Proof Theoretic Semantics for Questions

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In addition to $\mathcal{L}_e$, we construct another language, called $\mathcal{L}_Q$ which has two categories of well-formed expressions: *d-wffs* and *e-wffs*, that is, claims and questions. (For simplicity, we drop the '*e*' notation from our turnstile.) The syntax of $\mathcal{L}_Q$ is just that of $\mathcal{L}_e$ plus the following: ?, {, }, and the comma. D-wffs are just sentences of $\mathcal{L}_e$. E-wffs, on the other hand, are expressions of the following form: $?\{A_1, \ldots, A_n\}$ where $n > 1$ and $A_1 \ldots A_n$ are pairwise syntactically distinct d-wffs of $\mathcal{L}_e$. If $?\{A_1, \ldots, A_n\}$ is a question, then each of the d-wffs $A_1 \ldots A_n$ is a possible direct answer to the question, and these are the only possible direct answers. Not that any question in $\mathcal{L}_Q$ has at least two possible direct answers and that the set of those answers is always finite. We must stress that e-wffs are NOT sets of d-wffs, but rather singular expressions of a strictly defined form. We can paraphrase e-wffs as aksing *Which of the following claims am I entitled to:* $A_1, \ldots, A_n$*?* .

Here are possible rules for e-wffs:

Right rule for $?\{A_1, \ldots, A_n\}$

$$\frac{\Gamma,\, A_1 \mathrel{|\!\!\sim} B \ldots \Gamma,\, A_n \mathrel{|\!\!\sim} B}{\Gamma,\, B \mathrel{|\!\!\sim}\ ?\{A_1, \ldots, A_n\}} \qquad \text{where } \Gamma, B \mathrel{|\!\!\not\sim} A_1 \ldots \Gamma, B \mathrel{|\!\!\not\sim} A_n$$

Possible left rule for $?\{A_1, \ldots, A_n\}$

$$\frac{\Gamma \mathrel{|\!\!\sim} A_i}{\Gamma,\ ?\{A_1, \ldots, A_n\} \mathrel{|\!\!\sim} \bot} \qquad \text{where } 1 \leq i \leq n$$

(Negative) Partial Answer

$$\frac{\Gamma,\, B,\, A_i \mathrel{|\!\!\sim} \bot}{\Gamma,\, B,\ ?\{A_1, \ldots, A_n\} \mathrel{|\!\!\sim}\ ?\{A_1, \ldots, A_{n-1}\}} \qquad \text{where } n > 2 \text{ and } 1 \leq i \leq n$$

We can give the semantics for a non-factive why-question, *Why B?*, as follows:

$$?\{A_1 \twoheadrightarrow B, \ldots, A_n \twoheadrightarrow B\}$$

Once $\mathcal{L}_e$ is extended to include traditional logical operators, we can give the semantics for *factive* why questions, e.g. *Why B?*, as follows:

$$?\{(A_1 \twoheadrightarrow B) \wedge B, \ldots, (A_n \twoheadrightarrow B) \wedge B\}$$

Notice that once one has an answer to this question, one is entitled to the explanans.

$$\frac{\Gamma,\ A \mid\!\sim^{\twoheaduparrow}_{e} B}{\Gamma \ \mid\!\sim_{e} \ A \twoheadrightarrow B} \qquad \text{EER}$$

$$\frac{\Gamma \mid\!\sim_{e} \ A \twoheadrightarrow B}{\Gamma,\ A \ \mid\!\sim^{\twoheaduparrow}_{e} \ B} \qquad \text{EES}$$

$$\frac{\Gamma,\ A \mid\!\sim^{\twoheaduparrow}_{e} B \qquad \Gamma \ \mid\!\sim_{e} B}{\Gamma, B, A \twoheadrightarrow B \ \mid\!\sim_{e} A} \qquad \text{EEL}$$

Quantified Material Consequence (QMC):

$$\Gamma, A \ \mid\!\sim^{\uparrow W} B \Longleftrightarrow_{df} \begin{cases} 1.\ W \subseteq \mathcal{P}(\mathcal{L}) & \text{and} \\ 2.\ \forall \Delta \in W(\Gamma, A, \Delta \mid\!\sim\ B) \end{cases}$$

Modally Robust Consequence (MRC):

$$\Gamma, A \ \mid\!\sim^{\Uparrow W} B \Longleftrightarrow_{df} \begin{cases} 1.\ \Gamma, A \ \mid\!\sim^{\uparrow W} B & \text{and} \\ 2.\ \forall W'(\Gamma, A \ \mid\!\sim^{\uparrow W'} B \Longrightarrow W' \subseteq W) & \text{and} \\ 3.\ \forall \Delta(\Gamma, A, \Delta \ \mid\!\sim^{\uparrow} \perp \Longrightarrow \Delta \notin W) \end{cases}$$

Sturdy Consequence (SC): [Sets]

$$\Gamma,\ \Sigma \ \mid\!\sim^{\twoheaduparrow} B \Longrightarrow \begin{cases} 1.\ \Gamma,\ \Sigma \ \mid\!\sim^{\Uparrow W} B & \text{where } B \notin \Sigma\text{, and} \\ 2.\ \forall \Theta\, (\Gamma, \Theta \ \mid\!\sim^{\Uparrow W'} B \Longrightarrow W \not\subset W') & \text{where } B \notin \Theta \end{cases}$$