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Thermal Radiation from the Atmosphere[1]

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A theoretical analysis of atmospheric thermal radiation reveals that previous formulas relating this parameter to screen-level air temperature have lacked universal applicability. New considerations indicate that the effective emittance of the atmosphere is a minimum at 273°K and that it increases symmetrically to approach unity exponentially at higher and lower temperatures. A formula is developed that meets these standards and fits experimental data from Alaska, Arizona, Australia, and the Indian Ocean with a correlation coefficient of 0.992. The atmospheric radiation R integrated over all wavelengths, is specified solely in terms of the screen-level air temperature T as $R = \sigma T^4\{1 - c \exp [-d (273 - T)^2]\}$, where c and d are constants having values of 0.261 and 7.77×10^{-4}, respectively. It appears that the formula may be valid at all latitudes and seasons.

Introduction

The thermal radiation of the earth's atmosphere originates chiefly with the three major components water vapor, carbon dioxide, and ozone. A coarse emittance spectrum for these three gases as constituted in the atmosphere is shown in Figure 1. Because of the sharp breaks or separations between the various bands, it is not feasible to seek an analytic description of radiation emitted therefrom at terrestrial temperatures. Instead, we must resort to empirical relations.

Two of the earliest and most widely used relations of this type were derived by *Brunt* [1932] and *Ångström* [1918, 1936]. Each of them related atmospheric thermal radiation R to screen-level values of air temperature T and water vapor pressure e. Brunt's equation took the form

$$R = \sigma T^4(a + b \sqrt{e}) \tag{1}$$

and Ångström's

$$R = \sigma T^4(\alpha - \beta 10^{-\gamma e}) \tag{2}$$

where σ is the Stefan-Boltzmann constant and a, b, α, β, and γ are empirical coefficients.

Although equations 1 and 2 give fairly good regressions for most localities, the five empirical coefficients are somewhat variable. In an attempt to overcome this local specificity, *Swinbank* [1963] undertook a reevaluation of the subject. He first established a relation between R and σT^4 alone, exclusive of e, from certain measurements made at Benson in the United Kingdom. Then, from observations in Australia (Aspendale and Kerang) and in the Indian Ocean, he related R to the sixth power of T:

$$R = 5.31 \times 10^{-14} T^6 \tag{3}$$

Since the regression of R on T^6 extrapolated through the origin, and since the slope of the Benson data thus analyzed varied from that of (3) by only 0.10×10^{-14}, it appeared that (3) was a rather general formula and that it may provide the universality lacked by (1) and (2).

Indeed, equation 3 has been acknowledged to be a significant theoretical improvement over the relations of Brunt and Ångström. For one thing, if the effective emittance of the atmosphere is defined as

$$\epsilon = R/\sigma T^4 \tag{4}$$

as was done by *Swinbank* [1963], it may be seen that the Brunt (1) and Ångström (2) equations provide for no temperature dependence of ϵ, except through that inherent in e. As Swinbank noted, however, such could be the case only for an atmosphere of constant grayness—quite different from the real atmosphere depicted in Figure 1.

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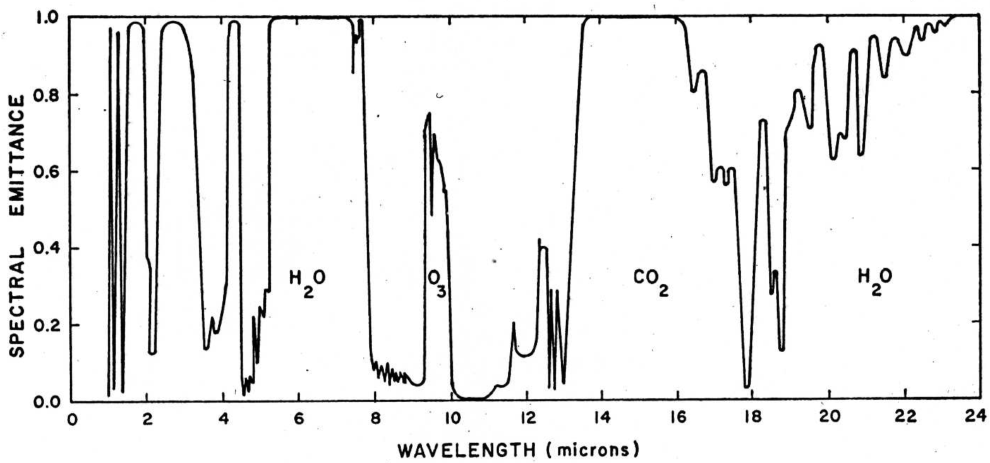

Fig. 1. The emission spectrum of the atmosphere at about 0°C. Adapted from *Gates* [1965] and *Specht* [1967].

Although Swinbank did not give a theoretical justification for the T^2 dependence of ϵ implied by (3), this has since come from *Gates* [1965]. He has stated that the reason the power of T in (3) is greater than 4 arises from the strength of the 6.3-μ water-vapor band. Stating that since the monochromatic emission of radiation varies with a higher power of T than 4 for wavelengths shorter than the modal and with a lower power for wavelengths longer than the modal, he has said that the strong water-vapor band at 6.3 μ, below the nominal mode of 10 μ for 300°K blackbody radiation, causes the whole spectrum to radiate at an average power of T greater than 4.

This paper presents theoretical considerations that demonstrate that the effective emittance of the atmosphere is temperature-dependent, but not in the manner implied by the relation of Swinbank nor by the reasoning of Gates. An equation is developed from relations among emittance, path length of precipitable water, saturation vapor pressure, and air temperature which describes atmospheric thermal radiation over the temperature range of $-45°$ to $+45°$C. This relation appears to be very general, since it accurately describes experimental data from such diverse locations as Alaska, Arizona, Australia, and the Indian Ocean.

Problems Associated with the Swinbank-Gates Analysis

Starting with the proposition of *Swinbank* [1963] that the effective emittance of the atmosphere should be temperature-dependent, we can immediately raise the question: Why should it be such as to result in a T^6 dependence for R? If the original data of Swinbank from Aspendale, Kerang, and the Indian Ocean are plotted as functions of T^N, where N varies from 1 to 10, for instance, the linear regression coefficients obtained are 0.986, 0.988, 0.988, 0.988, 0.988, 0.987, 0.987, 0.986, 0.986, and 0.985, respectively. Which plot is best? There is only one basis for deciding at this stage, and that is to compare the intercepts of each when $T = 0$; for theoretically there should be no radiation at that temperature. When this is done, it indeed is seen that the T^6 plot is the best. Swinbank's data do not, however, cover a large enough temperature range to make this conclusion of universal validity. This fact is well demonstrated by Figure 2. There a linear regression between R and T^4 is plotted for Swinbank's data (labeled 4). The other lines, labeled according to powers of T, are derived from this line by obtaining T for various values of R and forming the appropriate powers. The interpretation to be given them is that, if line 4 truly describes the relation between R and T, actual data plotted as another power should follow the appropriate derived line.

The two horizontal dashed lines of Figure 2 enclose the range of radiation values of Swinbank's data. Also, his actual data are plotted as a function of T^{16}. It is immediately obvious that there is no basis for choosing a T^6 relation over a T^4 relation, since in the range of available data the straight line (broken) implying a T^{16} relation and the curve implying a T^4 relation are practically coincident. The same could be said of the other lines also.

At this point, then, the problem is still unsettled. It is necessary to consider the comments of *Gates* [1965] to see if they may help in resolving the issue. To test his prediction that R should be described by a higher power of T than 4, we used the emission spectrum of Figure 1 and the Planck radiation law to calculate R and σT^4 as a function of T. These calculations were made numerically on a computer by dividing the radiation spectrum into 0.1-μ intervals and by considering the emittance of the atmosphere to be unity beyond 24 μ.

The results of our calculations are shown in Figure 3. The solid line covers a portion of the region of our calculations (223° to 353°K) and is essentially a straight line, implying a T^4 relation for R in this region. Since extending it to zero blackbody radiation yields a positive intercept, however, a concave upward curve must describe the relation for some distance

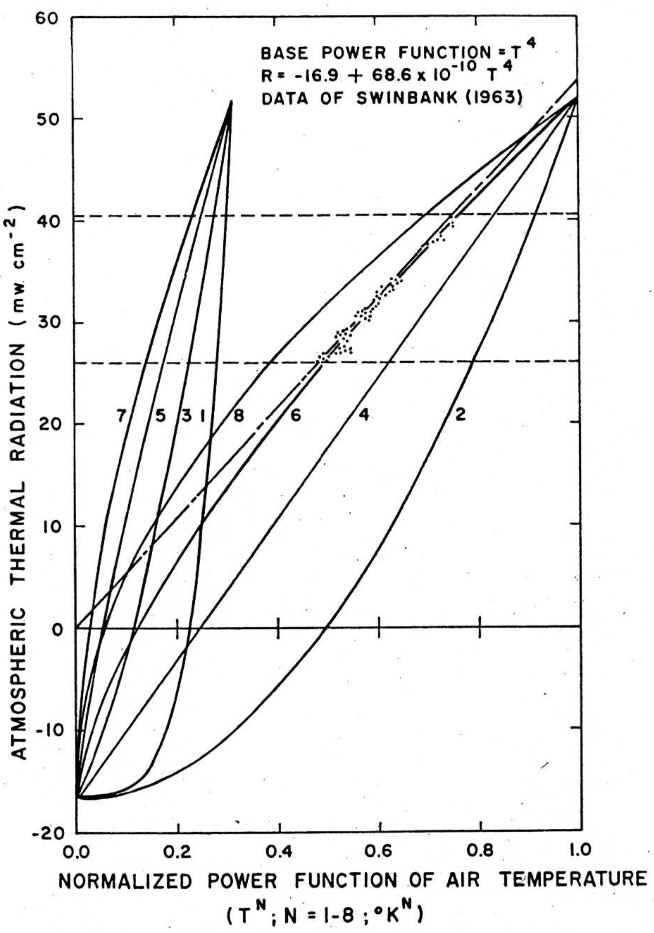

Fig. 2. Atmospheric thermal radiation as a function of eight different powers of air temperature at screen height, based on a T^4 relation published by *Swinbank* [1963] and including his data plotted as a function of T^6 with a linear relation in T^6 superimposed.

above 0°K. This in turn implies that for some very low temperature range R varies with a power of T less than 4. Thus, there appears to be no theoretical justification for the power of T being greater than 4 for any air temperature obtainable on earth, at least for the reasons tested.

New Theoretical Considerations

If we now consider the emittance spectrum of Figure 1 to represent the minimum water-vapor contribution of the atmosphere, since *Gates* [1965] has applied it at a temperature of 263°K, we may add a line to the plot of Figure 3 representative of the atmosphere as a blackbody and expect the true atmospheric radiation to lie somewhere between these two lines. This has been done in Figure 4, where the data of *Swinbank* [1963] are also included. As postulated, these data do fall between the two lines. Furthermore, it is evident that, although these data may be well described by a linear regression equation, this relation cannot be very general, for the plot intersects both of our upper and lower theoretical limits. Indeed, the relation is so far from being universal that for temperatures below about 223°K it predicts negative values of atmospheric radiation.

Now, a true description of atmospheric thermal radiation must not allow the upper and lower theoretical limits to be reached. There should be no problem thereby associated with the upper limit, since there is no way by which blackbody radiation may be exceeded. As water vapor is removed from the air at lower temperatures, however, the lower limit could be crossed. Normally, the water vapor in the air would change to a crystalline structure before this occurred. Since *Dorsey* [1940] has reported for radiation greater than 1 μ in wavelength, 'ice and snow will radiate nearly as an ideal blackbody radiator,' it may thus be postulated that as air temperature decreases below 273°K atmospheric radiation will again tend toward the blackbody curve.

The question thus naturally posed next is: What is the form of these variations? Since water vapor has been studied more than ice crystals in this regard, we will deal first with it. *Robinson* [1947] established a linear relation between the effective emittance of the atmos-

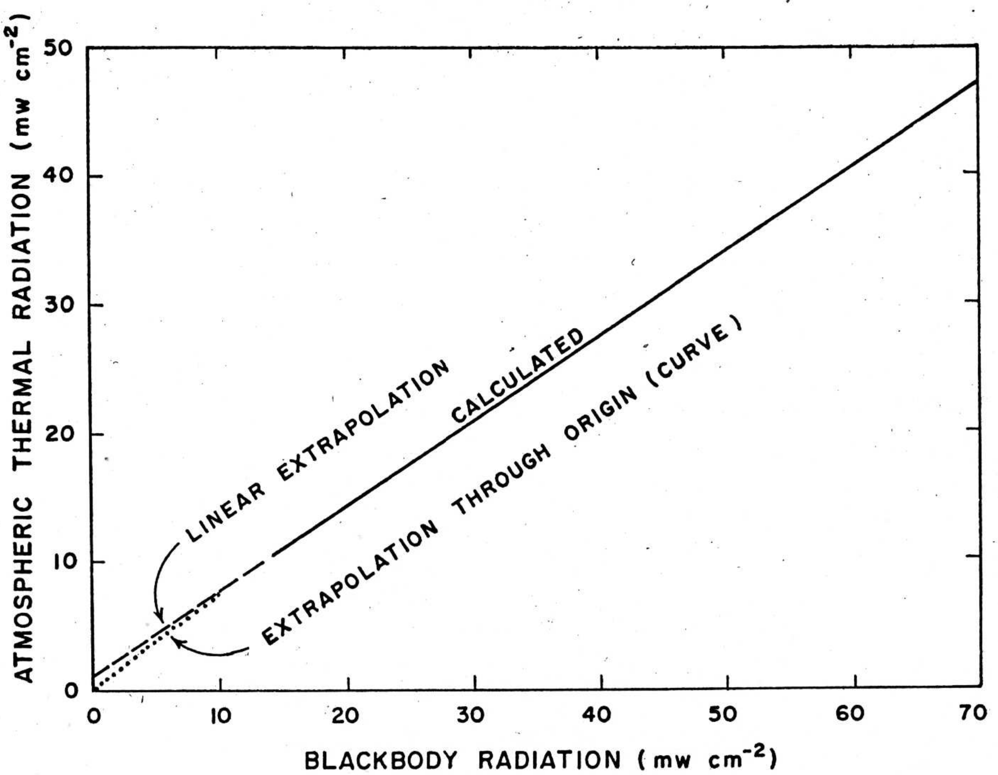

Fig. 3. Minimum atmospheric thermal radiation, calculated by means of the Planck equation in consideration of the emission spectrum of Figure 1, plotted as a function of blackbody radiation, with two different extrapolations to 0°K.

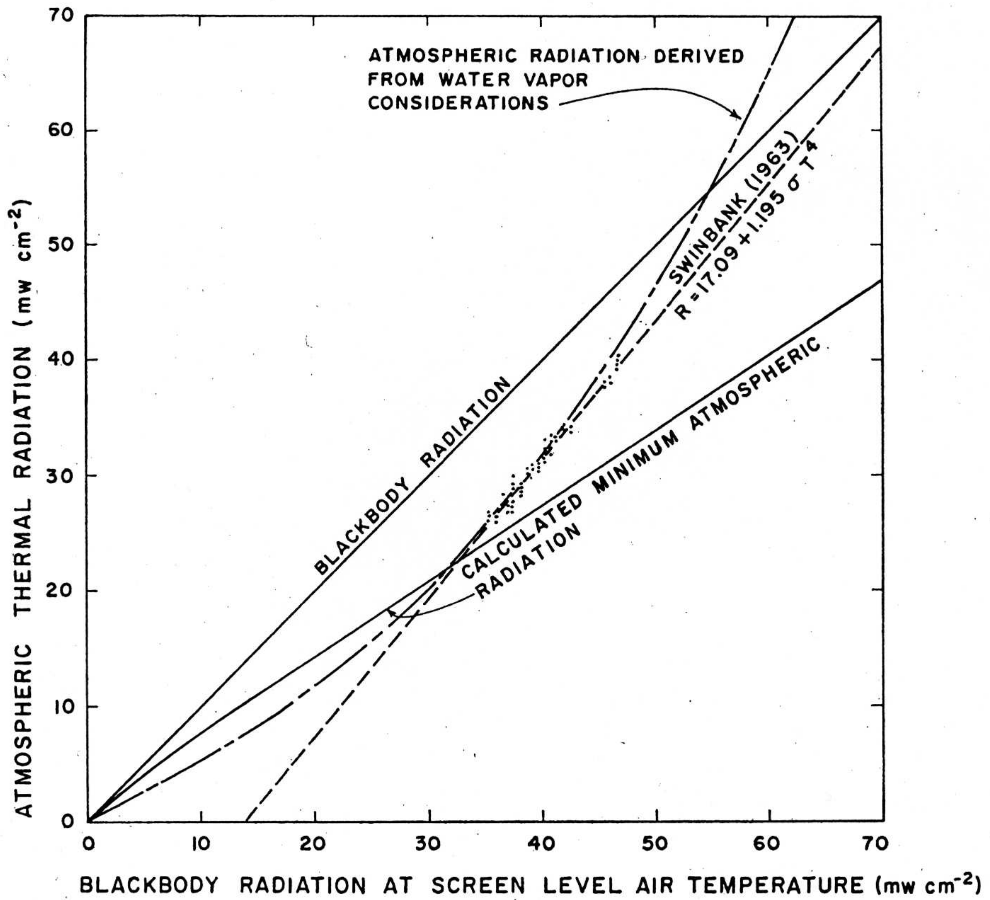

Fig. 4. Blackbody radiation, the radiation of Figure 3, *Swinbank*'s [1963] data, and atmospheric thermal radiation calculated from water-vapor variations all plotted as functions of blackbody radiation.

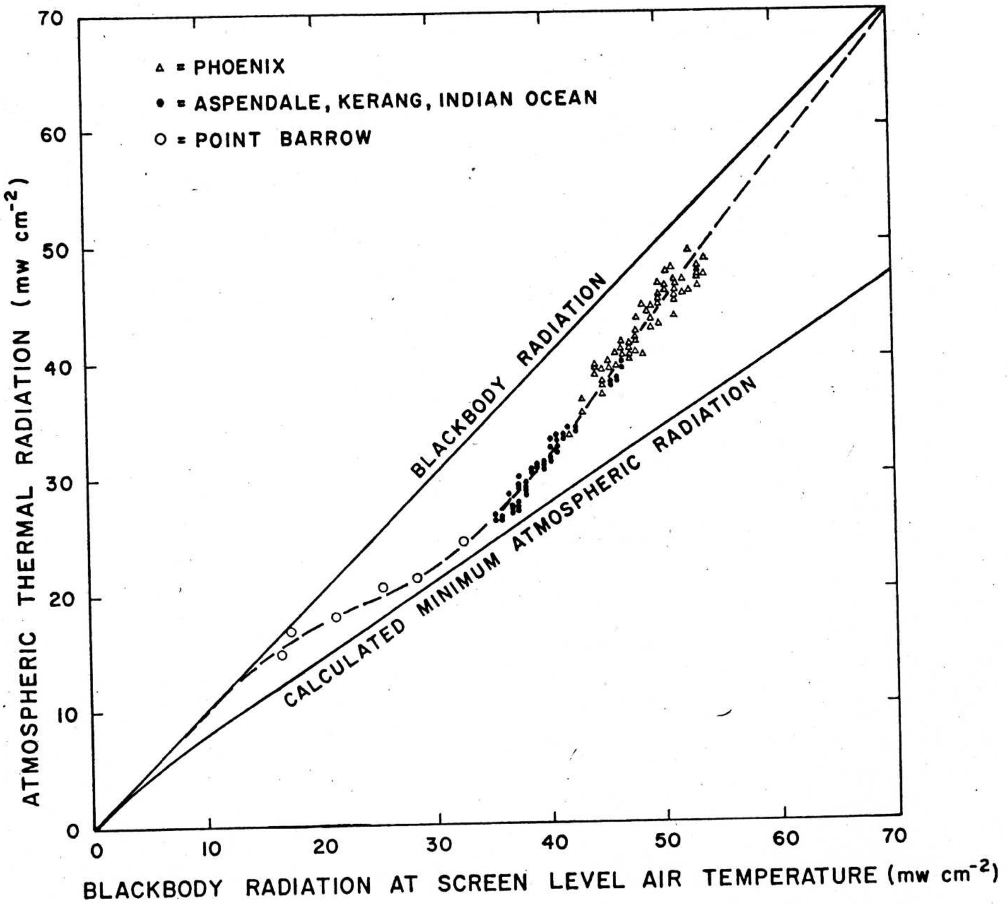

Fig. 5. Blackbody radiation, minimum atmospheric thermal radiation, and equation (12) all plotted as functions of blackbody radiation with data from Alaska and Arizona added to data of *Swinbank* [1963].

phere and the logarithm of the optical path length of precipitable water (m). *Monteith* [1961] determined that this relation at Kew could be expressed as

$$\epsilon = 0.704 + 0.220 \log_{10} m \tag{5}$$

He also determined the average year-round relation between m and e at Kew to be

$$\log m = 0.295 \sqrt{e} - 0.803 \tag{6}$$

whereupon ϵ became expressible as

$$\epsilon = 0.527 + 0.065 \sqrt{e} \tag{7}$$

To get ϵ in terms of T, we can utilize the Osborne-Meyers equation quoted by *Dorsey* [1940], if the actual vapor pressure is assumed to vary with temperature in a manner similar to saturation vapor pressure. Considering the first two dominating terms, we have

$$\ln e = A + (B/T) \tag{8}$$

from which it may be derived that

$$e = e_0 \exp \left[B\left(\frac{T_0 - T}{TT_0}\right)\right] \tag{9}$$

Since the constant B in (9) is a negative 4618, we may rewrite (9) giving e in millibars as

$$e = 6.11 \exp \left[16.9 \left(1 - \frac{273}{T}\right)\right] \tag{10}$$

Substitution of this expression for e into (7) then gives

$$\epsilon = 0.527 + 0.161 \exp \left[8.45\left(1 - \frac{273}{T}\right)\right] \tag{11}$$

Thus, the effective emittance of the atmosphere above 273°K could be expected to vary as described by (11). When R determined from this ϵ is plotted as a function of σT^4 (Figure 4), we see that, although it passes through the midst of Swinbank's data, it departs from Swinbank's relation at higher temperatures and crosses the blackbody curve; it also crosses our theoretical lower limit at 273°K.

In light of these considerations we could make the following postulations. Immediately above 273°K atmospheric thermal radiation may be described by an exponential function of temperature. Assuming as a first approximation that the variation of ϵ about 273°K is symmetrical, we could write

$$R = \sigma T^4 \{1 - c \exp[-d(273 - T)^2]\} \quad (12)$$

where c and d are constants. Squaring the T term in the exponential assures the symmetry, and subtracting the exponential from 1 relates it to the maximum blackbody radiation. It is also obvious that (12) keeps R from crossing over the blackbody curve at high as well as low temperatures. Thus, equation 12 meets all the theoretical limitations so far imposed on atmospheric thermal radiation. The next objective is to test the relation with a greater expanse of data.

New Experimental Data and Their Analysis

Figure 5 is a replica of Figure 4 with added data obtained from two other sources. The low-temperature data were obtained by *Lieske and Stroschein* [1968] near Point Barrow, Alaska; the high-temperature data were obtained from records of the U.S. Water Conservation Laboratory at Phoenix, Arizona. Although only six low-temperature points are shown, they were actually determined from 830 half-hourly means and are thus representative of atmospheric radiation at their respective air temperatures.

Lieske and Stroschein's data were acquired with a C.S.I.R.O. *Funk* [1959] radiometer[2] of the same type used by *Swinbank* [1963]. The Phoenix data were obtained, however, from a net radiometer of the *Fritschen* [1963] variety by adding nighttime ground emission calculated from surface temperature measurements to the net radiation. To get data at very high air

[2] Trade names and company names are included for the benefit of the reader only and do not imply an endorsement or preferential treatment of the product listed by the U.S. Department of Agriculture.

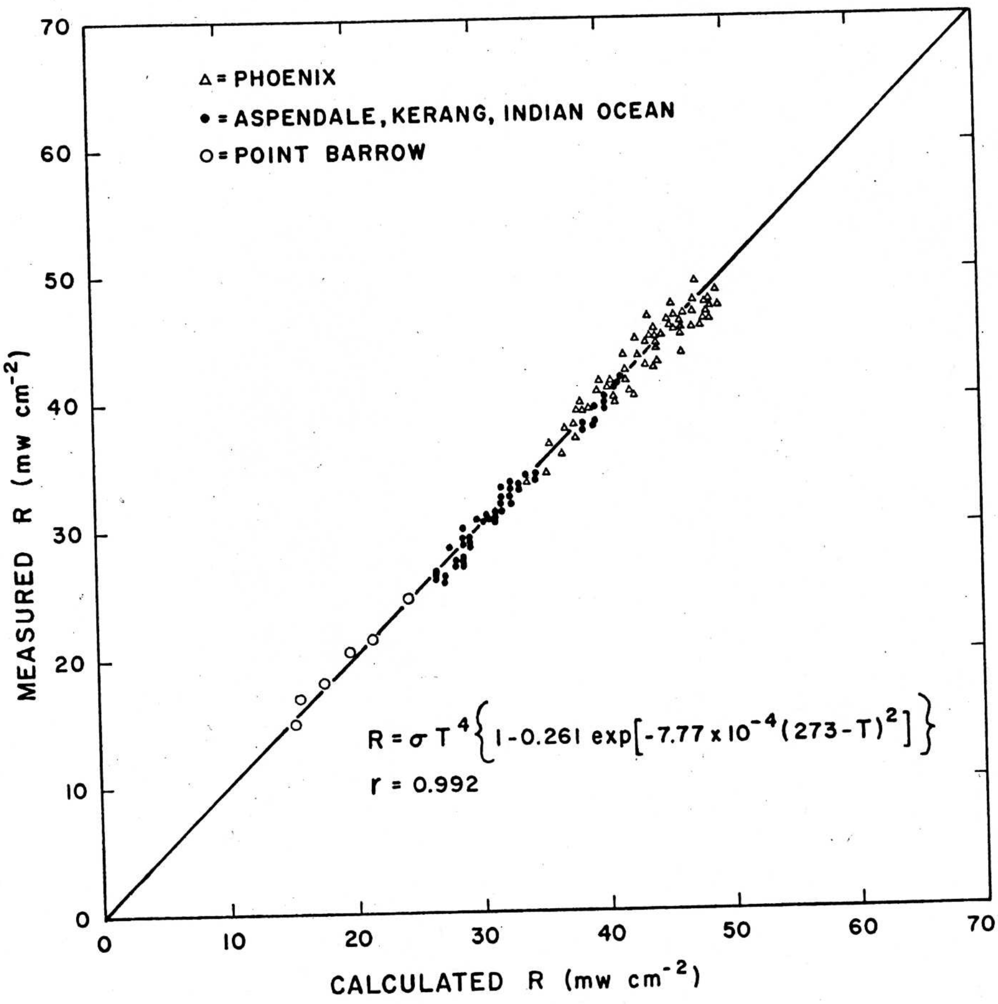

Fig. 6. Atmospheric thermal radiation as a function of radiation calculated by means of equation 12; all experimental data included.

temperatures, some of these measurements were also made during the day. At these times, incoming and reflected solar radiation also had to be considered, and these data were obtained with upright and inverted Kipp-Zonen solarimeters.

The new data combined with those of *Swinbank* [1963] clearly vindicate the theoretical prediction of the previous section. As air temperature increases from about 273°K, R increases as a result of the relation between T and e. Below 273°K, R again increases (relative to σT^4) in a similar fashion but must now do so as a result of increased conversion of water vapor to ice.

By statistical analysis of the data involved, we determined the coefficients c and d of (12) to be, respectively, 0.261 and 7.77×10^{-4}. The dashed line of Figure 5 represents this equation with these values of c and d. The goodness-of-fit of equation 12 is best portrayed, however, by plotting R as a function of the right-hand side of (12), for such a plot should give a regression line of slope 1 and intercept 0. This has been done in Figure 6. The pertinent statistics for this regression line are slope = 0.9993, intercept = − 0.2100, and correlation coefficient = 0.992.

It is our conclusion, based on this evidence, that equation 12 accurately describes a general relation between clear-sky atmospheric thermal radiation and screen-level air temperature that should be valid at any latitude and for any air temperature reached on earth.

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