The theoretical relationship between foliage temperature and canopy resistance in sparse crops

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SUMMARY

One-dimensional, sparse-crop interaction theory is reformulated to allow calculation of the canopy resistance from measurements of foliage temperature. A submodel is introduced to describe eddy diffusion within the canopy which provides a simple, empirical simulation of the reported behaviour obtained from a second-order closure model. The sensitivity of the calculated canopy resistance to the parameters and formulae assumed in the model is investigated. The calculation is shown to exhibit a significant but acceptable sensitivity to extreme changes in canopy aerodynamics, and to changes in the surface resistance of the substrate beneath the canopy at high and intermediate values of leaf area index. In very sparse crops changes in the surface resistance of the substrate are shown to contaminate the calculated canopy resistance, tending to amplify the apparent response to changes in water availability. The theory is developed to allow the use of a measurement of substrate temperature as an option to mitigate this contamination.

1. INTRODUCTION

In crops with well-established, closed canopies, most of the incident radiation is intercepted and partitioned by the vegetation. It is now common practice to represent such crops assuming a single source of energy flux positioned at an effective source position within the canopy (Shuttleworth 1976). It is further assumed that r_s^c, the so-called surface resistance operating at this position, is a reasonable measure of all the component stomatal resistances in the canopy acting in parallel (Monteith 1965). In this way surface resistance is a measure of the control exerted by the vegetation on the evaporation process and, as such, is presumably some measure of the water stress against which the crop is transpiring.

In the search to provide a simple diagnostic of the onset of large and possibly detrimental crop water stress, researchers have investigated the use of the surface resistance, deducing its value indirectly from standard meteorological measurements above the canopy and the surface temperature of the foliage as measured with an infrared thermometer (e.g. Jackson *et al.* 1981; Walker and Hatfield 1983; Hatfield 1985). Although the interrelationship between the radiometric foliage temperature and the required 'aerodynamic' foliage temperature is at present poorly understood, see for example Huband and Monteith (1986a, b), the prospect of applying this technique routinely in crop water management remains attractive. In part this is because the necessary measurements are comparatively simple; in part because they each provide some spatial average of whole-canopy behaviour.

One fundamental problem with the technique is the possibility that above-canopy fluxes are contaminated by fluxes from the substrate (usually soil) below, to the extent that the computed value of surface resistance is a poor measure of the average stomatal resistance of the canopy. This problem is particularly important at times of stress when canopy evaporation is reduced.

Shuttleworth and Wallace (1985) describe a resistance/energy combination model of sparse-crop energy partition with minimum complexity. This contains a more complicated resistance description of the surface and canopy than that for a complete-canopy cover, and also contains more than one source layer. The model, however, is still sufficiently simple to enable sensitivity studies to be performed to prescribed, typical values of the component resistances and to have practical utility. Choudhury and Monteith (1988) describe a somewhat similar but extended model which also gives a description of a sparse canopy. These models are still sufficiently simple to allow them to be manipulated analytically or with very simple numerical techniques, and thus to be used to gain an understanding of the interaction of the various model components.

This paper tackles the problem mentioned above, of providing some more theoretical background to estimating stress using surface temperature information. The paper is based on the expressions given in Shuttleworth and Wallace (1985), with some extensions. However, a theoretical yet simple description of the relationship between surface temperature and canopy behaviour in sparse canopies could be used both in more extensive monitoring using remotely sensed data and also as part of a land surface model in general circulation problems, as well as for the immediate problem of stress detection in sparse canopies.

Notwithstanding the above, there remain significant practical problems in applying the theory developed in this paper, the most significant being the precise estimation of surface temperature. Huband and Monteith (1986a, b) investigate the relationship between radiative and aerodynamic canopy temperatures for a wheat canopy, and make recommendations on sensor position to gain consistency in the temperature discrepancy. This discrepancy, they point out, can be accommodated by assigning apparent (rather than measured) values to the emissivity and reflectivity of the canopy. Smith *et al.* (1988) adopt these recommendations in practical application. These three papers are recommended to the reader interested in applying the theoretical concepts advanced here.

2. COMPLETE CANOPY COVER FORMULATION

(a) Nomenclature

For convenience the nomenclature used in later portions of this paper is listed here.

A, A_c, A_s	Total energy flux leaving the complete crop, the vegetation, the substrate, as sensible and latent heat per unit ground area (W m^{-2})
C	Extinction coefficient of the crop for net radiation (dimensionless)
c_0, c_1, c_2	Constants in parametrization of stomatal conductance (Eq. (34))
c_{ST}	Single leaf stomatal conductance (m s^{-1})
c_p	Specific heat at constant pressure (J kg^{-1}K^{-1})
d	Zero plane displacement of crop (m)
d_p	'Preferred value' of zero plane displacement ($0.63h$) (m)
D_r	Vapour pressure deficit at reference height, $[e_w(T_r) - e_r]$ (mb)
D_o	Vapour pressure deficit at canopy source height, $[e_w(T_o) - e_o]$ (mb)
e_o	Vapour pressure at canopy source height (mb)
e_r	Vapour pressure at reference height (mb)
$e_w(T)$	Saturated vapour pressure at temperature T ($T = T_r, T_o, T_s$) (mb)
G	Soil heat flux (W m^{-2})
h	Crop height (m)
H, H_c, H_s	Sensible heat flux from the complete crop, canopy, substrate (W m^{-2})

k	von Kármán's constant (dimensionless)
K	Eddy diffusion coefficient $(m^2 s^{-1})$
K_h	Eddy diffusion coefficient at top of canopy $(m^2 s^{-1})$
L	Projected area of leaf per unit ground area (leaf area index) (dimensionless)
m	Moisture stress factor (dimensionless)
n	Eddy diffusivity decay constant in the crop (dimensionless)
P	Biochemical storage of energy in the crop below reference height $(W\ m^{-2})$
r_a^a	Aerodynamic resistance between canopy source height and reference level $(s\ m^{-1})$
r_a^c	Bulk boundary layer resistance of the vegetative elements in the canopy $(s\ m^{-1})$
r_a^s	Aerodynamic resistance between the substrate and canopy source height $(s\ m^{-1})$
r_b	Mean boundary layer resistance per unit area of vegetation $(s\ m^{-1})$
r_s^c	Bulk stomatal resistance of the canopy $(s\ m^{-1})$
r_s^s	Surface resistance of the substrate $(s\ m^{-1})$
r_{ST}	Mean stomatal resistance $(s\ m^{-1})$
R_n	Net radiation flux into the complete crop $(W\ m^{-2})$
R_n^s	Net radiation flux into the substrate $(W\ m^{-2})$
S	Physical storage of energy in the atmosphere and crop below reference height $(W\ m^{-2})$
T_f	Temperature of canopy foliage (°C)
T_o	Air temperature at canopy source height (°C)
T_s	Temperature of the substrate surface (°C)
T_r	Air temperature at reference height (°C)
u	Wind speed at the reference height $(m\ s^{-1})$
u_h	Wind speed at crop height $(m\ s^{-1})$
u_*	Friction velocity $(m\ s^{-1})$
w	Leaf width (m)
z_r	Reference height above the crop where meteorological measurements are available (2 m in this analysis)
z	Height (variable) (m)
z_0	Roughness length of crop (m)
z_0'	Roughness length of the bare substrate (m)
Z_0	'Preferred value' of roughness length $(0 \cdot 13h)$ (m)
Δ	Mean rate of change of saturated vapour pressure with temperature, $\{e_w(T_r) - e_w(T_o)\}/(T_r - T_o)$ $(mb\ K^{-1})$
γ	Psychrometric 'constant' $(mb\ K^{-1})$
$\lambda E, \lambda E_c, \lambda E_s$	Latent heat flux from the complete crop, canopy, substrate $(W\ m^{-2})$
ρ	Density of air $(kg\ m^{-3})$

(b)   The complete-canopy surface resistance formula

Before proceeding to the description of the more complex sparse canopy, it is convenient and instructive first to consider a canopy represented in the simpler Penman–Monteith format, i.e. with no interaction at the substrate beneath. The equivalent resistance representation in this situation is illustrated in Fig. 1. It is emphasized that the equations and analysis advanced in this section are superseded by more general formulae later.
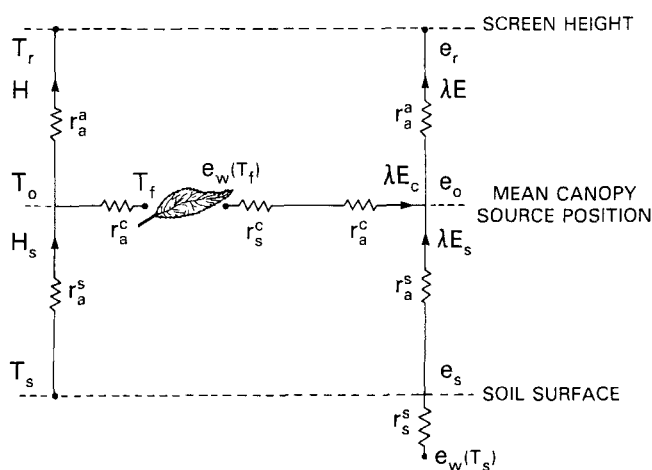
Figure 1.   Schematic diagram of a simple one-dimensional model of crop energy partition which ignores the interaction of the substrate.

The model shown in Fig. 1 draws up a single energy budget at the mean canopy source position such that

$$A_c = \lambda E_c + H_c \tag{1}$$

and analogy with Ohm's law gives

$$\lambda E_c = \frac{\rho c_p}{\gamma} \frac{e_w(T_f) - e_r}{(r_a^a + r_a^c) + r_s^c} \tag{2}$$

and

$$H_c = \rho c_p \frac{T_f - T_r}{r_a^a + r_a^c}. \tag{3}$$

Substituting (2) and (3) into (1) and rearranging yields the equation required to compute r_s^c, the canopy resistance of the crop, from meteorological measurements above the canopy and a measurement of T_f, the average surface temperature of the vegetation. This takes the form

$$r_s^c = \frac{(\rho c_p/\gamma)[e_w(T_f) - e_r]}{A - \rho c_p(T_f - T_r)/(r_a^a + r_a^c)} - (r_a^a + r_a^c). \tag{4}$$

The more complete description of the crop–atmosphere system given later yields a similar, though more complex, formula which must simplify to Eq. (4) when the substrate interaction is neglected.

3.   SPARSE-CANOPY COVER FORMULATION

The sparse-canopy resistance model adopted and developed in this paper is that analysed by Shuttleworth and Wallace (1985), which is illustrated in Fig. 2. In the context of the present discussion the computation of the surface resistance of the sparse canopy r_s^c is complicated by the energy fluxes contributed by the substrate beneath. The energy partition of this substrate is described by a subsidiary resistance framework of the Penman–Montieth type. The resistances involved in this submodel are here prescribed and the sensitivity with respect to this prescription investigated.

(a)   The energy budgets

Two energy budgets are drawn up, one at the substrate surface and one for the complete crop. The sum of the above-canopy fluxes of sensible heat, H, and latent heat,

λE, is the available energy, A, and is given by

$$A = \lambda E + H \tag{5}$$

$$= R_n - S - P - G \tag{6}$$

where R_n is the incoming net radiation, S and P are the physical and biochemical energy storage terms, and G is the heat conduction into the substrate. In a similar way, the energy available at the substrate, A_s, is given by

$$A_s = \lambda E_s + H_s \tag{7}$$

$$= R_n^s - G \tag{8}$$

where R_n^s is the net radiation at the substrate surface. In the limit of bare substrate A and A_s are equal.

(b) Ohm's law relationships

By analogy with Ohm's law for the electrical analogue shown in Fig. 2, the differences in vapour pressure and temperature across the component resistances can be written in terms of the relevant resistance and flux, firstly for latent heat flux as

$$e_o - e_r = \lambda E r_a^a \gamma / \rho c_p \tag{9}$$

$$e_w(T_f) - e_o = \lambda E_c (r_a^c + r_s^c)\gamma / \rho c_p \tag{10}$$

$$e_w(T_s) - e_o = \lambda E_s (r_a^s + r_s^s)\gamma / \rho c_p \tag{11}$$

and secondly for sensible heat flux as

$$T_o - T_r = H r_a^a / \rho c_p \tag{12}$$

$$T_f - T_o = H_c r_a^c / \rho c_p \tag{13}$$

$$T_s - T_o = H_s r_a^s / \rho c_p. \tag{14}$$

It is convenient here to perform some algebraic manipulation to provide a relationship, which has later application, for the in-canopy vapour pressure deficit. Substituting Eqs.

Figure 2. Schematic diagram of the one-dimensional model of crop energy partition which includes the interaction of the substrate and is the framework of the Shuttleworth and Wallace (1985) theory of sparse-crop evaporation.

(9) and (12) into Eq. (5), and rearranging gives

$$\rho c_p(e_o - e_r) = \gamma r_a^a A - \gamma \rho c_p(T_o - T_r). \tag{15}$$

The vapour pressure deficit of the canopy airstream is defined by the equation

$$D_o = e_w(T_o) - e_o. \tag{16}$$

Substituting the identity $e_w(T_r) - e_w(T_r) \equiv 0$ into Eq. (16) and then incorporating the definition of Δ from section 2(a) yields

$$D_o = e_w(T_r) - e_o + \Delta(T_o - T_r) \tag{17}$$

which, on substituting Eq. (15) and incorporating the definition for D_r, becomes

$$D_o = D_r + (\Delta + \gamma)(T_o - T_r) - \gamma r_a^a A / \rho c_p. \tag{18}$$

(c)   Substrate evaporation

The present analysis prescribes an effective value to the surface resistance for the substrate beneath the crop. In this way the substrate evaporation can be calculated by combining Eqs. (11) and (14) with Eq. (7) to yield a combination equation with the form

$$\lambda E_s = (\Delta A_s + \rho c_p D_o / r_a^s)\{\Delta + \gamma(1 + r_s^s / r_a^s)\}^{-1}. \tag{19}$$

(Note. In the above equations, and in the analysis which follows, Δ, the mean rate of change of saturated vapour pressure as a function of temperature evaluated above canopy, is assumed to have the same value above, within and below the canopy. Small temperature-dependent changes are ignored.)

(d)   Flux conservation

Conservation of sensible heat in the canopy air stream gives

$$H = H_c + H_s. \tag{20}$$

It can be shown (see appendix A), that substituting Eqs. (7), (12), (13) and (19) into Eq. (20), then introducing Eq. (18), ultimately gives an expression for the difference between within- and above-canopy temperature required in later analysis. The relationship takes the form

$$(T_o - T_r) = f r_a^a(T_f - T_r)/(r_a^a + r_a^c) + F r_a^a / \rho c_p \tag{21}$$

where

$$f = [1 + \eta(\Delta + \gamma)]^{-1} \tag{22}$$

and

$$F = f\eta[\gamma A + \gamma A_s(r_s^s + r_a^s)/r_a^a - \rho c_p D_r / r_a^a] \tag{23}$$

in which

$$\eta = (r_a^a r_a^c)/\{R_s(r_a^a + r_a^c)\}. \tag{24}$$

R_s retains the definition given by Shuttleworth and Wallace (1985), namely

$$R_s = (\Delta + \gamma)r_a^s + \gamma r_s^s. \tag{25}$$

The second canopy air stream flux conservation equation, that for latent heat, requires

$$\lambda E = \lambda E_c + \lambda E_s \tag{26}$$

and it can be shown (see appendix B), that substituting Eqs. (9) and (10) into Eq. (26), and then substituting Eq. (15), ultimately gives

$$r_s^c = \frac{(\rho c_p/\gamma)[e_w(T_f) - e_r] - r_a^a \lambda E_s}{(A - \lambda E_s) - \rho c_p(T_o - T_r)/r_a^a} - (r_a^a + r_a^c).$$ (27)

(e) The sparse-canopy surface resistance formula

Substituting Eq. (21) into Eq. (27) yields the formula required to compute the canopy resistance of a sparse crop growing above a substrate with prescribed (resistance) properties given meteorological measurements above the crop and a measurement of the surface temperature of the vegetation. The equation has the form

$$r_s^c = \frac{(\rho c_p/\gamma)[e_w(T_f) - e_r] - r_a^a \lambda E_s}{(A - \lambda E_s) - f\rho c_p(T_f - T_r)/(r_a^a + r_a^c) - F} - (r_a^a + r_a^c).$$ (28)

Evaluating Eq. (28) requires a preliminary calculation of the below-canopy evaporation flux, λE_s. This is made by first computing $(T_o - T_r)$ from Eq. (21), and then substituting the result into Eq. (18) to calculate D_o. The below-canopy evaporation then follows from Eq. (19).

Equation (28) has the required closed-canopy limit. If there is no substrate interaction $A_s = 0$, while r_s^a, r_s^s and hence R_s, from Eq. (25), are all infinite. In consequence, $\eta = 0$, from Eq. (24), $f = 1$, from Eq. (22), implying $F = 0$, from Eq. (23). $(T_o - T_r)$, from Eq. (21), and hence D_o, from Eq. (18), remain finite; so that $\lambda E_s = 0$, from Eq. (19). Introducing $\lambda E_s = F = 0$ and $f = 1$ into Eq. (28) yields Eq. (4), thereby regaining the simpler, closed-canopy description.

4. VEGETATION DENSITY VARIATIONS

In this section the response of Eq. (28) is investigated when applied to crops of the same height but with different leaf area, which are subject to a specified atmospheric demand. For the purpose of illustration and to preserve consistency, the remainder of the paper generally follows Shuttleworth and Wallace (1985) with regard to the choice of typical atmospheric conditions and crop characteristics and, in most respects, with regard to model specification. However, for simplicity we here select a single value of vapour pressure deficit, $D_r = 15$ mb, while the submodel of eddy diffusion resistance is significantly modified and upgraded as described later. Following Shuttleworth and Wallace, all the meteorological variables are assumed to be measured at a height $z_r = 2$ m, above a crop of height $h = 0.3$ m, and calculations are carried out with $R_n = 400$ W m^{-2}, $T_r = 25\,^\circ$C and $u = 2$ m s^{-1}. The soil heat flux is arbitrarily set to 20% of the radiation reaching the soil. In some of the calculations the measured foliage temperature, T_f, is set to two values, 28 and 30 $^\circ$C.

The present analysis necessarily differs in another respect. It is in the nature of the problem addressed here that leaf area index cannot be prescribed, as it was in Shuttleworth and Wallace, or even directly measured by labour-intensive crop sampling techniques. A simple estimate must be made which is consistent with the objective of providing a measure of surface resistance as a diagnostic of crop stress from simply made measurements which are representative of the whole canopy. Accordingly, we assume such an estimate is possible based on above- and below-canopy measurements of net

radiation, and assuming a Beer's law relationship of the form

$$R_n^s = R_n \exp(-CL) \tag{29}$$

with the extinction coefficient, C, arbitrarily prescribed as 0.7, see for example Monteith (1973). In practice, in order to reduce the effect of spatial variability in Eq. (29) it would probably be necessary to use measurements of below-canopy radiation made with several 'tube' net radiometers and average over at least one day to incorporate different sun angles. The sensitivity of the model to the prescribed value of C is investigated later: it affects the value of the surface resistance indirectly through the model's leaf-area-index-dependent formulation of aerodynamic resistance.

(a) Resistance specification

(i) *Mean boundary layer resistance.* The present paper provides an opportunity to improve on the simple formulation for boundary layer resistance assumed by Shuttleworth and Wallace (1985). However, no attempt is made to distinguish between the fluxes of vapour and heat exchange in this context: slight differences due to differing diffusion coefficients across the laminar boundary layers of leaves are negligible in comparison with other uncertainties.

The effective value of boundary layer resistance for the transfer of momentum between individual leaves and the canopy air stream is less than for energy transfer because of the additional, more efficient form drag mechanism. When describing closed canopies, it is often convenient to accommodate this difference in overall aerodynamic resistance by assigning a different (lower) value to the roughness length for energy exchange. This convenience is not appropriate in the context of the present sparse-canopy model. Here we follow Choudhury and Monteith (1988). Although we assume an approximate equivalence between the eddy diffusion of momentum and energy through the canopy, we write the boundary layer resistance for energy transfer assuming exchange only occurs by molecular diffusion through a laminar layer around the leaves. Accordingly, boundary layer resistance is given by

$$r_a^c = r_b/2L \tag{30}$$

where

$$r_b = (100/n)(w/u_h)^{1/2}[1 - \exp(-n/2)]^{-1}. \tag{31}$$

This equation is adopted as analogous to Eq. (29) of Choudhury and Monteith, but in addition assumes that the wind speed and eddy diffusivity decay at the same rate. In Eq. (31), w is the representative leaf width and u_h is the wind speed at the top of the canopy.

(ii) *Mean stomatal resistance.* In some of the analysis which follows the bulk stomatal resistance, r_s^c, is not prescribed, rather it is derived as the value consistent with a specified foliage temperature. On other occasions, a prescribed value is required and, in particular, a plausible description of the variation in this resistance in response to changing leaf area. The relationships assumed by Shuttleworth and Wallace (1985) and by Choudhury and Monteith (1988) are both discarded as providing unrealistically simple representations of this variation.

The bulk stomatal resistance at each level in the canopy is inversely related to the leaf area index there; but the very common observation of a radiation dependence in stomatal resistance complicates the synthesis of a whole-canopy average value, since local radiation is related to the integrated leaf area above. In many types of canopy the

irradiance below a leaf area index of L' can be written as

$$S'(L') = S \exp(-CL') \tag{32}$$

where S is the short-wave flux at the top of the canopy, and C is an attenuation constant here assumed equal to that for net radiation. At any (leaf area) depth in the canopy, the mean irradiance of a leaf, S_l, is equivalent to the attenuation of the vertical flux and, from Eq. (32), is

$$S_l = - \partial S'(L')/\partial L' = CS(L'). \tag{33}$$

In many problems it is more convenient to work with the reciprocal of stomatal resistance, the stomatal conductance, which is represented by c_{ST} for a single leaf and C_{ST} for the whole canopy. *In the absence of any other environmental stress* the stomatal conductance has a response to solar irradiance which often takes the form

$$c_{ST} = c_0 + \frac{c_1 S_l}{1 + c_2 S_l} \tag{34}$$

though the constants c_0, c_1 and c_2 may change significantly between canopies and species. In this equation, c_0 is the cuticular resistance, assumed light-independent, and is typically at least an order of magnitude less than the second term for daytime radiation levels in the field.

In the simplest case, when c_0, c_1 and c_2 are assumed independent of leaf position and the bulk stomatal conductance of the complete canopy, C_{ST}, is set equal to that of all the leaves acting in parallel, then, with a total leaf area of L,

$$C_{ST}(S, L) = \int_0^L c_{ST}(L') \, dL' \tag{35}$$

which, by combining Eqs. (33), (34) and (35) and integrating, means

$$C_{ST}(S, L) = c_0 L + \left(\frac{c_1}{c_2 C}\right) \ln\left[\frac{1 + c_2 CS}{1 + c_2 CS \exp(-\alpha' L)}\right]. \tag{36}$$

Figure 3 illustrates the typical form of the radiation-dependent functions c_{ST} and C_{ST} with $C = 0.6$. The light response of a single leaf usually involves an initial, rapid rise from the cuticular conductance, followed by levelling off at solar irradiances greater than (say) 400 W m^{-2}, see Fig. 3(a). Equation (36) requires that a canopy made up of similar leaves has a less moderated response at high radiation since more leaves deeper in the canopy achieve light saturation. Figure 3(b) illustrates the response of C_{ST} to increasing leaf area. Since a lower *proportion* of the leaves have light-limited stomatal conductance at higher leaf area, the *average* stomatal conductance of the leaves falls and the relationship is not a linear function of L.

(iii) *Surface resistance of the substrate.* Shuttleworth and Wallace (1985) assign the values 0, 500 and 2000 s m^{-1} to r_s^s, the surface resistance of the substrate. The two extremes represent wet soil (or free water) and fairly dry soil respectively: the third is selected and retained in this analysis merely as an intermediate, preferred value. In section 5 a prescribed time series of r_s^s is adopted to illustrate the possible effect of changing soil moisture on apparent canopy resistance during a drying/wetting sequence.

(iv) *Eddy diffusion resistance.* In this paper the scheme suggested by Shuttleworth and Wallace (1985) to describe in-canopy diffusion, hereafter referred to as submodel 'SW', is superseded by a new description, hereafter referred to as submodel 'SG'. This new
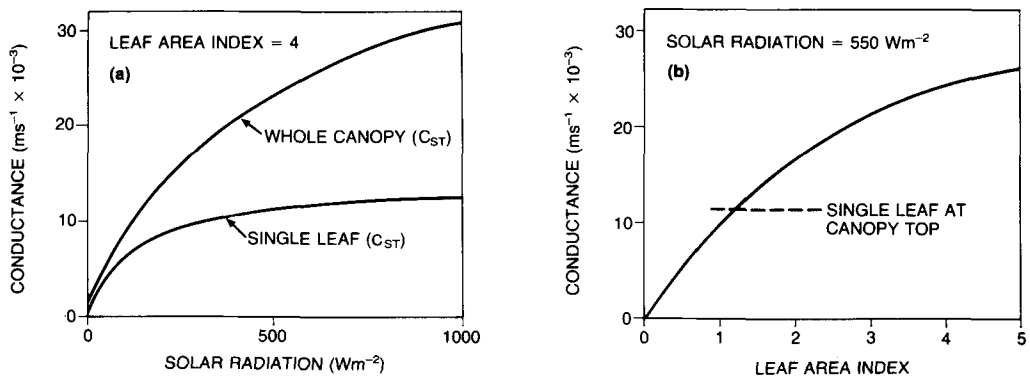
Figure 3. (a) The assumed form of the radiation-dependent, single-leaf stomatal conductance, see Eq. (34), and ensuing variation in whole-canopy stomatal conductance calculated from Eq. (36) with $L = 4$. (b) The variation in whole-canopy stomatal conductance as a function of leaf area index from Eq. (36) for an incident solar radiation of 550 W m^{-2}.

scheme is still pragmatically simple but is arguably superior to SW in providing at least a first-order simulation of the changes in z_0 and d predicted by a more complex second-order closure model of in-canopy turbulent transfer. Since the present submodel is considered better in this respect, its use is recommended both in 'diagnostic' mode, as here, and in 'predictive' mode, i.e. within the framework and application previously described by Shuttleworth and Wallace. This is true despite the fact that it is later demonstrated that this new submodel, though theoretically better, makes limited numerical difference.

The new submodel SG retains many of the assumptions of its predecessor. In particular, both submodels assume

(a) that above the crop the eddy diffusion coefficient, K, is given by

$$K = ku_*(z - d) \qquad z > h \tag{37}$$

where k is von Kármán's constant, z is height and u_* is the friction velocity, which, in conditions of neutral atmospheric stability, is given by the expression

$$u_* = ku/\ln\{(x - d)/z_0\} \tag{38}$$

where u is the wind speed at the reference height x;

(b) that the eddy diffusion coefficient decreases exponentially through the canopy, thus

$$K = K_h \exp\{-n(1 - z/h)\} \tag{39}$$

where K_h is the value of K at the top of the crop; we use a value of $n = 2.5$ which is typical of the agricultural crop being specified (Monteith 1973). (Note, there are well established weaknesses in the use of 'K theory' to describe through-canopy transfer, particularly for crops with elevated foliage; this theory is merely a working simplification of a more realistic but complex theoretical model);

(c) and that the *effective source of energy flux remains fixed*, regardless of leaf area, at the height of the sink of momentum relevant to 'closed-canopy' conditions. In this context the canopy is assumed closed when L is greater than 4 and the source/sink height is set to $(Z_0 + d_p)$, where Z_0 and d_p are preferred values of z_0 and d, chosen in both submodels as a fixed fraction of crop height following Monteith (1973), thus

$$Z_0 = 0.13h \tag{40}$$

$$d_p = 0.63h. \tag{41}$$

The two submodels SG and SW differ in the way they describe the variation in z_0 and d as a function of the leaf area index, L. In submodel SW their value is implicit (see Shuttleworth and Wallace 1985): in the new submodel SG they are explicitly defined, indeed this definition is crucial to the description of the transition between a closed canopy and a bare substrate.

Submodel SG. In common with Choudhury and Monteith (1988), we specify the values of z_0 and d as functions of leaf area index by fitting simple functions to curves in Shaw and Pereira (1982), these last being derived from second-order closure theory. In this way we set

$$d = 1 \cdot 1h \ln(1 + X^{1/4}) \tag{42}$$

and

$$z_0 = z_0' + 0 \cdot 3h\, X^{1/2} \qquad 0 < X < 0 \cdot 2 \tag{43a}$$

$$= 0 \cdot 3h(1 - d/h) \qquad 0 \cdot 2 < X < 1 \cdot 5 \tag{43b}$$

where z_0' is the roughness length of the substrate, and

$$X = c_d L. \tag{44}$$

Here c_d is the effective value of the mean drag coefficient for the individual vegetative elements making up the canopy.

At this point, the present analysis differs from that of Choudhury and Monteith (1988). In particular, we select an effective value of c_d, for general use, with a chosen value *which as far as possible* reproduces Eqs. (40) and (41) in closed-canopy conditions, i.e. with $L = 4$. The present analysis also differs from that of Choudhury and Monteith by selecting a fixed source height (see above), and by retaining the use of Eq. (39) in the canopy both above and below this source height.

The selection of an effective, general value of c_d is complicated by the fact that Eqs. (42) and (43) are not (in detail) numerically consistent with Eqs. (40) and (41). Substituting $d = 0 \cdot 63h$ into Eq. (42) gives an effective value of $c_d = 0 \cdot 09$ with $L = 4$, but the equivalent value of z_0 from Eq. (43) is $0 \cdot 11h$. At the same time, substituting $z_0 = 0 \cdot 13h$ into Eq. (43) gives $d = 0 \cdot 57h$, with a consequent effective value of $c_d = 0 \cdot 05$ from Eq. (42). Bearing in mind the approximate and empirical nature of all the equations involved, this problem is really no more than numerical semantics. Accordingly, we define $c_d = 0 \cdot 07$ as the required effective value for general use, intermediate to the two values above. The ensuing predicted variations of z_0 and d as functions of L are illustrated in Fig. 4, together with the equivalent variations with $c_d = 0 \cdot 05$ and $c_d = 0 \cdot 09$ to provide perspective.

Using the several assumptions given above, and performing an integration of Eqs. (37) and (39) over the height ranges 0 to $(Z_0 + d_p)$, and $(Z_0 + d_p)$ to z_r respectively, it is possible to write

$$r_a^s = \frac{h \exp(n)}{n\, K_h} [\exp(-nz_0'/h) - \exp\{-n(Z_0 + d_p)/h\}] \tag{45}$$

$$r_a^a = \frac{1}{ku_*} \ln\left[\frac{z_r - d}{h - d}\right] + \frac{h}{n\, K_h} [\exp\{n[1 - (Z_0 + d_p)/h]\} - 1] \tag{46}$$

where

$$K_h = ku_*(h - d) \tag{47}$$
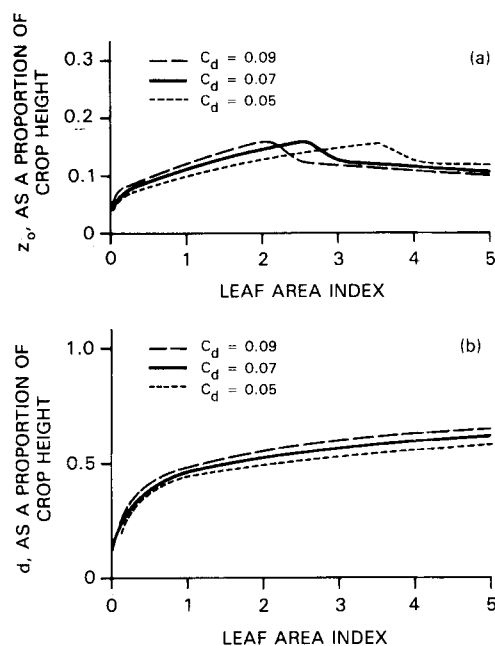
with u_* given by Eq. (38).

Figure 4. The calculated variation as a function of leaf area index of, (a), the roughness length and, (b), the zero plane displacement, in the SG aerodynamic submodel for values of the individual element exchange coefficient, $c_d = 0 \cdot 05$, $0 \cdot 07$, $0 \cdot 09$. $c_d = 0 \cdot 07$ is the preferred value in this analysis.

(b) Sensitivity to aerodynamic resistance formulation

(i) *Different diffusion submodels.* It is of interest to investigate the difference in calculated surface resistance given using the two submodels SW and SG. Figure 5 shows the variation in the value of r_s^c as a function of leaf area index given by Eq. (28), with r_a^s and r_a^a determined by Eqs. (45) and (46) on the one hand, and by Eqs. (30) and (31) of Shuttleworth and Wallace (1985) on the other. Calculations are made for the atmospheric conditions and crop characteristics given earlier, and for $r_s^s = 500 \, \text{s m}^{-1}$. Figures 5(a) and (b) correspond to foliage temperatures of 28 and 30 °C respectively. Figure 5 also shows the values of surface resistance which would be calculated were the presence of the substrate ignored and the complete canopy cover assumed, i.e. Eq. (4) used instead of Eq. (28). This is equivalent to assuming the Penman–Monteith equation (Monteith 1965).

The difference between the calculations made with the two submodels in Eq. (28) is, in general, small, but neglecting the presence of an evaporating substrate can give rise to considerable error, particularly with sparse crops. The presence of a (physically plausible) enhancement of turbulent transfer given by the new model SG at intermediate leaf area index, see Fig. 5(a), means that the canopy would have to have a slightly higher canopy conductance than with SW, in order to maintain the fixed foliage temperature arbitarily (and unrealistically) imposed for the purpose of the calculation shown in Fig. 5.

(ii) *Indirect sensitivity to net radiation absorption.* In the present model the value of L is deduced from Eq. (29) assuming $C = 0 \cdot 7$. Since this value is then used in the calculation of z_0 and d in Eqs. (42) and (43), a wrongly assigned value of C will alter the values of r_a^s and r_a^a from Eqs. (45) and (46), and in this way indirectly affect the value of r_s^c calculated from Eq. (28).
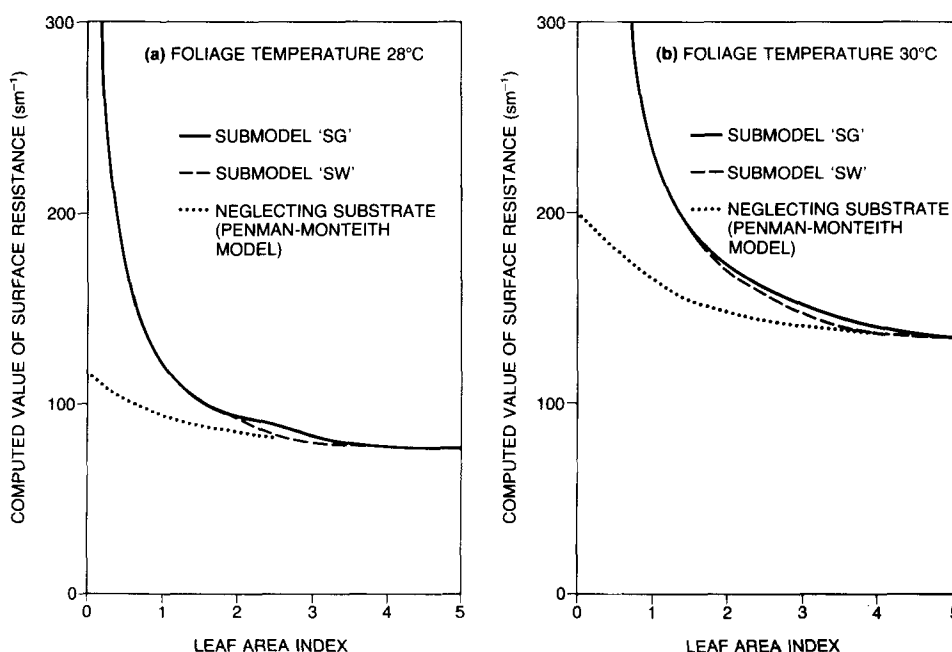
Figure 5. The calculated variation in canopy resistance as a function of leaf area index for the meteorological conditions given in the text, and with foliage temperature (a) 28 °C and (b) 30 °C. Calculations are made with the new submodel introduced in this paper, SG, and submodel, SW, previously described by Shuttleworth and Wallace (1985). Also shown is the miscalculated canopy resistance given by Eq. (4) in which substrate interaction is ignored and the Penman–Monteith model therefore assumed.

Figure 6 illustrates the changes in the calculated values of L, z_0 and d as functions of the fraction of net radiation penetrating the canopy, with C set to the value 0·7, as elsewhere in the analysis, and with $C = 0·5$ and 0·9. There is some systematic change in the calculated aerodynamic properties of the crop, and this is reflected in the computed values of r_s^c which are shown in Fig. 7. With $C = 0·5$, z_0 starts off lower but ends up higher than for $C = 0·7$, as the fraction of net radiation observed beneath the canopy increases, i.e. as L decreases. This behaviour, and the complementary behaviour when $C = 0·9$, helps reduce the sensitivity of the r_s^c calculation with respect to the assigned value of C. The magnitude of the response to changes in C over this range is between 2 and 10%, i.e. typically 5%.

(iii) *Parametrization of aerodynamic resistance.* The calculated values of r_s^c shown as a function of L in Tables 1 and 2 illustrate the effect of making extreme changes in the parametrization of aerodynamic resistance for two different foliage temperatures, 28 and 30 °C respectively. Tables 1(a) and 2(a) show the effect of halving and doubling the assumed value of w, the leaf width; Tables 1(b) and 2(b), the effect of halving and doubling the constant n, which describes the exponential decay in eddy diffusivity; and Tables 1(c) and 2(c), the effect of halving and doubling the value of c_d used to derive z_0 and d from Eqs. (42), (43) and (44).

In general, the consequences of these extreme changes are noticeable in the calculated value of r_s^c, giving changes in the order 20% and sometimes greater. Such sensitivity is most consistently apparent at higher values of leaf area index. In one sense the fact that the calculated canopy resistance shows sensitivity to these dramatic changes in the assumed formulation of aerodynamic resistance is neither unique to the present model, nor novel in the current analysis. In fact, the largest response occurs for closed
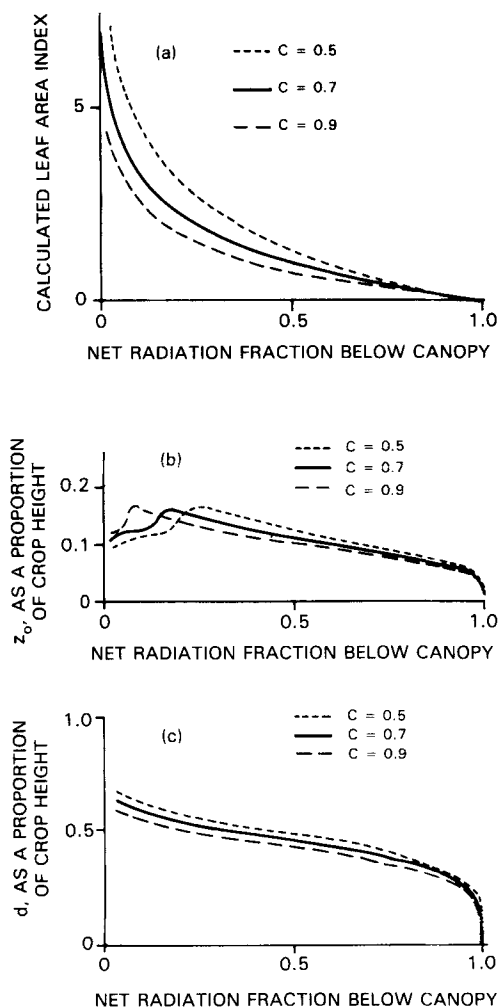
Figure 6. The variation in (a), the leaf area index, (b), the roughness length, and (c), the zero plane displacement calculated in the SG submodel as a function of the proportion of net radiation measured beneath the canopy for three different values of C, the net radiation extinction coefficient. $C = 0.7$ is the preferred value.

canopies in response to changes in c_d, i.e. in response to changes in d and, in particular, z_o. *Such sensitivity is, in fact, already present in the simpler, single-source, Penman–Monteith model of canopy interaction which the present theory supersedes.*

The level of sensitivity with respect to aerodynamic resistance demonstrated in Tables 1 and 2 does not necessarily preclude the identification of a *significant change* in canopy resistance, possibly associated with the onset of canopy water stress. To illustrate this last point, we investigate an arbitrary and idealized 12-day time series of moisture stress factor, m, imposed on (midday) values of C_{ST} calculated from Eq. (36) with S set to 550 W m^{-2}. The value of r_s^c is in this case given by

$$r_s^c = m/C_{ST}. \tag{48}$$

This time series, which is also used in a different context and illustrated later in Fig. 9(a), is meant to simulate a hypothesized increase in the stomatal resistance of a crop in
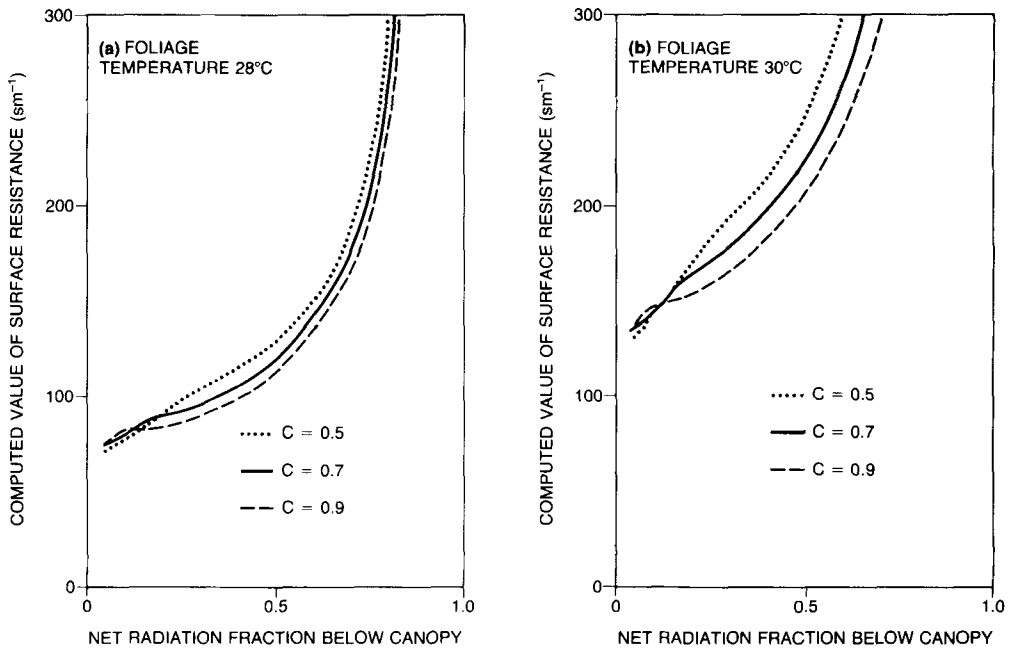
Figure 7. The variation in surface resistance as a function of the proportion of net radiation measured beneath the canopy calculated for the meteorological conditions described in the text, and with foliage temperature (a) 28 °C and (b) 30 °C. Calculations are made with submodel SG for three different values of C, the net radiation extinction coefficient. $C = 0.7$ is the preferred value.

response to increasing water stress, which is then reversed by rain (or irrigation) overnight between days 8 and 9. We concentrate on three of the worst-case sensitivities given in Tables 1 and 2, namely halving and doubling w with $L = 1$, and n and c_d with $L = 4$. Figures 8(a), (b) and (c) correspond to these three situations.

In calculating this figure the sparse-canopy theory of Shuttleworth and Wallace (1985) is used predictively to calculate the canopy exchange for the preferred, halved and doubled values of the relevant model parameter using the SG submodel of aerodynamic resistances, i.e. with Eqs. (45) and (46) and associated equations in the present paper replacing the original Eqs. (30) and (31) of Shuttleworth and Wallace. Using this theory implicitly calculates T_f: its value is explicitly available from the equation

$$T_f = T_r + (A - \lambda E)r_a^a/\rho c_p + (A - A_s - \lambda E_c)r_a^c/\rho c_p. \qquad (49)$$

Substituting this foliage temperature into Eq. (28) of the present paper then allows a calculation of surface resistance assuming the preferred values ($w = 0.02$, $n = 2.5$, $c_d = 0.07$) used elsewhere in this analysis. The above procedure therefore simulates the consequences of assuming these preferred values for canopies where the actual values differ by a factor two.

Inspection of Fig. 8 certainly confirms that there is a significant miscalculation of surface resistance with these extreme changes in these three worst-case examples. This in part takes the form of a systematic shift in the offset of the calculated value. However, even in these worst cases, the wrongly specified canopy aerodynamics is not sufficient to mask a factor two change in r_s^c. At the same time, it is emphasized that the indiscriminate use of the aerodynamic model and preferred parameters used in this paper to calculate the canopy resistance of sparse crops of diverse structure clearly could be prone to systematic error in the order of 20%.

TABLE 1.   CALCULATED CANOPY RESISTANCE IN s m^{-1} FOR $D_r = 15$ mb, $T_c = 25$ °C AND $T_f = 28$ °C, WITH CHANGES IN PARAMETRIZATION OF AERODYNAMIC RESISTANCE (SEE TEXT)

Model change	Leaf area index					
	0·5	1·0	1·5	2·0	3·0	4·0
(a) $w = 0·01$ m	189	130	110	101	87	80
No change	179	121	102	93	82	76
$w = 0·04$ m	170	113	95	87	78	73
	(11)	(14)	(15)	(15)	(11)	(9)
(b) $n = 1·25$	181	127	109	101	90	84
No change	179	121	102	93	82	76
$n = 5·00$	162	102	84	75	65	61
	(11)	(21)	(25)	(28)	(30)	(30)
(c) $c_d = 0·035$	179	122	104	95	89	87
No change	179	121	102	93	82	76
$c_d = 0·140$	177	117	94	82	70	62
	(1)	(4)	(10)	(14)	(23)	(33)

Numbers in brackets are the full range difference in the two perturbed values expressed as a percentage of the unperturbed value.

TABLE 2.   AS TABLE 1 BUT WITH $T_f = 30$ °C

Model change	Leaf area index					
	0·5	1·0	1·5	2·0	3·0	4·0
(a) $w = 0·01$ m	370	249	208	188	161	147
No change	337	224	188	171	149	139
$w = 0·04$ m	316	209	175	160	142	133
	(16)	(18)	(18)	(16)	(13)	(10)
(b) $n = 1·25$	348	242	207	190	167	156
No change	337	224	188	171	149	139
$n = 5·00$	310	192	154	137	119	110
	(11)	(22)	(28)	(31)	(32)	(33)
(c) $c_d = 0·035$	339	228	192	176	163	160
No change	337	224	188	171	149	139
$c_d = 0·140$	333	217	173	151	128	115
	(2)	(5)	(10)	(15)	(23)	(32)

Numbers in brackets are the full range difference in the two perturbed values expressed as a percentage of the unperturbed value.

5.   SUBSTRATE RESISTANCE

The present application of the Shuttleworth and Wallace (1985) sparse-crop model makes assumptions regarding aerodynamic transfer which are explored in the previous section and which necessarily limit its descriptive accuracy. However, the most significant inhibition on the utility of the present extension to Shuttleworth and Wallace's theory is likely to be the poor definition of the substrate resistance, r_s^s, whose value will remain unknown or poorly measured in many practical applications. The presence of a finite but unmeasured subsurface contribution to the above-canopy energy fluxes, which is probably the dominant contribution in very sparse crops, must appear in the equations derived here, and restrict their effectiveness in reinterpreting (as canopy resistance) those variables which are measured.
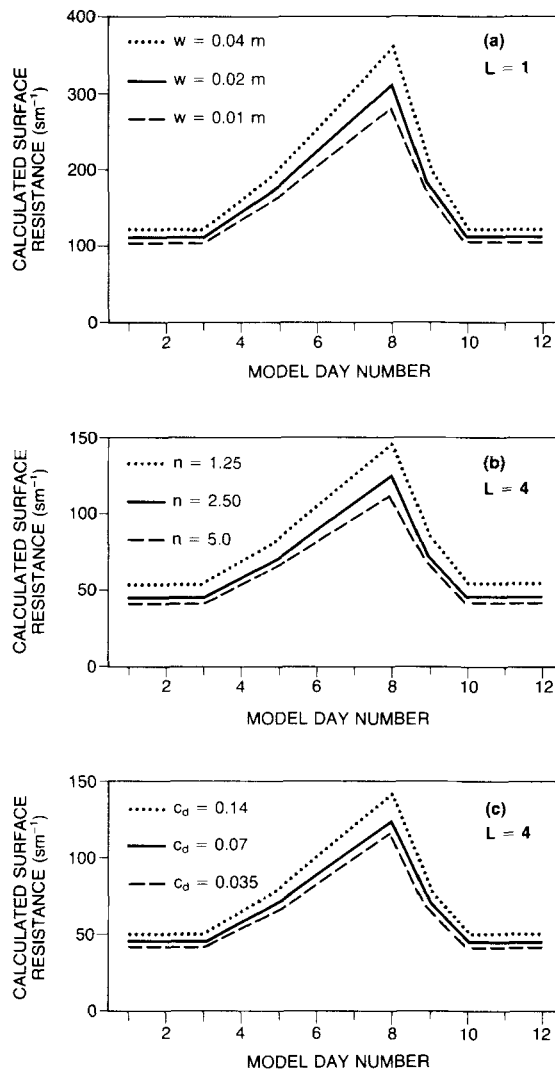
Figure 8. Changes in the calculated value of surface resistance in response to halving and doubling the preferred values of (a) w, leaf width, (b) n, the eddy correlation extinction coefficient, and (c) c_d, the individual element exchange coefficient, at values of leaf area index, $L = 1$ and 4 respectively. These values are selected, from Tables 1 and 2, to exhibit worst-case sensitivity. The time series of surface conductance values described in the text is used in each case to calculate energy partition and foliage temperature: this foliage temperature is then substituted into Eq. (28) to recalculate surface conductance with the preferred values of w, n and c_d.

To remain consistent with the philosophy adopted in previous sections we investigate the performance of the present theory in the face of the most exacting assumption available, namely that r_s^* retains a fixed and arbitrary value, of $500 \, \text{s} \, \text{m}^{-1}$. The consequences of this assumption are tested in a manner analogous to that used in the previous section. In this case the moisture stress factor and subsurface resistance are both prescribed a 12-day time series of midday values, which are shown in Figs. 9(a) and (b) respectively. These are meant to be an idealized representation of a drying sequence, with the onset of crop water stress and soil drying from days 3 to 8, followed by rain (or irrigation) overnight between days 8 and 9, which relieves crop stresses and (initially) saturates the soil.
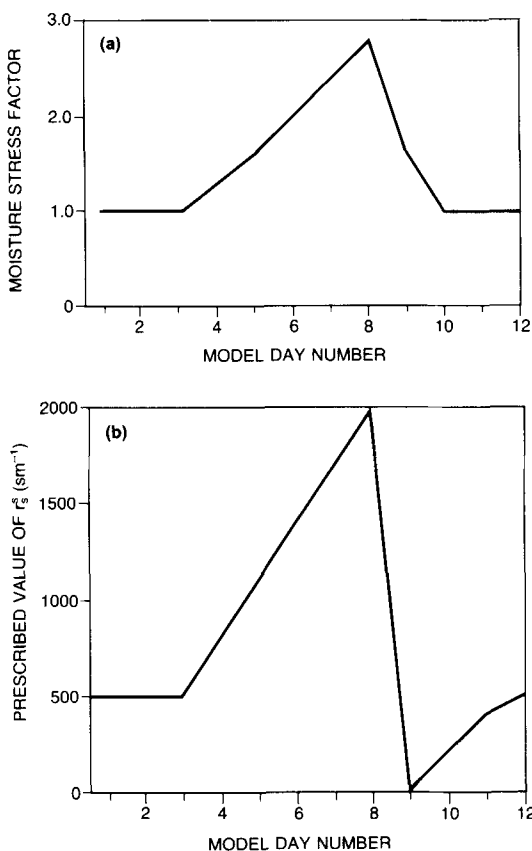
Figure 9. The prescribed time series of midday moisture stress factor (m) and subsurface resistance used in demonstrating the sensitivity to the surface resistance of the substrate. The series represents possible changes in these resistances during a drying cycle, from day 3 to day 8, which is immediately followed by rain (or irrigation) overnight between days 8 and 9.

These prescribed time series are again used to calculate the crop's energy partition (assuming the constant midday climate described earlier) following Shuttleworth and Wallace (1985), with the new aerodynamic submodel SG. The foliage temperature given by Eq. (49) is then reintroduced into Eq. (28) and associated equations, and the canopy resistance calculated under the assumption that r_s^c is unknown but arbitrarily set to $500 \, s \, m^{-1}$.

The net result of this procedure is shown in Fig. 10 for three different values of leaf area index, $L = 4$, 2·5 and 1. For progressively sparser canopies, they illustrate the consequences of assuming a constant r_s^c when the actual value changes, in a hopefully plausible way, in the range 0 to $2000 \, s \, m^{-1}$ during a drying/wetting event. Obviously there is perfect agreement between the canopy resistance given by the present theory and that input to the Shuttleworth and Wallace (1985) model when the latter calculates energy partition with $r_s^c = 500 \, s \, m^{-1}$ on days 1, 2, 3 and 12. This merely confirms the consistency of the two sets of equations. There is in general also a worthwhile, though imperfect, level of agreement at high ($L = 4$) and intermediate ($L = 2·5$) values of leaf area index. However, there is a dramatic miscalculation of canopy resistance at low canopy cover ($L = 1$), when, in effect, changes in the large subsurface contribution to the evaporation are misinterpreted as changes in canopy control. Perhaps the only
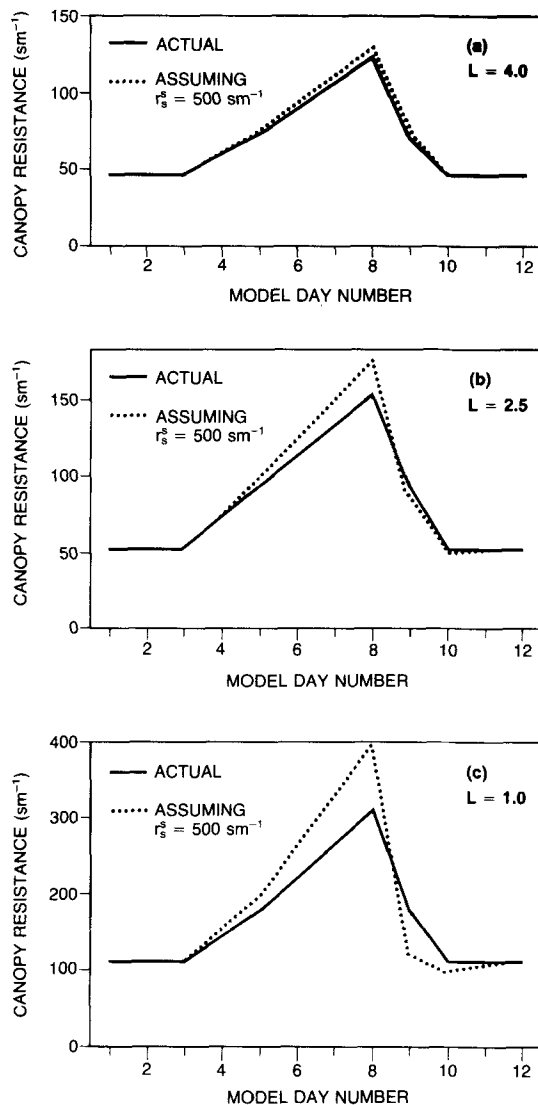
Figure 10. The canopy resistance calculated from Eq. (28) (wrongly) assuming a fixed value of $500 \, \text{s m}^{-1}$ for r_s^s, for the fixed meteorological conditions described in the text, but with the foliage temperature determined from the Shuttleworth and Wallace (1985) sparse-crop theory modified to include submodel SG. The time series of rm and r_s^s illustrated in Fig. 9 were used as input to the energy partition calculation.

consolation is that in the real world the correlation between changes in soil and crop components is well defined, i.e. drying soils are associated with increasing crop stress and *vice versa*. For this reason the amplification of changes in the apparent value of r_s^c is probably always greater than unity. Some possible applications of the present theory (e.g. diagnosing crop stress from canopy resistance and taking corrective action through irrigation) may even benefit from this, in that the system will err on the side of early intervention. In practice, field research would soon follow to allow optimized water use by providing an empirical description of this 'amplification factor'.

At a more basic level, the only way to determine canopy conductance better from the present theory is to provide it with a better value of r_s^s. The sensitivity at low leaf

area index is such that quite coarse measurements or estimates (perhaps using simple models based on the number of days since rain) are likely to give a marked improvement in the estimate of r_s^c.

Perhaps such information could be provided indirectly. The most consistent indirect information would be a measurement of the surface temperature of the soil. Clearly there are practical problems involved in providing this, not least the sampling problems associated with any below-canopy measurement. However, given the feasibility of such a measurement, it is possible to rework the sparse-crop model of Shuttleworth and Wallace and incorporate the aerodynamic assumptions introduced earlier in this paper to exploit the additional measurement.

Starting from Eq. (20), substituting for H, H_c and H_s from Eqs. (12), (13) and (14) and then rearranging, yields an equation for T_o in terms of the (now) measured values of T_r, T_f and T_s thus

$$T_o = (r_a^a r_a^s T_f + r_a^a r_a^c T_s + r_a^c r_a^s T_r)/(r_a^a r_a^c + r_a^a r_a^s + r_a^c r_a^s). \tag{50}$$

Substituting Eqs. (9) and (12) into Eq. (5) and rearranging then yields an equation for e_o thus

$$e_o = e_r + \gamma[r_a^a A - (T_o - T_r)]. \tag{51}$$

Subtracting Eq. (7) from Eq. (5), and then substituting Eqs. (20) and (26) gives

$$(A - A_s) = \lambda E_c + H_c \tag{52}$$

which, on substituting for λE_c and H_c from Eqs. (10) and (13) and rearranging, yields an equation analogous to Eq. (4), thus

$$r_s^c = \frac{(\rho c_p/\gamma)[e_w(T_f) - e_o]}{(A - A_s) - \rho c_p(T_f - T_o)/r_a^c} - r_a^c. \tag{53}$$

Clearly an analogous equation exists for r_s^s with the form

$$r_s^s = \frac{(\rho c_p/\gamma)[e_w(T_s) - e_o]}{A_s - \rho c_p(T_s - T_o)/r_a^s} - r_a^s. \tag{54}$$

6. CONCLUDING REMARKS

This paper investigates the use of the (predictive) sparse-crop model of Shuttleworth and Wallace (1985), reinterpreted into diagnostic mode to deduce the canopy resistance of vegetation of varying density from measurements of foliage temperature. The necessary algebraic manipulation (given in section 3 and appendices A and B) is fairly complex but gives rise to a simple expression, Eq. (28), which correctly reduces to Eq. (4), the form previously derived ignoring the interaction with the substrate beneath the crop.

Though it is not convenient to consider the practical difficulties involved in applying Eq. (28) into the present discussion, these should not be underestimated. The use of a below-canopy net radiation measurement to estimate leaf area index from Eq. (29) requires adequate, time-average sampling with tube net radiometers. More problematic than this is the precise estimation of surface temperature, as mentioned in the introduction. Perhaps these practical difficulties are such that use of this theory will be in studying or monitoring *change* in canopy resistance, rather than in providing an accurate estimate of its absolute value.

In section 4 a new submodel of aerodynamic transfer, SG, is introduced to supersede that of Shuttleworth and Wallace (1985). This empirically and simply simulates the more important results of a second-order closure theory of in-canopy turbulence (Shaw and Pereira 1982) and, for this reason, is arguably superior to the earlier submodel SW. Its use is recommended in both diagnostic and predictive applications of Shuttleworth and Wallace's (1985) theory merely on these grounds, though the numerical consequences of its introduction seem limited. In this same section a new submodel is also used to describe the leaf area and radiation dependence of r_s^c. This novel model is of potential use outside the present analysis, and merits the reader's attention. Equation (36) is a formula which is based on rather simple physical principles, yet captures much of both the intuitive and observational behaviours of field crops.

Later sections of this paper investigate the sensitivity of the theory to the assumed magnitude and form of individual model components. Realistic changes in the value of C, the parameter controlling radiation extinction in the canopy, generate changes in the calculated canopy resistance in the order of 5%. More extreme changes in canopy aerodynamics, i.e. halving and doubling the value of the controlling parameters, generate changes in the order of 20%. This sensitivity is therefore significant but probably would not in itself preclude the use of Eq. (28) to monitor any large *change* in canopy resistance as a hypothetical diagnostic of the onset of crop water stress.

A more significant and outstanding problem in this context is the poorly specified surface resistance of the substrate. Section 5 investigates the sensitivity with respect to this parameter. The conclusion is that assuming a constant value (of 500 s m^{-1}) does not catastrophically contaminate the calculated value of r_s^c for canopies of high ($L = 4$) or intermediate ($L = 2 \cdot 5$) leaf area index. However, for sparse canopies ($L = 1$) the likely behaviour of r_s^s during drying and wetting cycles is such as to greatly amplify the apparent change in the calculated canopy resistance of the vegetation. In practical applications such amplification might even prove useful, but at a more basic level a better specification of substrate interaction is identified as the primary problem in using this theory in very sparse canopies. Simple models might suffice, but a measurement of substrate temperature used in conjunction with Eqs. (50), (51) and (52) represents an attractive and simple alternative.

APPENDIX A

Substituting Eq. (7) into Eq. (20) gives

$$A_s + H_c - H = \lambda E_s \qquad (A1)$$

which, on substituting Eqs. (12), (13) and (19), and then (18) becomes

$$A_s + \rho c_p (T_f - T_o)/r_a^c - \rho c_p (T_o - T_r)/r_a^a$$

$$= \frac{\Delta A_s + \rho c_p [D_r + (\Delta + \gamma)(T_o - T_r) - \gamma r_a^a A/\rho c_p]/r_a^s}{\Delta + \gamma(1 + r_s^s/r_a^s)}. \qquad (A2)$$

Multiplying both numerator and denominator on the right-hand side of Eq. (A2) by r_a^s, adopting the Shuttleworth and Wallace (1985) definition

$$R_s = (\Delta + \gamma)r_a^s + \gamma r_s^s \qquad (A3)$$

and multiplying Eq. (A2) by the product $(r_a^a r_a^c R_s)$ gives

$$r_a^a r_a^c R_s A_s + \rho c_p r_a^a R_s (T_f - T_o) - \rho c_p r_a^c R_s (T_o - T_r)$$

$$= \Delta r_a^a r_a^c r_a^s A_s + \rho c_p r_a^a r_a^c D_r + \rho c_p r_a^a r_a^c (\Delta + \gamma)(T_o - T_r) - \gamma r_a^a r_a^a r_a^c A. \qquad (A4)$$

Now

$$\rho c_p r_a^a R_s (T_f - T_o) = \rho c_p r_a^a R_s (T_f - T_r) - \rho c_p r_a^a R_s (T_o - T_r) \qquad (A5)$$

and introducing Eq. (A5) into Eq. (A4), then collecting terms in $(T_o - T_r)$ and A_s gives

$$\rho c_p (T_o - T_r)[r_a^a r_a^c (\Delta + \gamma) + r_a^a R_s + r_a^c R_s]$$

$$= r_a^a r_a^c A_s (R_s - \Delta r_a^s) + \rho c_p r_a^a R_s (T_f - T_r) - \rho c_p r_a^a r_a^c D_r + \gamma r_a^a r_a^a r_a^c A. \qquad (A6)$$

Dividing Eq. (A6) by $(r_a^a R_s)$ and rearranging gives

$$\frac{\rho c_p (T_o - T_r)}{r_a^a}(r_a^a + r_a^c) + (\Delta + \gamma)(r_a^a r_a^c/R_s)$$

$$= \rho c_p (T_f - T_r) - \frac{\rho c_p D_r}{r_a^a}(r_a^a r_a^c/R_s) + \gamma A(r_a^a r_a^c/R_s) + \frac{A_s}{r_a^a}(R_s - \Delta r_a^s)(r_a^a r_a^c/R_s). \quad (A7)$$

Dividing Eq. (A7) by $(r_a^a + r_a^c)$, and setting

$$\eta = (r_a^a r_a^c)/\{R_s(r_a^a + r_a^c)\} \qquad (A8)$$

gives

$$\rho c_p \frac{(T_o - T_r)}{r_a^a}[1 + \eta(\Delta + \gamma)] = \frac{\rho c_p(T_f - T_r)}{(r_a^a + r_a^c)} + \eta\{\gamma A + A_s(R_s - \Delta r_a^s)/r_a^a - \rho c_p D_r/r_a^a\}$$

$$(A9)$$

which can be written as

$$(T_o - T_r) = f\frac{r_a^a(T_f - T_r)}{(r_a^a + r_a^c)} + r_a^a F/(\rho c_p) \qquad (A10)$$

where

$$f = [1 + \eta(\Delta + \gamma)]^{-1} \qquad (A11)$$

and

$$F = f\eta(\gamma A + \gamma A_s(r_s^s + r_a^s)/r_a^a - \rho c_p D_r/r_a^a) \qquad (A12)$$

APPENDIX B

Substituting Eqs. (9) and (10) into Eq. (26), and rearranging, gives

$$r_s^c = \frac{\rho c_p r_a^a [e_w(T_f) - e_o]}{\rho c_p (e_o - e_r) - \gamma r_a^a \lambda E_s} - r_a^a. \tag{B1}$$

Substituting Eq. (15) into Eq. (B1) then gives

$$r_s^c = \frac{r_a^a \{\rho c_p [e_w(T_f) - e_r] - [\gamma r_a^a A - \gamma \rho c_p (T_o - T_r)]\}}{\gamma r_a^a (A - \lambda E_s) - \gamma \rho c_p (T_o - T_r)} - r_a^a. \tag{B2}$$

Introducing the identity $\gamma r_a^a \lambda E_s - \gamma r_a^a \lambda E_s \equiv 0$ into the numerator of (B2) gives

$$r_s^c = \frac{r_a^a \{\rho c_p [e_w(T_f) - e_r] - \gamma r_a^a \lambda E_s - [\gamma r_a^a (A - \lambda E_s) - \gamma \rho c_p (T_o - T_r)]\}}{\gamma r_a^a (A - \lambda E_s) - \gamma \rho c_p (T_o - T_r)} - r_a^a \tag{B3}$$

which after further manipulation becomes

$$r_s^c = \frac{(\rho c_p / \gamma)[e_w(T_f) - e_r] - r_a^a \lambda E_s}{(A - \lambda E_s) - \rho c_p (T_o - T_r)/r_a^a} - (r_a^a + r_a^c). \tag{B4}$$

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