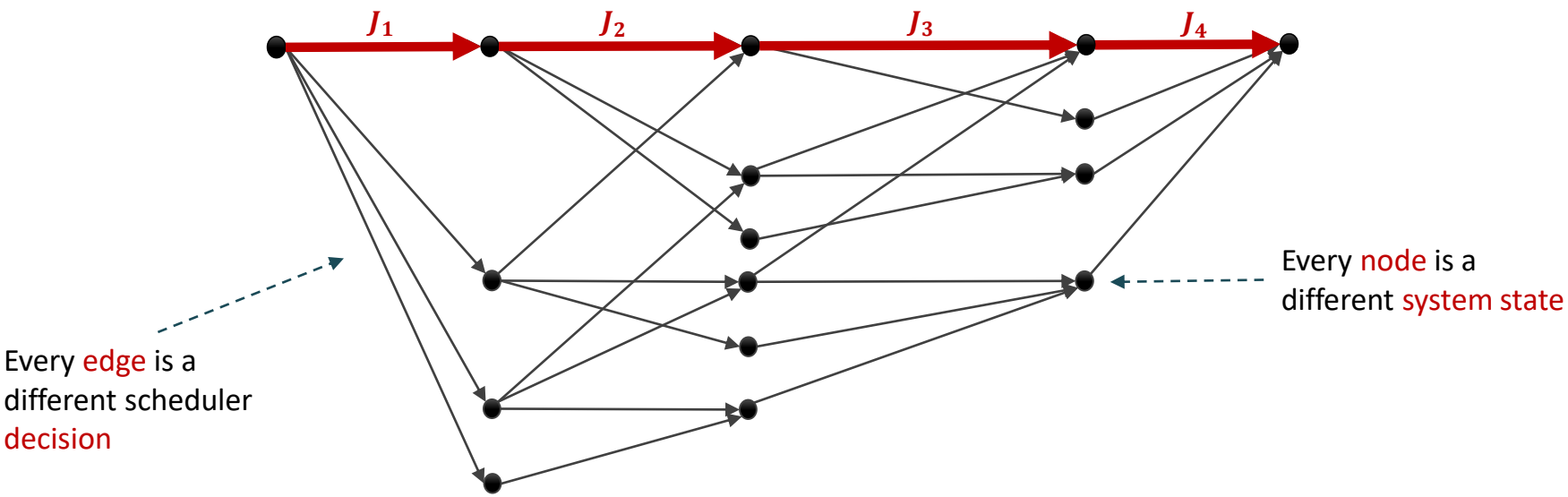

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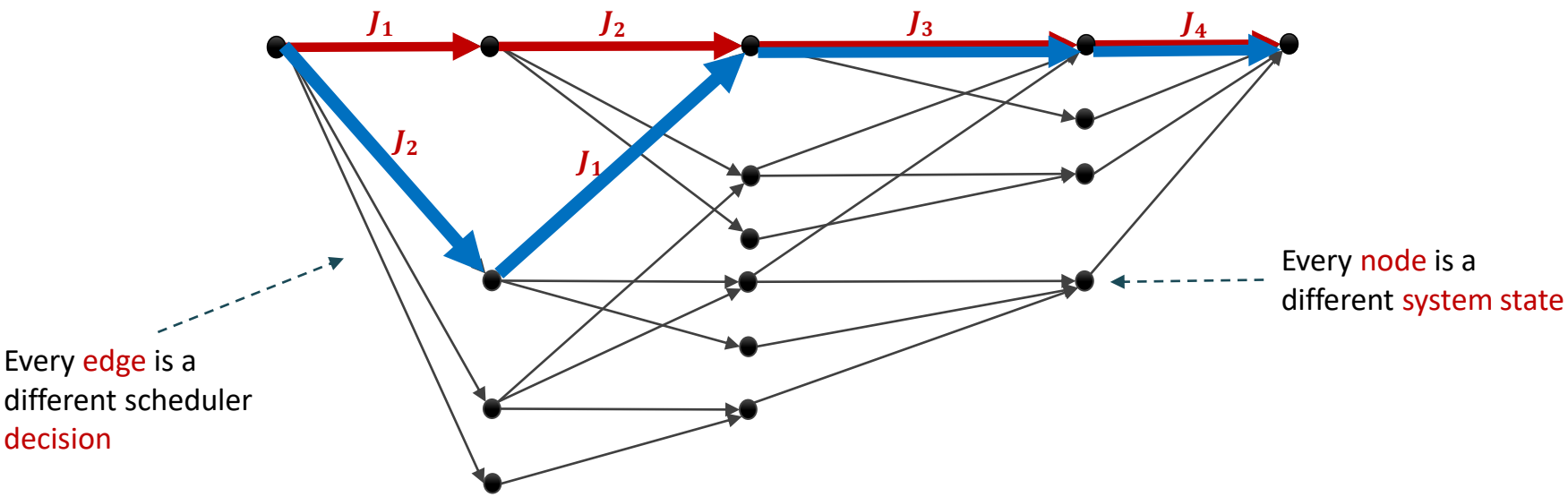
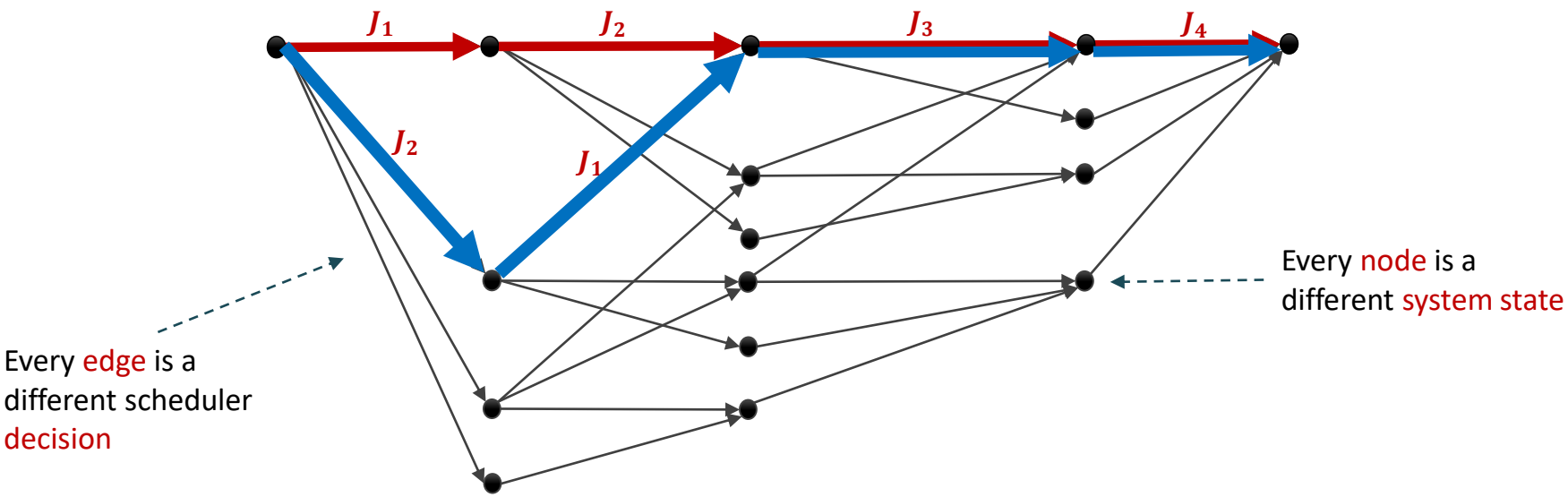


Schedule abstraction graph

- It is a technique that allows:
  - Search for all possible schedules
  - Aggregate “similar” schedules



Extending SAG

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- Update system state representation

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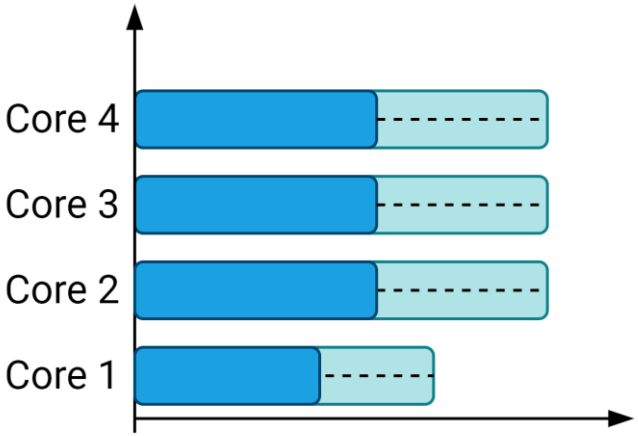
- Update expansion rules
- Update merge rules

New state representation

New state representation

- We already had core availabilities



New state representation

- We already had core availabilities
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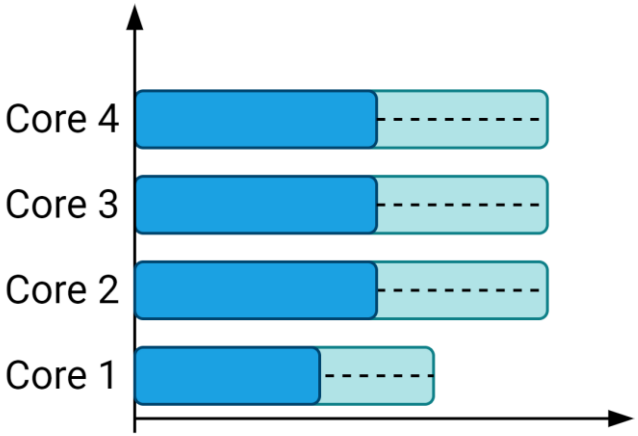

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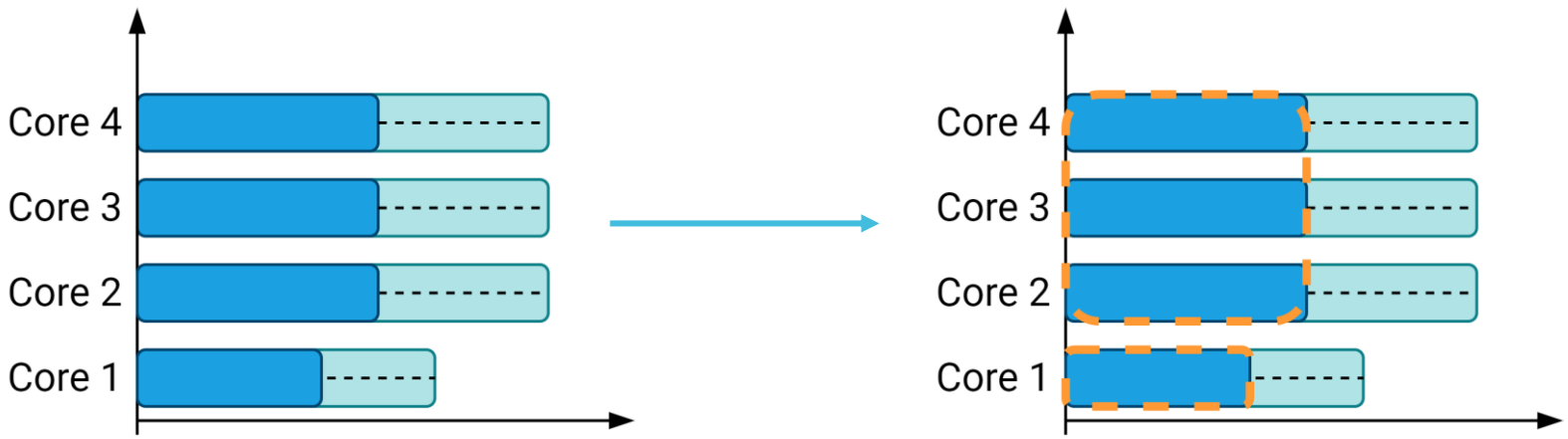

New state representation

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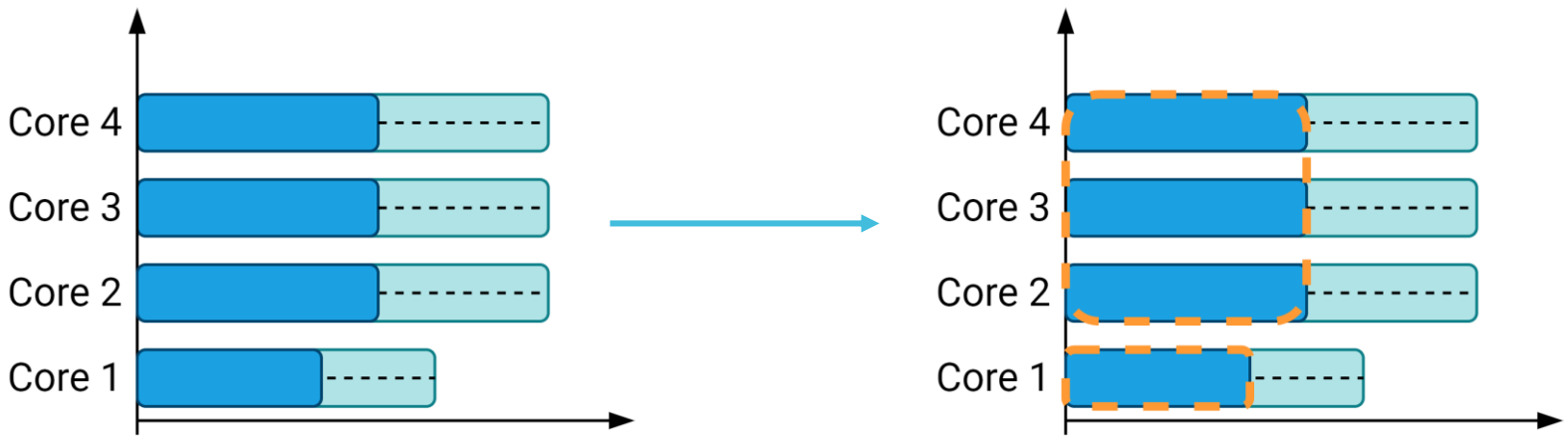

New SAG expansion rules

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S_0

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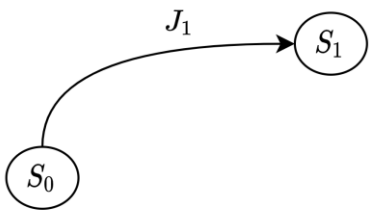

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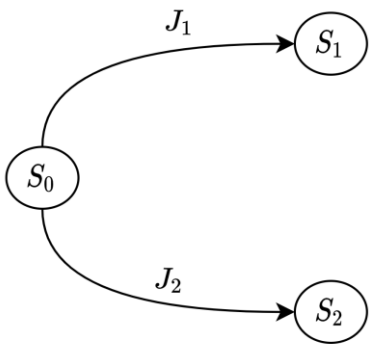

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- Previously a state was created for every schedulable job
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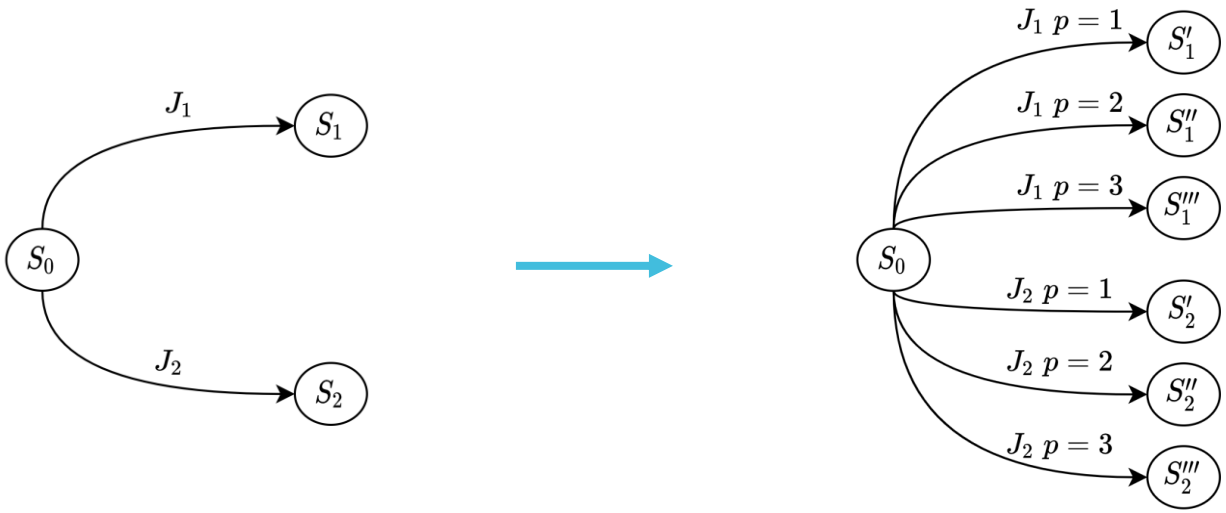
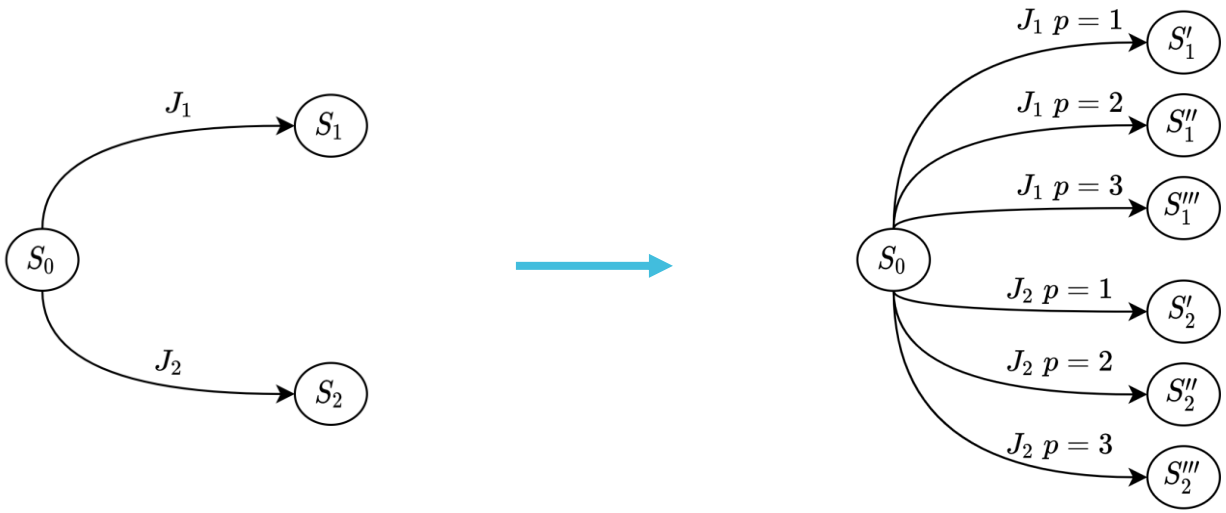


New SAG expansion rules

- Previously a state was created for every schedulable job
- Now a state is created for every job and possible number of cores
- Can stimulate state-space explosion



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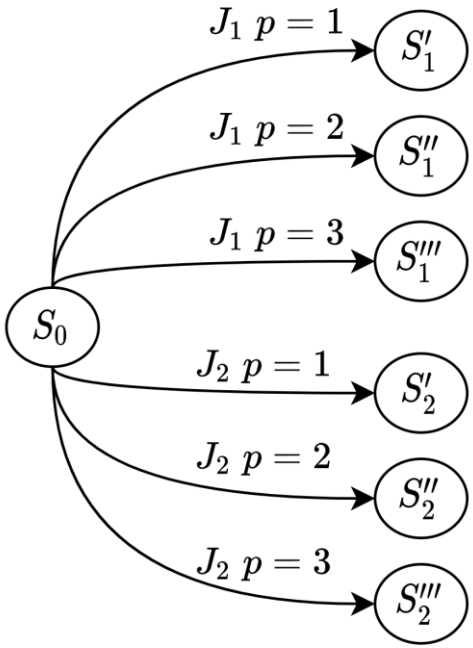
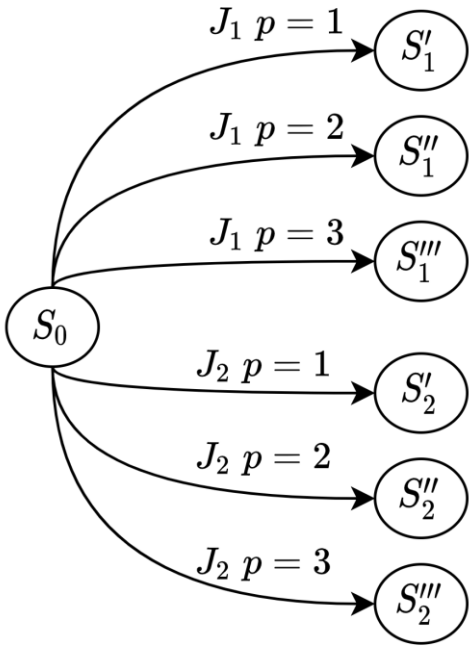


New SAG expansion rules

- Purge states by checking that



New SAG expansion rules

- Purge states by checking that
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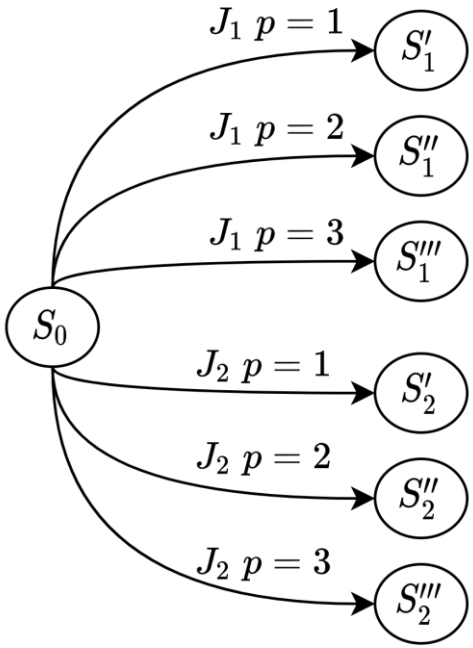
  -



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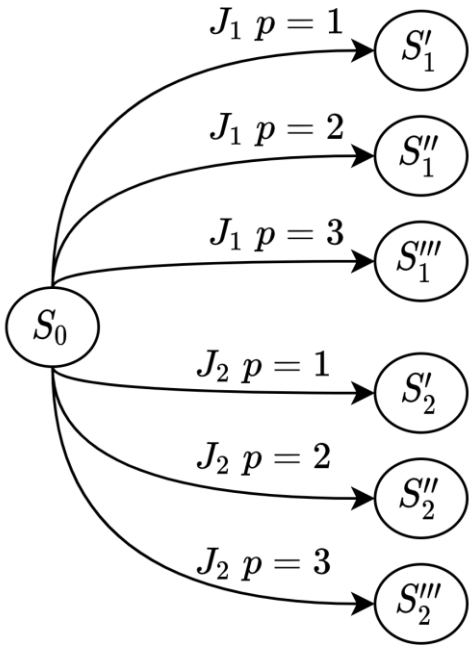

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- Purge states by checking that
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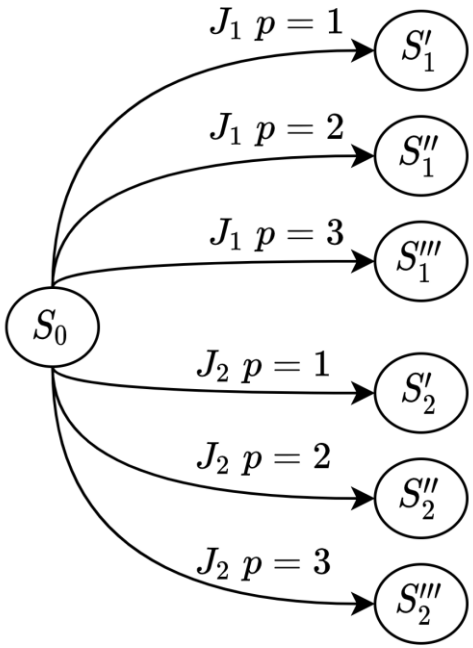

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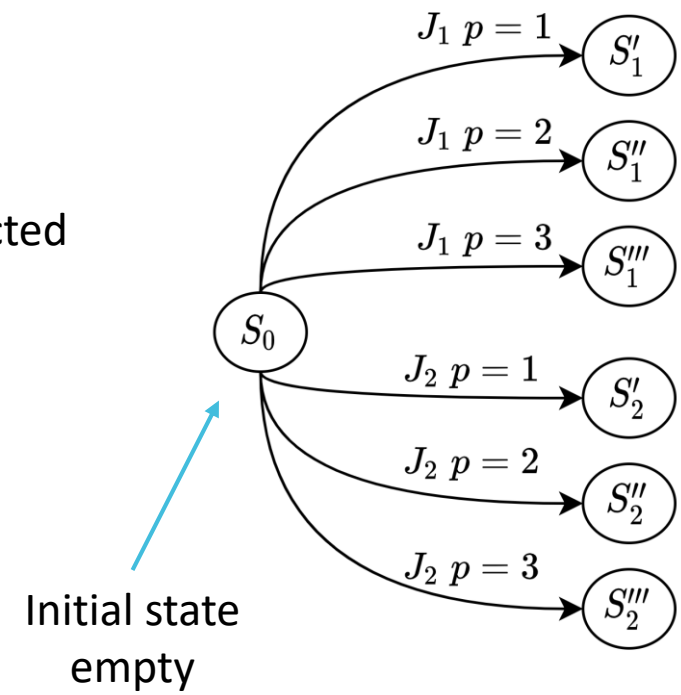

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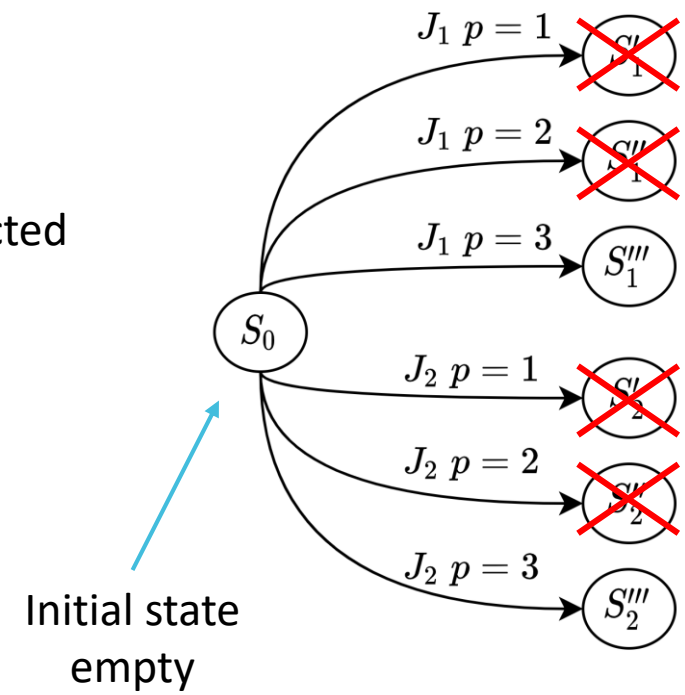

New SAG expansion rules

- Purge states by checking that
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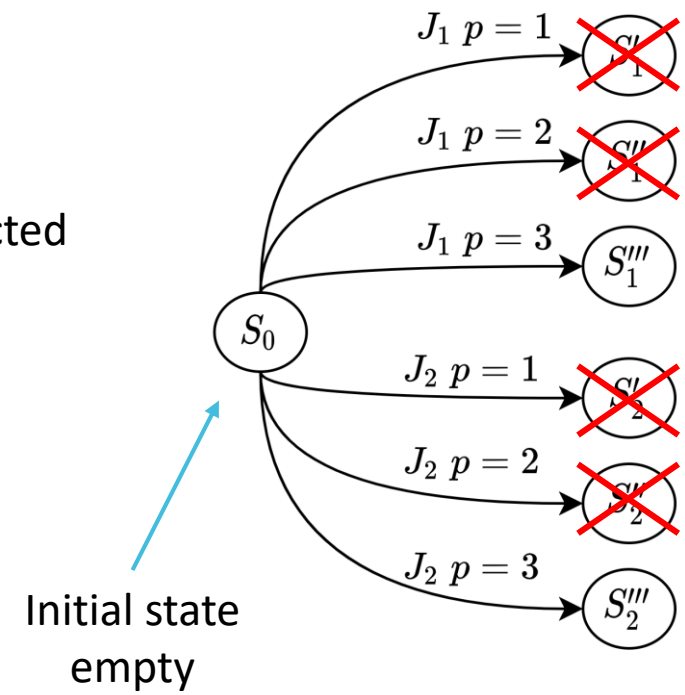
- Exploring more states



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- Purge states by checking that
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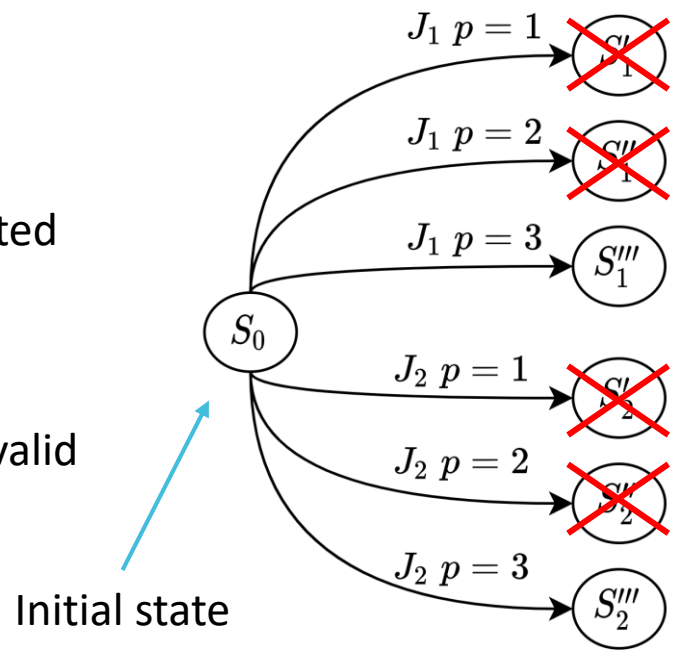
  - Is safe, does not make analysis invalid



Initial state empty

New SAG expansion rules

- Purge states by checking that
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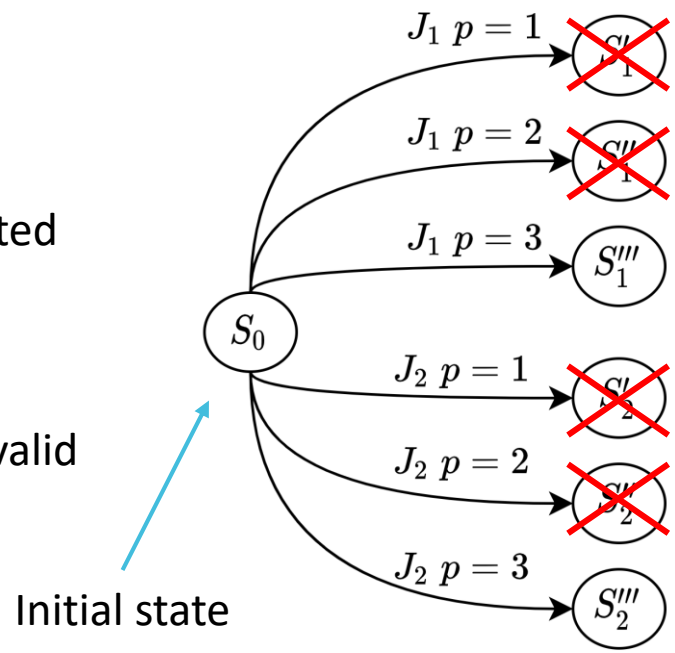
- Exploring more states
  - Is safe, does not make analysis invalid
  - Slower and more pessimistic



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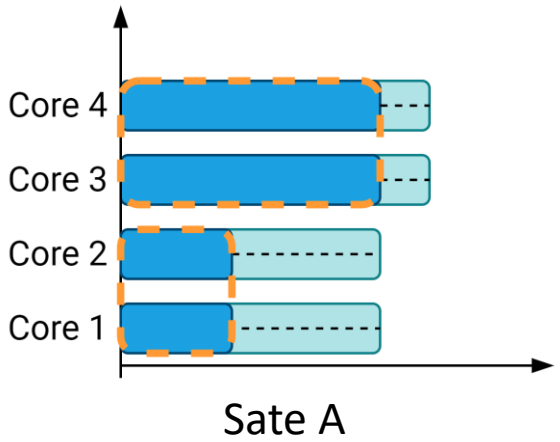

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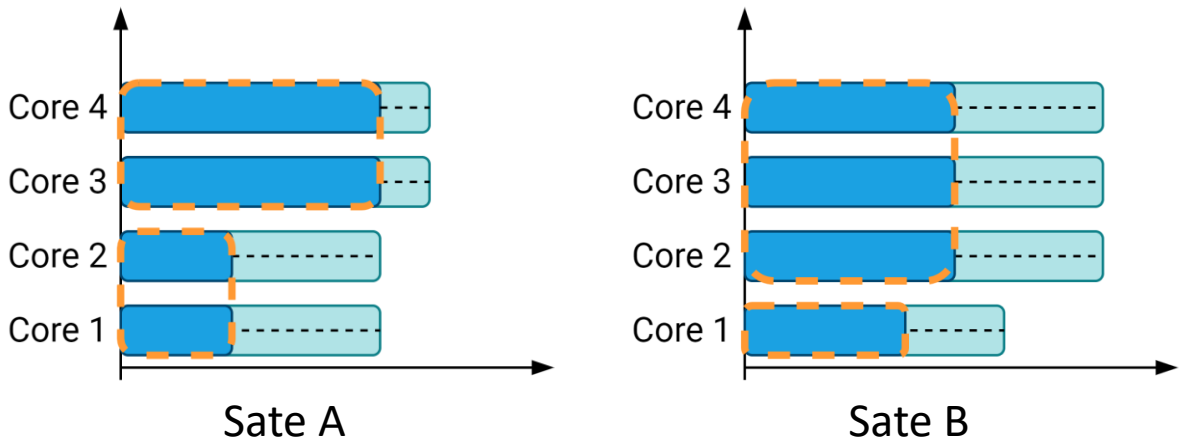

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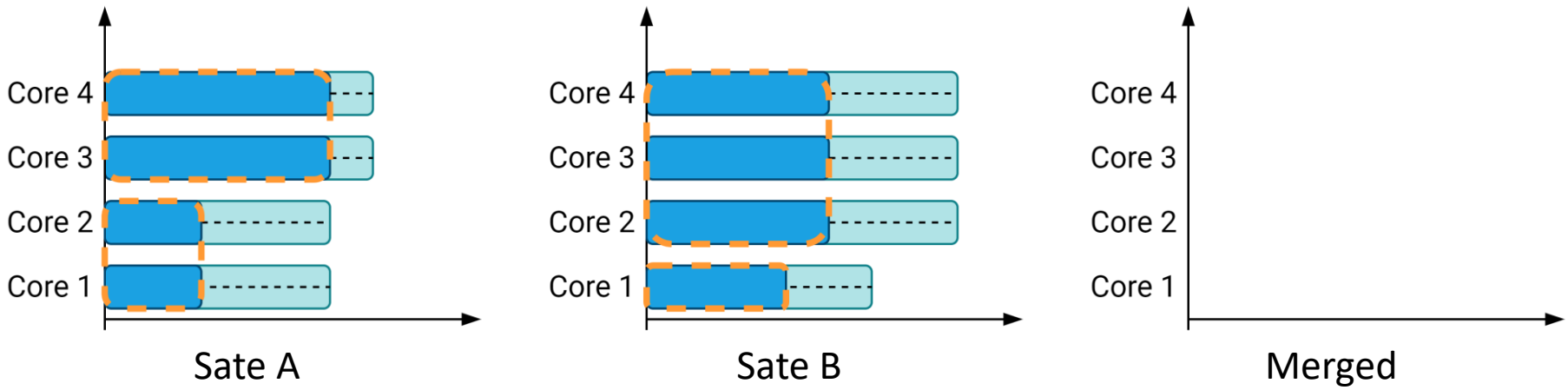

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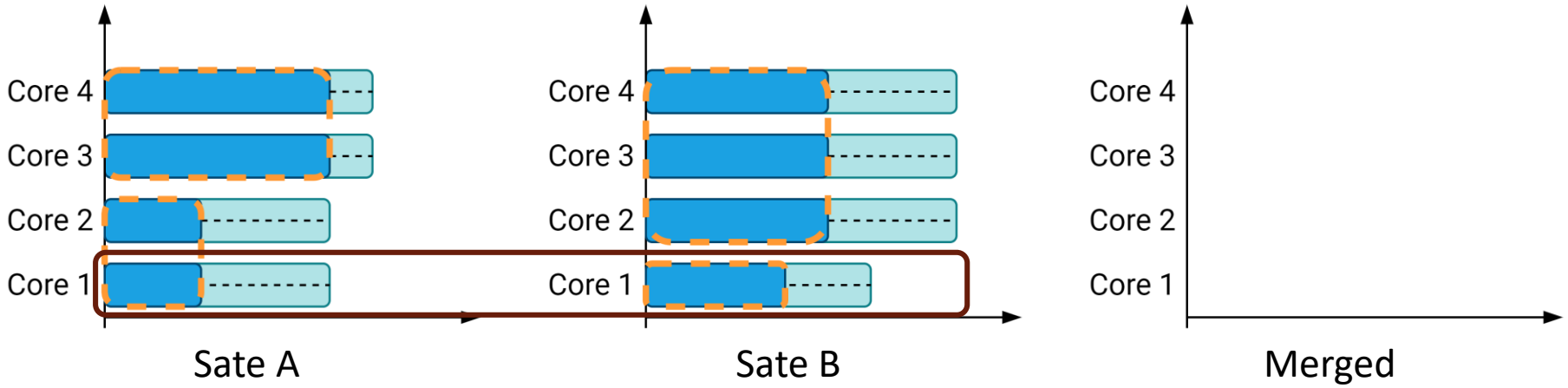

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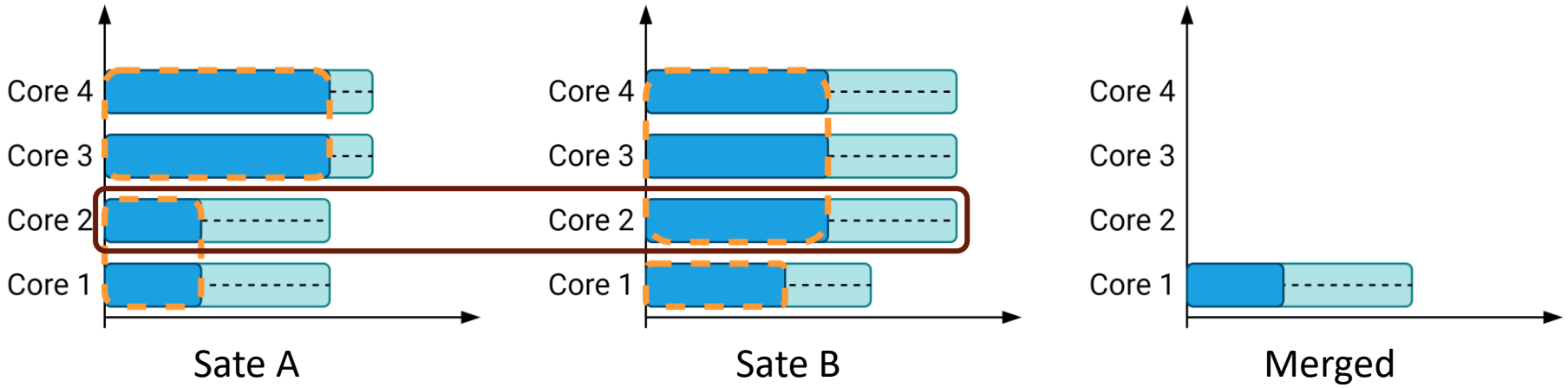

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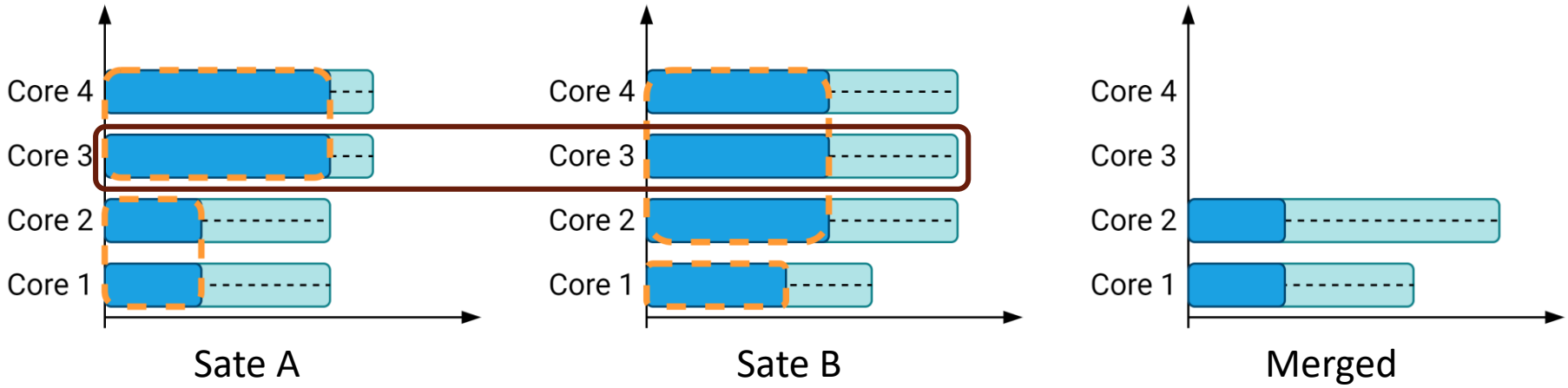

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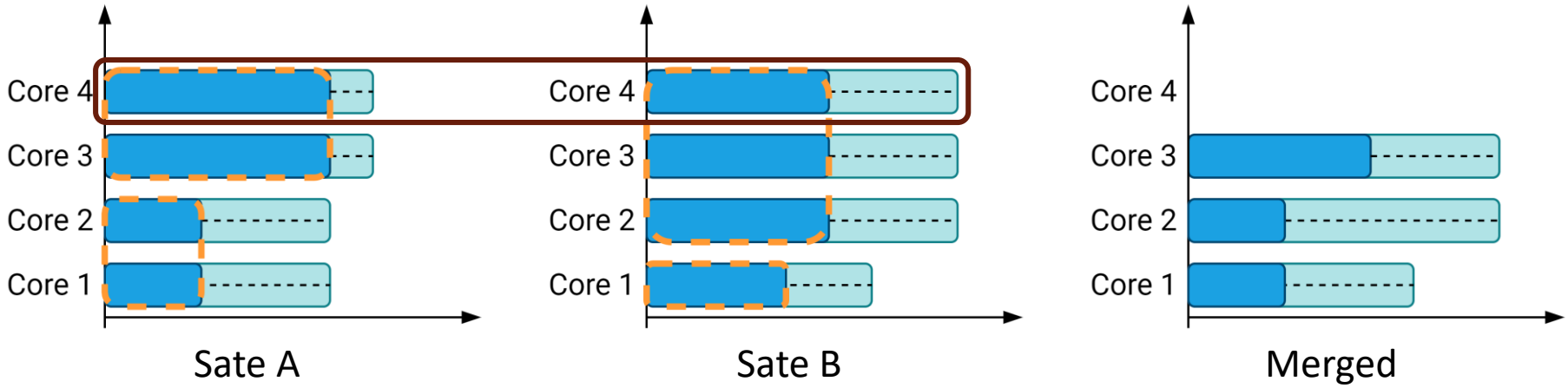

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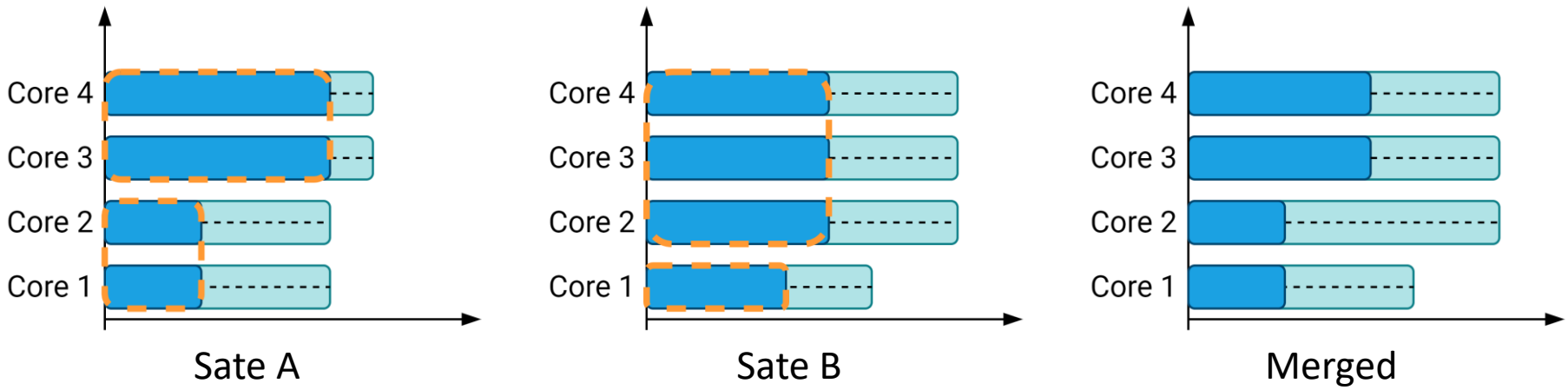

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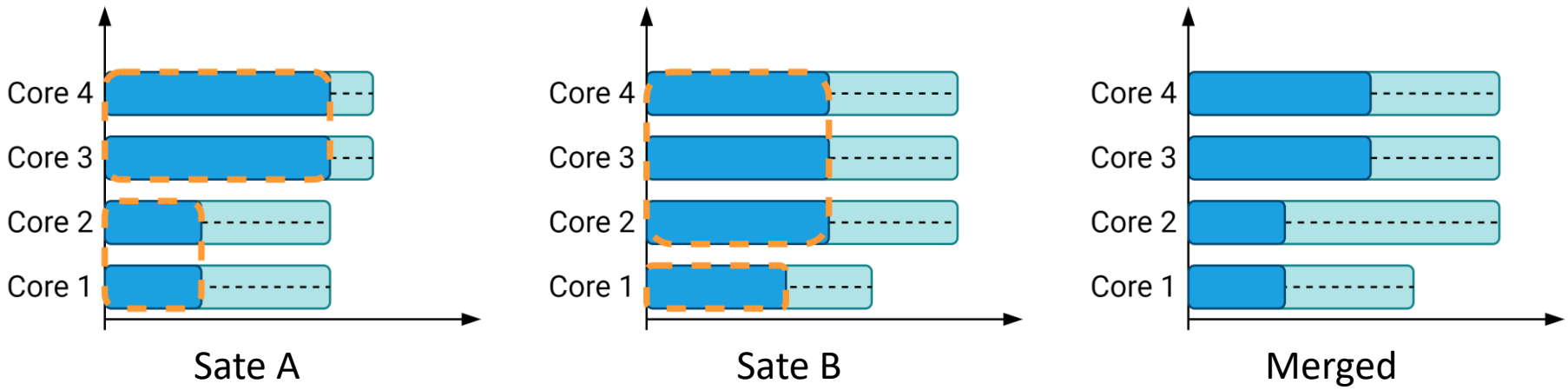

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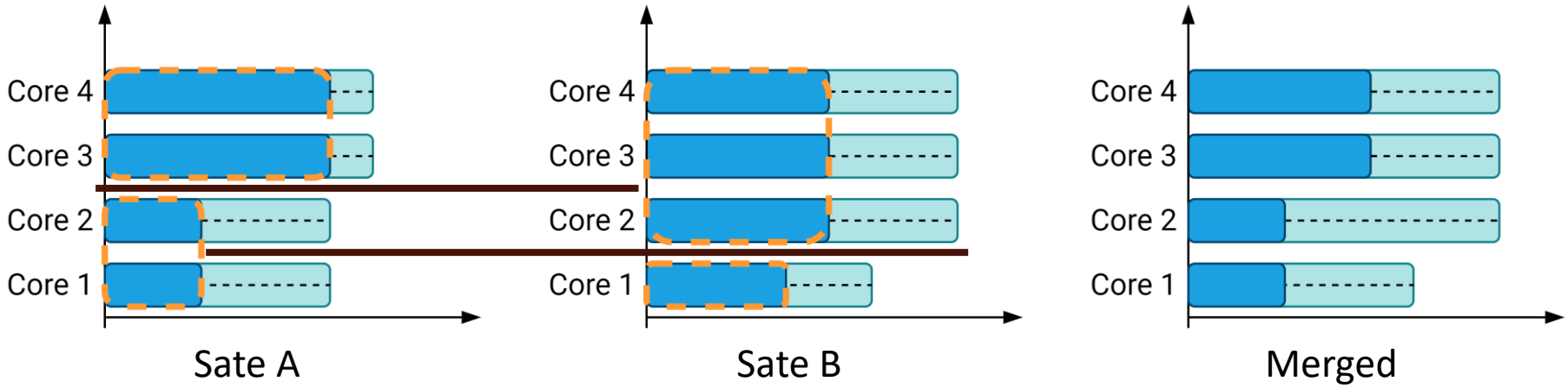

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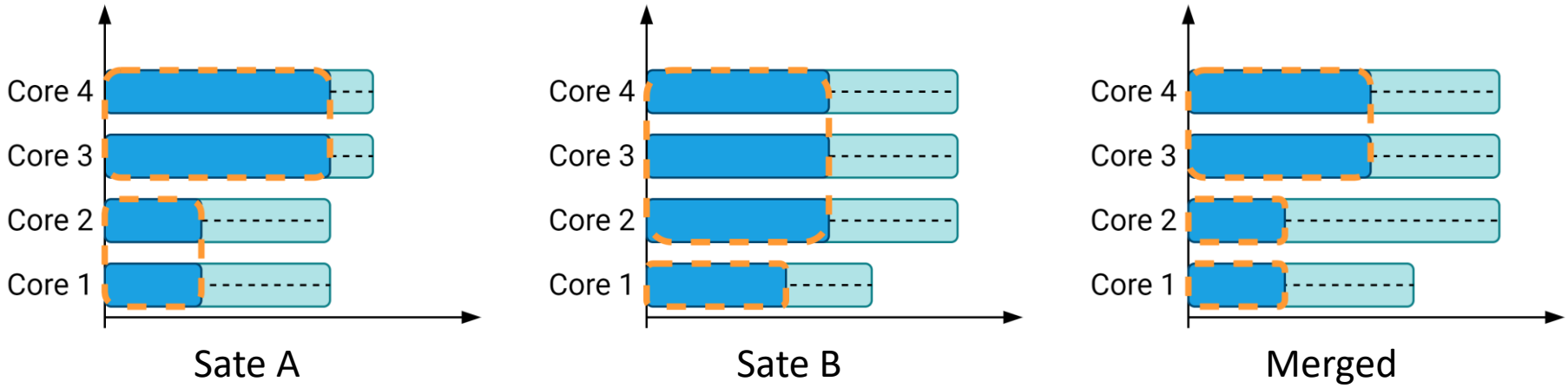

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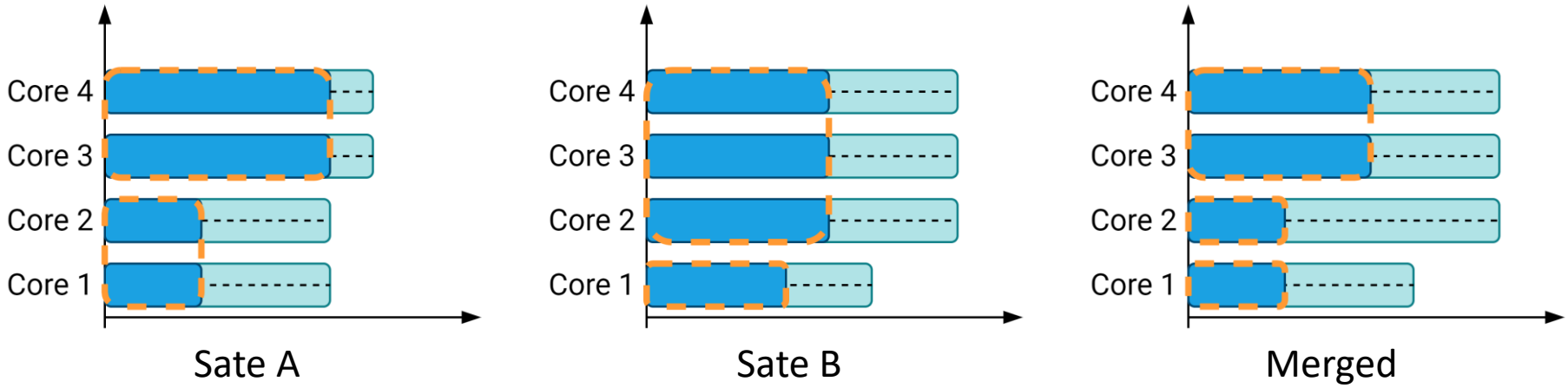
  - Certainly running jobs → Keep only jobs running in both states (intersect)



Our analysis results

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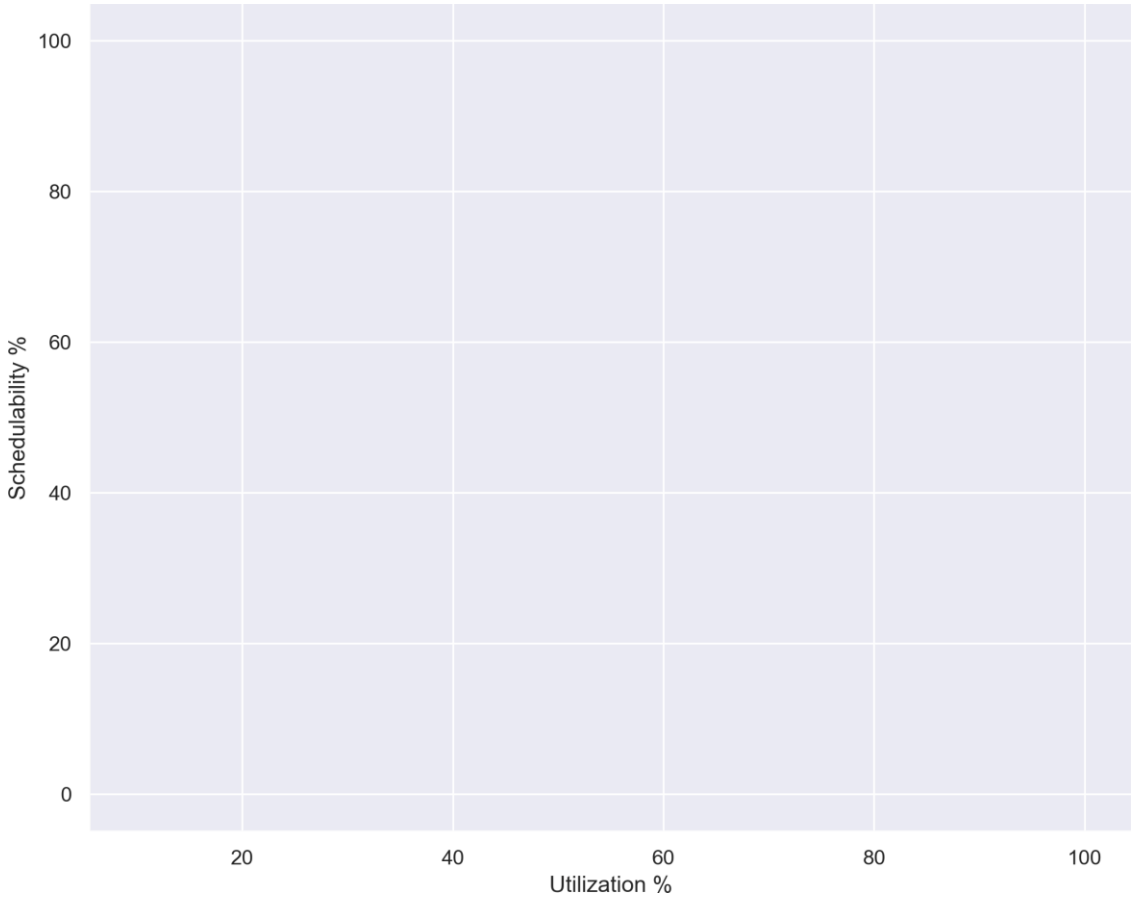
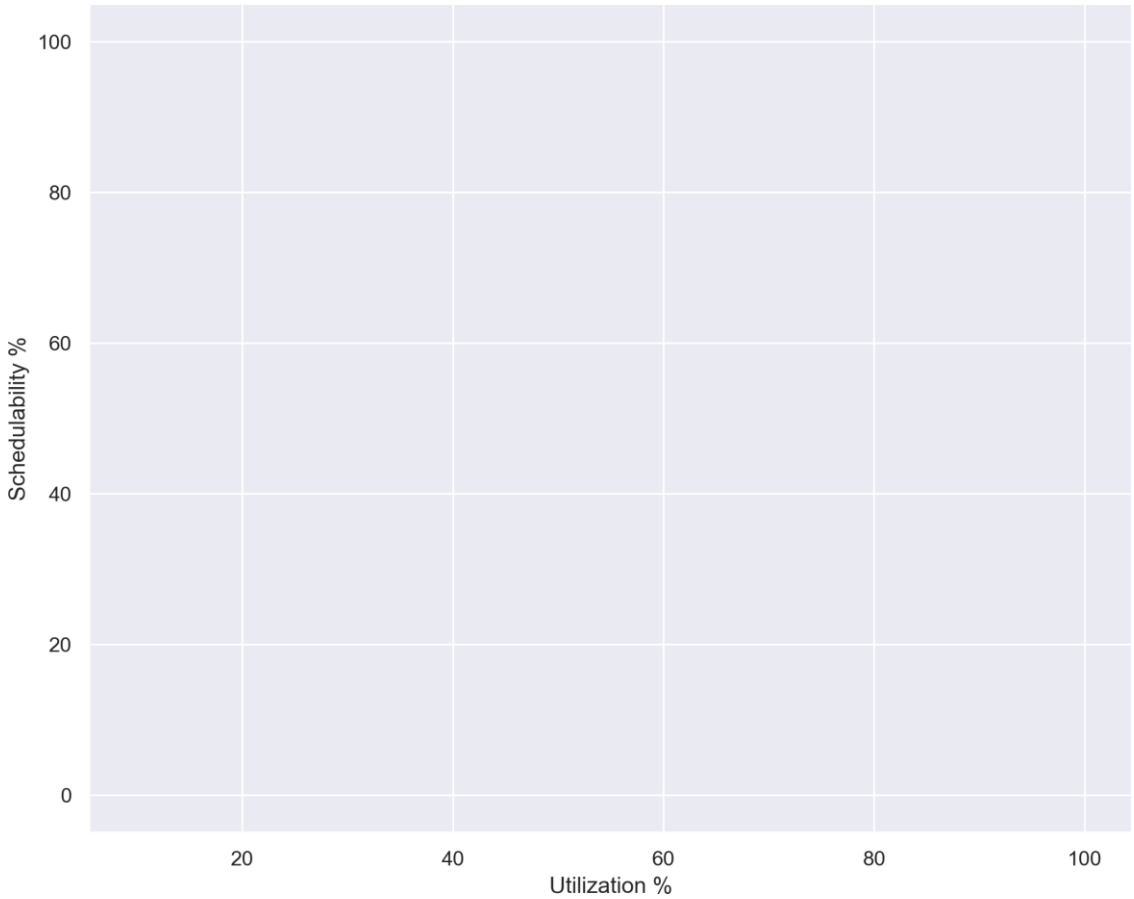


Our analysis results

Randomly generated task sets

- System processors: 8
- System tasks: 20 rigid tasks
- Execution time variation: 50%



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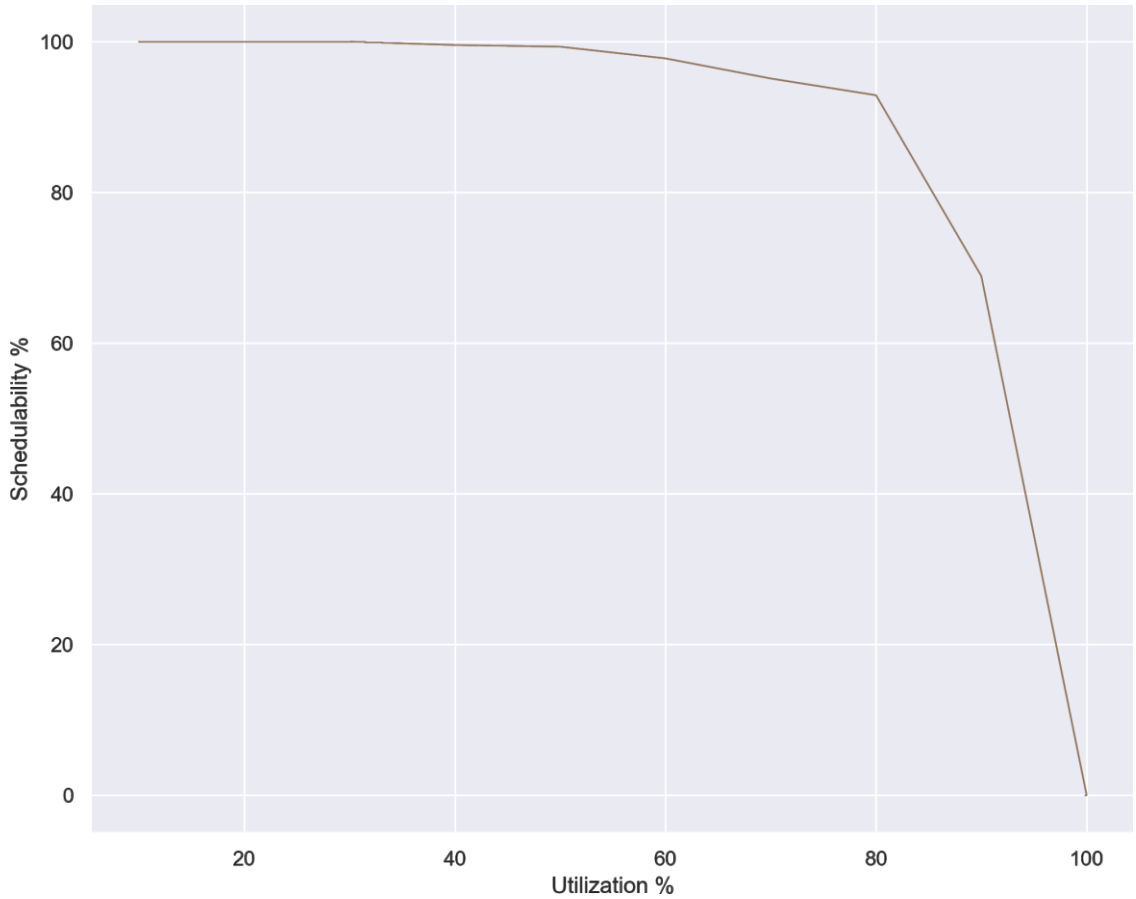

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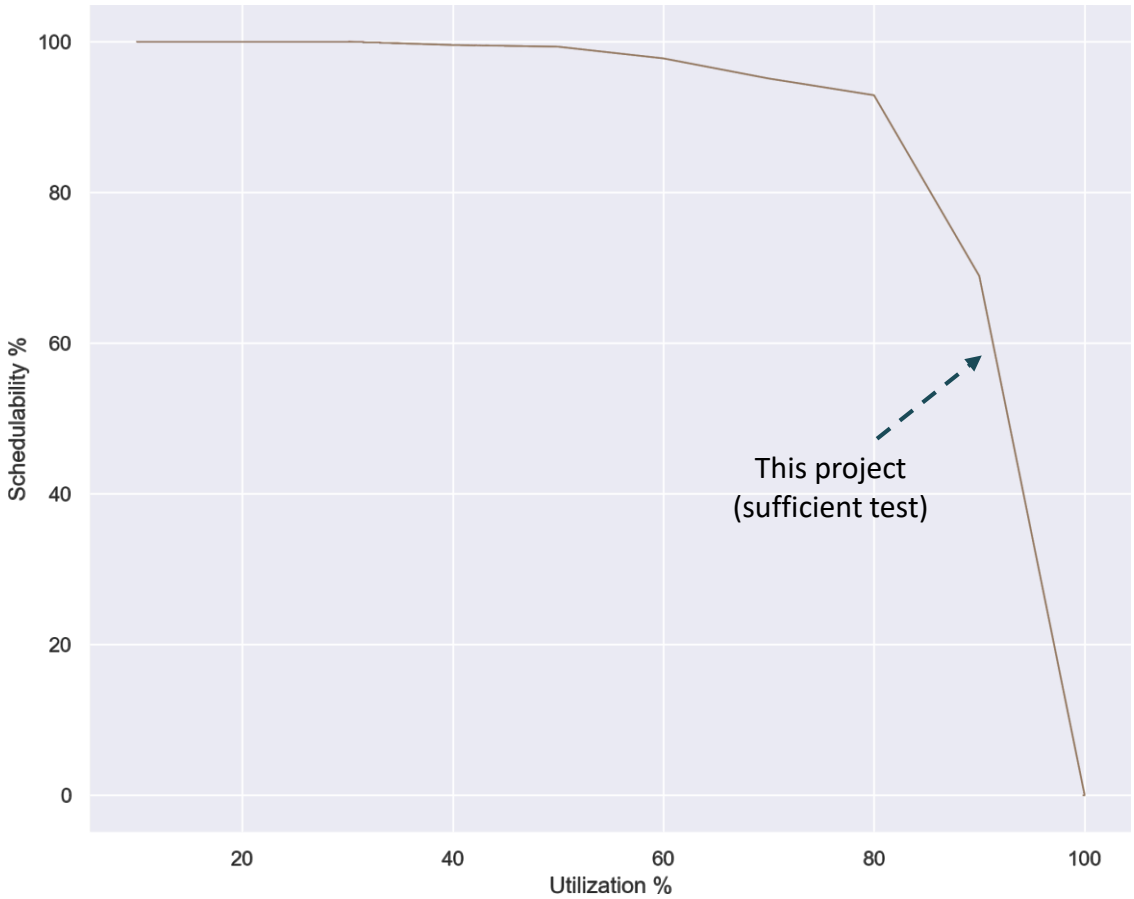

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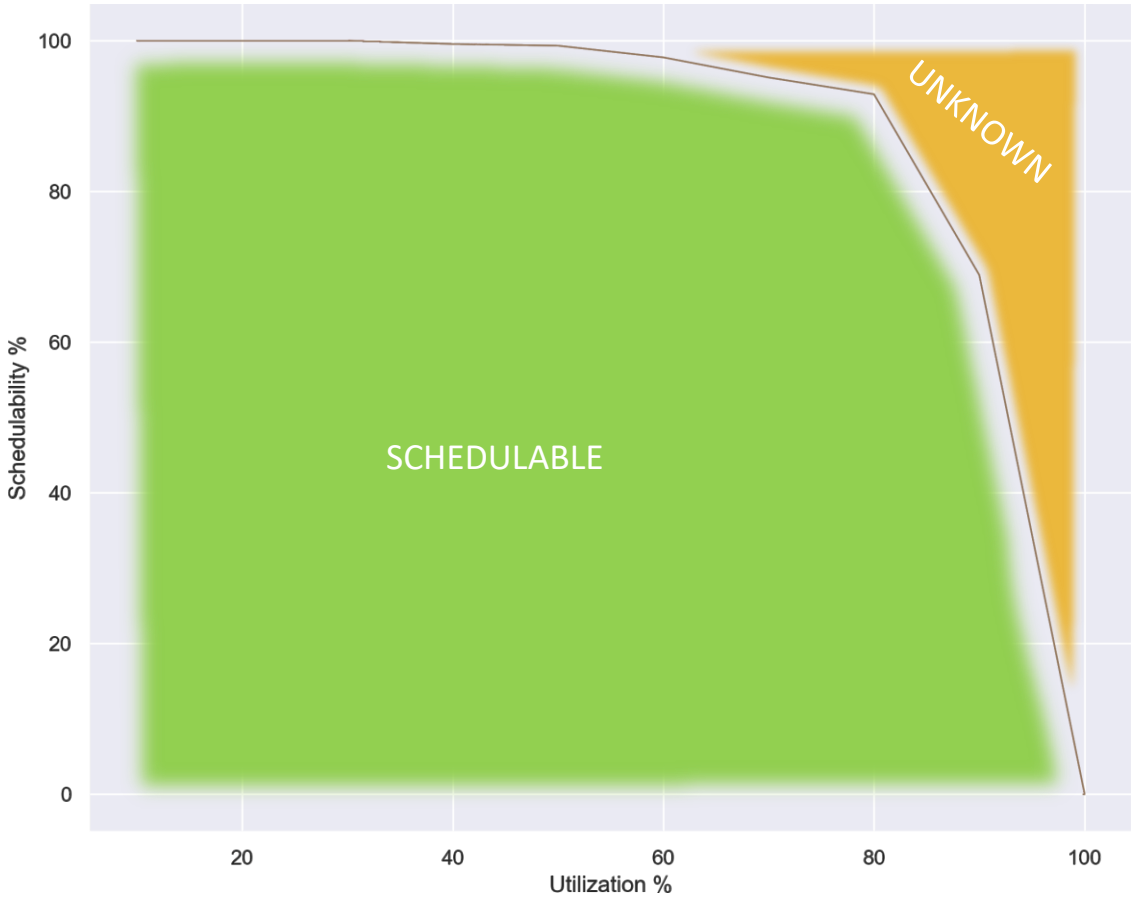

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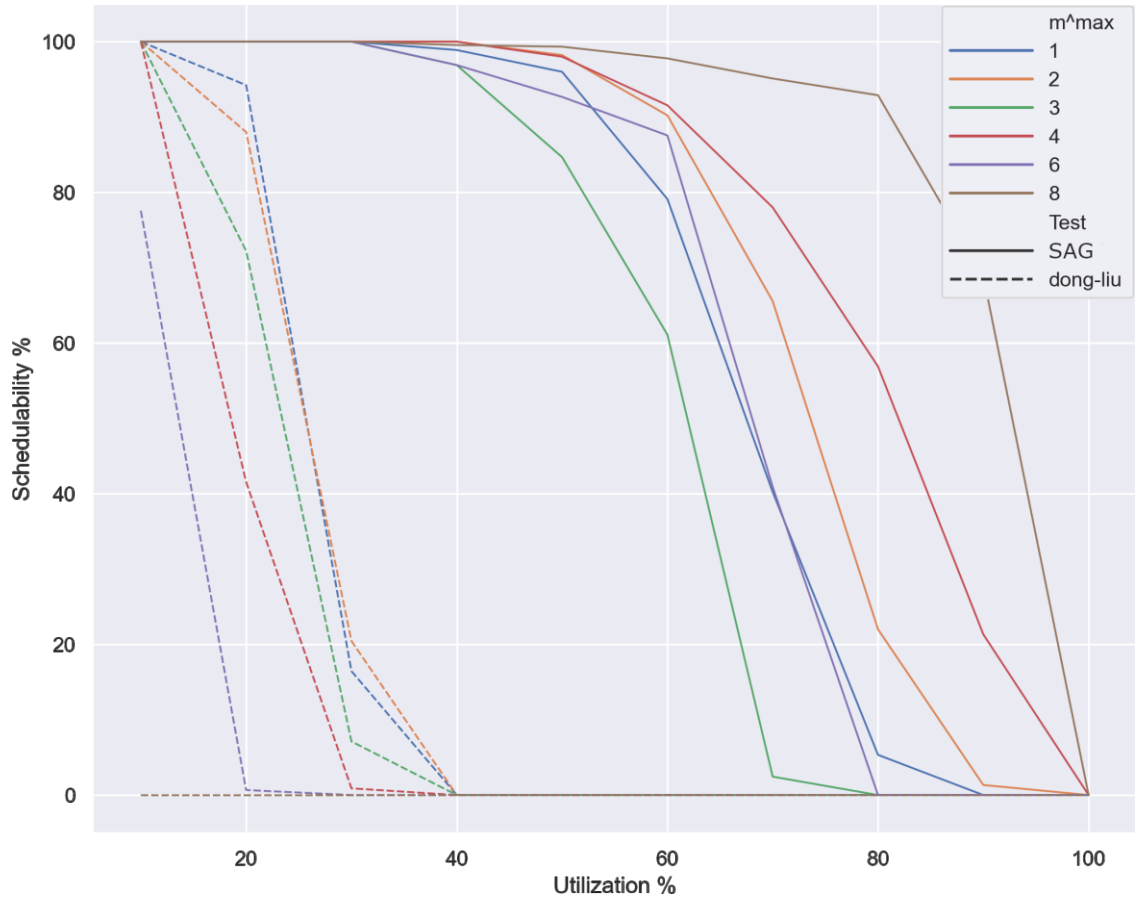

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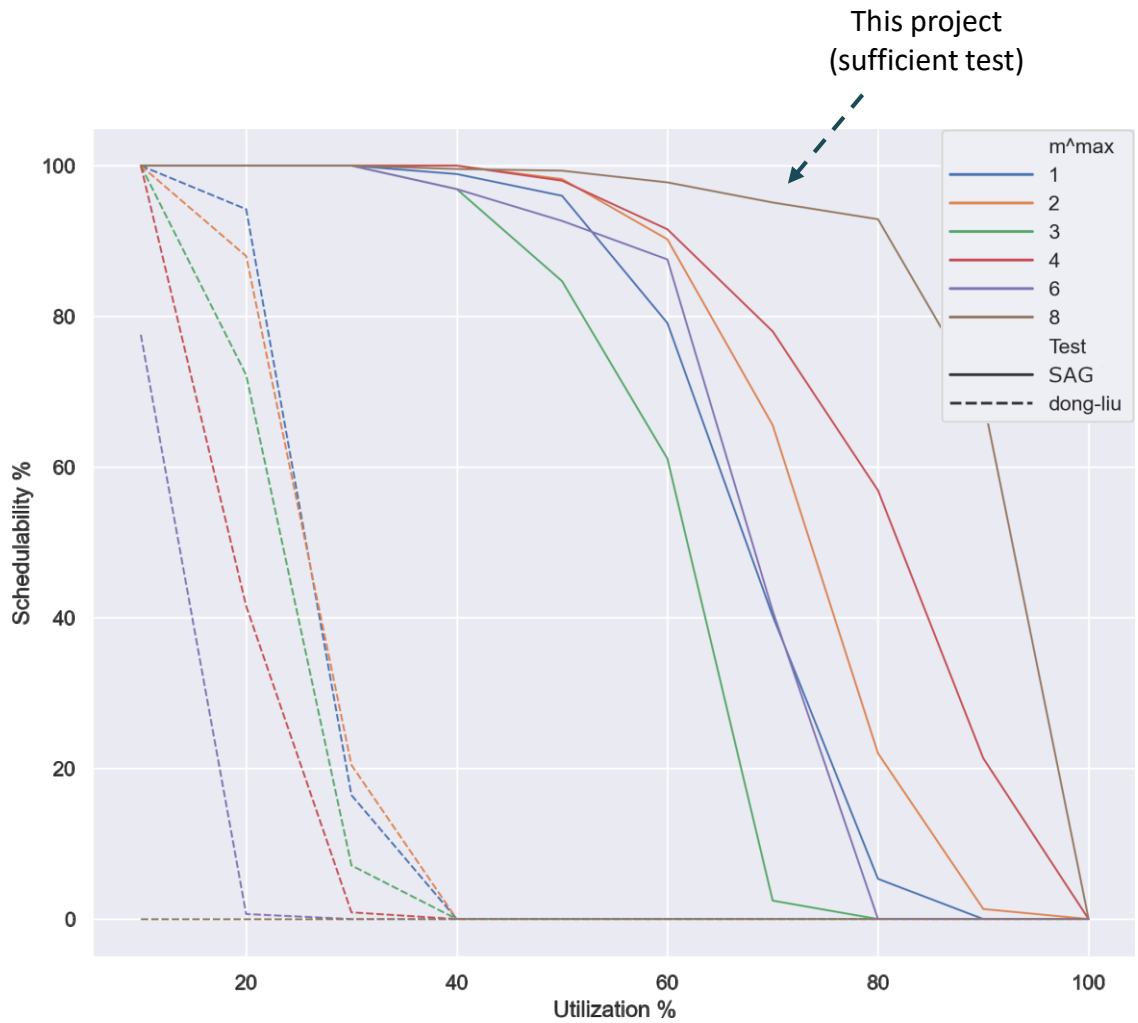

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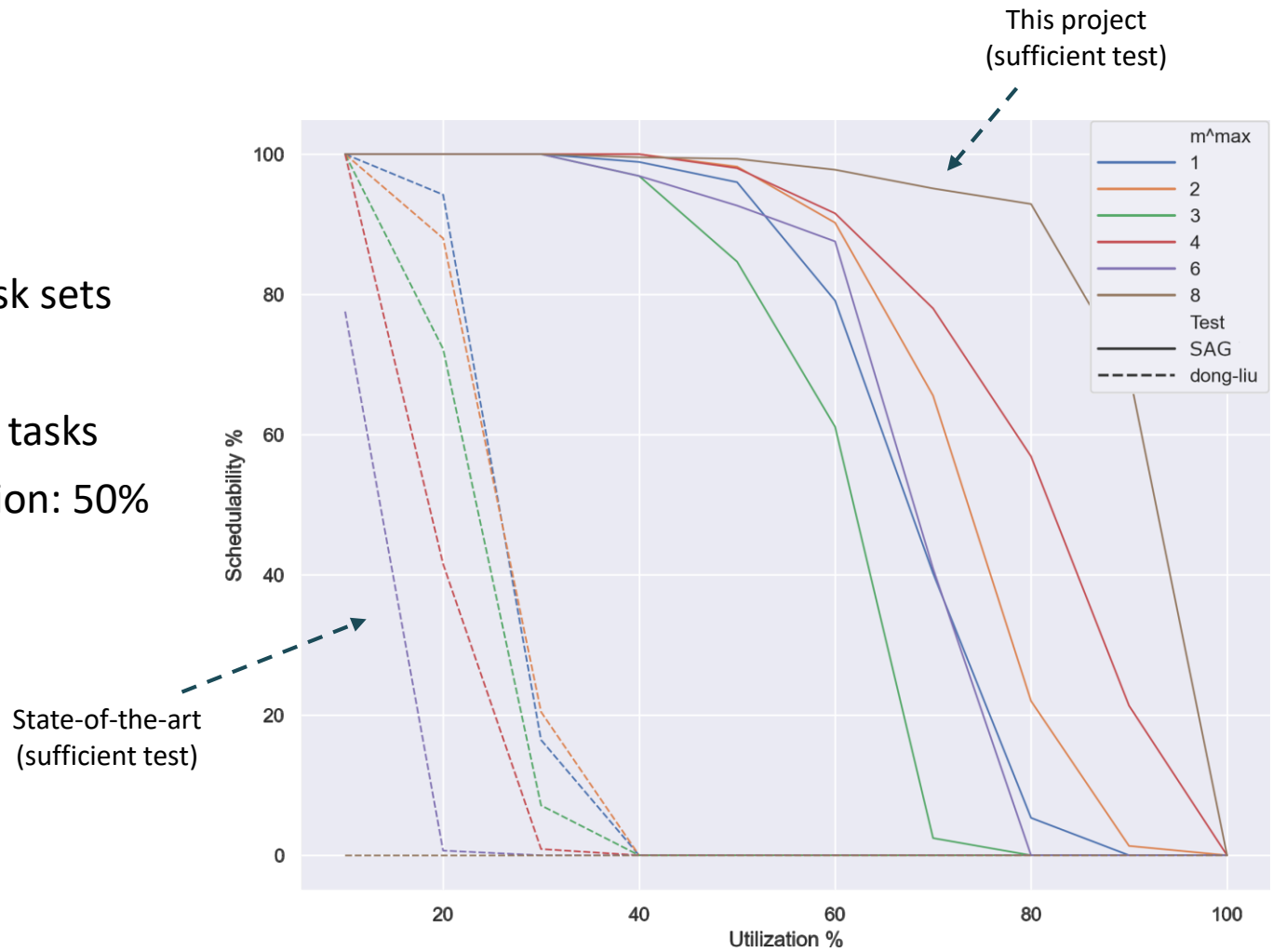
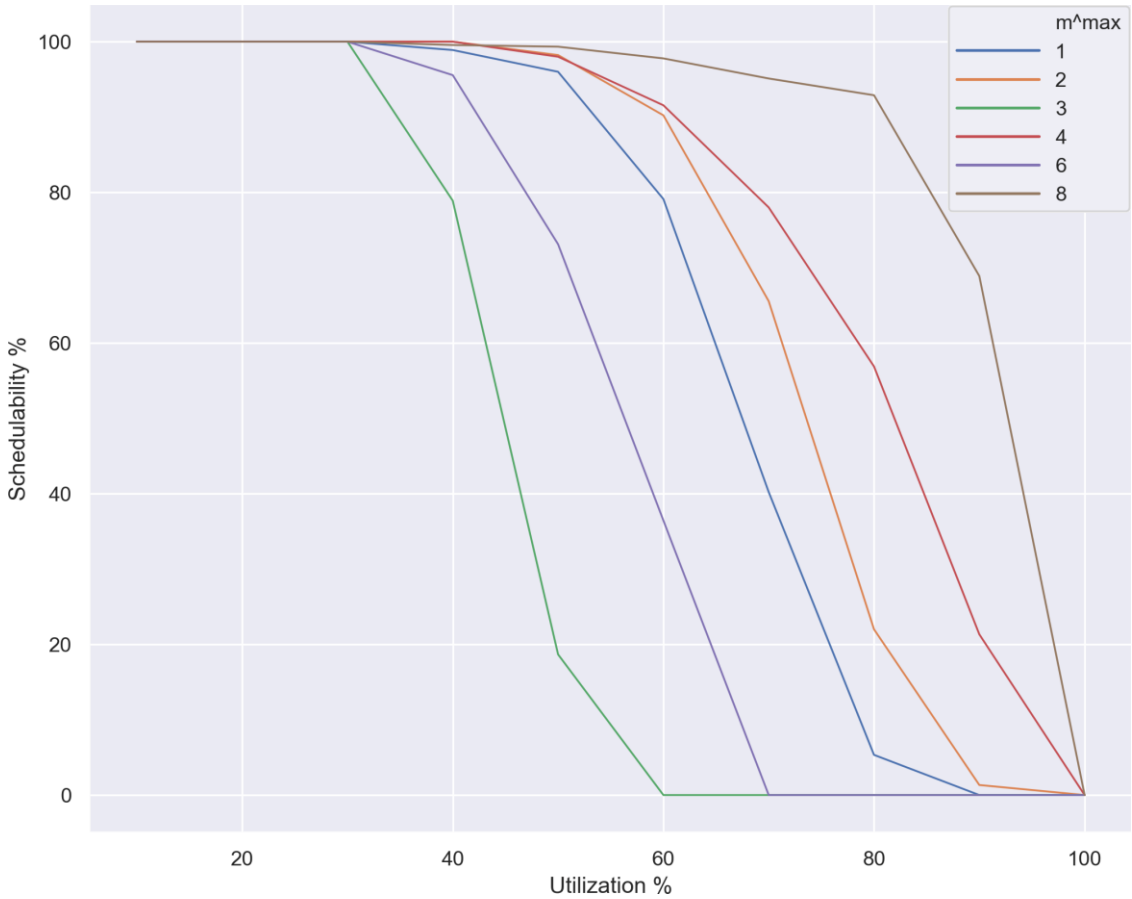


Our analysis results

Randomly generated task sets

- System processors: 8
- System tasks: 20 moldable tasks
- Execution time variation: 50%

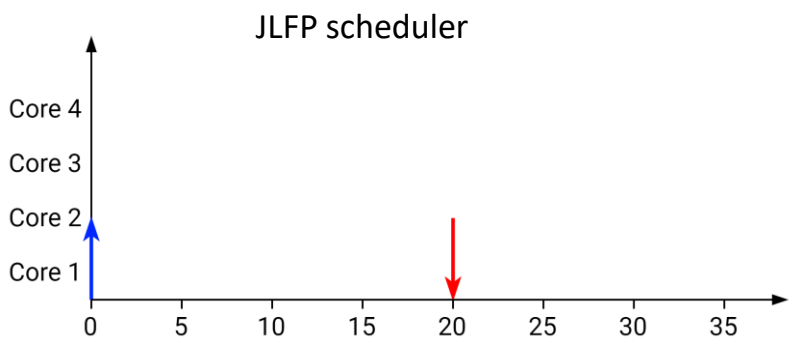


Agenda

- ~~Gang schedulability analysis~~
- New scheduling policy

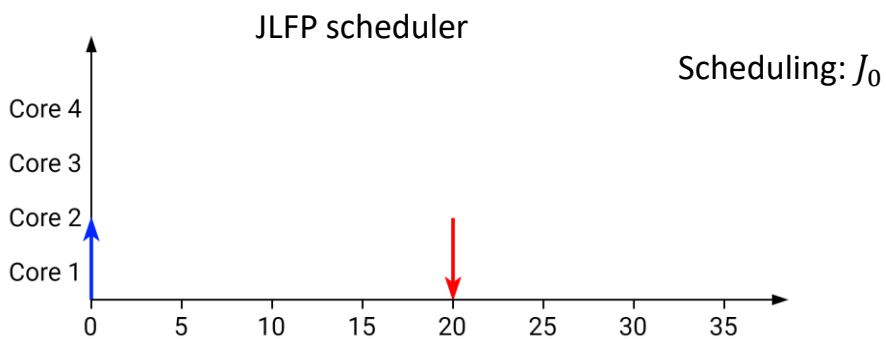
JLFP limitations with moldable gang

JLFP limitations with moldable gang



	Priority	Cores	Deadline	Execution time
J_0	High	2	∞	10
J_1	Mid-high	3	20	5
J_2	Mid-low	1	∞	20
J_3	Low	1	∞	20

JLFP limitations with moldable gang



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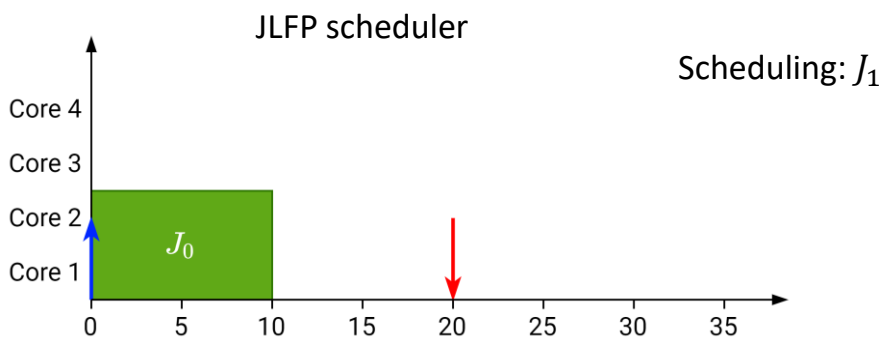

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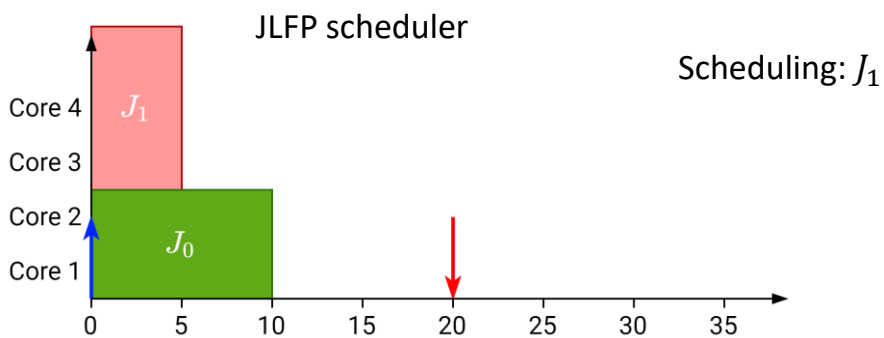

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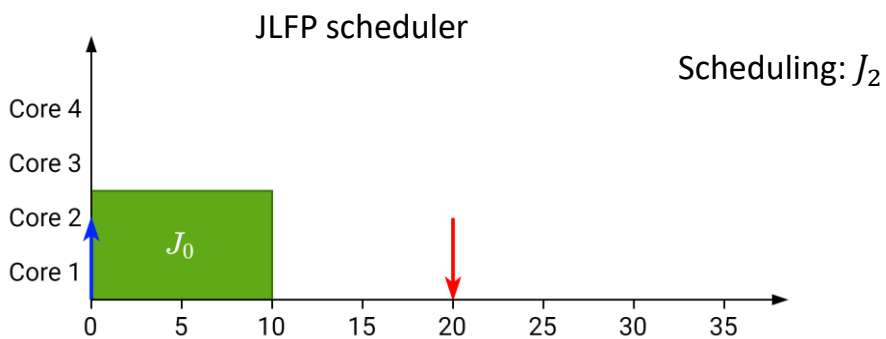

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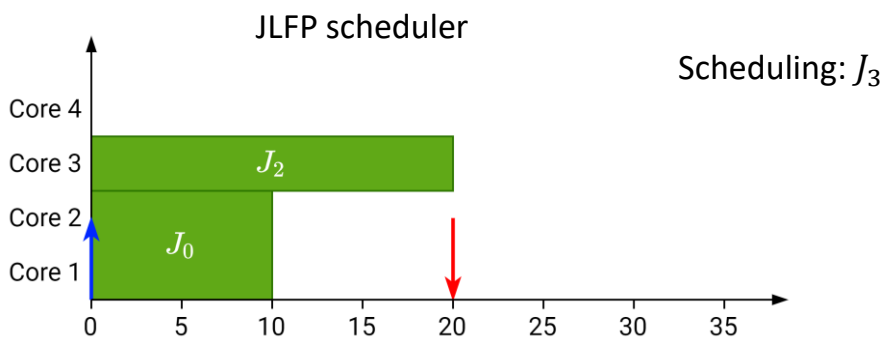

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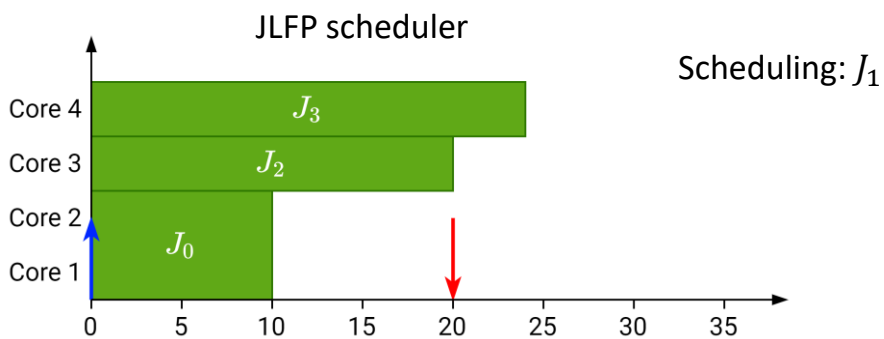

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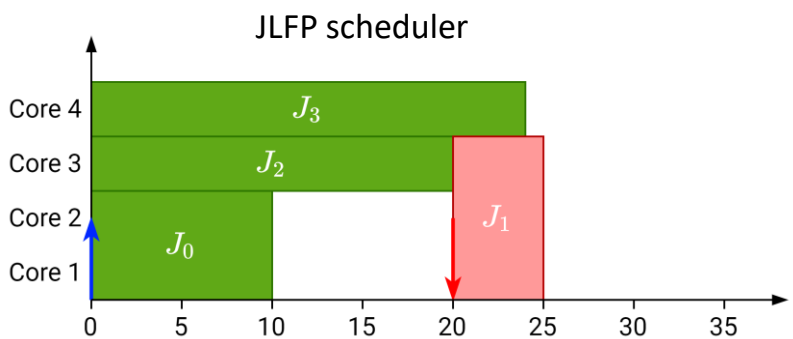

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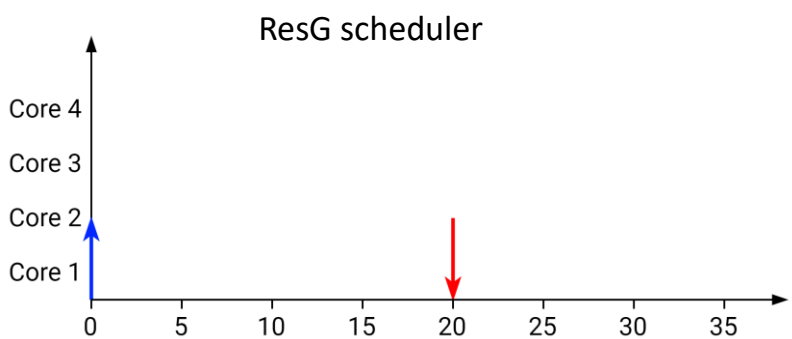
Reservation-based gang scheduler

- Reservation-based



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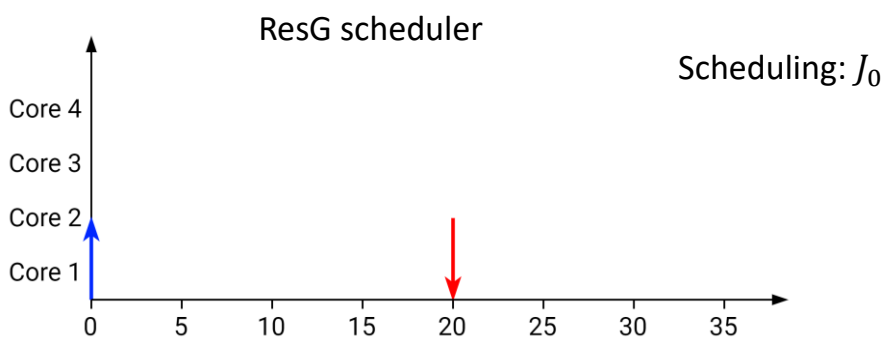

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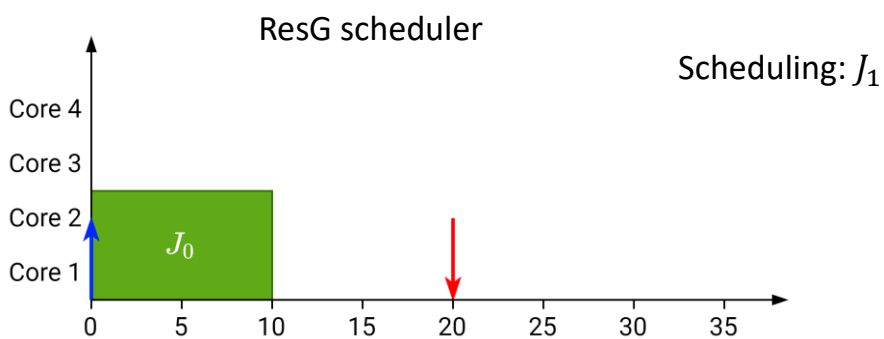

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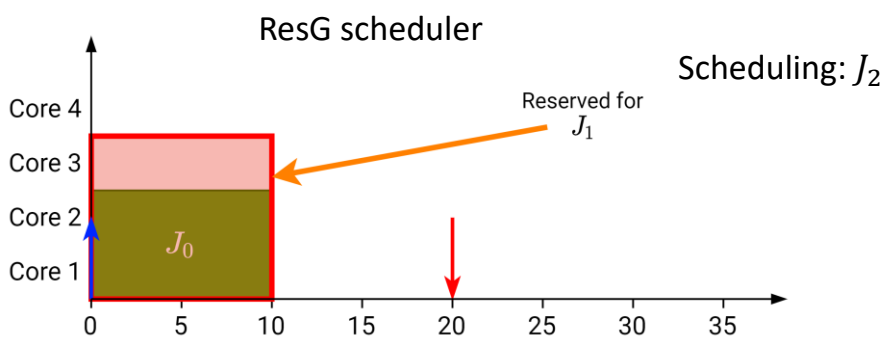
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



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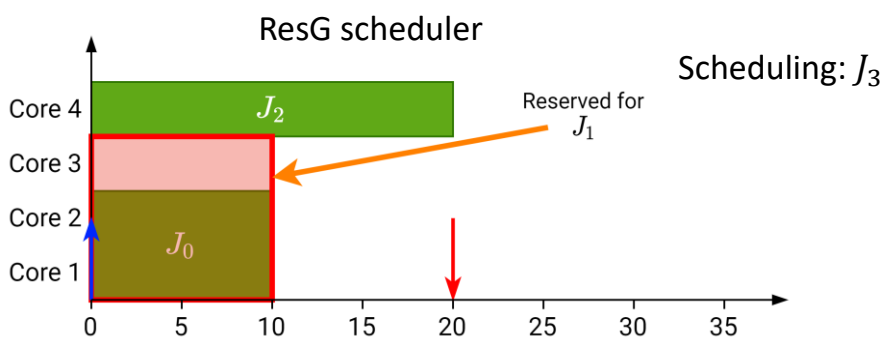

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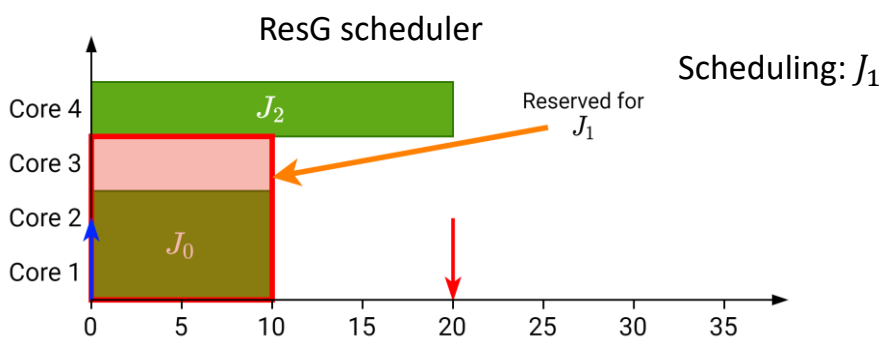

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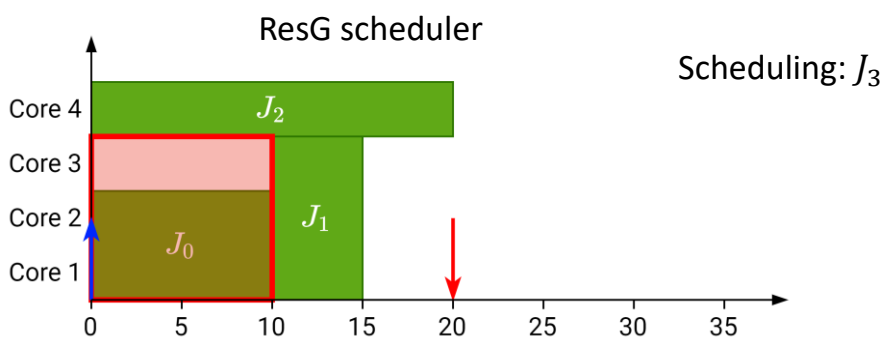

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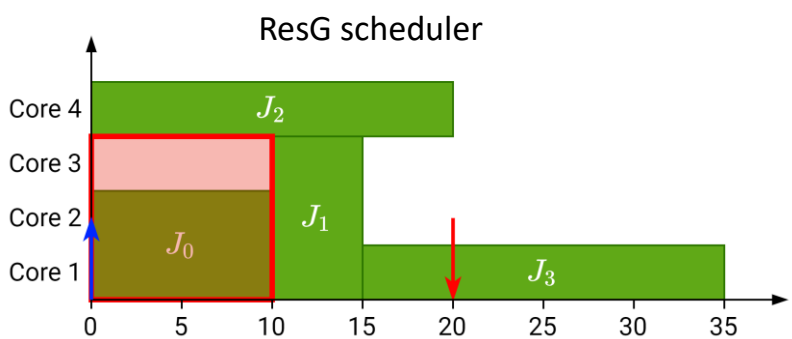

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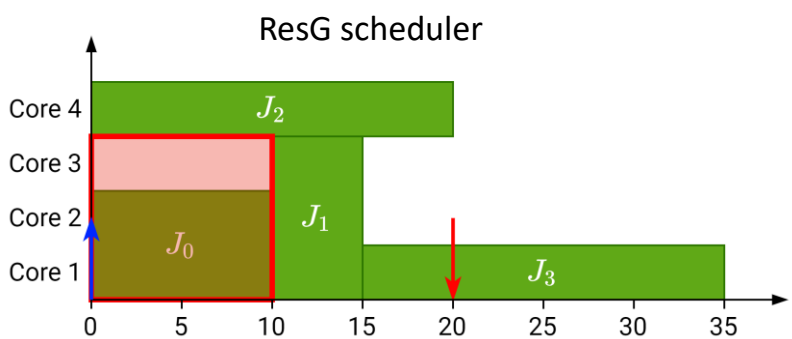
Reservation-based gang scheduler

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks
- Non-work conserving scheduler



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Results JLFP vs ResG in simulator

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- Evaluated in simulator

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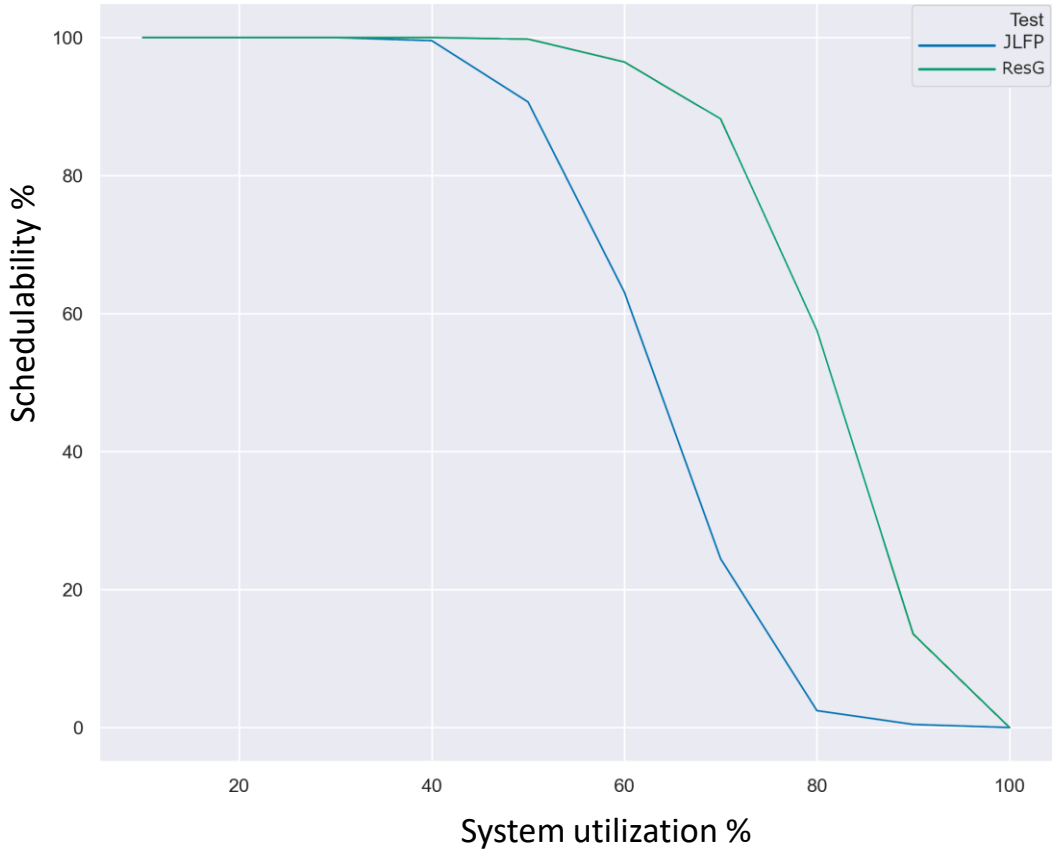

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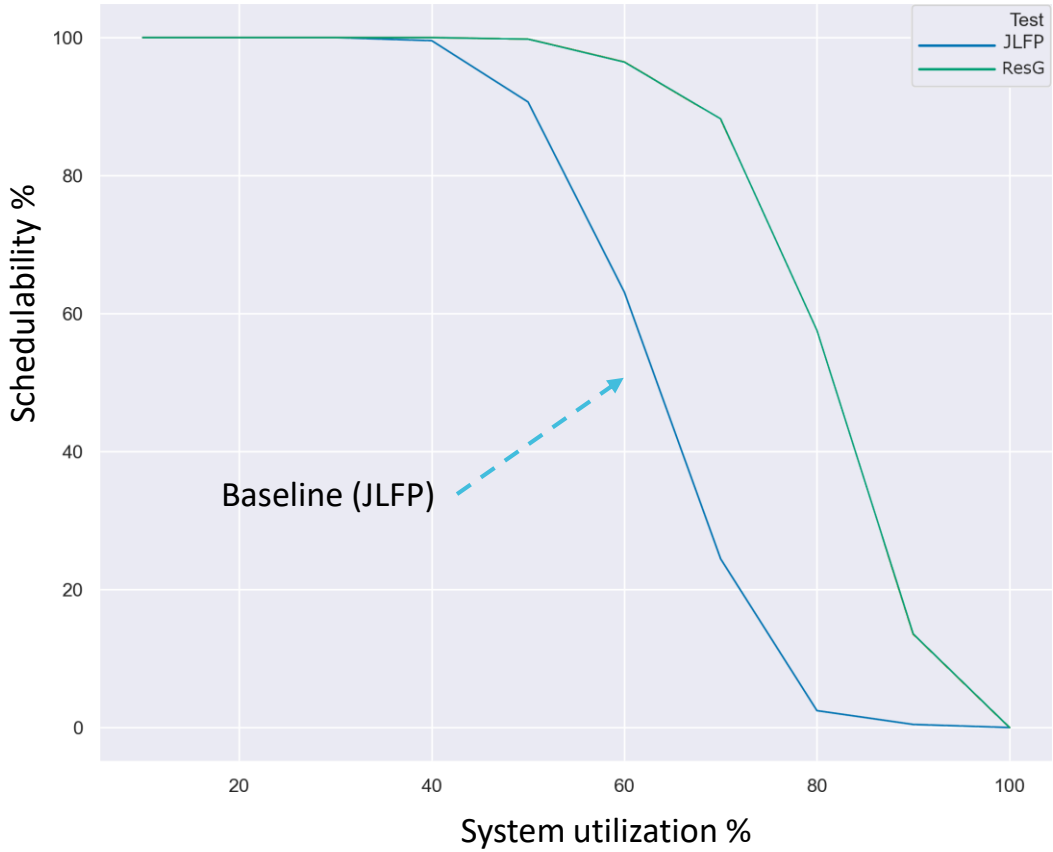

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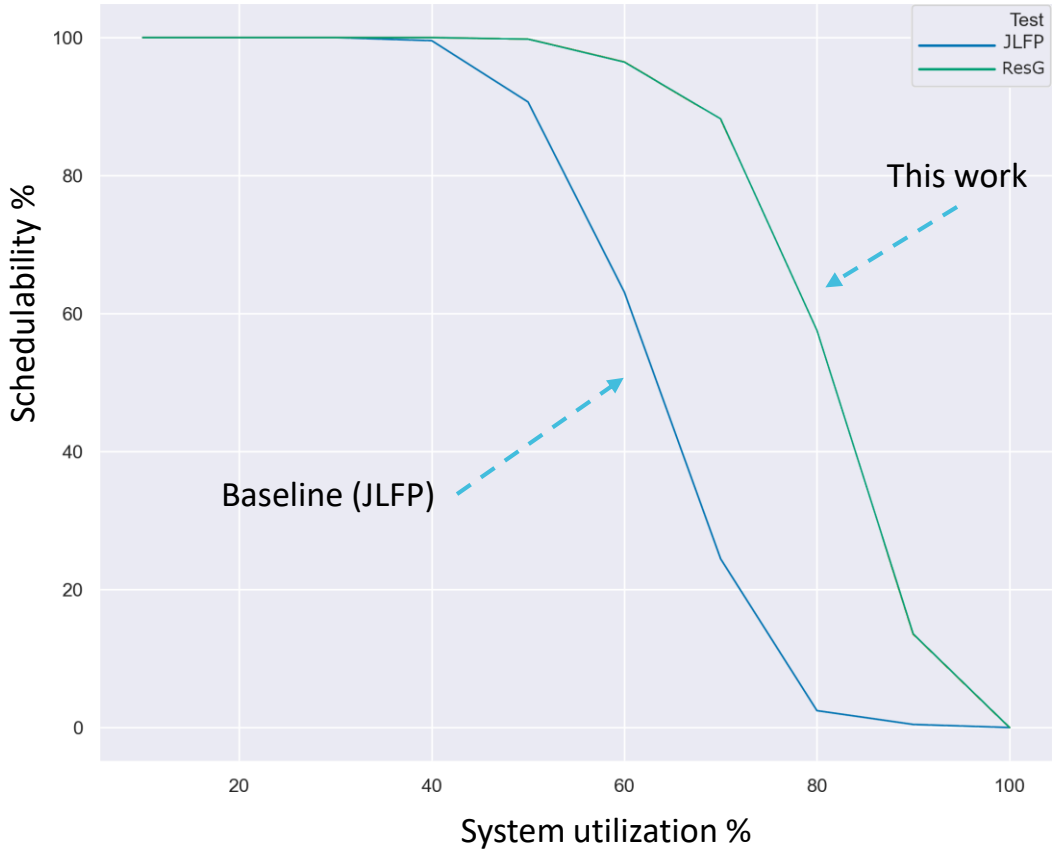

Results JLFP vs ResG in simulator

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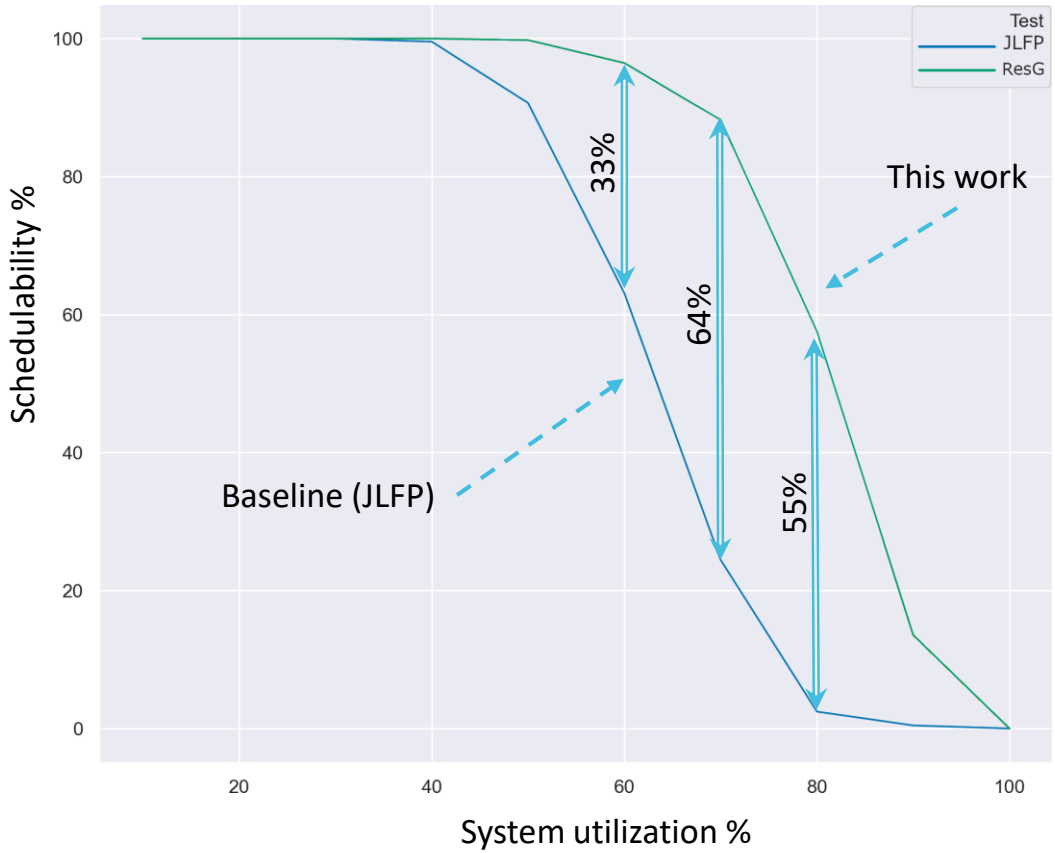


Conclusions

Conclusions

- With a better scheduling policy one can improve the schedulability of moldable gang tasks

Summary

Summary

- A new analysis for gang tasks using SAG has been defined

Summary

- A new analysis for gang tasks using SAG has been defined
- A new scheduling policy that uses gang moldable properties has been created

Next steps

Next steps

- Further reduce sources of pessimism

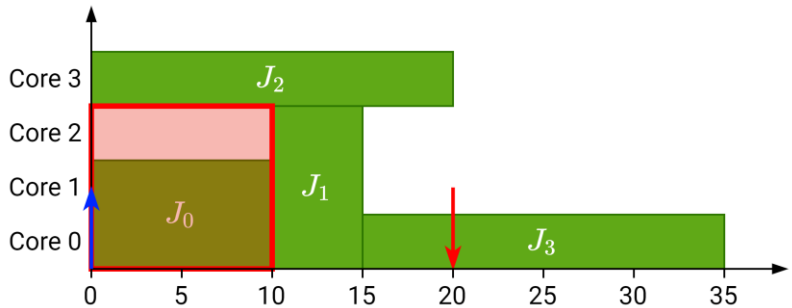
Next steps

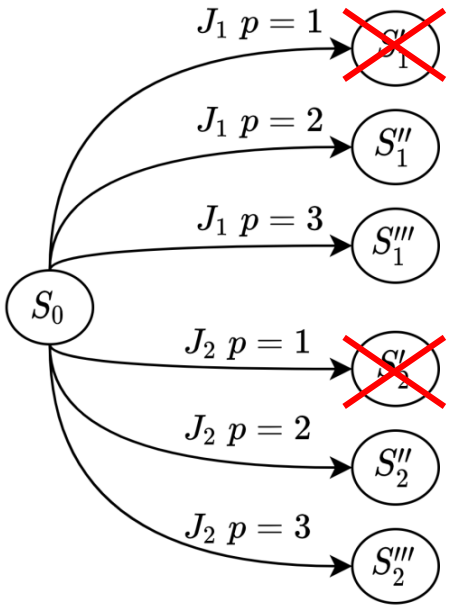
- Further reduce sources of pessimism
- Provide analysis for ResG scheduler and respective proofs

Next steps

- Further reduce sources of pessimism
- Provide analysis for ResG scheduler and respective proofs
- Thorough evaluation of results using SURFSara cluster

Questions?

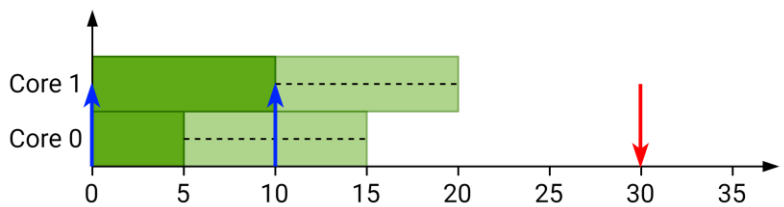

Core 3
Core 2
Core 1
Core 0
J_2
J_1
J_0
J_3
0
5
10
15
20
25
30
35


S_0
$J_1\ p = 1$
$J_1\ p = 2$
$J_1\ p = 3$
$J_2\ p = 1$
$J_2\ p = 2$
$J_2\ p = 3$
S_1''
S_1'''
S_2''
S_2'''

How does SAG work?

How does SAG work?

J_i	C_i^{min}	C_i^{max}	r_i	d_i	P_i
J_1	10	15	10	30	1
J_2	5	5	0	100	2



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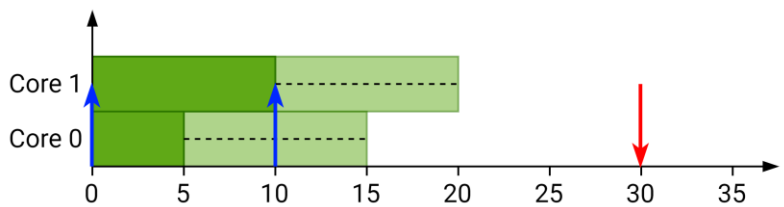

$[5, 15]$
$[10, 20]$

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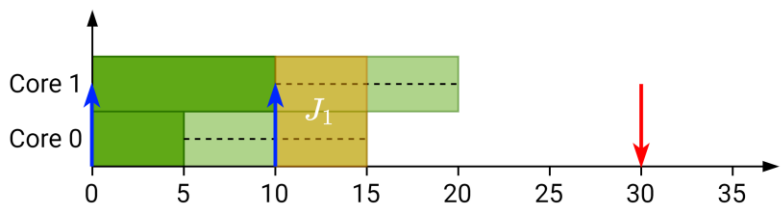

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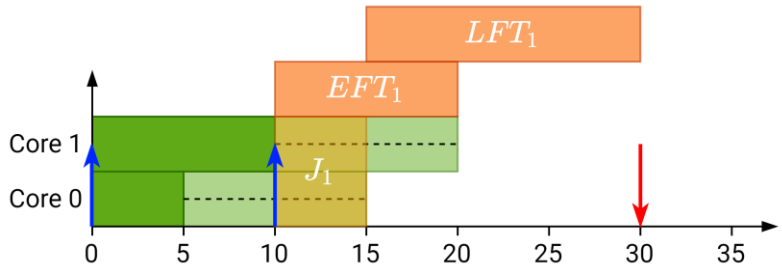

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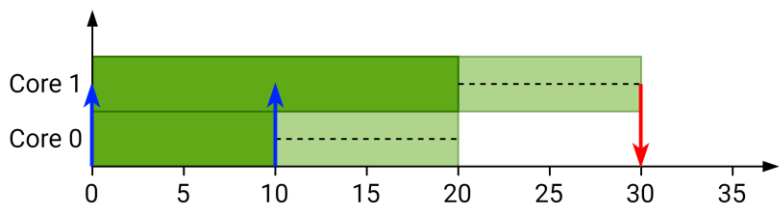

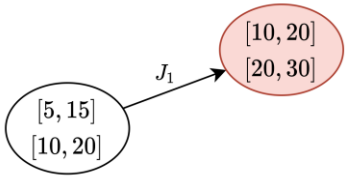

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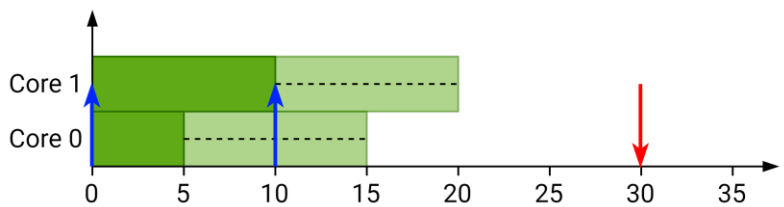

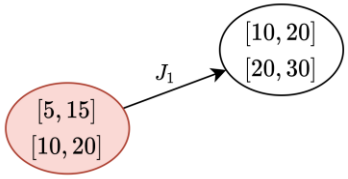

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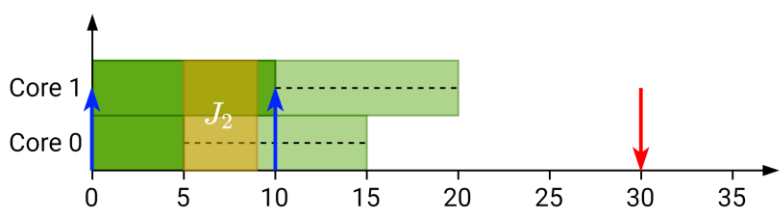

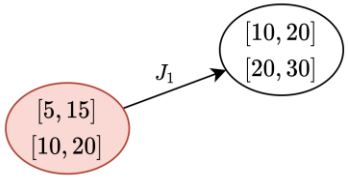

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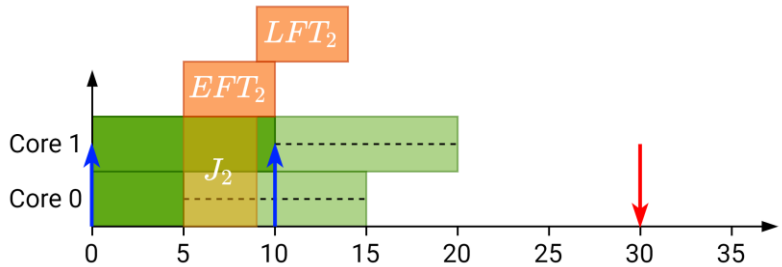

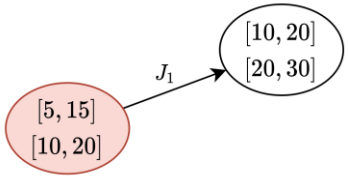

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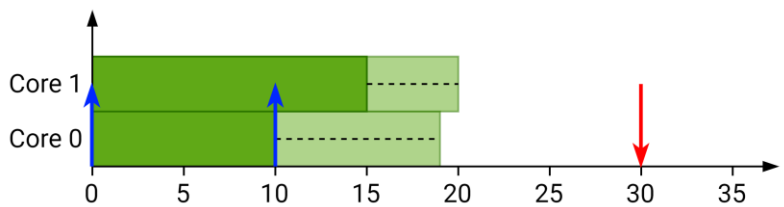

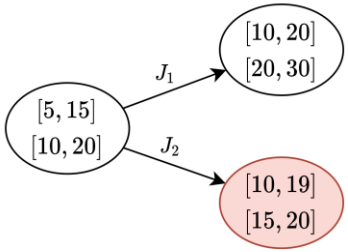

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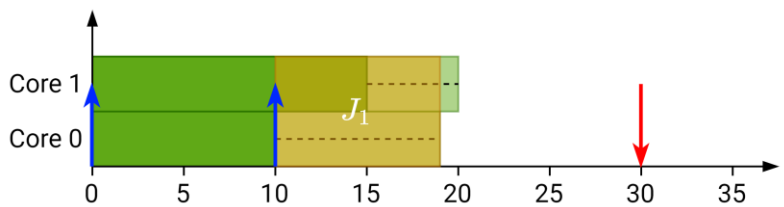

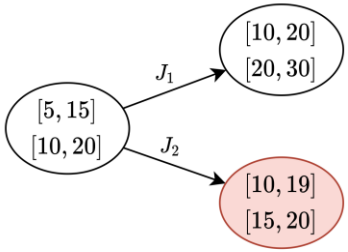

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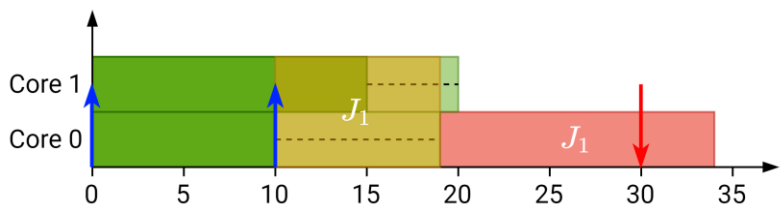
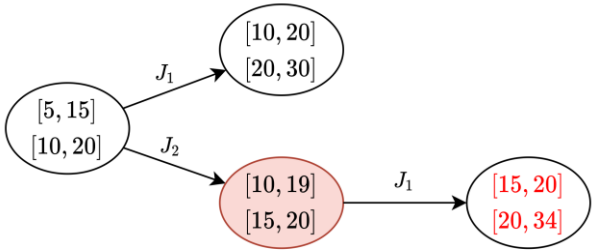
J_1	10	15	10	30	1
J_2	5	5	0	00	2





SAG analysis changes for gang

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$$EST_i = \max\{R_i^{\min}, A_1^{\min}\}$$

$$LST_i = \min\{t_{wc}, t_{high} - 1\}$$

$$t_{wc} = \max\left\{A_1^{\max}, \overset{\infty}{\min}\{R_x^{\max} \mid J_x \in \mathcal{R}^p\}\right\}$$

$$t_{high} = \overset{\infty}{\min}\{th_x(J_i) \mid J_x \in \mathcal{R}^p \wedge p_x < p_i\}$$

$$th_x(J_i) = \max\left\{r_x^{\max}, \overset{0}{\max}\{LFT_y^* \mid J_y \in pred(J_x) \setminus pred(J_i)\}\right\}$$

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$$EST_i^p = \max\{R_i^{\min}, t_{gang}\}$$

$$LST_i^p = \min\{t_{avail}, t_{wc}, t_{high} - 1\}$$

$$t_{wc} = \min_{J_j \in \mathcal{R}^v}\left\{\max\left\{R_j^{\max}, A_{m_j^{\min}}^{\max}\right\}\right\}$$

$$t_{high} = \overset{\infty}{\min}_{J_j \in \{hp_i \cap \mathcal{R}^v\}}\left\{th_x(J_i, J_j), \overset{0}{\max}\{LFT_y^* \mid J_y \in pred(J_j) \setminus pred(J_i)\}\right\}$$

$$t_h(J_i, J_j) = \begin{cases} r_j^{\max} & \text{if } m_j^{\min} \leq p \\ \max\left\{r_j^{\max}, A_{m_j^{\min}}^{\max}\right\} & \text{otherwise} \end{cases}$$

$$t_{gang} = \begin{cases} A_p^{\min} & \text{if } p = m_i^{\max} \\ A_p^{exact} & \text{otherwise} \end{cases} \qquad t_{avail} = \begin{cases} A_{p+1}^{\max} - 1 & \text{if } p < m_i^{\max} \\ +\infty & \text{otherwise} \end{cases}$$

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Check if execution with p cores is possible

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p cores available

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$p + 1$ cores **not** available

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