

Quantified Material Consequence (QMC):

$$\Gamma, \varphi \vdash^W \psi \iff_{df} \begin{cases} 1. W \subseteq \mathcal{P}(\mathcal{L}) \\ 2. \forall \Delta \in W (\Gamma, \varphi, \Delta \vdash \psi) \end{cases} \quad \text{and}$$

Modally Robust Consequence (MRC):

$$\Gamma, \varphi \vdash^W \psi \iff_{df} \begin{cases} 1. \Gamma, \varphi \vdash^W \psi \\ 2. \forall W' (\Gamma, \varphi \vdash^{W'} \psi \implies W' \subseteq W) \\ 3. \forall \Delta (\Gamma, \varphi, \Delta \vdash \perp \implies \Delta \notin W) \end{cases} \quad \begin{matrix} \text{and} \\ \text{and} \end{matrix}$$

Sturdy Consequence (SC):

$$\Gamma, \varphi \vdash^{\dagger} \psi \implies \begin{cases} 1. \Gamma, \varphi \vdash^W \psi \\ 2. \forall \vartheta (\Gamma, \vartheta \vdash^{W'} \psi \implies W \not\subseteq W') \end{cases} \quad \begin{matrix} \text{where } \varphi \neq \psi, \text{ and} \\ \text{where } \psi \neq \vartheta \end{matrix}$$

Sturdy Consequence (SC): [Sets]

$$\Gamma, \Sigma \vdash^{\dagger} \varphi \implies \begin{cases} 1. \Gamma, \Sigma \vdash^W \varphi \\ 2. \forall \Theta (\Gamma, \Theta \vdash^{W'} \varphi \implies W \not\subseteq W') \end{cases} \quad \begin{matrix} \text{where } \varphi \notin \Sigma, \text{ and} \\ \text{where } \varphi \notin \Theta \end{matrix}$$

$$\sigma \in \Sigma$$

$$\frac{\Gamma, \Sigma \vdash_e^{\dagger} \varphi}{\Gamma \vdash_e \Sigma \twoheadrightarrow \varphi} \quad \text{EER}$$

$$\frac{\Gamma \vdash_e \Sigma \twoheadrightarrow \varphi}{\Gamma, \Sigma \vdash_e \varphi} \quad \text{EES}$$

$$\frac{\Gamma, \Sigma \vdash_e^{\dagger} \varphi \quad \Gamma \vdash_e B}{\Gamma, \varphi, \Sigma \twoheadrightarrow \varphi \vdash_e \Sigma} \quad \text{EEL}$$