

# Leaf-scale experiments reveal important omission in Penman-Monteith equation

Stan J. Schymanski\* and Dani Or

Department of Environmental Systems Science, Soil and Terrestrial Environmental Physics (STEP), Swiss Federal Institute of Technology (ETH) Zurich, Switzerland.

\*stan.schymanski@env.ethz.ch

## 1 Motivation and objectives

### (a) Motivation

1. A new wind tunnel enables detailed energy and mass balance evaluation of transpiring leaves.
2. Observations show substantial and unexplained underestimation of leaf transpiration ( $E$ ) by PM-equation.
3. Re-evaluation of the PM-equation reveals an important shortcoming in the formulation

### (b) Objectives

- Systematically evaluate transpiration and energy balance of artificial leaves in wind tunnel
- Propose and evaluate correction to Penman-Monteith equation

Penman-Monteith (PM) equation

$$E_l = \frac{\Delta_e T_a (R_s - R_{ll}) + \rho_a c_{pa} (P_{was} - P_{wa}) / r_a}{\Delta_e T_a + \gamma_v \left(1 + \frac{r_s}{r_a}\right)}$$

## 2 PM-Equation and correction

### (a) PM-Equation

Penman (1948) formulated an analytical solution for evaporation from a wet surface ( $E_w$ ):

Penman equation

$$E_w = \frac{\Delta_e T_a (R_s - R_{ll}) + f_u \gamma_v (P_{was} - P_{wa})}{\Delta_e T_a + \gamma_v}$$

Monteith (1965) introduced an additional stomatal resistance ( $r_s$ ) and formulated PM-equation for leaf transpiration ( $E_l$ ).

### (b) Corrected analytical solution

- Explicit consideration of 2-sided exchange of radiative and sensible heat (Fig. 1)

- Consideration of dependence of radiative exchange on leaf temperature, using linearised equation:

$$R_{ll} = 4a_{sh}\epsilon_l\sigma T_a^3 T_l - a_{sh}\epsilon_l\sigma(T_w^4 + 3T_a^4)$$

- Following Penman (1952), but eliminating  $P_{wl}$  instead of  $T_l$ , we get latent heat flux as a function of leaf temperature and an equation for steady-state leaf temperature:

New analytical solution

$$E_l = \frac{c_H (\Delta_e T_a (T_l - T_a) + P_{was} - P_{wa})}{\gamma_v}$$

$$T_l = \frac{R_s + c_H T_a + c_E (\Delta_e T_a T_a + P_{wa} - P_{was}) + a_{sh}\epsilon_l\sigma (3T_a^4 + T_w^4)}{c_H + \Delta_e T_a + 4a_{sh}\epsilon_l\sigma T_a^3}$$

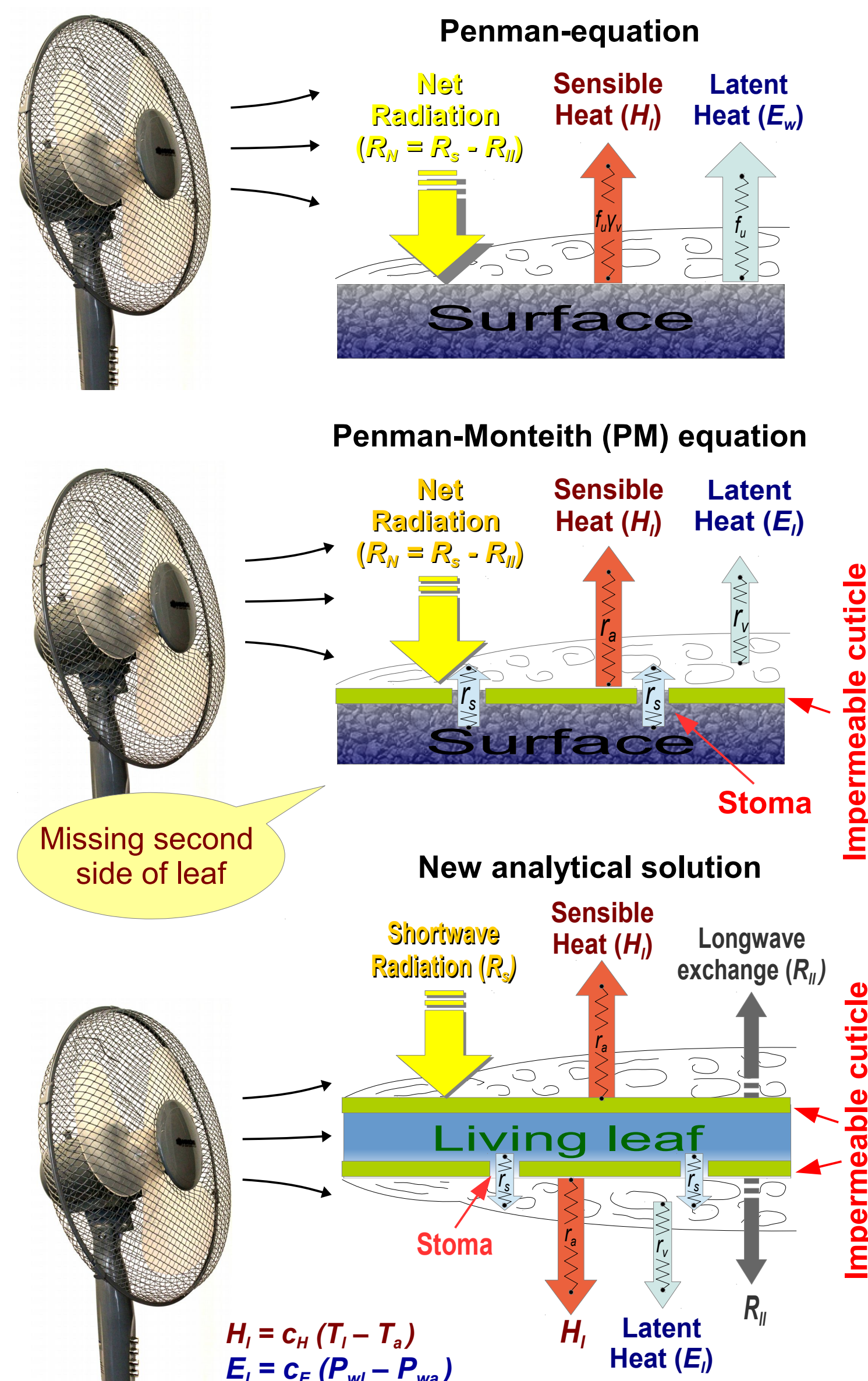


Fig. 1: Processes considered in different analytical solutions. Top: Penman equation for wet surface; middle: PM-equation with additional resistance; bottom: New analytical solution, illustrated for a hypostomatous leaf, with sensible heat exchange on both sides of the leaf, but latent heat flux only from the lower side. Here,  $c_r$  represents the two-sided sensible heat transfer coefficient, while  $c_e$  represents the one-sided total latent heat transfer coefficient, depending on both boundary layer and stomatal conductances. (Note that "latent heat" refers to the energy equivalent of transpiration.)

## 2 Experimental setup

- Insulated wind tunnel with fully controlled energy and mass exchange (Figs. 2, 3)
- Artificial leaf with laser-perforated foil ensuring constant stomatal resistance, embedded thermocouple and monitored water flow (Fig. 3)
- Stomatal conductance calculated from pore dimensions of laser perforations (Fig. 3)
- Sensible heat flux is computed from total chamber heat exchange, using monitored flow rate and temperature of incoming and outgoing chamber air (Fig. 2).

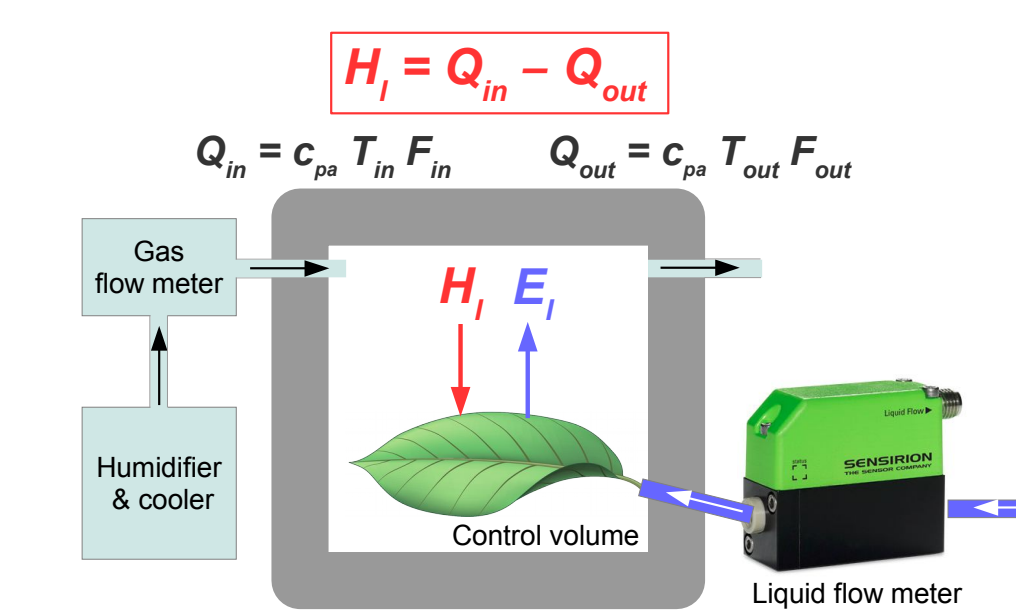
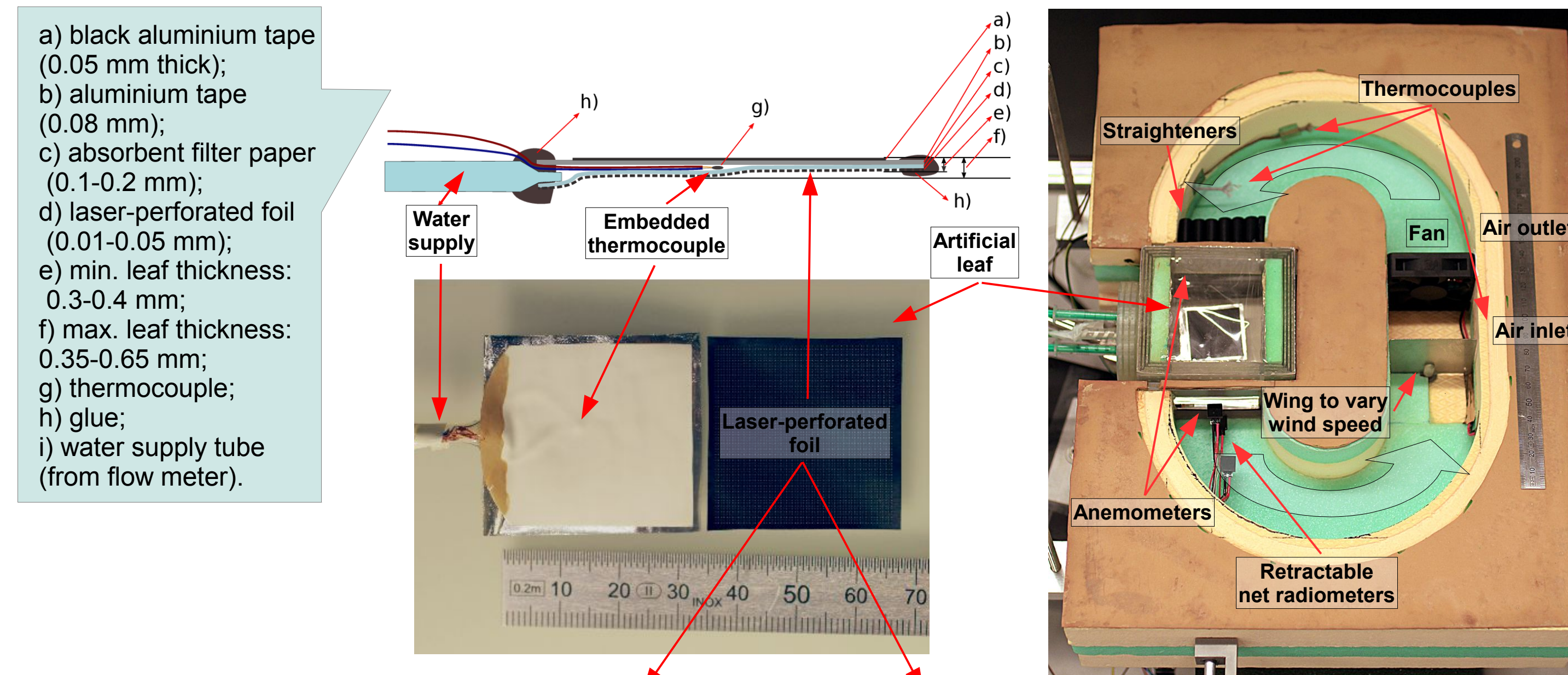


Fig. 2: Conceptual drawing of insulated leaf chamber. Latent heat flux ( $E$ ) is calculated from liquid flow rate into leaf, sensible heat flux ( $H$ ) is calculated from difference in heat content of incoming and outgoing air.



Dimensions of perforations (diameter, number per area, depth) were measured using a confocal laser scanning microscope and used to compute stomatal conductance ( $g_{sw}$ ) following Lehmann and Or (2015). Sub-area of the artificial leaf with 35 perforations  $\text{mm}^2$ ; resulting  $g_{sw}$ -values between 0.028 and 0.051  $\text{m s}^{-1}$ .

Fig. 3: Artificial leaf (left) in mini-wind tunnel (right, lid removed) with transparent chamber, fan and sensors. Top left: Cross section of artificial leaf; center left: Bottom view of artificial leaf, prior to attachment of laser-perforated foil; bottom left: confocal laser scanning microscope topography of perforated foil used in experiment described here.

## 3 Numerical experiment (varying $T_a$ and $R_s$ )

**Symbols & Units:**

- $\Delta_e T_a$ :  $dP_{was}/dT_a$  at  $T_a$  (Pa K<sup>-1</sup>)
- $\gamma_v$ : Psychrometric constant (Pa K<sup>-1</sup>)
- $\rho_a$ : Air density (kg m<sup>-3</sup>)
- $c_{pa}$ : Air heat capacity (J K<sup>-1</sup> kg<sup>-1</sup>)
- $E_w$ : Wet surface latent heat flux (W m<sup>-2</sup>)
- $E_l$ : Leaf latent heat flux (W m<sup>-2</sup>)
- $f_u$ : Penman's wind function (W m<sup>-2</sup> Pa<sup>-1</sup>)
- $H_i$ : Leaf sensible heat flux (W m<sup>-2</sup>)
- $R_{ll}$ : Net longwave exchange (W m<sup>-2</sup>)
- $R_s$ : Net radiation (W m<sup>-2</sup>)
- $R_{sw}$ : Absorbed SW radiation (W m<sup>-2</sup>)
- $P_{wa}$ : Ambient vapour pressure (Pa)
- $P_{ws}$ : Sat. vapour pressure at  $T_s$  (Pa)
- $r_s$ : Boundary layer resistance (s m<sup>-1</sup>)
- $r_{sa}$ : Stomatal resistance (s m<sup>-1</sup>)
- $T_a$ : Air temperature (K)
- $T_l$ : Leaf temperature (K)
- $T_w$ : Temperature of chamber walls (K)

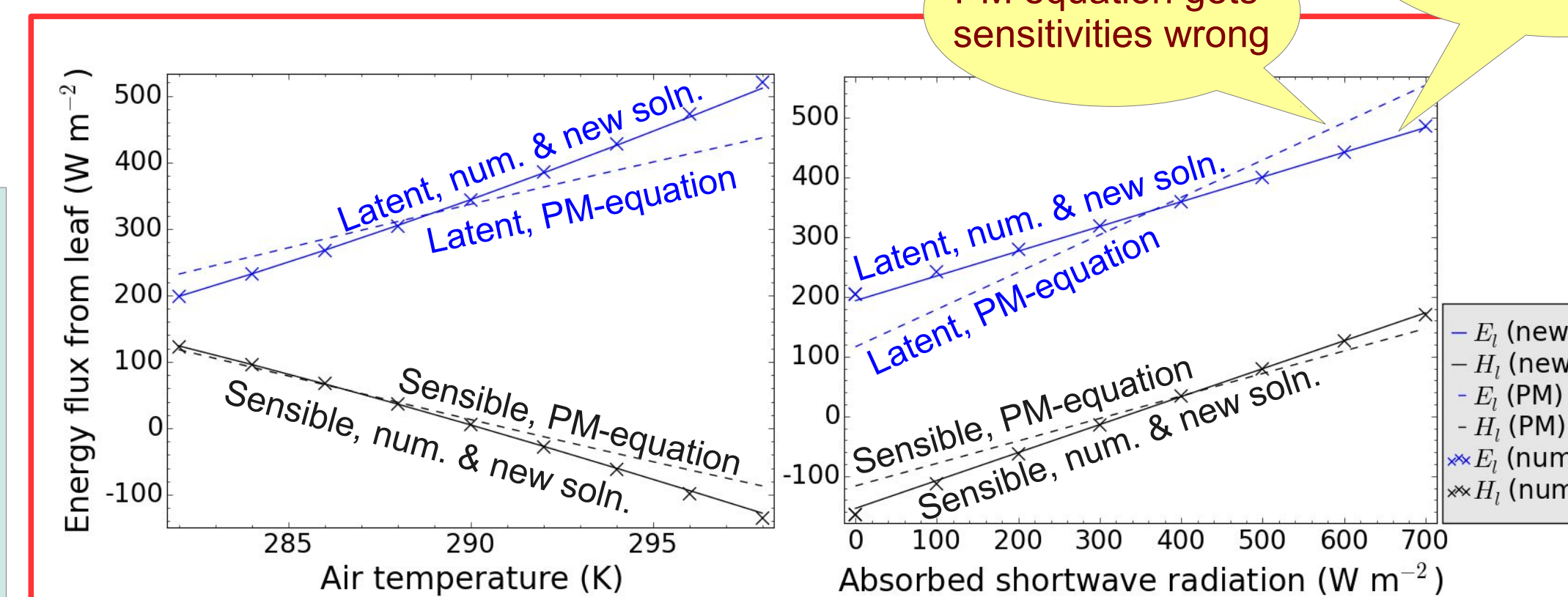


Fig. 4: Simulations of sensitivity of latent and sensible heat fluxes to air temperature (left) and absorbed shortwave radiation (right). Crosses represent numerical solution of leaf energy balance model (Schymanski & Or, 2016), solid lines our new analytical solution (see Block 4 on the right) and dashed lines represent Penman-Monteith equation. Simulation conditions:  $g_{sw}=0.045 \text{ m s}^{-1}$ , 1300 Pa vapour press., 1  $\text{m s}^{-1}$  wind speed, 350  $\text{W m}^{-2}$  irradiance (left) and 295 K air temperature (right).

## 4 Wind tunnel experiments ( $R_s = 0$ )

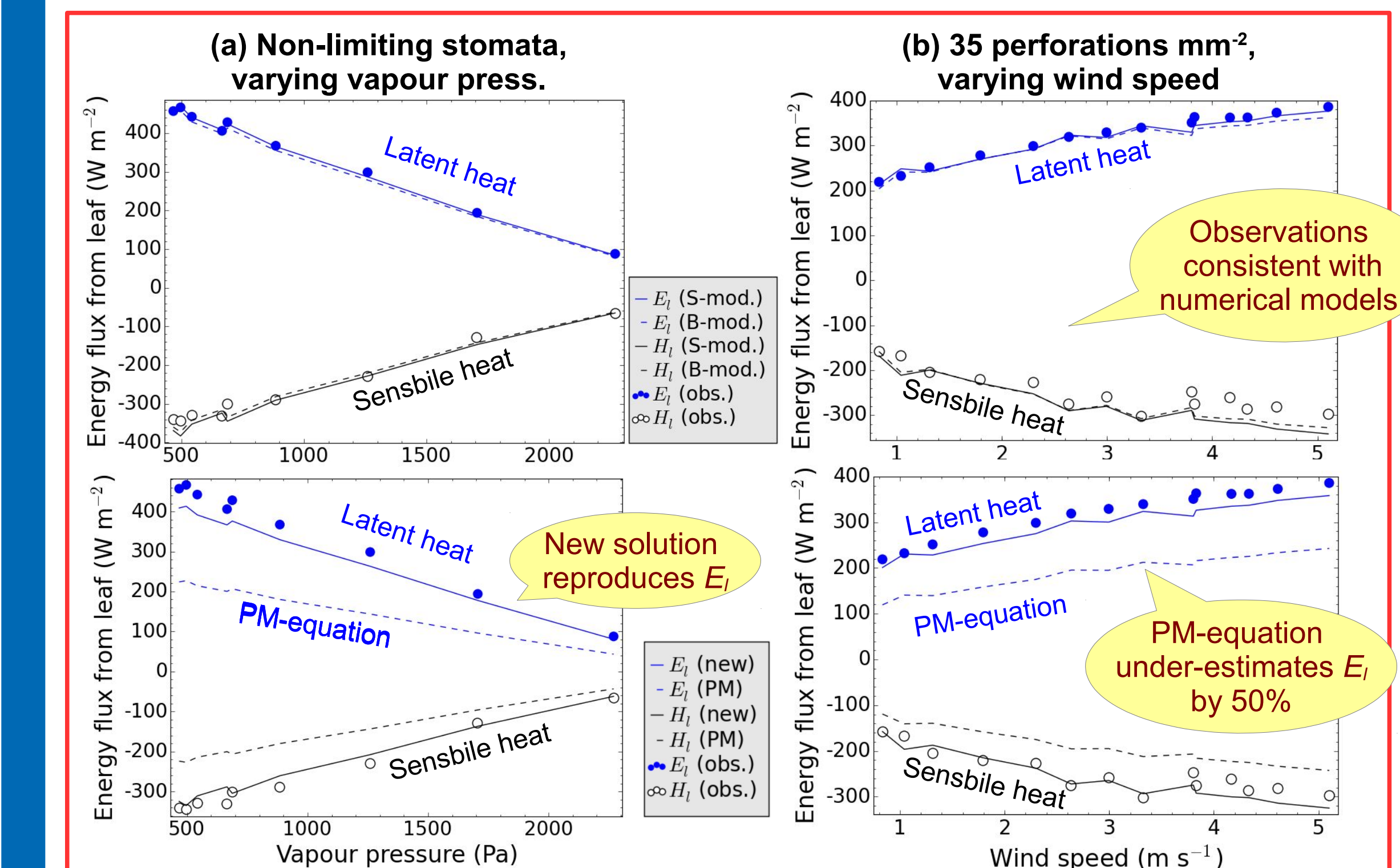


Fig. 5: Observed and simulated latent and sensible heat fluxes. Experimental data shown as symbols, simulations as lines. **Top row:** numerical models following Schymanski and Or (2016, solid lines) and Ball et al. (1988, dashed lines). **Bottom row:** Analytical solutions representing new analytical solution (solid lines) and PM-equation (dashed lines). **Left column:** Artificial leaf with wet filter paper on lower side. Exp. cond.: 1.0-1.05  $\text{m s}^{-1}$  wind speed, 295.4-296.6 K air temp. **Right column:** Artificial leaf with 35 perforations  $\text{mm}^2$ , equivalent to stomatal conductance of  $g_{sw} = 0.028$  to  $0.051 \text{ m s}^{-1}$  (simulations for  $g_{sw} = 0.045 \text{ m s}^{-1}$ ). Exp. cond.: 1200-1300 Pa vapour press., 295-296.5 K air temp.

## 5 Conclusions

- **Experimental setup enables research on the leaf energy balance at entirely controlled conditions.**
- **Elimination of calibration need (stomatal conductance known a priori) enabled rigorous evaluation of PM-equation at the leaf scale.**
- **PM equation does not consider two-sided exchange of sensible heat and hence fails to reproduce magnitude of leaf transpiration and its sensitivity to air temperature and irradiance.**
- **New analytical solution presented here is consistent with experimental results**

## 6 Literature and acknowledgements

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