# Schedulability analysis of limitedpreemptive moldable gang tasks

Joan Marcè i Igual



**Daily Supervisor** 

Geoffrey Nelissen

Co-supervisor

Mitra Nasri

27<sup>th</sup> of August, 2020





 Systems of which correctness does depends not only on logical results but also on timing constraints

 Systems of which correctness does depends not only on logical results but also on timing constraints



 Systems of which correctness does depends not only on logical results but also on timing constraints

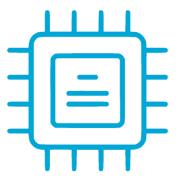




- Systems of which correctness does depends not only on logical results but also on timing constraints
- Multicore systems









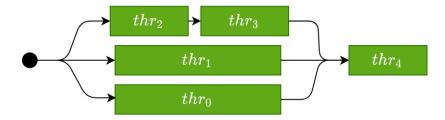
- Task
  - A functionality of the system

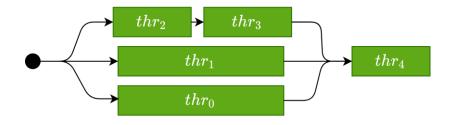
- Task
  - A functionality of the system
- Job
  - Instance of a task

- Task
  - A functionality of the system
- Job
  - Instance of a task
- Schedule
  - A particular assignment of jobs to the processors and time intervals

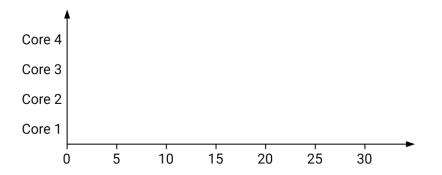
- Task
  - A functionality of the system
- Job
  - Instance of a task
- Schedule
  - A particular assignment of jobs to the processors and time intervals
- Scheduling policy
  - Algorithm that produces a schedule
  - FIFO, Round-Robin, JLFP, EDF

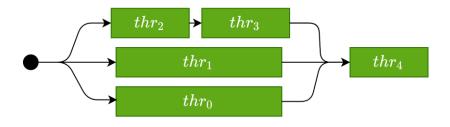


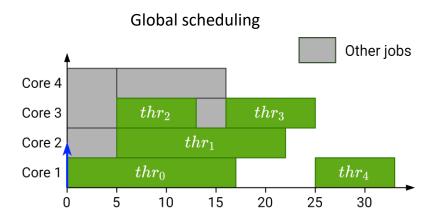


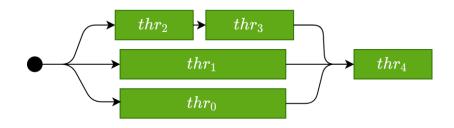


#### Global scheduling

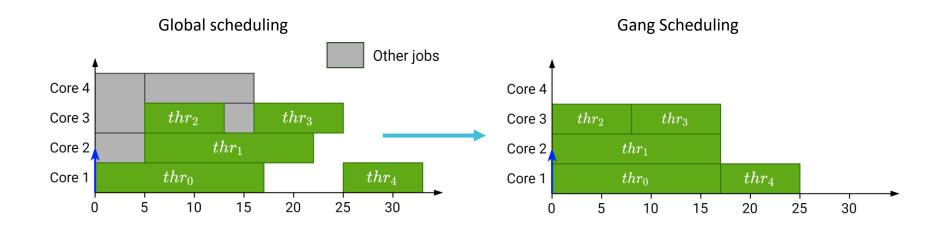


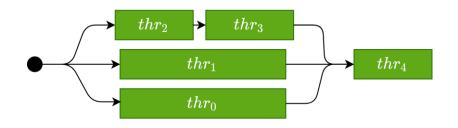




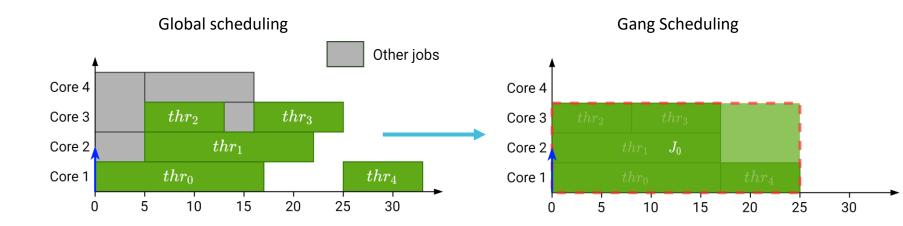


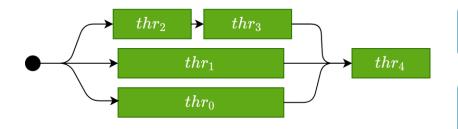
Parallel threads together as a "gang"





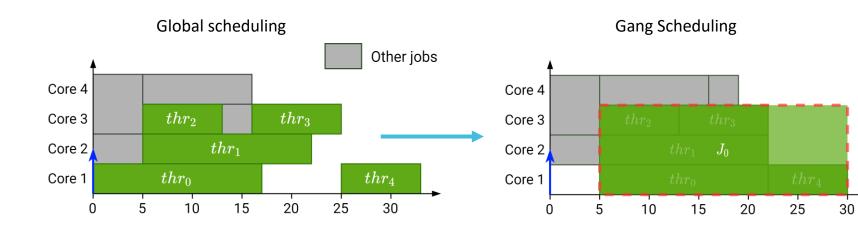
Parallel threads together as a "gang"

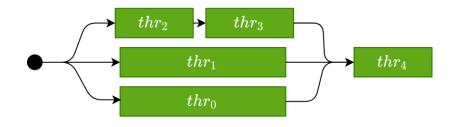




Parallel threads together as a "gang"

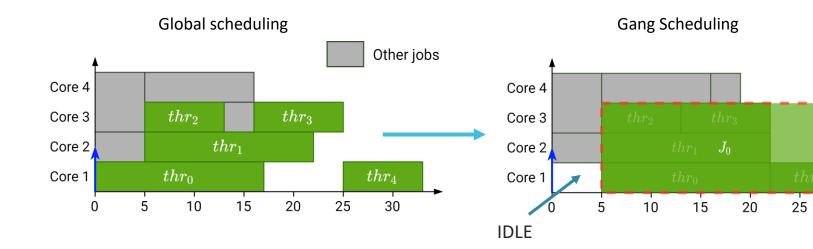
Execution does not start until there are enough cores



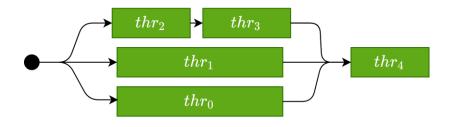


Parallel threads together as a "gang"

Execution does not start until there are enough cores

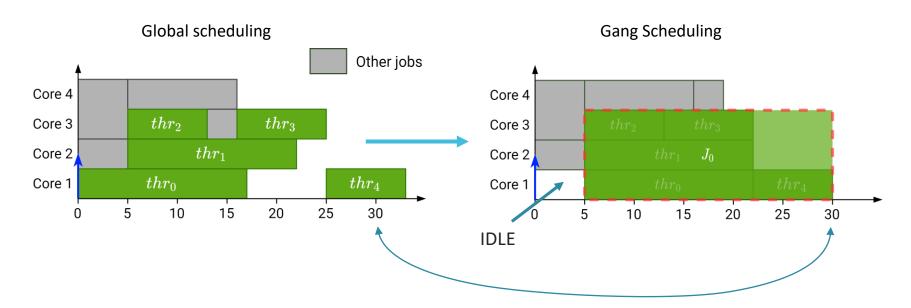


30



Parallel threads together as a "gang"

Execution does not start until there are enough cores



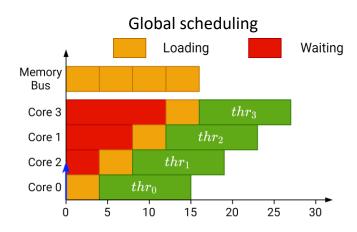


More efficient synchronization

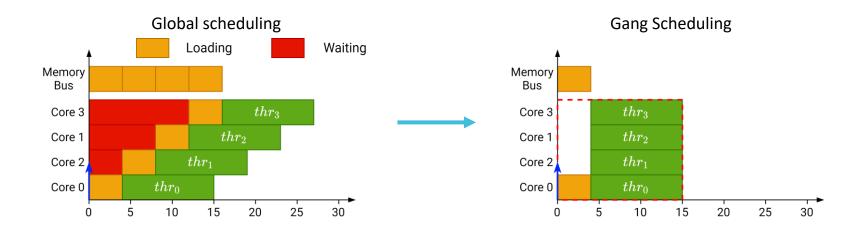
- More efficient synchronization
- Reduces variability in the execution

- More efficient synchronization
- Reduces variability in the execution
- Avoids overhead when loading initial data

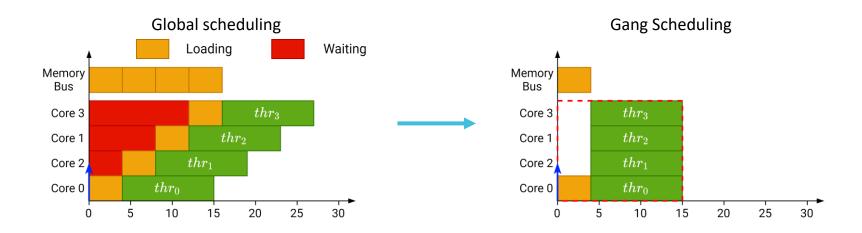
- More efficient synchronization
- Reduces variability in the execution
- Avoids overhead when loading initial data



- More efficient synchronization
- Reduces variability in the execution
- Avoids overhead when loading initial data

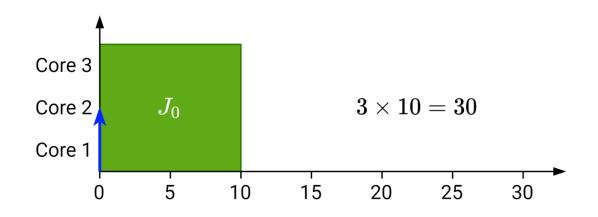


- More efficient synchronization
- Reduces variability in the execution
- Avoids overhead when loading initial data
- Shows its full potential when executed non-preemptively

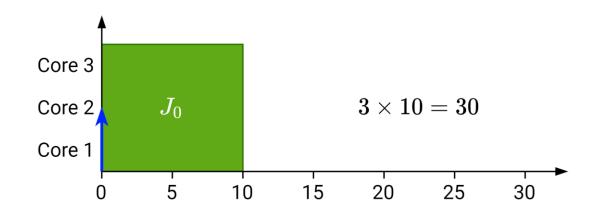




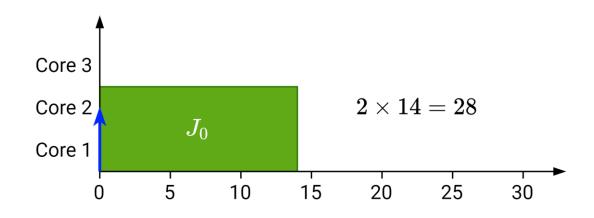
• Rigid: number of cores set by programmer



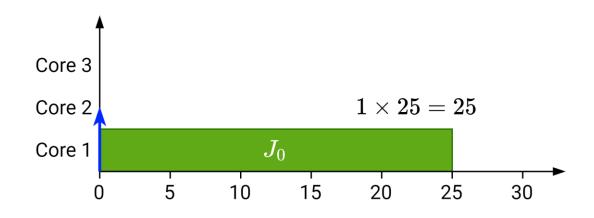
- Rigid: number of cores set by programmer
- Moldable: number of cores assigned when job is dispatched



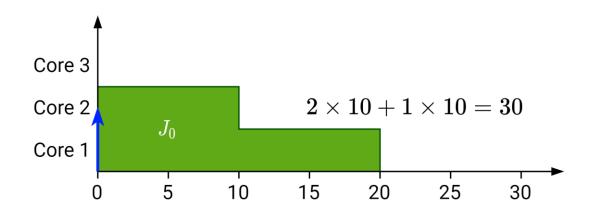
- Rigid: number of cores set by programmer
- Moldable: number of cores assigned when job is dispatched



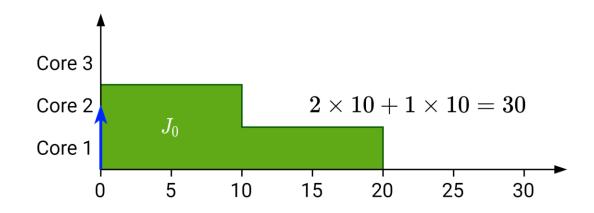
- Rigid: number of cores set by programmer
- Moldable: number of cores assigned when job is dispatched



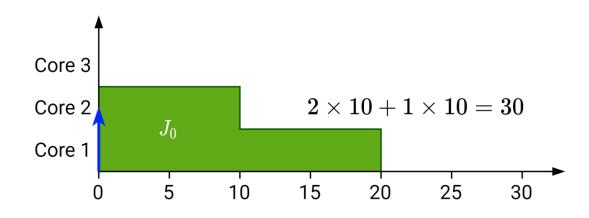
- **Rigid**: number of cores set by programmer
- Moldable: number of cores assigned when job is dispatched
- Malleable: number of cores can change during runtime

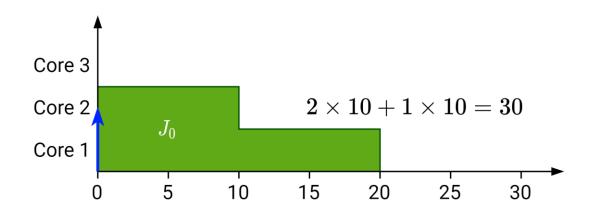


- **Rigid**: number of cores set by programmer
- Moldable: number of cores assigned when job is dispatched
- Malleable: number of cores can change during → ♥ Hard to implement runtime

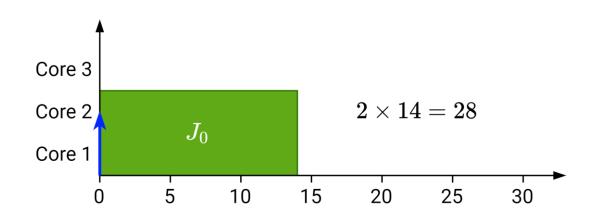


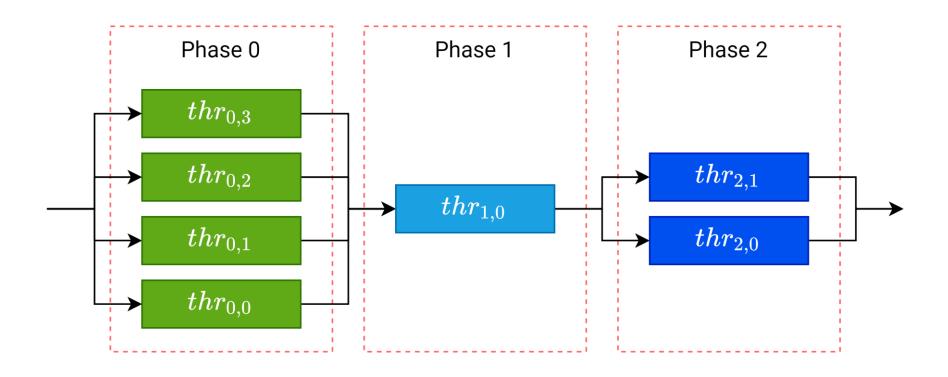
- Moldable: number of cores assigned when job is dispatched



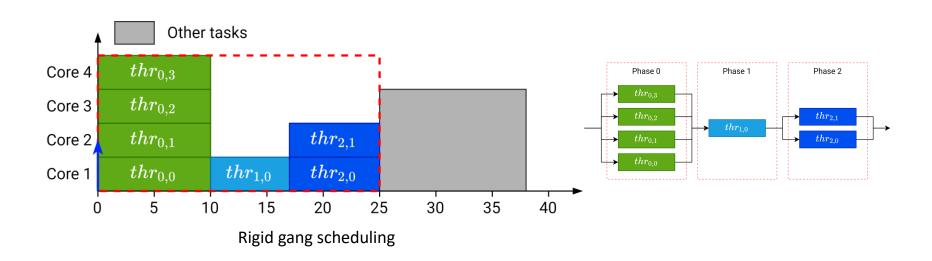


### Types of gang

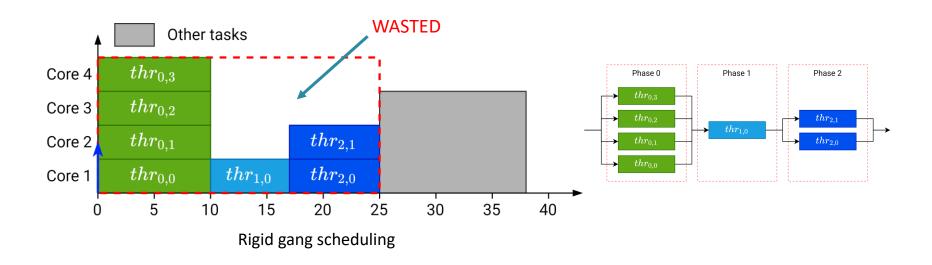




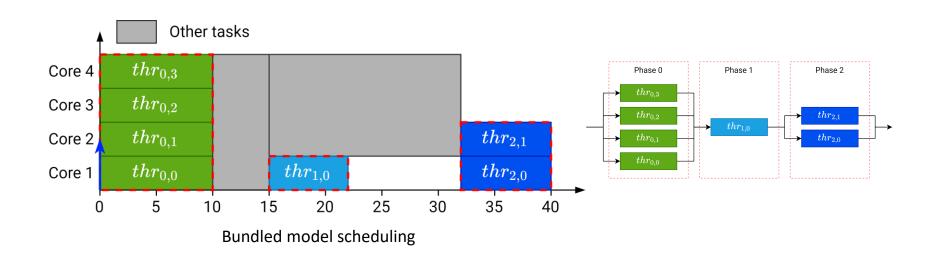
Rigid gang reserves the whole block



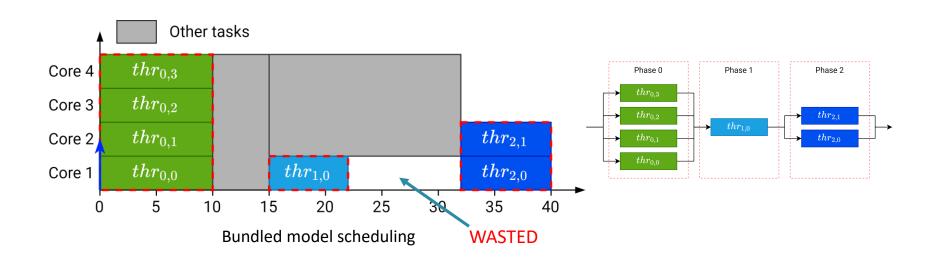
Rigid gang reserves the whole block



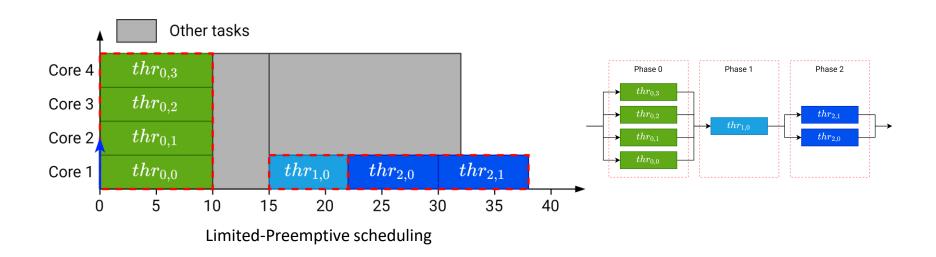
- Rigid gang reserves the whole block
- Bundled creates rigid blocks with dependencies



- Rigid gang reserves the whole block
- Bundled creates rigid blocks with dependencies



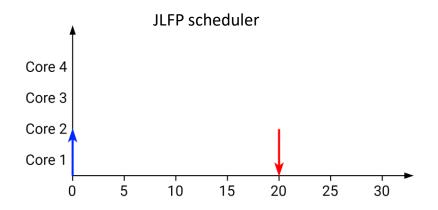
- Rigid gang reserves the whole block
- Bundled creates rigid blocks with dependencies
- Limited-Preemptive creates moldable blocks with dependencies



Based on global JLFP scheduler

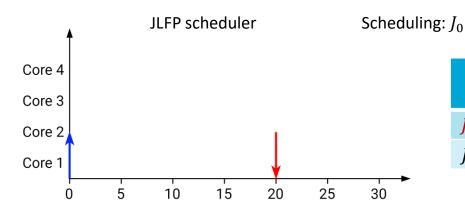
- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows

- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows



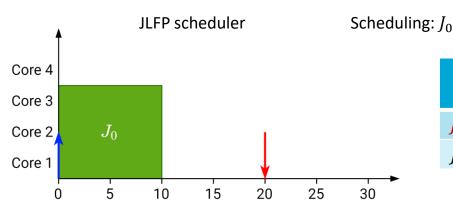
	the state of the s	Min cores		Deadline	Execution time
$J_0$	High	2	3	$\infty$	15, 10
$J_1$	Low	2	2	20	15

- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows



	Priority	Min cores		Deadline	Execution time
$J_0$	High	2	3	$\infty$	15, 10
$J_1$	Low	2	2	20	15

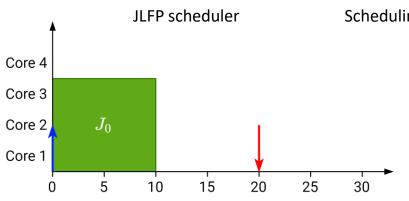
- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows



	Min cores		Execution time

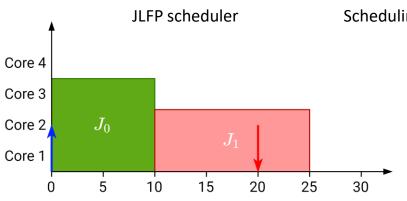
		cores	cores		time
$J_0$	High	2	3	∞	15, 10
$J_1$	Low	2	2	20	15

- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows



	Priority	Min cores		Deadline	Execution time
$J_0$	High	2	3	$\infty$	15, 10
$J_1$	Low	2	2	20	15

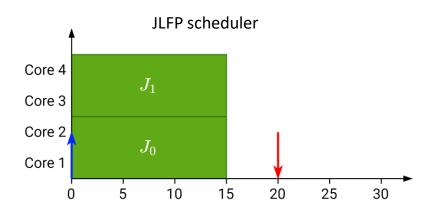
- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows



Scheduling: <i>J</i>	1
----------------------	---

		Priority	Min cores		Deadline	Execution time
J	0	High	2	3	$\infty$	15, 10
J	1	Low	2	2	20	15

- Based on global JLFP scheduler
- Assigns maximum number of available cores to a job between the range of cores that the job allows



		Min cores		Deadline	Execution time
$J_0$	High	2	3	$\infty$	15, 10
$J_1$	Low	2	2	20	15



Introduced in high-performance computing in 1982<sup>[1]</sup>

Introduced in high-performance computing in 1982<sup>[1]</sup>

Preemptive solutions

Introduced in high-performance computing in 1982<sup>[1]</sup>

# Schedulability tests

Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

• Job-level fixed-priority<sup>[2]</sup>

Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first<sup>[3]</sup>

Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first<sup>[3]</sup>

#### Schedulers



Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first<sup>[3]</sup>

#### **Schedulers**

Optimal for rigid gang (DP-Fair)<sup>[4]</sup>



[4]Goossens et al., 2016

Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first<sup>[3]</sup>

#### **Schedulers**

- Optimal for rigid gang (DP-Fair)<sup>[4]</sup>
- Moldable gang<sup>[5]</sup>



Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first[3]

#### **Schedulers**

- Optimal for rigid gang (DP-Fair)[4]
- Moldable gang<sup>[5]</sup>

#### Non-preemptive solutions



Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first[3]

#### Schedulers

- Optimal for rigid gang (DP-Fair)<sup>[4]</sup>
- Moldable gang<sup>[5]</sup>

#### Non-preemptive solutions

Schedulability tests



Introduced in high-performance computing in 1982<sup>[1]</sup>

#### Preemptive solutions

#### Schedulability tests

- Job-level fixed-priority<sup>[2]</sup>
- Earliest deadline first[3]

#### **Schedulers**

- Optimal for rigid gang (DP-Fair)<sup>[4]</sup>
- Moldable gang<sup>[5]</sup>

#### Non-preemptive solutions

#### Schedulability tests

Earliest deadline first for rigid gang<sup>[6]</sup>



[4]Goossens et al., 2016

[6] Dong et al. 2019

### Our work





1. Design an accurate schedulability analysis for limited-preemptive moldable gang tasks

- Design an accurate schedulability analysis for limited-preemptive moldable gang tasks
- 2. Evaluate the impact of the level of parallelism assigned to the jobs

- Design an accurate schedulability analysis for limited-preemptive moldable gang tasks
- 2. Evaluate the impact of the level of parallelism assigned to the jobs
- Propose a new scheduling algorithm to improve the schedulability of limited-preemptive moldable gang tasks

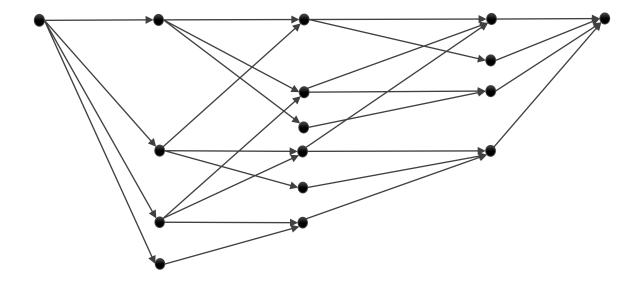
### Agenda

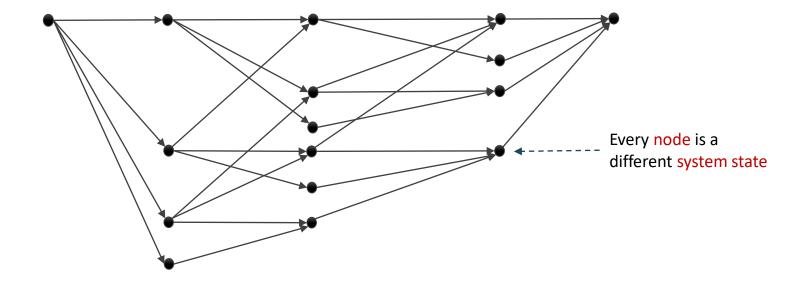
- Gang schedulability analysis
- New scheduling policy

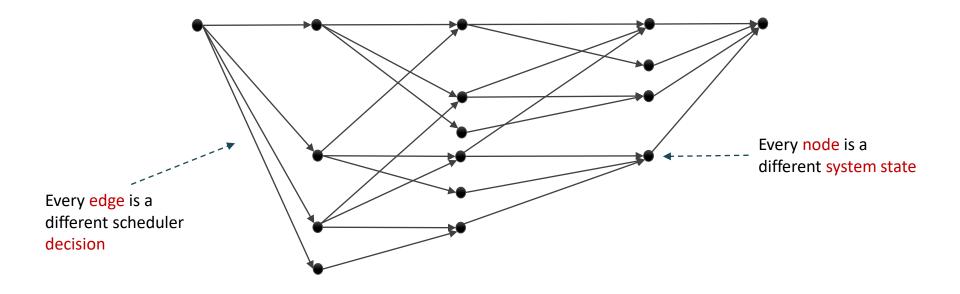
### Schedule abstraction graph

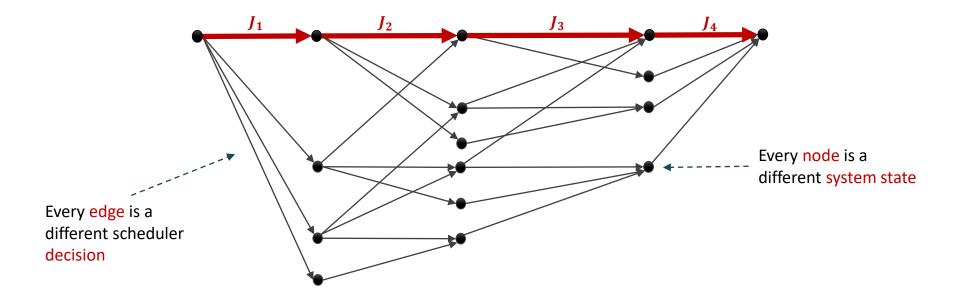


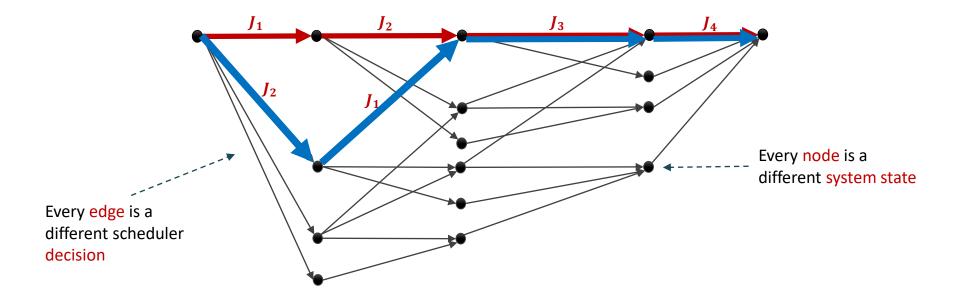
### Schedule abstraction graph



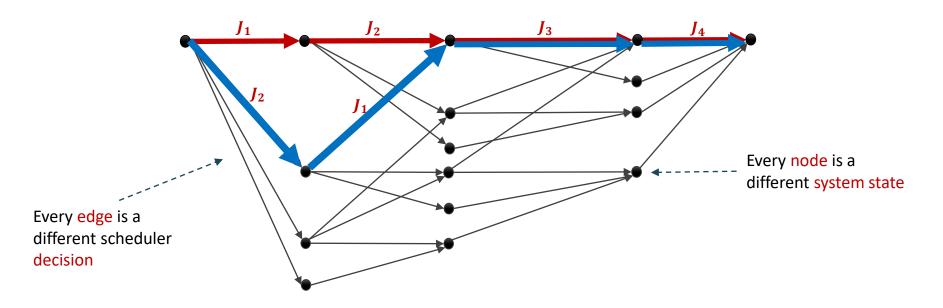








- It is a technique that allows:
  - Search for all possible schedules
  - Aggregate "similar" schedules





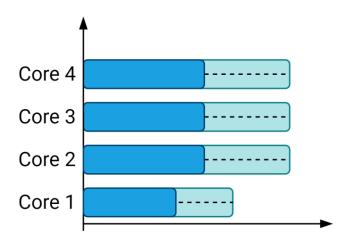
Update system state representation

- Update system state representation
- Update expansion rules

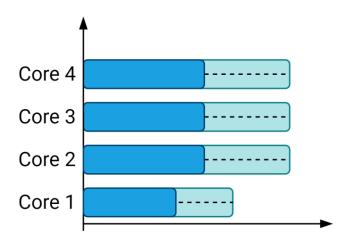
- Update system state representation
- Update expansion rules
- Update merge rules



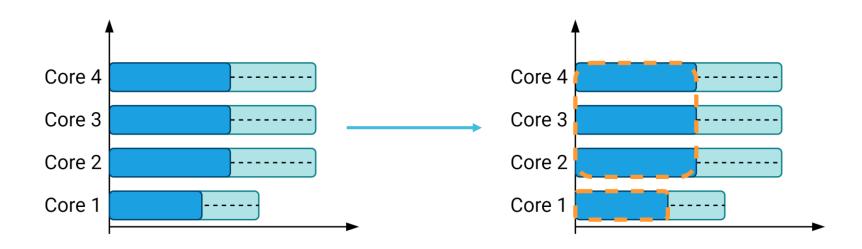
We already had core availabilities



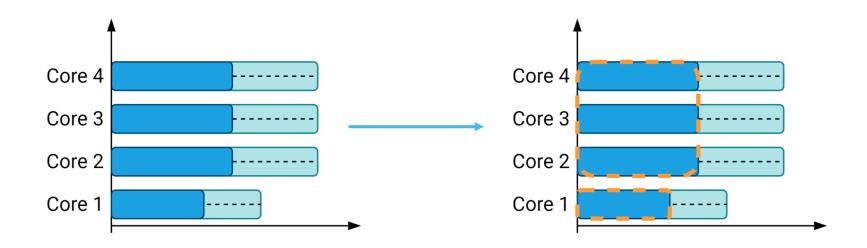
- We already had core availabilities
- We need to keep track of



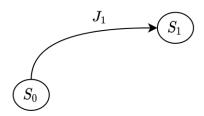
- We already had core availabilities
- We need to keep track of
  - Groups of cores

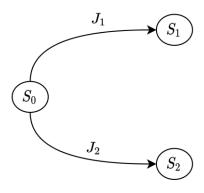


- We already had core availabilities
- We need to keep track of
  - Groups of cores
  - Certainly running jobs

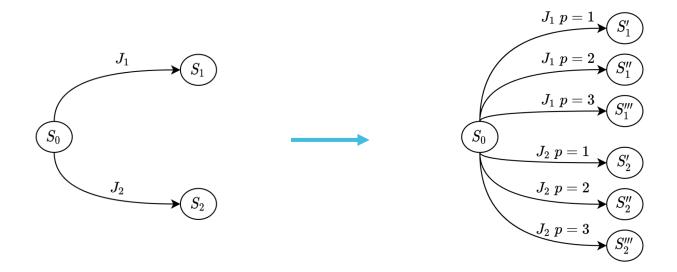




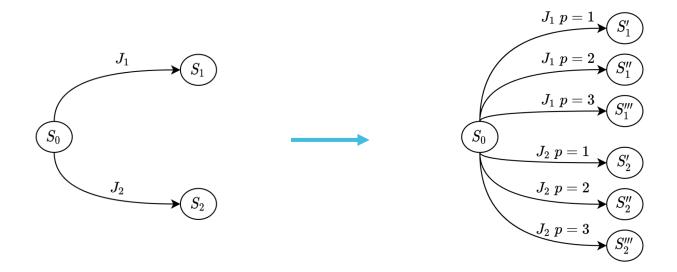


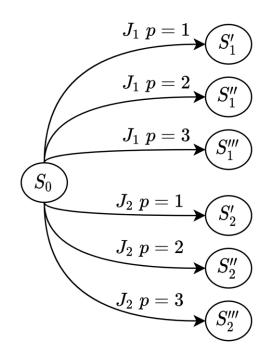


- Previously a state was created for every schedulable job
- Now a state is created for every job and possible number of cores

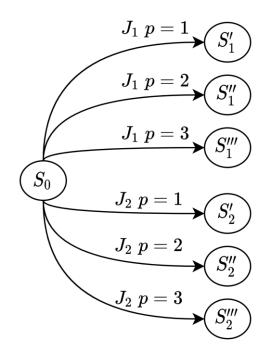


- Previously a state was created for every schedulable job
- Now a state is created for every job and possible number of cores
- Can stimulate state-space explosion



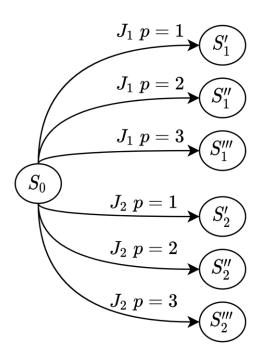


Purge states by checking that

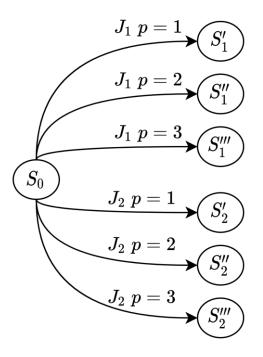


- Purge states by checking that
  - *p* cores available

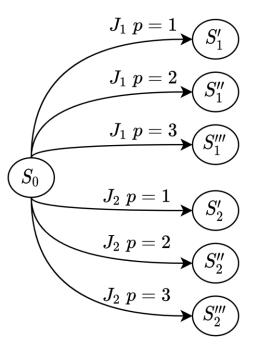
•



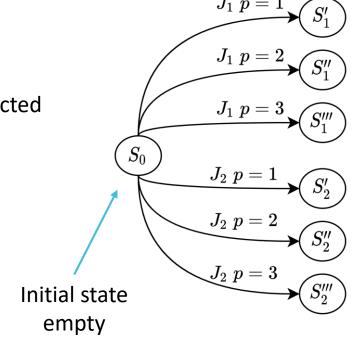
- Purge states by checking that
  - *p* cores available
  - More cores not available



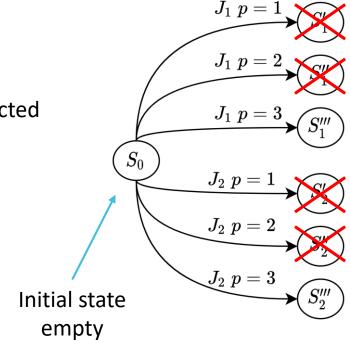
- Purge states by checking that
  - *p* cores available
  - More cores not available
  - Precedence constraints are respected



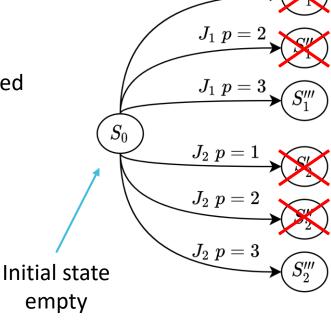
- Purge states by checking that
  - *p* cores available
  - More cores not available
  - Precedence constraints are respected



- Purge states by checking that
  - p cores available
  - More cores not available
  - Precedence constraints are respected

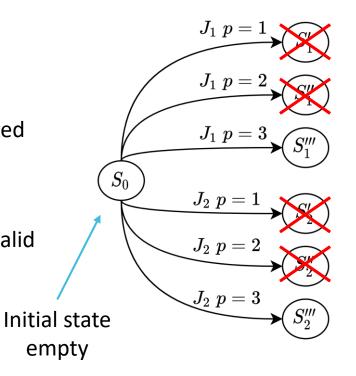


- Purge states by checking that
  - p cores available
  - More cores not available
  - Precedence constraints are respected
- Exploring more states

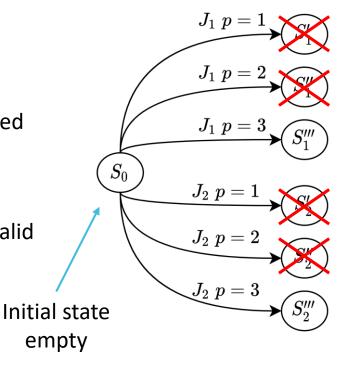


 $J_1 \ p = 1$ 

- Purge states by checking that
  - *p* cores available
  - More cores not available
  - Precedence constraints are respected
- Exploring more states
  - ls safe, does not make analysis invalid



- Purge states by checking that
  - p cores available
  - More cores not available
  - Precedence constraints are respected
- Exploring more states
  - de la safe, does not make analysis invalid
  - ♥ Slower and more pessimistic





We have to merge

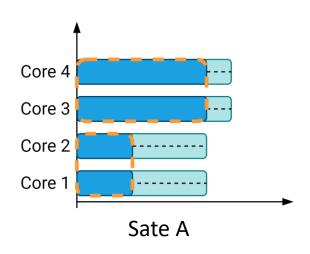
- We have to merge
  - Availability times

- We have to merge
  - Availability times
  - Groups of cores

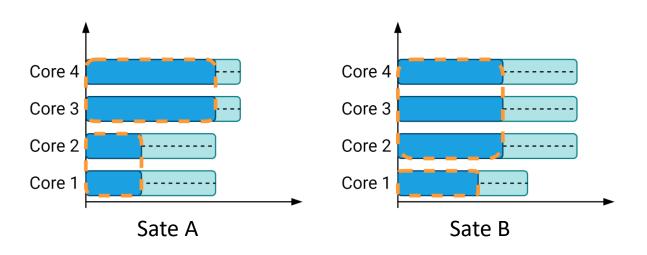
- We have to merge
  - Availability times
  - Groups of cores
  - Certainly running jobs

- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs

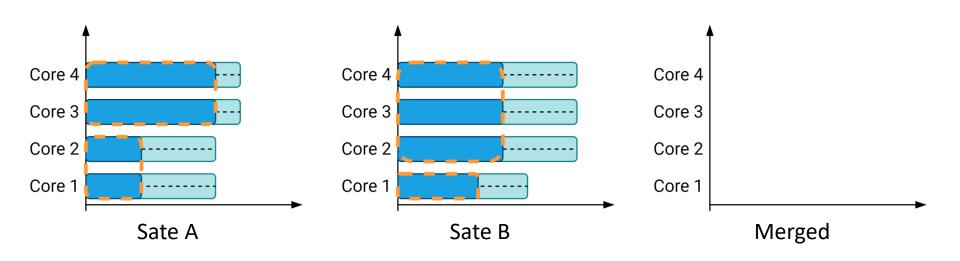
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



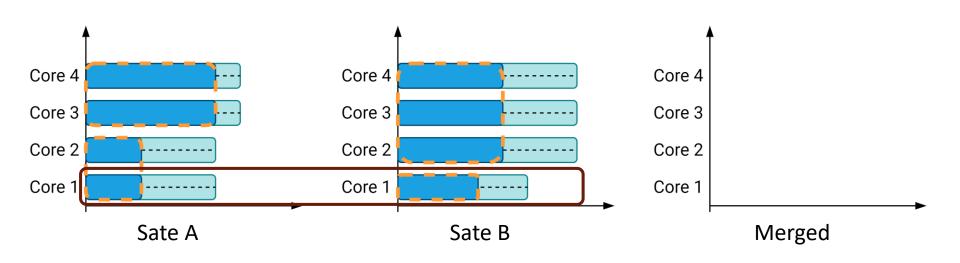
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



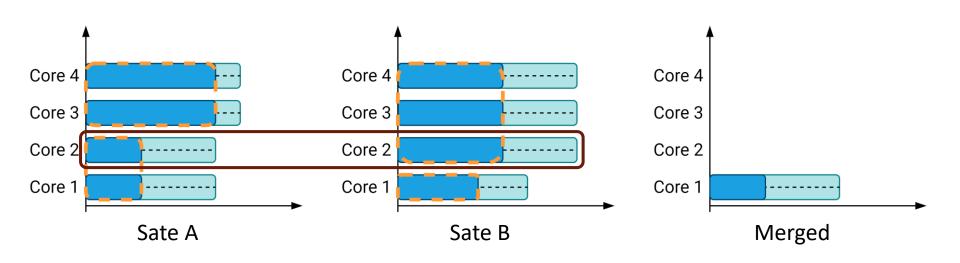
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



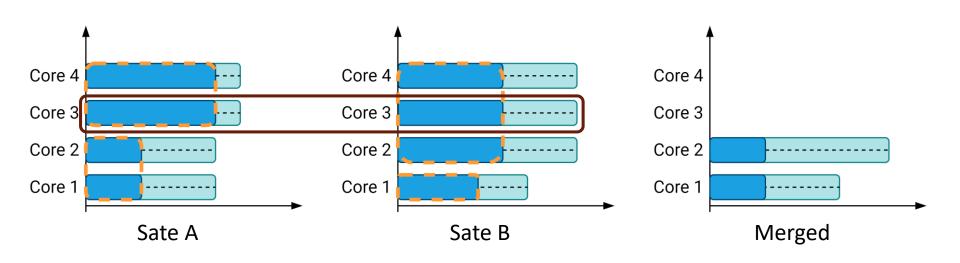
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



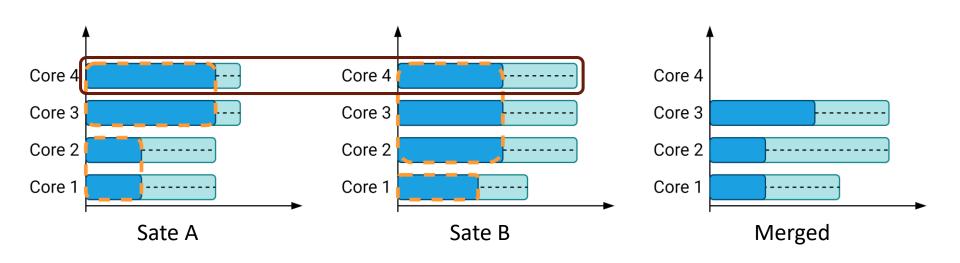
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



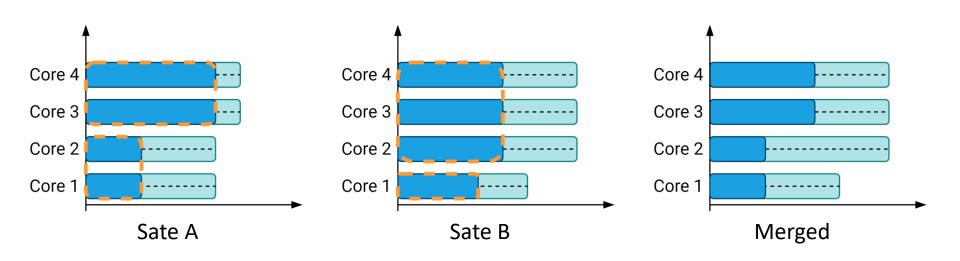
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



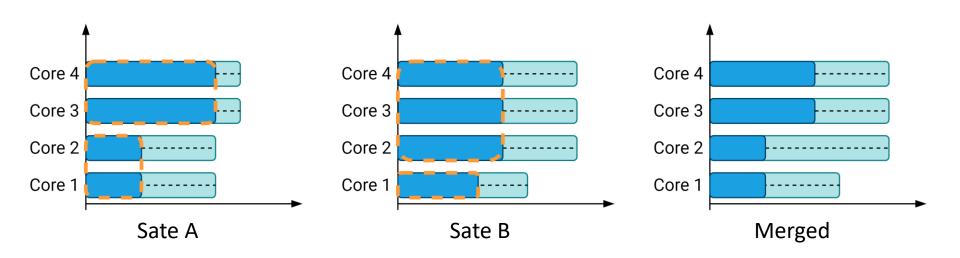
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



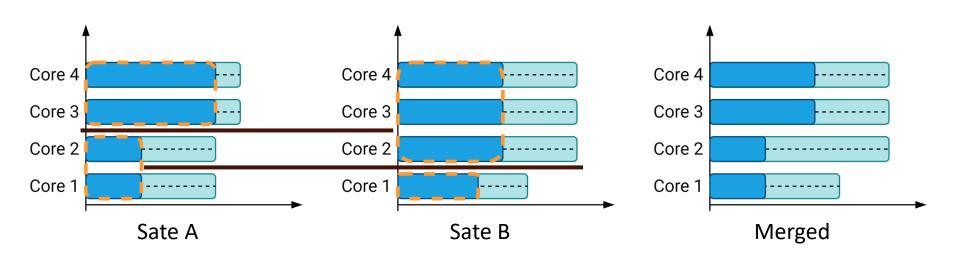
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
  - Certainly running jobs



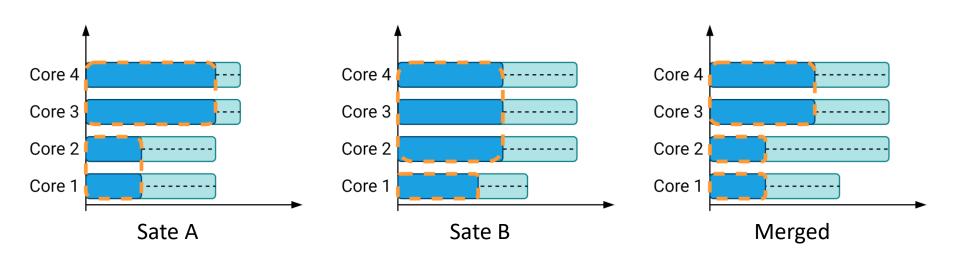
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
     Break the groups into same size
  - Certainly running jobs



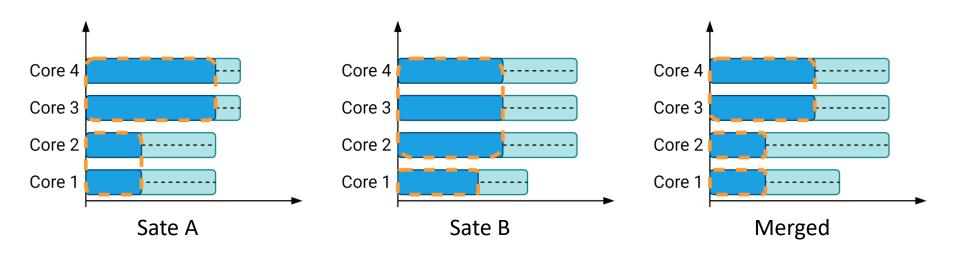
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
     Break the groups into same size
  - Certainly running jobs



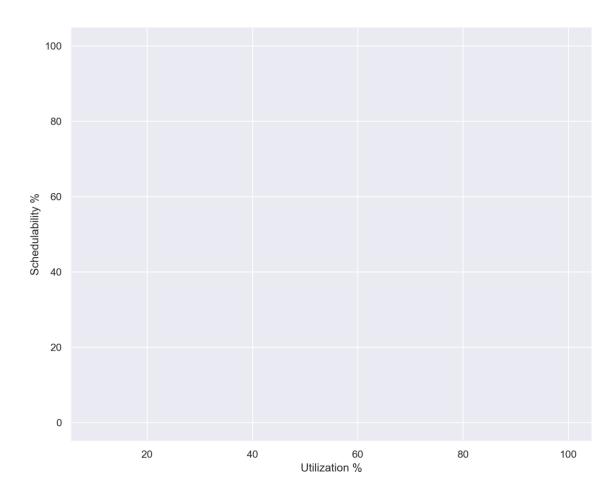
- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
     Break the groups into same size
  - Certainly running jobs



- We have to merge
  - Availability times ———— Extend the intervals
  - Groups of cores
     Break the groups into same size
  - Certainly running jobs ———— Keep only jobs running in both states (intersect)



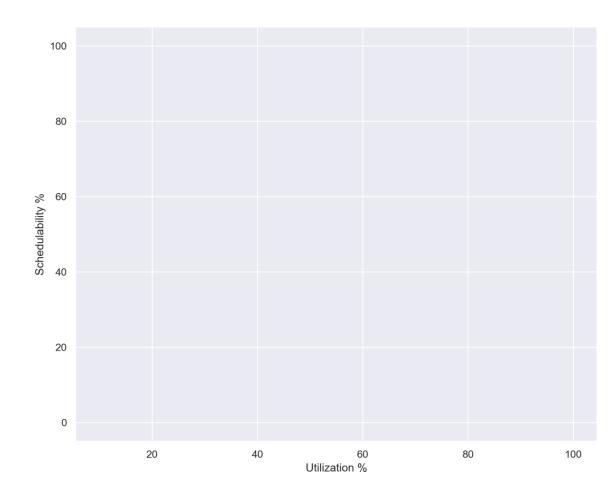




#### Randomly generated task sets

System processors: 8

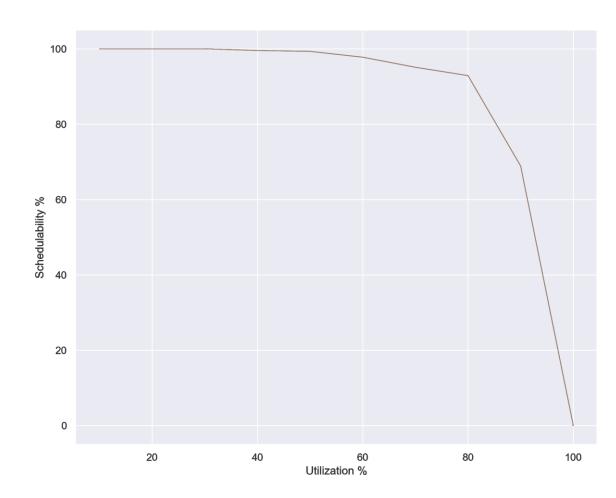
System tasks: 20 rigid tasks



#### Randomly generated task sets

System processors: 8

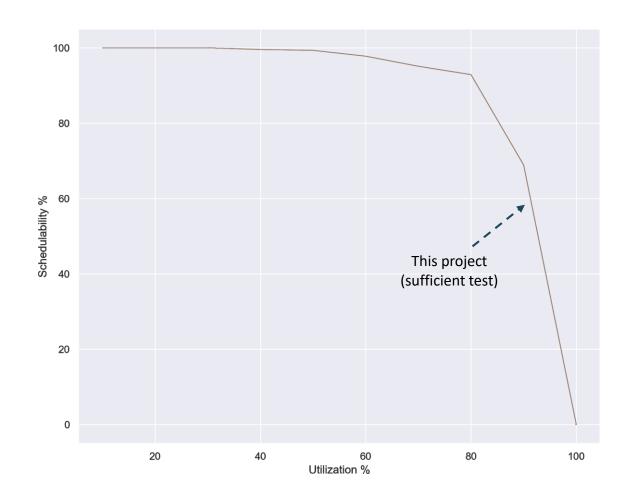
System tasks: 20 rigid tasks



#### Randomly generated task sets

System processors: 8

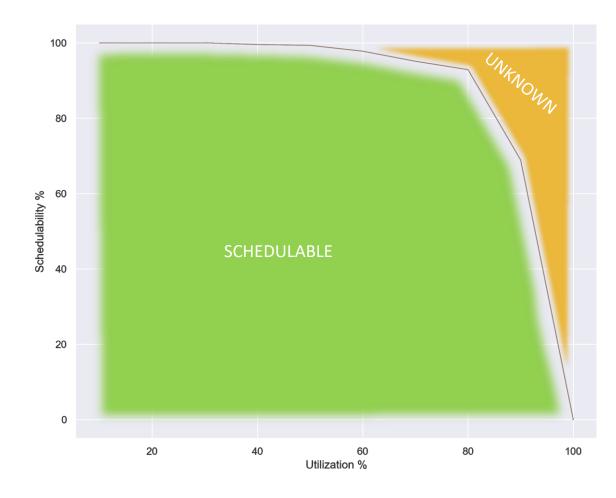
System tasks: 20 rigid tasks



#### Randomly generated task sets

System processors: 8

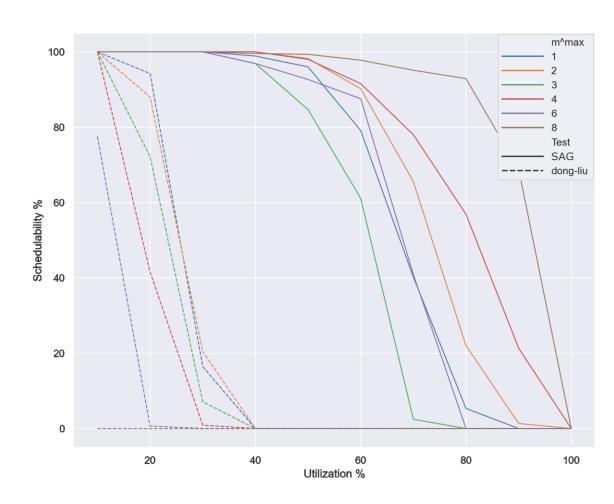
System tasks: 20 rigid tasks



#### Randomly generated task sets

System processors: 8

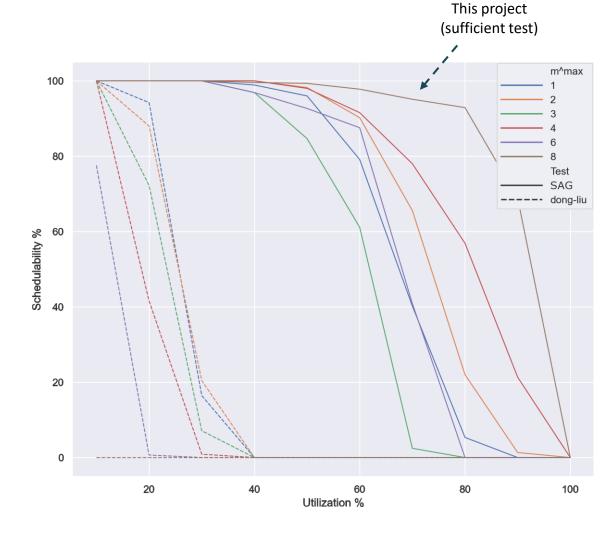
System tasks: 20 rigid tasks



#### Randomly generated task sets

System processors: 8

System tasks: 20 rigid tasks



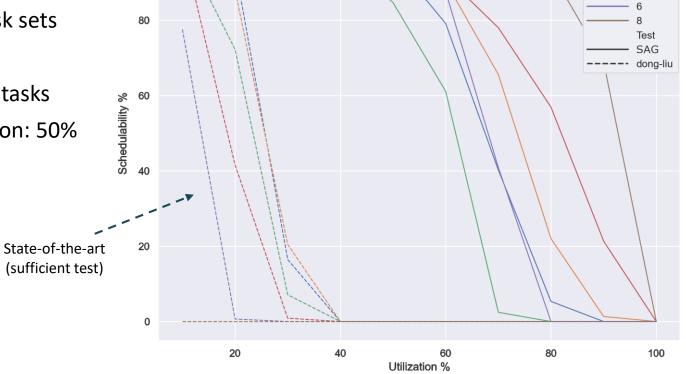
100

Randomly generated task sets

System processors: 8

System tasks: 20 rigid tasks

Execution time variation: 50%

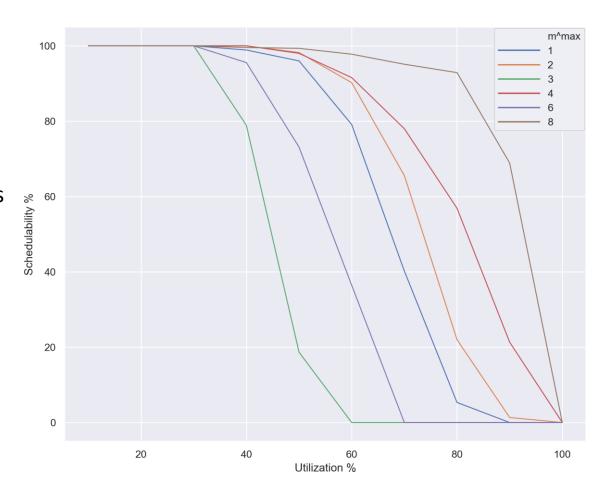


This project (sufficient test)

#### Randomly generated task sets

System processors: 8

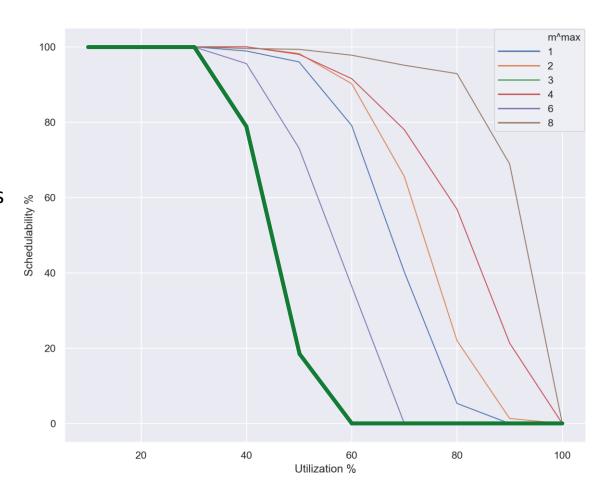
System tasks: 20 moldable tasks



#### Randomly generated task sets

System processors: 8

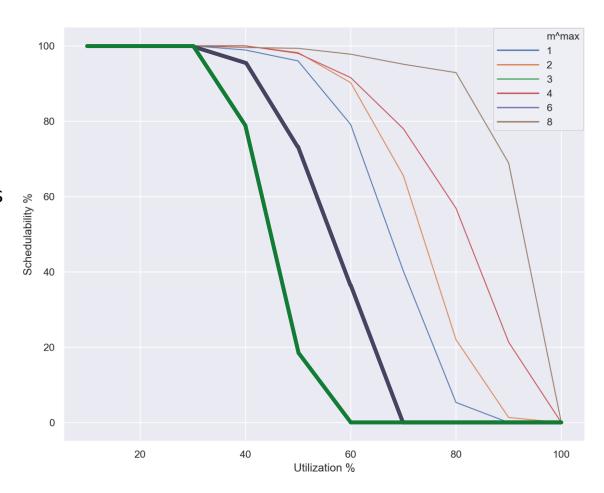
System tasks: 20 moldable tasks



#### Randomly generated task sets

System processors: 8

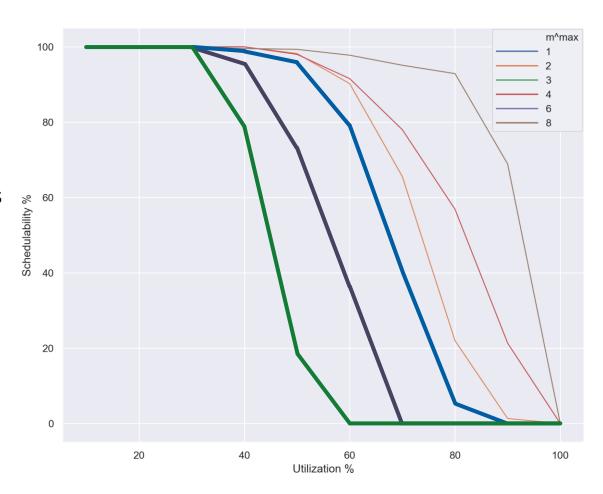
System tasks: 20 moldable tasks



#### Randomly generated task sets

System processors: 8

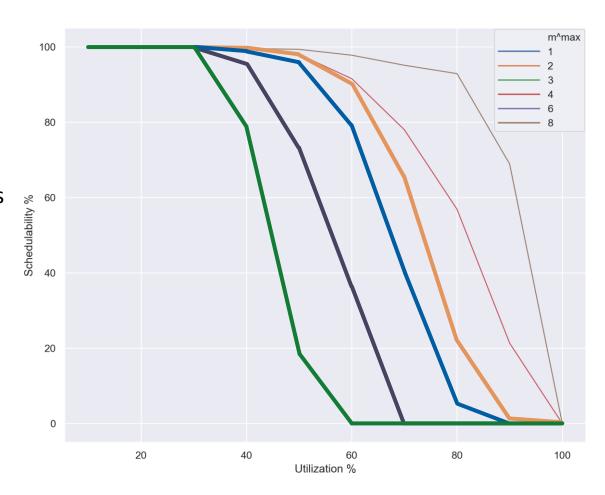
System tasks: 20 moldable tasks



#### Randomly generated task sets

System processors: 8

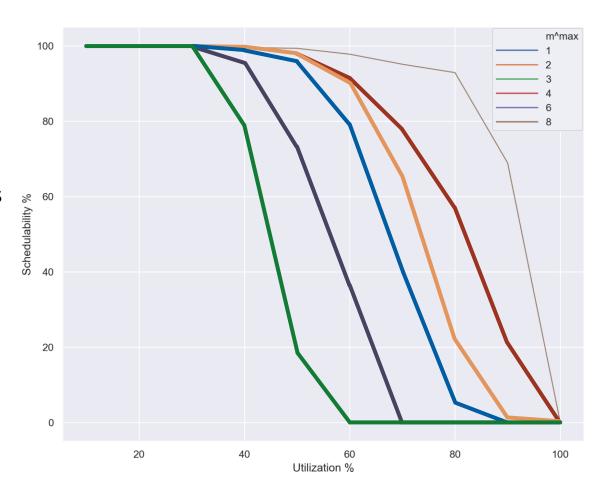
System tasks: 20 moldable tasks



#### Randomly generated task sets

System processors: 8

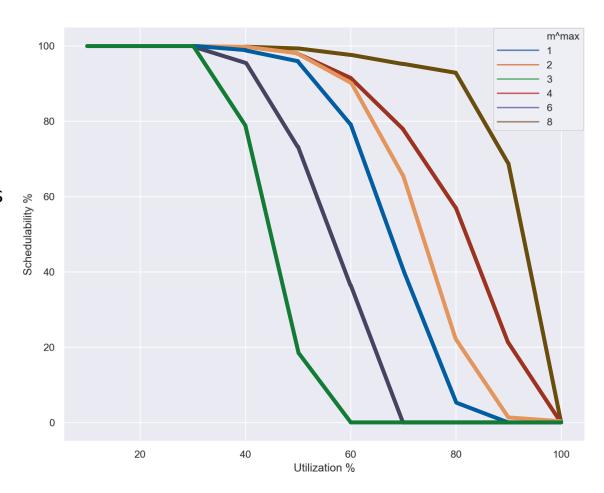
System tasks: 20 moldable tasks



#### Randomly generated task sets

System processors: 8

System tasks: 20 moldable tasks



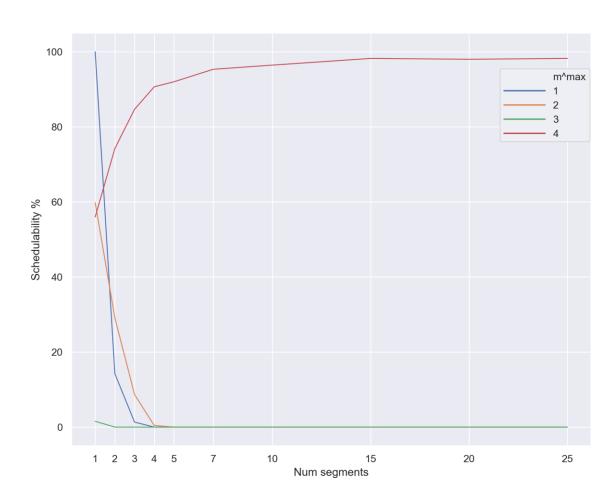
#### Randomly generated task sets

System processors: 4

System tasks: 4 moldable tasks

Execution time variation: 50%

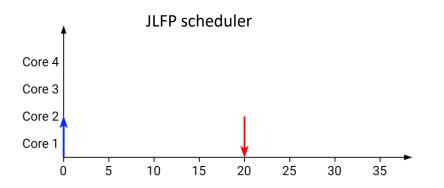
System utilization: 70%



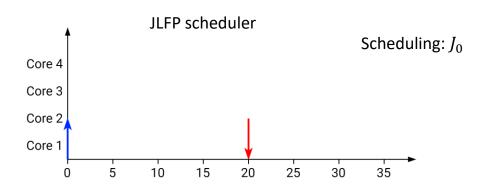
## Agenda

- Gang schedulability analysis
- New scheduling policy

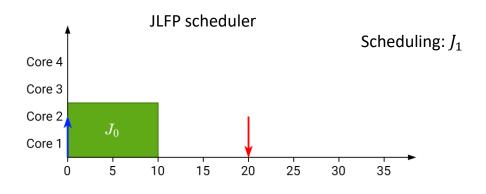




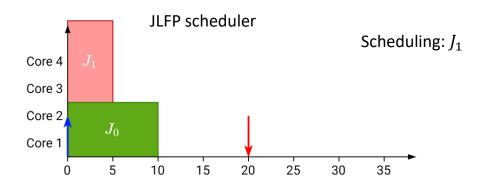
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	$\infty$	20



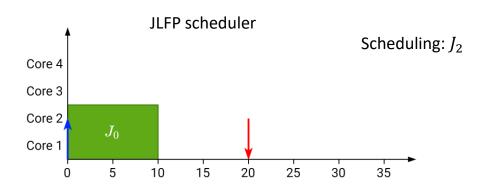
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	$\infty$	20



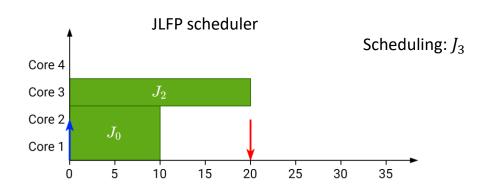
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	$\infty$	20



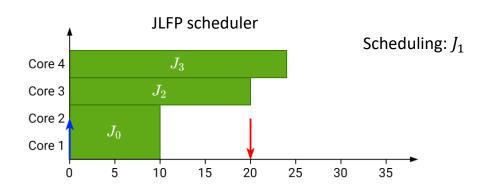
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20



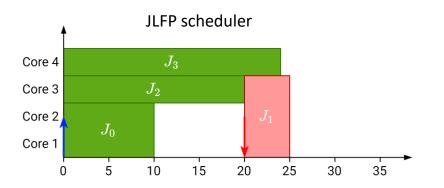
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20



	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

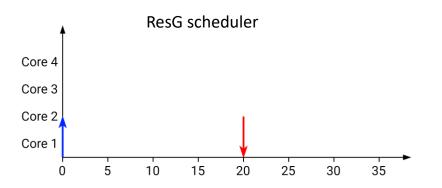


	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20



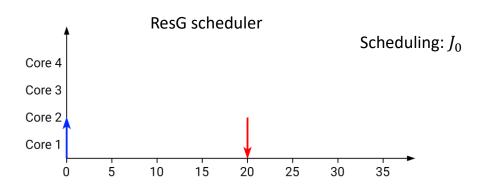
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

Reservation-based



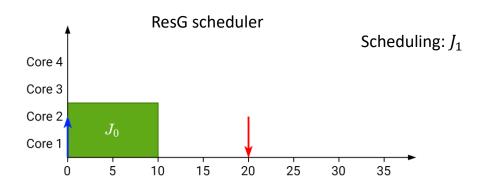
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

Reservation-based



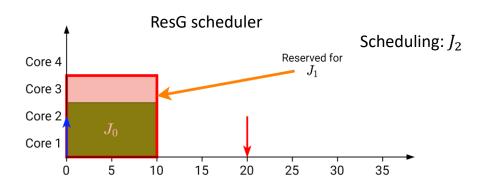
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

Reservation-based



	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



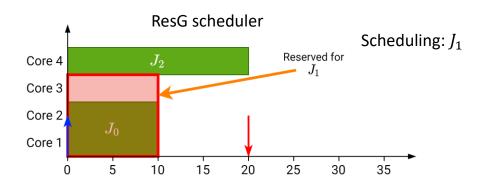
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



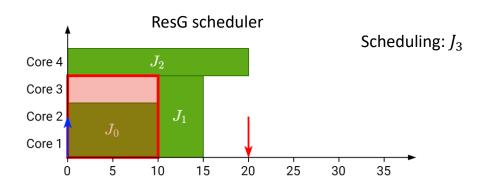
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



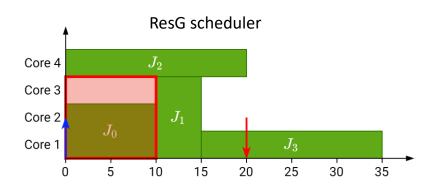
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



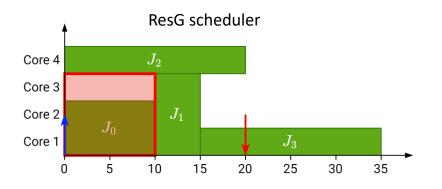
	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks



	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20

- Reservation-based
- Reserve cores of higher-priority tasks and distribute the remaining ones among lower priority tasks
- Non-work conserving scheduler



	Priority	Cores	Deadline	Execution time
$J_0$	High	2	∞	10
$J_1$	Mid-high	3	20	5
$J_2$	Mid-low	1	∞	20
$J_3$	Low	1	∞	20



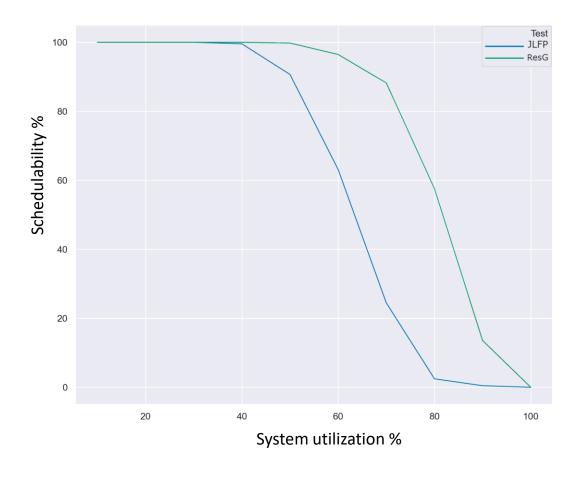
Evaluated in simulator



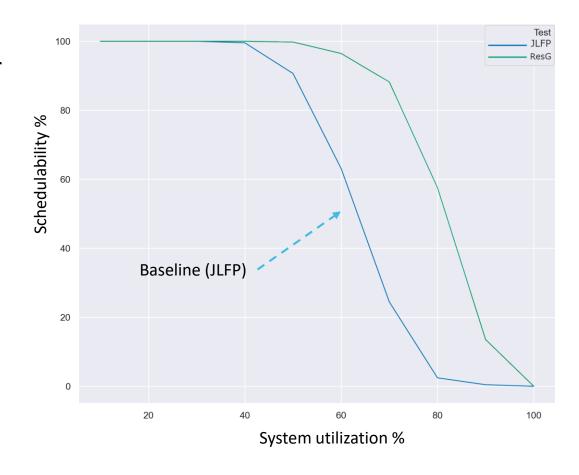
- Evaluated in simulator
- Randomly generated task sets



- Evaluated in simulator
- Randomly generated task sets

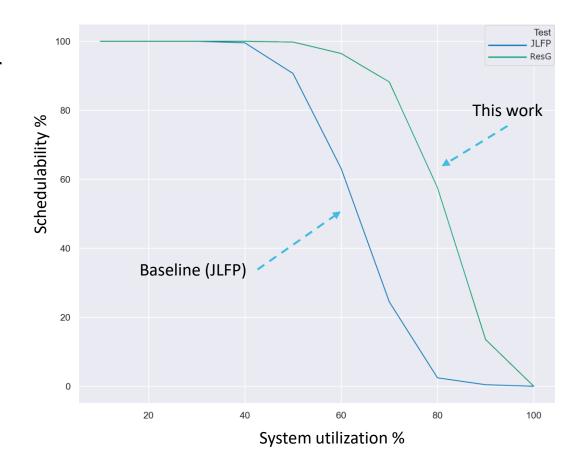


- Evaluated in simulator
- Randomly generated task sets



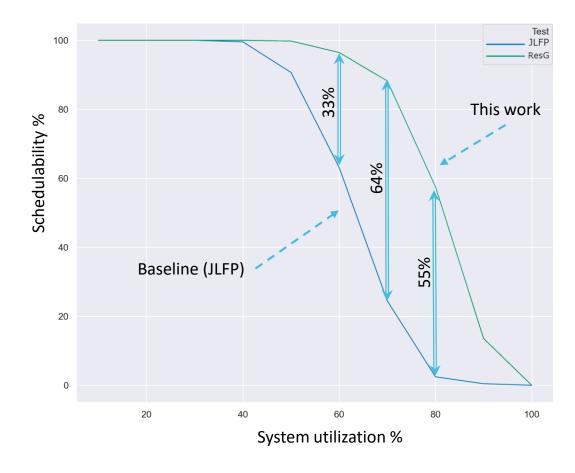


- Evaluated in simulator
- Randomly generated task sets





- Evaluated in simulator
- Randomly generated task sets





### Conclusions



#### Conclusions

 With a better scheduling policy one can improve the schedulability of moldable gang tasks

# Summary



### Summary

A new analysis for gang tasks using SAG has been defined

### Summary

- A new analysis for gang tasks using SAG has been defined
- A new scheduling policy that uses gang moldable properties has been created

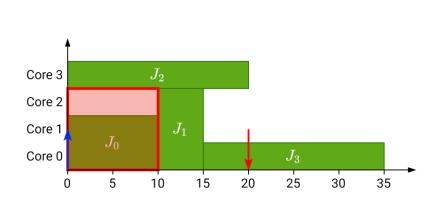


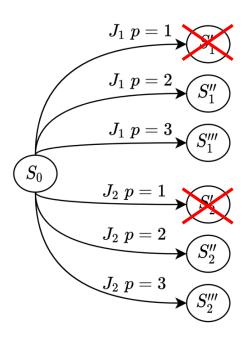
Further reduce sources of pessimism

- Further reduce sources of pessimism
- Provide analysis for ResG scheduler and respective proofs

- Further reduce sources of pessimism
- Provide analysis for ResG scheduler and respective proofs
- Thorough evaluation of results using SURFSara cluster

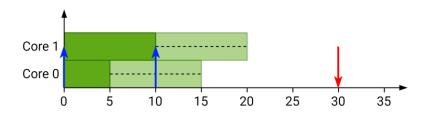
### Questions?



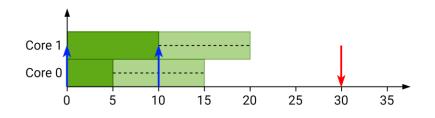




$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	100	2

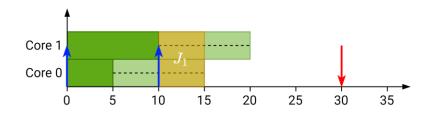


$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	100	2



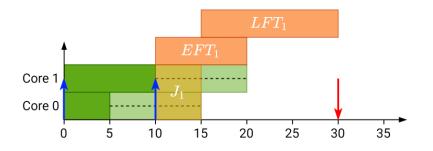
[5, 15] [10, 20]

$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	100	2



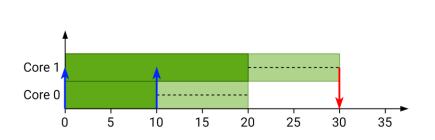
[5, 15] [10, 20]

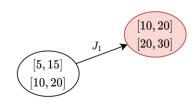
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	100	2



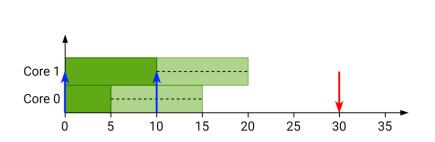
[5, 15] [10, 20]

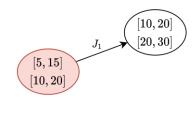
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2



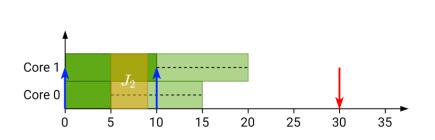


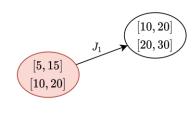
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2



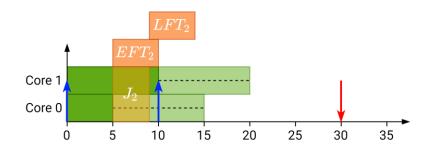


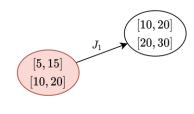
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2



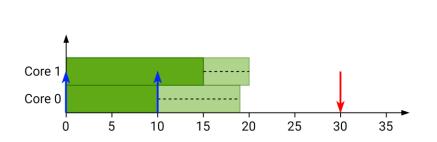


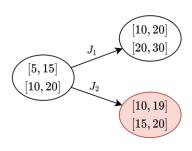
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2



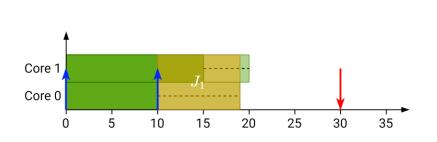


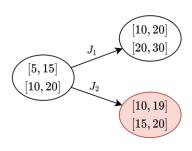
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2



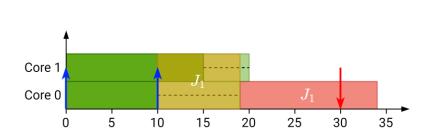


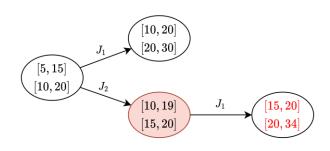
$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2





$J_i$	$C_i^{min}$	$C_i^{max}$	$r_i$	$d_i$	$P_i$
$J_1$	10	15	10	30	1
$J_2$	5	5	0	00	2







```
\begin{split} EST_i &= \max\{R_i^{\min}, A_1^{\min}\} \\ LST_i &= \min\{t_{wc}, t_{high} - 1\} \\ t_{wc} &= \max\left\{A_1^{\max}, \min\{R_x^{\max} \mid J_x \in \mathcal{R}^p\}\right\} \\ t_{high} &= \min\{th_x(J_i) \mid J_x \in \mathcal{R}^p \land p_x < p_i\} \\ th_x(J_i) &= \max\left\{r_x^{\max}, \right. \\ &\left. \max_{0} \left\{LFT_y^* \mid J_y \in pred(J_x) \setminus pred(J_i)\right\}\right\} \end{split}
```

$$EST_{i} = \max\{R_{i}^{\min}, A_{1}^{\min}\}$$

$$EST_{i}^{p} = \max\{R_{i}^{\min}, t_{gang}\}$$

$$LST_{i} = \min\{t_{wc}, t_{high} - 1\}$$

$$LST_{i}^{p} = \min\{t_{avall}, t_{wc}, t_{high} - 1\}$$

$$t_{wc} = \max\{A_{1}^{\max}, \min_{i}\{R_{x}^{\max}|J_{x} \in \mathcal{R}^{p}\}\}$$

$$t_{high} = \min_{i}\{th_{x}(J_{i})|J_{x} \in \mathcal{R}^{p} \land p_{x} < p_{i}\}$$

$$t_{high} = \min_{j}\{th_{x}(J_{i})|J_{x} \in \mathcal{R}^{p} \land p_{x} < p_{i}\}$$

$$t_{high} = \min_{j}\{th_{x}(J_{i},J_{j}), t_{high} = t_{high}(LFT_{y}^{*}|J_{y} \in pred(J_{i})\}$$

$$t_{high} = \min_{j}\{th_{x}(J_{i},J_{j}), t_{high}(J_{i},J_{j}), t_{high}(J_{i},J_{j}), t_{high}(J_{i},J_{j}), t_{high}(J_{i},J_{j}), t_{high}(J_{i},J_{j})$$

$$t_{high} = \min_{j}\{th_{x}(J_{i},J_{j}), t_{high}(J_{i},J_{j}), t_{high}(J_{i},J_{i}), t_{high}(J_{i},J_{i}), t_{high}(J_{i},J_{i}), t_{high}(J_{i},J_{i}), t_{high}(J_{i},J_{i},J_{i}), t_{high}(J_{i},J_{i}), t_{high}(J_{i},J_{i}$$

 $t_{gang} = \begin{cases} A_p^{\min} & \text{if } p = m_i^{\max} \\ A_p^{exact} & \text{otherwise} \end{cases} \quad t_{avail} = \begin{cases} A_{p+1}^{\max} - 1 & \text{if } p < m_i^{\max} \\ +\infty & \text{otherwise} \end{cases}$ 

Check if execution with 
$$p$$
 cores is possible 
$$LST_{i}^{p} = \max\{R_{i}^{\min}, t_{gang}\}$$

$$LST_{i}^{p} = \min\{t_{avail}, t_{wc}, t_{high} - 1\}$$

$$t_{wc} = \min_{J_{j} \in \mathbb{R}^{v}} \left\{ \max\left\{R_{j}^{\max}, A_{m_{j}^{\min}}^{\max} \right\} \right\}$$

$$t_{high} = \min_{J_{j} \in \{hp_{i} \cap \mathbb{R}^{v}\}} \left\{ th_{x}(J_{i}, J_{j}) \right., \\ \max\{LFT_{y}^{*}|J_{y} \in pred(J_{j}) \setminus pred(J_{i})\} \right\}$$

$$t_{h}(J_{i}, J_{j}) = \begin{cases} r_{j}^{\max} & \text{if } m_{j}^{\min} \leq p \\ \max\{r_{j}^{\max}, A_{m_{j}^{\min}}^{\max} \right\} & \text{otherwise} \end{cases}$$

$$t_{gang} = \begin{cases} A_{p}^{\min} & \text{if } p = m_{i}^{\max} \\ A_{p}^{exact} & \text{otherwise} \end{cases}$$

$$EST_{i} = \max\{R_{i}^{\min}, A_{1}^{\min}\}$$

$$EST_{i}^{p} = \max\{R_{i}^{\min}, t_{gang}\}$$

$$LST_{i} = \min\{t_{wc}, t_{high} - 1\}$$

$$t_{wc} = \max\{A_{1}^{\max}, \min_{min}^{\infty} \{R_{x}^{\max} \mid J_{x} \in \mathcal{R}^{p}\}\}$$

$$t_{high} = \min_{min}^{\infty} \{th_{x}(J_{i}) \mid J_{x} \in \mathcal{R}^{p} \land p_{x} < p_{i}\}$$

$$t_{high} = \min_{min}^{\infty} \{th_{x}(J_{i}) \mid J_{x} \in \mathcal{R}^{p} \land p_{x} < p_{i}\}$$

$$t_{high} = \min_{min}^{\infty} \{th_{x}(J_{i}, J_{j})$$

$$th_{x}(J_{i}) = \max\{r_{x}^{\max}, m_{x}^{\max}\}$$

$$max\{LFT_{y}^{*} \mid J_{y} \in pred(J_{x}) \land pred(J_{i})\}\}$$

$$t_{h}(J_{i}, J_{j}) = \begin{cases} r_{j}^{\max}, A_{min}^{\max} \\ max\{r_{j}^{\max}, A_{min}^{\max}\} \end{cases} \text{ otherwise}$$

$$t_{gang} = \begin{cases} A_{pin}^{\min}, t_{gang}, t_{gang$$

$$\begin{split} EST_i &= \max\{R_i^{\min}, A_1^{\min}\} \\ LST_i &= \min\{t_{wc}, t_{high} - 1\} \\ t_{wc} &= \max\left\{A_1^{\max}, \min\{R_x^{\max} \mid J_x \in \mathcal{R}^p\}\right\} \\ t_{high} &= \min\{th_x(J_i) \mid J_x \in \mathcal{R}^p \land p_x < p_i\} \\ th_x(J_i) &= \max\left\{r_x^{\max}, \right. \\ &\left. \max_{0} \left\{LFT_y^* \mid J_y \in pred(J_x) \setminus pred(J_i)\right\}\right\} \end{split}$$

$$EST_{i}^{p} = \max\{R_{i}^{\min}, t_{gang}\}$$

$$LST_{i}^{p} = \min\{t_{avail}, t_{wc}, t_{high} - 1\}$$

$$t_{wc} = \min_{J_{j} \in \mathbb{R}^{v}} \left\{ \max\left\{R_{j}^{\max}, A_{m_{j}^{\min}}^{\max}\right\} \right\}$$

$$t_{high} = \min_{J_{j} \in \{hp_{i} \cap \mathbb{R}^{v}\}} \left\{ th_{x}(J_{i}, J_{j}), max\{LFT_{y}^{*}|J_{y} \in pred(J_{j}) \setminus pred(J_{i})\} \right\}$$

$$t_{h}(J_{i}, J_{j}) = \begin{cases} r_{j}^{\max} & \text{if } m_{j}^{\min} \leq p \\ \max\{r_{j}^{\max}, A_{m_{j}^{\min}}^{\max} \} & \text{otherwise} \end{cases}$$

$$t_{gang} = \begin{cases} A_{p}^{\min} & \text{if } p = m_{i}^{\max} \\ A_{p}^{exact} & \text{otherwise} \end{cases}$$

$$t_{avail} = \begin{cases} A_{p+1}^{\max} - 1 & \text{if } p < m_{i}^{\max} \\ +\infty & \text{otherwise} \end{cases}$$