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A ONE-DIMENSIONAL THEORETICAL DESCRIPTION OF THE VEGETATION-ATMOSPHERE INTERACTION

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Abstract. An analytically continuous, one-dimensional model of the vegetation-atmosphere interaction is created which is 'multi-layer' in concept, and therefore similar in principle to previous numerical simulation models (except that it also includes the effect of surface water on the vegetation). Mathematical development of this model yields a 'combination equation' similar in form to that produced by single-source, Penman-Monteith models; and demonstrates that the interaction is indeed capable of conceptual representation as a simple electrical analogue, but that *all* property transfers are subject to a 'surface' resistance. In this way, it is shown that 'multi-layer' and 'single-source' descriptions are more similar than they appear at first sight: the differences between the two approaches become apparent and in this way the assumptions and approximations involved in the single-source hypothesis become explicit.

1. Introduction

It is more than a quarter of a century since Penman (1948) attempted to formulate a physically based, mathematical description of the vegetation-atmosphere interaction, one of the most important features of which is the way that the sun's radiation is partitioned into other forms of energy. The equation he produced describes long-term evaporation from short vegetation when this vegetation is not subject to physiological restraint. Subsequent work has attempted to generalize this initial study and two alternative approaches have usually been adopted, either to make a 'single-source' assumption, or to attempt a multi-layer 'simulation': they are sometimes considered as 'competing' descriptions.

In an attempt at generalization, Monteith (1965) formulated a combination equation which included some physiological dependence in the form of a parameter r_s, the 'surface' resistance to vapour flux. The equation assumed that both vapour and sensible heat fluxes experience the same aerodynamic resistance r_A, and that this resistance was proportional to the aerodynamic resistance seen by the momentum flux. The equation has the form:

$$\lambda E = \frac{\Delta A + \dfrac{\rho c_p (e_w(T_z) - e_z)}{r_A}}{\Delta + \gamma\left[1 + \dfrac{r_s}{r_A}\right]} \tag{1}$$

where e_z and T_z are the vapour pressure and temperature at a height z; $e_w(T_z)$ is the saturation vapour pressure at temperature T_z; Δ is the mean slope of the saturated vapour pressure curve between T_z and T_s (the surface temperature of the vegetation); A is sensible plus latent heat; and γ is the psychrometric 'constant'. In this equation r_s was assumed to be primarily dependent on stomatal resistance and to reflect, to a fair approximation, changes in that resistance.

Suspecting that any shortcomings in the interpretation of r_s as a stomatal resistance might primarily be a result of the assumption that r_A was applicable to all properties, Thom (1972) made a further attempt at generalization by acknowledging the existence of differences in the coefficients describing the transfer of vapour, sensible heat and momentum between individual vegetative elements and the canopy airstream. Thom's work represents a useful extension of the Penman-Monteith equation (Equation (1)), in that it defines a parameter, r_{ST}, called the 'bulk physiological resistance', which, since it acknowledges the differences mentioned above, might be expected to be more closely related to stomatal resistance than r_s. Nevertheless, the treatment maintains the assumption that each flux can be considered as being created or destroyed at one level in the canopy, though not necessarily at the same level; and that all the latent heat flux originates inside an average stomatal cavity at that level when the canopy is dry.

The 'single-source' approach has the advantage of simplicity and practicability (and there is some evidence for its adequacy in 'dry canopy' conditions, e.g., Black *et al.*, 1970). On the other hand, the multi-layer, numerical simulation technique (e.g., Waggoner and Reifsnyder, 1968; Lemon *et al.*, 1971) is superior in that it is physically more plausible.

The purpose of this paper is to attempt to unify current attitudes by demonstrating that it is analytically possible to start from an 'elemental' description of the interaction (similar to that used in numerical treatments, but including the effect of surface water on the vegetation) and derive a 'combination equation' similar to that produced by the single-source approach. It is hoped that this process will bring increased knowledge of the parameters present in the 'combination equation' and a better understanding of the assumptions involved in the single-source treatment.

2. The Concept of 'Diffusive' and 'Diversive' Resistance

In all that follows, horizontal homogeneity is assumed. The wind speed, temperature and vapour pressure at height z are represented by $u(z)$, $T(z)$, and $e(z)$, respectively; and the effective surface temperature of the vegetation at this level by $T_s(z)$. In a similar way, the fluxes of shearing stress and sensible and latent heat are represented by $\tau(z)$, $H(z)$, and $\lambda E(z)$, respectively. It is assumed that these fluxes can be related to the gradients of wind speed, temperature and vapour pressure by the diffusion equations:

$$\tau(z) = -\rho K_M(z) \frac{\partial u(z)}{\partial z} \tag{2}$$

$$H(z) = -\rho c_p K_H(z) \frac{\partial T(z)}{\partial z} \tag{3}$$

$$\lambda E(z) = \frac{-\rho c_p}{\gamma} K_V(z) \frac{\partial e(z)}{\partial z} \tag{4}$$

where $K_M(z)$, $K_H(z)$, and $K_V(z)$ represent the 'eddy diffusivities' for momentum, sensible heat and water vapour, respectively. The corresponding 'diffusive' resistances between levels z_1 and z_2 are defined by the equations:

$$r_M^{\mathrm{DIF}}(z_1, z_2) = \int_{z_1}^{z_2} K_M^{-1}(z)\, \mathrm{d}z \tag{5}$$

$$r_H^{\mathrm{DIF}}(z_1, z_2) = \int_{z_1}^{z_2} K_H^{-1}(z)\, \mathrm{d}z \tag{6}$$

$$r_V^{\mathrm{DIF}}(z_1, z_2) = \int_{z_1}^{z_2} K_V^{-1}(z)\, \mathrm{d}z. \tag{7}$$

There is a close similarity between this diffusive resistance and 'aerodynamic resistance', but in the sense assigned to this term by (say) Thom (1971; 1972), the two are not identical. The diffusive resistance does not include all the aerodynamical resistance seen by the flux, or parts of that flux. In particular, it does not include that resistance between the flux source (on an individual vegetative element) and the canopy air stream, where the transfer is not primarily controlled by eddy diffusion. It is merely the parameter relating total, vertical fluxes to their respective 'potential' differences.

At any level z within the canopy, there can be sources and sinks of flux, the strengths being related to the divergence of the total flux at that level. In a one-dimensional model, the source strength in the crop is presumably also related to the difference, at each level, between conditions in the mean canopy air stream, and at the flux source on the vegetative element, e.g.,

$$\nabla \cdot H \propto [T_s(z) - T(z)].$$

The 'diversive resistivity' puts this relationship, at this stage hypothetical, on a formal basis. Accordingly the 'diversive resistivities' for momentum, sensible heat and latent heat $\rho_M^{\mathrm{DIV}}(z)$, $\rho_H^{\mathrm{DIV}}(z)$, and $\rho_V^{\mathrm{DIV}}(z)$, respectively, are defined by the equations:

$$\frac{\partial(\tau(z))}{\partial z} = \rho \frac{-u(z)}{\rho_M^{\mathrm{DIV}}(z)} \tag{8}$$

$$\frac{\partial(H(z))}{\partial z} = \rho c_p \frac{T_s(z) - T(z)}{\rho_H^{\mathrm{DIV}}(z)} \tag{9}$$

$$\frac{\partial(\lambda E(z))}{\partial z} = \frac{\rho c_p}{\gamma} \frac{e_w(T_s(z)) - e(z)}{\rho_V^{\mathrm{DIV}}(z)}. \tag{10}$$

It is not convenient at this point to establish the physical entity of these resistivities in any formal or rigorous way: this is quite complex and forms the subject of Appendix 1. However, for purposes of illustration, it is useful to select from that work the almost intuitive, approximate results:

$$\rho_M^{\mathrm{DIV}}(z) = \frac{P^M(z)}{u(z)\overline{C^M(z)}L(z)} \tag{11}$$

$$\rho_H^{\mathrm{DIV}}(z) = \frac{p^H(z)}{u(z)\overline{C^H(z)}L(z)} \tag{12}$$

and

$$\rho_V^{\mathrm{DIV}}(z) = \frac{1}{L(z)}\left[r_{\mathrm{STO}}(z) + \frac{P^V(z)}{u(z)\overline{C^V(z)}}\right] \tag{13}$$

which apply for a dry canopy, and the result:

$$\rho_V^{\mathrm{DIV}}(z) = \frac{P^V(z)}{u(z)\overline{C^V(z)}L(z)} \tag{14}$$

which applies for a totally wet canopy. In these equations $\overline{C^M}$, $\overline{C^H}$, and $\overline{C^V}$ are average transfer coefficients between individual vegetative elements and the air stream for momentum, sensible heat and water vapour, respectively; and P^M, P^H and P^V are 'constants' of proportionality, called 'shelter factors', which, amongst other things, take account of aerodynamic interference between these elements. The parameter $r_{\mathrm{STO}}(z)$ is the average stomatal resistance at level z, while $L(z)$ is the leaf area index per unit height at that level (or more correctly leaf area *per unit volume*).

Although the formalism in Equations (8) to (14) is similar to that used by Thom (1972), conceptually the ideas expressed in these equations are very similar to those of the multi-layer model of (say) Waggoner and Reifsnyder (1968). However, expressing these concepts in 'continuous' rather than 'finite-difference' form gives rise to entities with the *relative* dimensions of resistivity rather than resistance. It is important to remember that the rigorous analogue of electrical *resistivity* is the reciprocal of eddy diffusivity (e.g., K_M^{-1}); and also that the now conventional use of the term 'aerodynamic resistance' is not dimensionally precise, since this entity has dimensions of (Resistance $\times\,\mathbf{L}^2$). The parameters here called 'diversive resistivity' are similarly poor electrical analogues in that they also contain a dimensional factor $\mathbf{L}^2$. In this way, the name 'diversive *resistivity*' is merely an expression of their dimensional content *with respect to* diffuse resistance.

Equations (8), (9), and (10) embody the fundamental difference between 'single-source' and 'multi-layer' models with regard to the rôle played by the aerodynamic transfer resistance between individual elements and the airstream. In single-source models, this resistance forms part of the Penman-Monteith 'aerodynamic' resistance. In the course of the following, it will become apparent that its rôle in a simple electrical analogue is more correctly that of a 'surface' resistance.

It is not hard to see how confusion could arise: the mathematical distinction that 'diffusive' aerodynamic resistance applies to the total vertical flux, while 'individual element' aerodynamic resistance and stomatal resistance together apply to the separate contributions to that flux, is lost in the simplification of the 'single-source' hypothesis. Identities may have been assigned by the implicit observation that both canopy flow and individual element resistances can be regarded as 'aerodynamic', since they are associated with air motion. However, the physical processes responsible for stomatal and 'individual element' resistance are to a certain extent similar, at least for properties other than momentum; and both are related to area, a blatantly 'surface'-like characteristic. The association between 'individual element' and stomatal resistance is explicit in equation (13). Obviously the reassignment of this 'individual element' resistance carries with it the implication that all property transfers are subject to some kind of surface resistance, which is additional to the intrinsic surface resistance at the surface-air interface (Shuttleworth, 1975), and usually dominates that resistance. The vapour flux, however, sees a further 'surface' resistance, the stomatal resistance, when the canopy is dry, and some portion of that resistance (Appendix 1) when partially wet.

The consequences of this reassignment at the single-leaf level appear trivial, and it is of course still possible to make a 'Monteith type' extrapolation to the leaf surface through the 'boundary-layer' part of the surface resistance in order to deduce the 'internal' or 'stomatal' part: the change is mere nomenclature. However, the author believes that even at this level a conceptual reassignment is worthwhile since it helps to clarify the process by which the behaviour of individual leaves can be combined to deduce the behaviour of the whole community.

3. A 'Height-Dependent' Available Energy

All models of the vegetation-atmosphere interaction rely on the conservation of energy: it is convenient here to apply this principle in its more general form as the continuity equation for total energy. In one dimension this takes the form:

$$\sum_k \left[\frac{\partial q_k(z)}{\partial t} + \frac{\partial i_k(z)}{\partial z}\right] = 0 \tag{15}$$

where, for each form of energy k, q_k represents the 'charge' or energy storage at level z, while i_k represents the 'current' or energy flux at that level.

Included in this summation are:

(1) the latent heat of the water vapour in the air, for which:

$$q_1 = F_A(z) \frac{\rho c_p}{\gamma} e(z) \tag{16}$$

and

$$i_1 = \lambda E(z) \tag{17}$$

where F_A is the fractional volume of air per unit volume of crop;

(2) the sensible heat in the air, with:

$$q_2 = F_A(z)\rho c_p T(z) \tag{18}$$

and

$$i_2 = H(z); \tag{19}$$

(3) the sensible heat in the biomass, for which:

$$q_3 = (1 - F_A)\rho_B c_B T_B(z) \tag{20}$$

$$i_3 = b(z) \tag{21}$$

where ρ_B and c_B are the density and specific heats of the biomass, respectively, and T_B its average temperature, while b is the vertical flux of sensible heat in the biomass;

(4) the net, all-wavelength radiation, R_N (defined positive *into* the surface), for which

$$q_4 = 0 \tag{22}$$

$$i_4 = -R_N(z); \tag{23}$$

(5) the chemical energy in the biomass, with:

$$q_5 = C(z) \tag{24}$$

$$i_5 = c(z) \tag{25}$$

where C is the energy stored chemically in the biomass and c is the flux of chemical energy through the biomass.

Neglecting other contributions, e.g., the sensible heat of any liquid water present within, and possibly moving through, the crop gives:

$$F_A\rho c_p\left[\frac{1}{\gamma}\frac{\partial e}{\partial t} + \frac{\partial T}{\partial t}\right] + (1 - F_A)\rho_B c_B \frac{\partial T_B}{\partial t} + \frac{\partial C}{\partial t} = -\frac{\partial(\lambda E)}{\partial z} - \frac{\partial H}{\partial z} - \frac{\partial b}{\partial z} + \frac{\partial R_N}{\partial z} - \frac{\partial c}{\partial z}. \tag{26}$$

It is convenient to collect the first two terms in this equation as:

$$s = F_A\rho c_p\left[\frac{1}{\gamma}\frac{\partial e}{\partial t} + \frac{\partial T}{\partial t}\right] + (1 - F_A)\rho_B c_B \frac{\partial T_B}{\partial t}, \tag{27}$$

calling the resulting parameter, s, 'energy storage'. In addition, familiarity is enhanced by relabelling the third, thus:

$$p = \frac{\partial C}{\partial t} \tag{28}$$

with p regarded as the net rate of energy uptake *per unit height* (volume) resulting from the combined processes of photosynthesis and respiration. The fluxes b and c (or more correctly their first spatial derivatives) are regarded as negligible, so that Equation (26) simplifies to the form:

$$-s-p+\frac{\partial R_N}{\partial z}=\frac{\partial(\lambda E)}{\partial z}+\frac{\partial H}{\partial z}. \tag{29}$$

The terms on the left-hand side of this equation can conveniently be combined as the parameter a, the available energy *per unit height* (volume), that is:

$$a(z)=-s-p+\frac{\partial R_N}{\partial z} \tag{30}$$

and Equation (29) written as:

$$a(z)=\frac{\partial(\lambda E)}{\partial z}+\frac{\partial H}{\partial z}. \tag{31}$$

To obtain A, the total available energy, Equation (30) is integrated through the vegetation to the level h, the top of the crop, i.e.,

$$\begin{aligned} A &= \int_0^h a(z)\,\mathrm{d}z \\ &= -S-P+[R_N-R_N(0)] \end{aligned} \tag{32}$$

where S is the total energy flux into storage, given by:

$$S=\int_0^h \left\{F_A\rho c_p\left[\frac{1}{\gamma}\frac{\partial e}{\partial t}+\frac{\partial T}{\partial t}\right]+(1-F_A)\rho_B c_B\frac{\partial T_B}{\partial t}\right\}\mathrm{d}z \tag{33}$$

and P is the rate at which energy is absorbed chemically by the whole crop; R_N is the downward flux of net radiation into the crop from above; and $R_N(0)$ is the flux out below. (Hereafter, R_N should be regarded as a number and not a function.)

The equivalent integral form of Equation (31) is:

$$A=\lambda E_h+H_h-A_0 \tag{34}$$

where

$$A_0=\lambda E_0+H_0 \tag{35}$$

and λE_h and H_h are the total fluxes of latent and sensible heat out of the crop above, and λE_0 and H_0 the equivalent fluxes into the crop from the soil. Very often Equation (34) has been written as:

$$A'=\lambda E_h+H_h \tag{36}$$

where A' is given by:

$$A'=-S-P+R_N-G \tag{37}$$

with G, the heat flux into the soil, equal to:

$$G = R_N(0) - [\lambda E_0 + H_0].$$

4. The One-Dimensional Combination Equation

Once the divergence of the latent heat flux inside the vegetation has been assigned the analytic form of Equation (10), it is possible to integrate this divergence through the crop to produce an expression for the total flux. If local energy conservation and vertical diffusion are introduced into this integration through Equations (31), and (3), and (4), respectively, a 'combination equation' results which, within the limits of a one-dimensional approach, describes the net evaporation produced in the vegetation-atmosphere interaction. The mathematical details involved in the derivation of this equation are outlined in Appendix 2; the resulting equation takes the form:

$$\lambda E_h = \frac{\Delta A' + (\rho c_p D + \delta)/r_H}{\Delta + \gamma_E(1 + r_c/r_V)} \tag{38}$$

where

$$\delta = \gamma[r_c' I_1(A' - A_0) + r_c \lambda E_0] \tag{39}$$

$$\gamma_E = \left[\frac{r_V}{r_H}\right]\gamma \tag{40}$$

with

$$I_1 = \left[\frac{r_c}{A r_c'}\right] \int_0^h \frac{\rho'_{\mathrm{DIV}} a}{\rho_{\mathrm{DIV}}} \, dz \tag{41}$$

$$[r_c']^{-1} = \int_0^h [\rho'_{\mathrm{DIV}}]^{-1} \, dz \tag{42}$$

$$[r_c]^{-1} = \int_0^h [\rho_{\mathrm{DIV}}]^{-1} \, dz \tag{43}$$

$$r_H = -\rho c_p \frac{(\delta T)'}{H_h} \tag{44}$$

$$r_V = -\frac{\rho c_p}{\gamma} \frac{(\delta e)'}{\lambda E_h} \tag{45}$$

where $(\delta T)'$ and $(\delta e)'$, the weighted means of the differences between the values of temperature and humidity in the canopy and the values at the top of the canopy, are defined by:

$$(\delta T)' = \frac{\int_0^h [T(h) - T(z)][\rho_{\mathrm{DIV}}]^{-1}\,\mathrm{d}z}{\int_0^h [\rho_{\mathrm{DIV}}]^{-1}\,\mathrm{d}z} \tag{46}$$

and

$$(\partial e)' = \frac{\int_0^h [e(h) - e(z)][\rho_{\mathrm{DIV}}]^{-1}\,\mathrm{d}z}{\int_0^h [\rho_{\mathrm{DIV}}]^{-1}\,\mathrm{d}z}\,. \tag{47}$$

The functions $\rho_{\mathrm{DIV}}(z)$ and $\rho'_{\mathrm{DIV}}(z)$ are defined in terms of the diversive resistivities for vapour and sensible heat from the equations:

$$\rho_{\mathrm{DIV}}(z) = \rho_V^{\mathrm{DIV}}(z) + \frac{\Delta'}{\gamma}\,\rho_H^{\mathrm{DIV}}(z) \tag{48}$$

$$\rho'_{\mathrm{DIV}}(z) = \frac{\Delta'}{\gamma}\,\rho_H^{\mathrm{DIV}}(z). \tag{49}$$

The value of D, the vapour pressure deficit, used in Equation (38) is that applicable at the top of the vegetation, which is in this respect the 'screen' height. When the deficit is monitored at a new (higher) level, z_R, providing there is no flux divergence between h and z_R, the combination equation preserves its general form (see Appendix 3), and extending Equation (38) to a higher level is equivalent to a simple and fairly obvious redefinition of the parameters Δ, r_H, r_V, and γ_E, thus:

$$r_H \rightarrow r''_H = r_H + r_{AH} \tag{50}$$

$$r_V \rightarrow r''_V = r_V + r_{AV} \tag{51}$$

$$\gamma_E \rightarrow \gamma''_E = \left[\frac{r''_V}{r''_H}\right]\gamma \tag{52}$$

$$\Delta \rightarrow \Delta'' = \frac{\Delta r_H + \Delta''' r_{AH}}{r_H + r_{AH}} \tag{53}$$

where

$$r_{AH} = \int_h^{z_R} [K_H(z)]^{-1}\,\mathrm{d}z \tag{54}$$

$$r_{AV} = \int_h^{z_R} [K_V(z)]^{-1}\,\mathrm{d}z \tag{55}$$

and Δ''' is the mean gradient of the saturation vapour-pressure curve between $T(z_R)$ and $T(h)$.

In this respect, a change in screen height is of no fundamental significance, and within the constant flux layer all heights are equally valid. It is convenient here to retain a screen height coincident with the top of the vegetation. It is worth noticing that it is possible to extend the generality of definitions (42), (43), (46), and (47) to higher screen heights by making the replacement $h \to z_R$ and setting $\rho'_{\mathrm{DIV}}(z) = \rho_{\mathrm{DIV}}(z) = \infty$ for $h < z \leq z_R$.

The combination equation, Equation (38), can of course be written in its diagnostic form as:

$$r_c = \frac{\lambda E_h}{\lambda E_h - \lambda E_0}\left[\left(\frac{\Delta\beta}{\gamma} r_H - r_V\right) + (1+\beta) r_I + (1+\beta) r'_c I_1 \left(1 - \frac{A_0}{A'}\right)\right] \tag{56}$$

in which β is the Bowen Ratio above the crop and r_I is the 'isothermal resistance', defined by:

$$r_I = \frac{\rho c_p}{\gamma}\frac{D}{A'}. \tag{57}$$

The similarity between Equation (56) and that deduced by Thom for the bulk physiological resistance, r_{ST} (Stewart and Thom, 1973, Equation (6)), is striking, if not unexpected. However this equation contains the extra factor $[\lambda E_h/(\lambda E_h - \lambda E_0)]$, to take account of the flux from beneath the vegetation; and an extra term to take account of differences in the spatial distribution of the sources of latent and sensible heats. It also differs fundamentally in that r_c is not a pure measure of bulk stomatal resistance but depends to some extent on the size of the aerodynamic resistance between the vegetative elements and the canopy air stream, i.e.,

$$\frac{1}{r_c} = \int_0^h L\left\{\left[r_{\mathrm{STO}} + \frac{p^V}{u\overline{C^V}}\right] + \frac{\Delta'}{\gamma}\left[\frac{p^H}{u\overline{C^H}}\right]\right\}^{-1} \mathrm{d}z \tag{58}$$

in dry canopy conditions. Obviously the extent to which Thom's parameter, r_{ST}, is a measure of bulk stomatal resistance depends not only on the assumptions he makes about values of r_H and r_V, but also in a complicated way on the extent to which the inclusion of this extra resistance on the left-hand side of Equation (56) is compensated by its inclusion as the extra term $\left[(1+\beta) r'_c I_1\left(1 - \frac{A_0}{A'}\right)\right]$ on the right-hand side.

5. Equivalent Electrical Analogues

We have now shown that it is possible to start from an elemental, multi-layer, one-dimensional description of the vegetation interaction and deduce a 'combination equation' containing parameters very similar (but not identical) to those

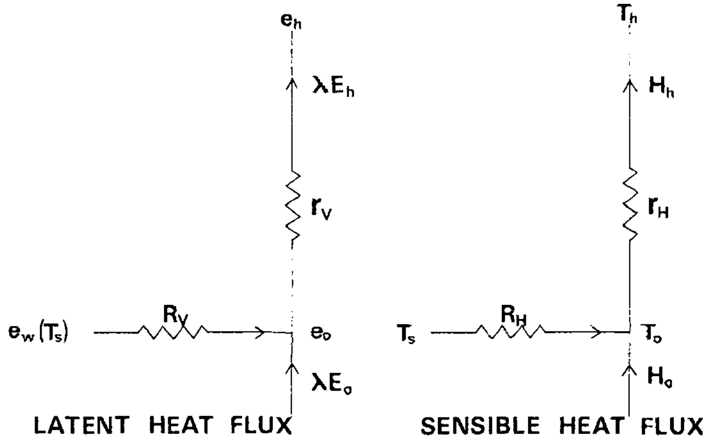

Fig. 1. Schematic diagram representing electrical analogues of the exchange of latent and sensible heat fluxes at the vegetation-atmosphere interface which give rise to the 'generalized combination equation'.

occurring in the Penman-Monteith equation (or more particularly Thom's version of that equation). All previous models can be considered as describing the interaction in terms of an equivalent electrical analogue: the difference between 'single-source' and 'multi-layer' models is merely the complexity of the analogue used. With this in mind it is conceptually interesting to consider what equivalent circuit produces Equation (38), the combination equation derived in Appendix 2.

Figure 1 describes an analogue model in which currents of both latent and sensible heat leave a moist vegetative surface at temperature T_s, pass through respective 'surface' resistances R_V and R_H, and then, in confluence with the currents λE_0 and H_0 (from below the vegetation), pass through 'aerodynamic' resistances r_V and r_H. Direct analogy with Ohm's Law gives the immediate results:

$$T_h - T_0 = \frac{-H_h r_H}{\rho c_p} \tag{59}$$

$$T_0 - T_s = -\frac{(H_h - H_0)R_H}{\rho c_p} \tag{60}$$

$$e_h - e_w(T_s) = -\frac{\lambda E_h r_V \gamma}{\rho c_p} - \frac{(\lambda E_h - \lambda E_0)R_V \gamma}{\rho c_p}. \tag{61}$$

Obviously the left-hand side of (61) can be written in the form:

$$\begin{aligned} e_h - e_w(T_s) &= -D + e_w(T_h) - e_w(T_0) + e_w(T_0) - e_w(T_s) \\ &= -D + \Delta[T_h - T_0] + \Delta'[T_0 - T_s] \end{aligned} \tag{62}$$

where D is vapour-pressure deficit at height h and Δ and Δ' are the mean gradients of the saturated vapour-pressure curve between T_h and T_0, and T_o and T_s respectively. Combining Equations (59), (60), (61), and (62), and rearranging, yields an equation identical to (38) *providing* R_H and R_V are assigned the values:

$$R_H = \frac{\gamma}{\Delta'} r_c' I_1 \tag{63}$$

$$R_V = r_c - \frac{\Delta' R_H}{\gamma}. \tag{64}$$

In this way the complexities of the one-dimensional description of the vegetation-atmosphere interaction are capable of simple representation in terms of an elementary electrical analogue. It is important to remember however that *the analytic complexity is still present* in the assignment of the resistances used.

This electrical analogue differs fundamentally from single-source models in that a finite surface resistance is used for both energy fluxes: indeed it is the essence of the multi-layer approach, embodied here in 'continuous' form, that the transfer of *all* properties is subject to this type of resistance. Further, it is to be anticipated that the effective resistances seen by other types of flux should be formally similar in definition to r_H, r_V, r_c', and, with this in mind, it is convenient at this stage to consider momentum flux.

To preserve the comparison, a screen height is chosen which is coincident with the top of the vegetation; the bulk resistance to the exchange of momentum between this height and the surface is then defined from the equation:

$$r_D = -\rho \frac{u_h}{\tau_h}. \tag{65}$$

(In this analysis the sign of τ is negative.) Integrating Equation (8) through the crop gives the result:

$$\tau_h - \tau_0 = -\rho \int_0^h \frac{u(z)}{\rho_M^{\mathrm{DIV}}} \, \mathrm{d}z.$$

It is easily seen (by direct substitution of this last equation) that the bulk exchange resistance, r_D, can be written in the form:

$$r_D = \frac{-\rho}{\tau_h} \frac{\int_0^h [u_h - u(z)][\rho_M^{\mathrm{DIV}}]^{-1} \, \mathrm{d}z}{\int_0^h [\rho_M^{\mathrm{DIV}}]^{-1} \, \mathrm{d}z} + \frac{(\tau_h - \tau_0)}{\tau_h} \frac{1}{\int_0^h [\rho_M^{\mathrm{DIV}}]^{-1} \, \mathrm{d}z}. \tag{66}$$

Figure 2 describes an electrical analogue model of the momentum exchange in a vegetation-atmosphere interaction which is identical to that for latent and sensible heat exchanges. According to this model, u_h is given by:

$$u_h = \frac{-\tau_h r_M}{\rho} - \frac{(\tau_h - \tau_0) R_M}{\rho}$$

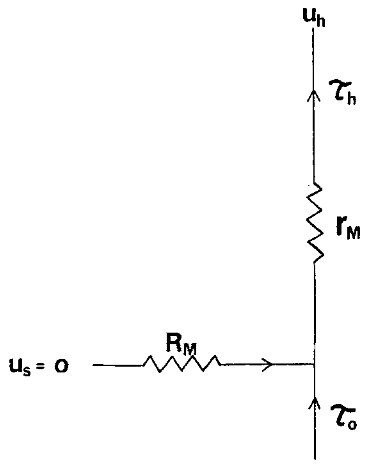

Fig. 2. Schematic diagram representing an electrical analogue of the exchange of momentum at the vegetation-atmosphere interface.

which substituted into (65) gives:

$$r_D = r_M + \frac{(\tau_h - \tau_0)}{\tau_h} R_M.$$

Comparing this with Equation (66) allows identification of 'surface' and 'aerodynamic' resistances to momentum fluxes, given by the expressions:

$$[R_M]^{-1} = \int_0^h [\rho_M^{\mathrm{DIV}}]^{-1}\, \mathrm{d}z \tag{67}$$

and

$$r_M = -\rho \frac{(\delta u)'}{\tau_h} \tag{68}$$

where

$$(\delta u)' = \frac{\int_0^h [u(h) - u(z)][\rho_M^{\mathrm{DIV}}]^{-1}\, \mathrm{d}z}{\int_0^h [\rho_M^{\mathrm{DIV}}]^{-1}\, \mathrm{d}z} \tag{69}$$

which are similar to Equations (42) to (47). Thus, momentum exchange can be treated like latent and sensible heat exchanges but the effective surface resistance is simpler: the treatment of energy fluxes is complicated by the extra inter-relationship of total energy conservation.

It is emphasised that subdividing the resistance to momentum flux in this way is not new in concept, e.g., see Thom (1971); what is new is the interpretation of the resulting fractions. Here, the 'uniform canopy wind speed', $\bar{u}$, is assigned the analytic form:

$$\bar{u} = u_h - (\delta u)' \tag{70}$$

6. Discussion

It has been shown that both 'single-source' and 'multi-layer' models of the vegetation-atmosphere interaction are similar in as much as both can be shown to yield combination equations with a broadly similar form, and that both can be regarded as equivalent to simple electrical analogues (although in the multi-layer case this simplicity is more apparent than actual).

There remain, however, important differences, firstly with respect to fluxes from the soil, and secondly in the way the aerodynamic 'boundary-layer' resistance is introduced into the model. The consequences of these differences determine the plausibility of the simple single-source assumption in a particular application. It is not of course possible to make general statements about the practical importance of these differences; this will depend on particular meteorological conditions and crop characteristics. However, *within the limits of the one-dimensional assumption*, it should be possible, by comparing (say) Equation (56) with its single-source counterpart (e.g., Stewart and Thom, 1973, Equation (6)), to determine whether the single-source hypothesis is valid in any particular case.

At the risk of denegrating the generality of this analysis, it is perhaps worthwhile considering this procedure in more detail. In this context the equation

$$r_{\mathrm{ST}} = \left(\frac{\Delta\beta}{\gamma} r_H^{\mathrm{ST}} - r_V^{\mathrm{ST}}\right) + (1+\beta) r_{\mathrm{I}} \tag{71}$$

introduced by Stewart and Thom (1973) is regarded as a *definition* of the 'bulk physiological resistance', r_{ST}, used in their work where r_H^{ST} and r_V^{ST} are the 'aerodynamic' resistances to the transfers of sensible heat and water vapour as defined in that analysis. The object is to estimate how reliable a measure r_{ST} is of r_{STOM}, the resistance of all the stomata acting in parallel, i.e., to test the hypothesis:

$$\frac{1}{r_{\mathrm{ST}}} = \frac{1}{r_{\mathrm{STOM}}} = \int_0^h \frac{L(z)}{r_{\mathrm{STO}}(z)}\,\mathrm{d}z. \tag{72}$$

To do this we combine Equations (56) and (71) in the form:

$$r_{\mathrm{ST}} = \left[\left(\frac{\lambda E_h - \lambda E_0}{\lambda E_h}\right) r_c\right] - \left[(1+\beta) r_c' I_1 \left(1 - \frac{A_0}{A'}\right)\right] + \left[\frac{\Delta\beta}{\gamma}(r_H^{\mathrm{ST}} - r_H) - (r_V^{\mathrm{ST}} - r_V)\right]. \tag{73}$$

To assist in understanding how the several terms of Equation (73) interact, it is perhaps instructive to place this equation in some actual numerical framework. The data given by Stewart and Thom (1973) are probably adequate for this purpose, especially since, in this case, the sizes of the terms allow us to make fairly arbitrary approximations and assumptions. For the purpose of this illustration, it is probably adequate to assume (on the basis of the figures presented in Table 1 of that paper) that

$$r_H^{\mathrm{ST}} = r_V^{\mathrm{ST}} = 6\ \mathrm{s\ m^{-1}}.$$

It is further assumed that r_H and r_V can be set equal to the 'eddy diffusive' part of these resistances, and (on the basis of the same Table) that the 'eddy diffusive' and 'boundary-layer' parts are of equal size ($3\ \mathrm{s\ m^{-1}}$). We make the additional assumption that the functions $p^V/\overline{uC^V}$ and $p^H/\overline{uC^H}$ are constant (independent of z) through the active canopy, and that they are equal to each other; while, for the purposes of illustration, we set $\Delta/\gamma = \Delta'/\gamma = 2$ and $\beta = 1.5$.

These several assumptions are equivalent to setting:

$$r_H = r_V = 3\ \mathrm{s\ m^{-1}}$$

$$r_c' = 2 \times 3 = 6\ \mathrm{s\ m^{-1}}$$

$$\rho'_{\mathrm{DIV}}(z) = \frac{6(\mathrm{LAI})}{L(z)}$$

and

$$\rho_{\mathrm{DIV}}(z) = \frac{1}{L(z)}\, r_{\mathrm{STO}}(z) + 9(\mathrm{LAI})$$

inside the active canopy, where (LAI) is the leaf area index.

Within these assumptions, the degree of equivalence between r_{ST} and r_{STOM} depends on the form of the functions $L(z)$, $r_{\mathrm{STO}}(z)$ and $a(z)$, and on the size of the ratios (A_0/A') and *particularly* $(\lambda E_h - \lambda E_o)/\lambda E_h$. For Thetford Forest in clear sky, mid-day radiation conditions, similar to those described by Stewart and Thom (1973) (viz., $A' = \lambda E_h + H_h \simeq 500-600\ \mathrm{W\ m^{-2}}$), the net radiation measured beneath the canopy, $R_N(0)$, is typically $50\ \mathrm{W\ m^{-2}}$; while the measured soil heat flux is typically $20\ \mathrm{W\ m^{-2}}$: these correspond to a value of $30\ \mathrm{W\ m^{-2}}$ for A_0. If (say) half of this energy appears as latent heat flux from the soil, then, bearing in mind that the measured latent heat flux above the canopy is typically $200\ \mathrm{W\ m^{-2}}$ in these conditions, it is possible to make the following estimates:

$$\left(\frac{\lambda E_h - \lambda E_0}{\lambda E_h}\right) = 0.925$$

and

$$\frac{A_0}{A'} = 0.06.$$

Variations in the particular form of $L(z)$, $r_{STO}(z)$ and $a(z)$ affect Equation (73) through r_c but their major effect is through the value of the integral I_1 whose value might be altered by a factor of two or three. However, *for the purposes of this illustration*, it is convenient to assume that there is no variation in stomatal resistance through the canopy. This simplifies the evaluation of the terms in Equation (73) considerably, and makes their values independent of the particular forms of $L(z)$ and $a(z)$. With this assumption

$$\rho_{DIV}(z) = \frac{(r_{STOM} + 9)(\text{LAI})}{L(z)}$$

so that

$$r_c = r_{STOM} + 9 \quad \text{s m}^{-1}$$

and

$$I_1 = 1.$$

Substituting all the values into Equation (73) yields:

$$r_{ST} = 0.925\, r_{STOM}.$$

The agreement between r_{ST} and r_{STOM} is therefore fairly good for the values of r_{ST} (in the region $100\ \text{s m}^{-1}$) presented in that paper.

7. Concluding Remarks

It is important to remember that the theoretical description of the vegetation-atmosphere interaction presented in this paper depends ultimately on the validity of a one-dimensional approach. The artificiality of this assumption gives rise to empirical 'shelter factors' and puts limits on the validity of the theoretical formalism produced. The analysis also depends on the assumed validity of the diffusion equation, which could also put limits on its applicability.

It is easily seen that, even in one dimension, the general description of the vegetation-atmosphere interaction is very complex, perhaps too complex for direct practical application in any predictive sense. This is of some importance since it implies that the *prediction* of evaporation might always have to rely on simplified and less precise treatments: the *precise determination* of evaporation is perhaps only possible (even as a long-term average) by direct measurement.

A large part of this paper has been devoted to demonstrating that the concept of a 'combination equation' has validity even in a multi-layer model of the vegetation interaction. The author sees danger in that this will add further to the stature of the 'combination equation' approach to evaporation '*measurement*', and foster the erroneous belief that the detailed measurement of meteorological variables, *specifically* for use as input to a combination equation (which *a priori* involves making assumptions about other parameters in the equation), represents a direct measurement of evaporation.

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Appendix 1. Formulation of Diversive Resistivity

The diversive resistivity is the parameter relating the divergence of a mean flux at a height z to the difference, at that height, between mean air conditions and average surface conditions. It controls the contribution made to the total fluxes by the portion of vegetation at height z. It is a function of a parameter called here the 'true surface resistance', and denoted by R_s^M, R_s^H, and R_s^V, for momentum, sensible heat and vapour, respectively. In general, the true surface resistance consists of two parts: firstly, the 'intrinsic surface resistance' (Shuttleworth, 1975); and secondly, the aerodynamic resistance seen by the flux in diffusing from the surface into the air stream at height z. This is more complicated for the vapour flux from a 'dry' canopy, because most of the surface responsible for that flux is separated from the visible surface of the vegetation by the additional aerodynamic resistance it sees in the stomatal fissure. It can of course be made to appear more simple by the introduction of the term 'stomatal resistance'.

(a) The flux from individual elements

Consider an individual vegetative element i of area a_i, surface temperature T_{si} and wetness factor w_i, this being the ratio of the area of the free water on the surface to the total surface area of the element. Consider first the resistance to vapour flux, and assume that the vegetative element has a 'local' surface resistance which is equal to the stomatal resistance r_i^{STO} over the 'dry' fraction of the element and equal to the intrinsic surface resistance to vapour flux R_I^V over the wet portion of the element. If these resistances are *assumed* to act in parallel, the effective resistance from the vapour source to the air in immediate contact with the vegetative element is given by:

$$\frac{1}{R_i^{V,S}}=\frac{w_i}{R_I^V}+\frac{(1-w_i)}{r_i^{STO}} \tag{A1.1}$$

or

$$R_i^{V,S}=\frac{r_i^{STO}}{1+w_i\mu_i(z)} \tag{A1.2}$$

where

$$\mu_i(z)=\frac{r_i^{\mathrm{STO}}-R_I^V}{R_I^V}. \tag{A1.3}$$

Obviously $R_i^{V,S}$ reduces to r_i^{STO} when the element is dry and to R_I^V when completely wet.

Similarly, but more simply, $R_i^{H,S}$ and $R_i^{M,S}$, the 'local' surface resistances to sensible heat flux and momentum flux, respectively, for element i are given by the intrinsic surface resistances to these parameters, that is, by:

$$R_i^{H,S}=R_I^H \tag{A1.4}$$

and

$$R_i^{M,S}=R_I^M. \tag{A1.5}$$

Once in the air, the fluxes diffuse into the air stream flowing over the particular element. In this context, the portion of flux originating on element i is regarded as being 'in the air stream', at level z, when its transfer begins to be primarily controlled by the eddy diffusion, at which point the effective vapour pressure, air temperature and wind speed are represented by e_i, T_i and u_i, respectively. In diffusing from the surface into the air stream, the fluxes see aerodynamic resistances $R_i^{V,A}$, $R_i^{H,A}$ and $R_i^{M,A}$ given by:

$$R_i^{V,A}=[u_iC_i^V(u_i,\phi_i)]^{-1} \tag{A1.6}$$

$$R_i^{H,A}=[u_iC_i^H(u_i,\phi_i)]^{-1} \tag{A1.7}$$

$$R_i^{M,A}=[u_iC_i^M(u_i,\phi_i)]^{-1} \tag{A1.8}$$

where C_i^V, C_i^H and C_i^M are the transfer coefficients for vapour, sensible heat and momentum for the element i exposed at an angle of incidence ϕ_i to a wind speed u_i. Hence the total resistances seen by the elemental fluxes of vapour, sensible heat and momentum between the internal origin on element i and the air stream at level z, R_{si}^V, R_{si}^H and R_{si}^M, respectively, are given by:

$$R_{si}^V=R_i^{V,S}+R_i^{V,A}$$

etc., that is by:

$$R_{si}^V=\frac{r_i^{\mathrm{STO}}}{1+w_i\mu_i}+[u_iC_i^V(u_i,\phi_i)]^{-1} \tag{A1.9}$$

$$R_{si}^H=R_I^H+[u_iC_i^H(u_i,\phi_i)]^{-1} \tag{A1.10}$$

$$R_{si}^M=R_I^M+[u_iC_i^M(u_i,\phi_i)]^{-1} \tag{A1.11}$$

while the total fluxes *leaving* the element i, f_i^V, f_i^H and f_i^M, are given by:

$$f_i^V = \frac{\rho c_p}{\gamma} a_i \left[\frac{e_w(T_{si}) - e_i}{R_{si}^V} \right] \tag{A1.12}$$

$$f_i^H = \rho c_p a_i \left[\frac{T_{si} - T_i}{R_{si}^H} \right] \tag{A1.13}$$

$$f_i^M = \rho a_i \left[\frac{-u_i}{R_{si}^M} \right]. \tag{A1.14}$$

(b) DEFINITIONS

At this stage it is convenient to define mean transfer coefficients $\overline{C^V}$, $\overline{C^H}$ and $\overline{C^M}$ at level z by the equations:

$$\overline{C^V(z)} = \underset{dz \to 0}{\text{Limit}} \left[\frac{\sum a_i C_i^V}{\sum a_i} \right] \tag{A1.15}$$

$$\overline{C^H(z)} = \underset{dz \to 0}{\text{Limit}} \left[\frac{\sum a_i C_i^H}{\sum a_i} \right] \tag{A1.16}$$

$$\overline{C^M(z)} = \underset{dz \to 0}{\text{Limit}} \left[\frac{\sum a_i C_i^M}{\sum a_i} \right] \tag{A1.17}$$

and the elemental area per unit height $L(z)$ by:

$$L(z) = \underset{dz \to 0}{\text{Limit}} \left[\frac{\sum a_i}{dz} \right] \tag{A1.18}$$

with all the summations extending to the elements i between the levels z and $(z + dz)$ and over unit area of crop. When the element is a leaf, and the plan area is used, then $L(z)$ is the Leaf Area Index *per unit height*. It is also convenient to define an average wetness factor $W(z)$ at level z, and an average stomatal resistance $r_{STO}(z)$ in a similar way by the equations:

$$W(z) = \underset{dz \to 0}{\text{Limit}} \left[\frac{\sum a_i w_i}{\sum a_i} \right] \tag{A1.19}$$

$$[r_{STO}(z)]^{-1} = \underset{dz \to o}{\text{Limit}} \left[\frac{\sum a_i / r_i^{STO}}{\sum a_i} \right] \tag{A1.20}$$

with the range of summation similarly restricted.

It is also convenient to introduce the 'elemental modifiers' γ_i^V, γ_i^H, γ_i^M, $\gamma_i^{V,C}$, $\gamma_i^{M,C}$ and γ_i^W which take account of the individuality of element i with respect to the mean parameters at level z. They are defined by the equations:

$$\left.\begin{aligned}(e_W(T_{si})-e_i)\gamma_i^V &= e_w(T_s(z))-e(z)\\ (T_{si}-T_i)\gamma_i^H &= T_s(z)-T(z)\\ (u_i)\gamma_i^M &= u(z)\\ C_i^V(u_i,\phi_i)\gamma_i^{V,C} &= \overline{C^V(z)}\\ C_i^H(u_i,\phi_i)\gamma_i^{H,C} &= \overline{C^H(z)}\\ C_i^M(u_i,\phi_i)\gamma_i^{M,C} &= \overline{C^M(z)}\\ \frac{r_i^{\mathrm{STO}}}{1+w_i\mu_i} &= \gamma_i^W\overline{R^{V,S}}\end{aligned}\right\} \tag{A1.21}$$

where

$$\overline{R^{V,S}}=\frac{r_{\mathrm{STO}}(z)}{1+W(z)\mu(z)} \tag{A1.22}$$

and

$$\mu(z)=\frac{r_{\mathrm{STO}}-R_I^V}{R_I^V}. \tag{A1.23}$$

(c) SHELTER FACTORS

Combining these definitions with Equations (A1.12), (A1.13) and (A1.14) gives the results:

$$f_i^V=\frac{\rho c_p}{\gamma}a_i\frac{e_w(T_s)-e}{\gamma_i^V[\gamma_i^W\overline{R^{V,S}}+\gamma_i^M\gamma^{V,C}\overline{R^{V,A}}]} \tag{A1.24}$$

$$f_i^H=c_pa_i\frac{T_s-T}{\gamma_i^H[R_I^H+\gamma_i^M\gamma_i^{H,C}\overline{R^{H,A}}]} \tag{A1.25}$$

$$f_i^M=\rho a_i\frac{-u}{\gamma_i^M[R_I^M+\gamma_i^M\gamma_i^{M,C}\overline{R^{M,A}}]} \tag{A1.26}$$

where

$$\overline{R^{V,A}}=[u(z)\overline{C^V(z)}]^{-1} \tag{A1.27}$$

$$\overline{R^{H,A}}=[u(z)\overline{C^H(z)}]^{-1} \tag{A1.28}$$

$$\overline{R^{M,A}}=[u(z)\overline{C^M(z)}]^{-1}. \tag{A1.29}$$

These equations can be made to appear more simple by the introduction of the 'product modifiers' p_i^V, p_i^H and p_i^M defined as:

$$p_i^V = \gamma_i^V \gamma_i^M \gamma_i^{V,C} \tag{A1.30}$$

$$p_i^H = \gamma_i^H \gamma_i^M \gamma_i^{H,C} \tag{A1.31}$$

$$p_i^M = \gamma_i^M \gamma_i^M \gamma_i^{M,C} \tag{A1.32}$$

and the 'hybrid modifier' ω_i defined from:

$$\omega_i = \gamma_i^V \gamma_i^W \tag{A1.33}$$

(bearing in mind that ω_i reduces to γ_i^V when the canopy is dry or completely wet). The resulting simplified equations are:

$$f_i^V = \frac{\rho c_p}{\gamma} a_i \frac{e_w(T_s) - e}{\omega_i \overline{\overline{R^{V,S}}} + p_i^V \overline{\overline{R^{V,A}}}} \tag{A1.34}$$

$$f_i^H = \rho c_p a_i \frac{T_s - T}{\gamma_i^H R_I^H + p_i^H \overline{\overline{R^{H,A}}}} \tag{A1.35}$$

$$f_i^M = \rho a_i \frac{-u}{\gamma_i^M R_I^M + p_i^M \overline{\overline{R^{M,A}}}}\,. \tag{A1.36}$$

Consider now a volume of crop of unit cross-sectional area between heights z and $z + dz$; and consider, for example, the momentum flux. There is a flux $\tau(z)$ flowing into the volume from below and a flux $\tau(z + dz)$ out above. The difference between these fluxes is the sum of the contributions f_i^M inside the volume, i.e.,

$$\begin{aligned} \tau(z + dz) - \tau(z) &= \sum_i f_i^M \\ &= -\rho u \sum_i \frac{a_i}{\gamma_i^M R_I^M + p_i^M \overline{\overline{R^{M,A}}}}\,. \end{aligned} \tag{A1.37}$$

It is convenient to define two height-dependent terms $\Gamma^M(z)$ and $P^M(z)$ such that:

$$\sum \frac{a_i}{\gamma_i^M R_I^M + p_i^M \overline{\overline{R^{M,A}}}} = \frac{\sum a_i}{\Gamma^M(z) R_I^M + P^M(z) \overline{\overline{R^{M,A}}}} \tag{A1.38}$$

which is an identity in the situation $\gamma_i^M = \gamma^M$, $p_i^M = P^M$ for all i, with equivalent terms $P^H(z)$ and $P^V(z)$ (corresponding to p_i^H and p_i^V), and $\Gamma^H(z)$ and $\Omega^V(z)$ (corresponding to γ_i^H and ω_i) for the sensible heat and vapour fluxes.

These functions have the effect of decoupling the individual contributions to the summation and replacing them by a fraction of the sum of an equal number of 'average' elements. They are identical in concept to the 'shelter factors' used by Thom (1971, 1972) in as much as they are (at worst) no more than a mathematical device in which to shroud the complexity of elemental interference by introducing apparent simplicity into the equations. Their conceptual similarity with the 'shelter factor' is emphasised by the fact that $P^M(z)$, $P^H(z)$ and $P^V(z)$

are identical to the shelter factors for momentum, sensible heat and vapour as used by Thom (1971, 1972) in the approximation $R_I^M = R_I^H = \overline{R^{S,V}} = 0$, apart from the fact that Thom makes the extra implicit assumption that $\gamma_i^M = 1$ (which is equivalent to setting $u_i = \bar{u}$) and that he also assumes the shelter factor is height independent. The term 'shelter factor' is adopted for the functions P^M, P^H and P^V in the following discussion.

The need to contain such essentially empirical factors is a result of using a model which is fundamentally based on horizontal homogeneity: their presence reflects the fact that this assumption is not perfect inside the crop. No doubt it would be possible in principle to evaluate the summation on the right-hand side of Equation (A1.38) explictly if the details of the canopy were known: it might even be possible to do this in practice, with a large digital computer. However, the value of the summation would be so particular in time and space as to be of trivial importance from a practical standpoint, and of no real interest theoretically, introducing no new understanding. These empirical factors are retained for this reason.

(d) Reduction to diversive resistivity

Making use of the definition:

$$\frac{\partial \tau}{\partial z} = \underset{dz \to 0}{\text{Limit}} \left[\frac{\tau(z + dz) - \tau(z)}{dz} \right] \tag{A1.39}$$

with Equations (A1.37) and (A1.38) gives:

$$\frac{\partial \tau}{\partial z} = \rho \frac{-u}{\Gamma^M(z) R_I^M + P^M(z) \overline{R^{M,A}}} \underset{dz \to 0}{\text{Limit}} \left[\frac{\sum a_i}{dz} \right]$$

which combined with Equation (A1.18) becomes:

$$\frac{\partial \tau}{\partial z} = \rho \frac{-uL(z)}{\Gamma^M(z) R_I^M + P^M(z) \overline{R^{M,A}}}. \tag{A1.40}$$

Equations describing the divergences of sensible and latent heat fluxes follow by direct analogy thus:

$$\frac{\partial H}{\partial z} = \rho c_p \frac{[T_s(z) - T(z)]L(z)}{\Gamma^H(z) R_I^H + P^H(z) \overline{R^{H,A}}} \tag{A1.41}$$

$$\frac{\partial (\lambda E)}{\partial z} = \left(\frac{\rho c_p}{\gamma} \right) \frac{[e_w(T_s(z)) - e(z)]L(z)}{\Omega(z) \overline{R^{V,S}} + P^V(z) \overline{R^{V,A}}}. \tag{A1.42}$$

Comparing these equations with Equations (8), (9), (10) gives immediately:

$$\rho_M^{\text{DIV}}(z) = \frac{\Gamma^M(z) R_I^M + P^M(z) \overline{R^{M,A}}}{L(z)} \tag{A1.43}$$

$$\rho_H^{\text{DIV}}(z) = \frac{\Gamma^H(z) R_I^H + P^H(z) \overline{R^{H,A}}}{L(z)} \tag{A1.44}$$

$$\rho_V^{\mathrm{DIV}}(z)=\frac{\Omega(z)\overline{R^{V,S}}+P^V(z)\overline{R^{V,A}}}{L(z)} \tag{A1.45}$$

which are the most general results.

Using the assumptions $R_I^M \ll R^{M,A}$ and $R_I^H \ll R^{H,A}$, which are usually valid, and incorporating (A1.27), and (A1.28), reduces Equations (A1.43) and (A1.44) to the more useful form:

$$\rho_M^{\mathrm{DIV}}(z)=\frac{P^M(z)}{u(z)\overline{C^M(z)}L(z)} \tag{A1.46}$$

and

$$\rho_H^{\mathrm{DIV}}(z)=\frac{P^H(z)}{u(z)\overline{C^H(z)}L(z)}. \tag{A1.47}$$

It is of course possible to write a simplified equation for the diversive resistivity for vapour flux with the assumption $\overline{R^{V,S}} \ll \overline{R^{V,A}}$ thus:

$$\rho_V^{\mathrm{DIV}}(z)=\frac{P^V(z)}{u(z)C^V(z)L(z)} \tag{A1.48}$$

but it must be remembered that this can *only* be used when the canopy is completely wet. If the crop is not completely wet, and if it is assumed that over a horizontal plane inside the crop the fractional variation in vapour-pressure deficit and stomatal resistance is small, and further that the crop is equally 'wet' throughout, then $\omega_i=\gamma_i^V\gamma_i^W$ can be set equal to one for all i. Further making the assumption that all the elements in the crop are aerodynamically identical, gives the approximate result:

$$\rho_V^{\mathrm{DIV}}(z)=\frac{1}{L(z)}\left[\frac{r_{\mathrm{STO}}(z)}{1+W(z)\mu(z)}+\frac{P^V(z)}{u(z)\overline{C^V(z)}}\right] \tag{A1.49}$$

where

$$\mu(z)=\frac{r_{\mathrm{STO}}-R_I^V}{R_I^V} \tag{A1.50}$$

and where $W(z)$ is the average Wetness Factor at height z. When the crop is completely dry, $W(z)=0$ and (A1.49) reduces to:

$$\rho_V^{\mathrm{DIV}}(z)=\frac{1}{L(z)}\left[r_{\mathrm{STO}}(z)+\frac{P^V(z)}{u(z)\overline{C^V(z)}}\right]. \tag{A1.51}$$

Appendix 2. The Derivation of a Generalized Combination Equation

Most of the concepts are introduced and assigned mathematical formalism elsewhere but before proceeding, it is convenient to introduce the identities:

$$e_w(T(h)) - e_w(T(z)) = \Delta[T(h) - T(z)] \tag{A2.1}$$

and

$$e_w(T_s(z)) - e_W(T(z)) = \Delta'[T_s(z) - T(z)] \tag{A2.2}$$

in which h is the level of the top of the vegetation, where the temperature and vapour pressure are $T(h)$ and $e(h)$, respectively, and Δ and Δ' are the *mean* slopes of the saturated vapour-pressure curve between the temperature $T(z)$ and $T(h)$, and $T_s(z)$ and $T(z)$, respectively.

Rewriting Equation (4) as:

$$\frac{\partial e}{\partial z} = -\frac{\gamma}{\rho c_p} \frac{\lambda E(z)}{K_V(z)}$$

and integrating from z to h gives the result:

$$e(z) = e(h) + \frac{\gamma}{\rho c_p} \int_z^h \frac{\lambda E(z')}{K_V(z')} \mathrm{d}z' \tag{A2.3}$$

while rewriting and integrating Equation (3) gives:

$$T(h) - T(z) = \frac{-1}{\rho c_p} \int_z^h \frac{H(z')}{K_H(z')} \mathrm{d}z' \tag{A2.4}$$

which combined with (A2.1) gives:

$$e_w(T(h)) - e_w(T(z)) = -\frac{\Delta}{\rho c_p} \int_z^h \frac{H(z')}{K_H(z')} \mathrm{d}z'. \tag{A2.5}$$

Taking the identity:

$$\begin{aligned} e_w(T_s(z)) - e(z) = {} & [e_w(T_s(z)) - e_w(T(z))] - [e_w(T(h)) - e_w(T(z))] \\ & - [e(z) - e_w(T(h))] \end{aligned}$$

and introducing (A2.2), (A2.5) and (A2.3) gives:

$$e_w(T_s(z)) - e(z) = \Delta'[T_s(z) - T(z)] + D + \frac{\Delta}{\rho c_p}\int_z^h \frac{H(z')}{K_H(z')}\,dz' - \frac{\gamma}{\rho c_p}\int_z^h \frac{\lambda E(z')}{K_V(z')}\,dz' \quad \text{(A2.6)}$$

where D is the vapour-pressure deficit at the top of the vegetation, i.e.,

$$D = e_w(T(h)) - e(h).$$

Combining Equations (9) and (31) gives the result:

$$T_s(z) - T(z) = \frac{\rho_H^{\mathrm{DIV}}(z)}{\rho c_p}\left[a(z) - \frac{\partial(\lambda E)}{\partial z}\right] \quad \text{(A2.7)}$$

while (10) can be rewritten in the form:

$$e_w(T_s(z)) - e(z) = \frac{\rho_V^{\mathrm{DIV}}(z)}{\rho c_p}\,\gamma\,\frac{\partial(\lambda E)}{\partial z}. \quad \text{(A2.8)}$$

After some mathematical manipulation, introducing Equations (A2.7) and (A2.8) into Equation (A2.6), yields the result:

$$\rho_{\mathrm{DIV}}\frac{\partial(\lambda E)}{\partial z} = [\rho'_{\mathrm{DIV}}a(z)] + \frac{\rho c_p D}{\gamma} + \frac{\Delta}{\gamma}\int_z^h \frac{H(z')}{K_H(z')}\,dz' - \int_z^h \frac{\lambda E(z')}{K_V(z')}\,dz' \quad \text{(A2.9)}$$

where

$$\rho'_{\mathrm{DIV}} = \frac{\Delta'}{\gamma}\rho_H^{\mathrm{DIV}} \quad \text{(A2.10)}$$

and

$$\rho_{\mathrm{DIV}} = \rho_V^{\mathrm{DIV}} + \rho'_{\mathrm{DIV}}. \quad \text{(A2.11)}$$

Dividing Equation (A2.9) by ρ_{DIV} and integrating from ground level through the crop gives the result:

$$\lambda E_h - \lambda E_0 = \int_0^h \frac{\rho'_{\mathrm{DIV}}a(z)}{\rho_{\mathrm{DIV}}}\,dz + \frac{\rho c_p D}{\gamma}\int_0^h \frac{dz}{\rho_{\mathrm{DIV}}} + \frac{\Delta}{\gamma}\int_0^h \frac{dz}{\rho_{\mathrm{DIV}}}\int_z^h \frac{H(z')}{K_H(z')}\,dz' - \int_0^h \frac{dz}{\rho_{\mathrm{DIV}}}\int_z^h \frac{\lambda E(z')}{K_V(z')}\,dz'. \quad \text{(A2.12)}$$

It is convenient to define the parameters r'_c, r_c, r_H and r_V from the equations:

$$r'_c = \left[\int_0^h \frac{\mathrm{d}z}{\rho'_{\mathrm{DIV}}}\right]^{-1} \tag{A2.13}$$

$$r_c = \left[\int_0^h \frac{\mathrm{d}z}{\rho_{\mathrm{DIV}}}\right]^{-1} \tag{A2.14}$$

$$r'_H = \int_0^h \frac{\mathrm{d}z}{K_H} \tag{A2.15}$$

$$r'_V = \int_0^h \frac{\mathrm{d}z}{K_V} \tag{A2.16}$$

and to introduce $\hat{\rho}'_{\mathrm{DIV}}$, $\hat{\rho}_{\mathrm{DIV}}$, k_V, k_H and $\hat{a}$, normalized versions of the parameters ρ'_{DIV}, ρ_{DIV}, K_V, K_H and a, which are defined by:

$$\hat{\rho}'_{\mathrm{DIV}} = \frac{\rho'_{\mathrm{DIV}}}{r'_c} \tag{A2.17}$$

$$\hat{\rho}_{\mathrm{DIV}} = \frac{\rho_{\mathrm{DIV}}}{r_c} \tag{A2.18}$$

$$k_V = r'_V K_V \tag{A2.19}$$

$$k_H = r'_H K_H \tag{A2.20}$$

$$\hat{a} = \frac{a}{A}. \tag{A2.21}$$

It is also convenient to introduce $\hat{H}(z)$ and $\hat{E}(z)$, renormalized versions of $H(z)$ and $E(z)$, defined by:

$$\hat{H}(z) = \frac{H(z)}{H_h} \tag{A2.22}$$

$$\hat{E}(z) = \frac{E(z)}{E_h}. \tag{A2.23}$$

Introducing Equations (A2.17) to (A2.23) into Equation (A2.12) produces the result:

$$\lambda E_h - \lambda E_0 = \frac{r'_c}{r_c} A I_1 + \frac{\rho c_p D}{\gamma r_c} + \frac{\Delta}{\gamma} \frac{r'_H}{r_c} H_h I_2 - \frac{r'_V}{r_c} \lambda E_h I_3 \tag{A2.24}$$

in which I_1, I_2 and I_3 are defined by:

$$I_1 = \int_0^h \frac{\hat{\rho}'_{\mathrm{DIV}}\hat{a}}{\hat{\rho}_{\mathrm{DIV}}} \mathrm{d}z \tag{A2.25}$$

$$I_2 = \int_0^h \frac{\mathrm{d}z}{\hat{\rho}_{\mathrm{DIV}}} \int_z^h \frac{\hat{H}}{k_H} \mathrm{d}z' \tag{A2.26}$$

$$I_3 = \int_0^h \frac{\mathrm{d}z}{\hat{\rho}_{\mathrm{DIV}}} \int_z^h \frac{\hat{E}}{k_V} \mathrm{d}z'. \tag{A2.27}$$

Introducing Equations (34) and (36) into (A2.24) gives:

$$\lambda E_h - \lambda E_0 = \frac{r'_c}{r_c}(A' - A_0)I_1 + \frac{\rho c_p D}{\gamma r_c} + \frac{\Delta}{\gamma}\frac{r'_H}{r_c} I_2(A' - \lambda E_h) - \frac{r'_V}{r_c}\lambda E_h I_3 \tag{A2.28}$$

where

$$A_0 = H_0 + \lambda E_0. \tag{A2.29}$$

Rearranging (A2.28) gives:

$$\lambda E_h = \frac{\Delta A' + (\rho c_p D + \delta)/r_H}{\Delta + \gamma_E(1 + r_c/r_V)} \tag{A2.30}$$

where

$$\delta = \gamma[r'_c I_1(A' - A_0) + r_c \lambda E_0] \tag{A2.31}$$

$$\gamma_E = \left[\frac{r_V}{r_H}\right]\gamma \tag{A2.32}$$

$$r_H = I_2 r'_H \tag{A2.33}$$

$$r_V = I_3 r'_V. \tag{A2.34}$$

These later definitions of r_H and r_V are not particularly convenient: it is fairly easily shown, by combining Equations (A2.18), (A2.20), (A2.22), (A2.26) and (A2.33) and then introducing Equation (3), that r_H has the alternative form:

$$r_H = -\rho c_p \frac{(\delta T)'}{H_h} \tag{A2.35}$$

where

$$(\delta T)' = \frac{\int_0^h [T(h) - T(z)][\rho_{\mathrm{DIV}}]^{-1}\, \mathrm{d}z}{\int_0^h [\rho_{\mathrm{DIV}}]^{-1}\, \mathrm{d}z} . \tag{A2.36}$$

Combining the equivalent equations for vapour flux yields:

$$r_V = -\frac{\rho c_p}{\gamma} \frac{(\delta e)'}{\lambda E_h} \tag{A2.37}$$

where

$$(\delta e)' = \frac{\int_0^h [e(h) - e(z)][\rho_{\mathrm{DIV}}]^{-1}}{\int_0^h [\rho_{\mathrm{DIV}}]^{-1}\, \mathrm{d}z} \; \mathrm{d}z. \tag{A2.38}$$

It is perhaps worth noting that it is possible (by combining Equations (A2.2), (A2.7) and (A2.8)) to obtain an equation similar to Equation (A2.30) which applies at each level in the canopy, namely:

$$\frac{\partial(\lambda E)}{\partial z} = \frac{\Delta' \dfrac{\partial A}{\partial z} + \dfrac{\rho c_p D(z)}{\rho_H^{\mathrm{DIV}}(z)}}{\Delta' + \gamma_{\mathrm{E}}^* \left[1 + \dfrac{r_{\mathrm{STOM}}}{\bar{r}_V}\right]} \tag{A2.39}$$

where

$$D(z) = e_w(T(z)) - e(z)$$

$$\Delta' = [e_w(T_s(z)) - e_w(T(z))]/[T_s(z) - T(z)]$$

$$\gamma_{\mathrm{E}}^* = \frac{\bar{r}_V}{\bar{r}_H}$$

$$\bar{r}_V = \frac{P^V(z)}{C^V(z)}$$

$$\bar{r}_H = \frac{P^H(z)}{C^H(z)}$$

which equation is very similar to that applicable to a single leaf.

Appendix 3. 'Screen Height' Changes in the Generalized Combination Equation

It is necessary to know how Equation (38) is changed when the 'screen height' is changed from h, the top of the vegetation, to a new (higher) level, z_R. It is assumed that there is no flux divergence between the levels h and z_R, i.e.,

$$\left.\begin{aligned}\lambda E(z) &= \lambda E_h = \lambda E \\ H(z) &= H_h = H\end{aligned}\right\} \text{ for } \quad h \leqslant z \leqslant z_R.$$

The diffusion equations, Equations (3) and (4), yield after rearrangement and integration:

$$e(z_R) - e(h) = -\lambda E \frac{\gamma}{\rho c_p} r_{AV} \tag{A3.1}$$

and

$$T(z_R) - T(h) = -H \frac{1}{\rho c_p} r_{AH} = -\frac{(A' - \lambda E)}{\rho c_p} \tag{A3.2}$$

where:

$$r_{AV} = \int_h^{z_R} K_V^{-1}\,dz \tag{A3.3}$$

and

$$r_{AH} = \int_h^{z_R} K_H^{-1}\,dz. \tag{A3.4}$$

Introducing the identity,

$$e_w(T(z_R)) - e_w(T(h)) = \Delta'''[T(z_R) - T(h)]$$

where Δ''' is the mean gradient of the saturated vapour-pressure curve between $T(z_R)$ and $T(h)$, and combining this identity with Equations (A3.2) and (A3.1) yields:

$$\rho c_p D = \rho c_p D' + \Delta''' A' r_{AH} - \lambda E[\Delta''' r_{AH} - \gamma r_{AV}]$$

which, when introduced into (38), yields, after some manipulation,

$$\lambda E = \frac{\Delta'' A' + \dfrac{(\rho c_p D' + \delta)}{r''_H}}{\Delta'' + \gamma''_E\left[1 + \dfrac{r_c}{r''_V}\right]} \tag{A3.5}$$

where

$$\Delta'' = \frac{\Delta r_H + \Delta''' r_{AH}}{r''_H} \tag{A3.6}$$

$$r''_H = r_{AH} + r_H \tag{A3.7}$$

$$r''_V = r_{AV} + r_V \tag{A3.8}$$

$$\gamma''_E = \left[\frac{r''_V}{r''_H}\right]\gamma. \tag{A3.9}$$

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