Proof Theoretic Semantics for Questions

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1 Interrogatives in NM-MS

We start with a language of atomic sentences, $\mathcal{L}_0 = \{p_1, \ldots, p_n\}$ and define a consequence relation over it such that $\mathrel{|\!\sim}_0 \subseteq \mathcal{P}(\mathcal{L}_0) \times \mathcal{P}(\mathcal{L}_0)$. The structure $\langle \mathrel{|\!\sim}_0, \mathcal{L}_0 \rangle$ has the following properties:

1. $\emptyset \not\mathrel{|\!\sim}_0 \emptyset$
2. $\mathcal{L}_0 \mathrel{|\!\sim}_0 \emptyset$
3. $\forall \Delta, \Lambda, p\,(\Delta, p \mathrel{|\!\sim}_0 p, \Lambda)$ (**Containment**)

We quantify over our consequence relations by sets of 2-tuples $W \subseteq \mathcal{P}(\mathcal{L}_0) \times \mathcal{P}(\mathcal{L}_0)$ in accordance with the following axiom: if $\forall \langle \Delta, \Lambda \rangle \in W(\Delta, \Gamma \mathrel{|\!\sim}_0 \Theta, \Lambda)$ then $\Gamma \mathrel{|\!\sim}^{\updownarrow W}_0 \Theta$.

We extend $\langle \mathrel{|\!\sim}_0, \mathcal{L}_0 \rangle$ to $\langle \mathrel{|\!\sim}, \mathcal{L} \rangle$ where $\mathcal{L} = \mathcal{L}_0 \cup \{?\}$. The latter is an erotetic operator that maps sets of sentences in $\mathcal{L}_0$ to formulas in $\mathcal{L}$, i.e $\Delta \mapsto ?\Delta$. We call formulas of $\mathcal{L}_0$ d-wffs and call the formulas in $\{\mathcal{L}\} - \{\mathcal{L}_0\}$ e-wffs.

Syntax of $\mathcal{L}$: $\phi ::= p \mid ?\{\phi_1, \ldots, \phi\}$

Possible rules for e-wffs:

Right rule for $?\Delta$

$$\frac{\Gamma \mathrel{|\!\sim} \Theta}{\Gamma \mathrel{|\!\sim} ?\Theta} \qquad \text{where } \Gamma \not\mathrel{|\!\sim} \theta \text{ for any } \theta \in \Theta$$

Possible left rule for $?\{A_1, \ldots, A_n\}$

$$\frac{\Gamma \mathrel{|\!\sim} \{A_i\}}{\Gamma, ?\{A_1, \ldots, A_n\} \mathrel{|\!\sim} \emptyset} \qquad \text{where } 1 \leq i \leq n$$

(Negative) Partial Answer

$$\frac{\Gamma, \Delta \mathrel{|\!\sim} \emptyset}{\Gamma, ?(\Delta \cup \Theta) \mathrel{|\!\sim} ?\Theta}$$

2 Interrogatives in NM-SS

In addition to $\mathcal{L}_e$, we construct another language, called $\mathcal{L}_Q$ which has two categories of well-formed expressions: *d-wffs* and *e-wffs*, that is, claims and questions. (For simplicity, we drop the '*e*' notation from our turnstile.) The syntax of $\mathcal{L}_Q$ is just that of $\mathcal{L}_e$ plus the following: ?, {, }, and the comma. D-wffs are just sentences of $\mathcal{L}_e$. E-wffs, on the other hand, are expressions of the following form: $?\{A_1, \ldots, A_n\}$ where $n > 1$ and $A_1 \ldots A_n$ are pairwise syntactically distinct d-wffs of $\mathcal{L}_e$. If $?\{A_1, \ldots, A_n\}$ is a question, then each of the d-wffs $A_1 \ldots A_n$ is a possible direct answer to the question, and these are the only possible direct answers. Not that any question in $\mathcal{L}_Q$ has at least two possible direct answers and that the set of those

answers is always finite. We must stress that e-wffs are NOT sets of d-wffs, but rather singular expressions of a strictly defined form. We can paraphrase e-wffs as aksing *Which of the following claims am I entitled to:* $A_1, \ldots, A_n$*?* .
Here are possible rules for e-wffs:

Right rule for $?\{A_1, \ldots, A_n\}$

$$\frac{\Gamma,\, A_1 \mid\!\sim B \ldots \Gamma,\, A_n \mid\!\sim B}{\Gamma,\, B \;\mid\!\sim\; ?\{A_1, \ldots, A_n\}} \qquad \text{where } \Gamma, B \not\mid\!\sim A_1 \ldots \Gamma, B \not\mid\!\sim A_n$$

Possible left rule for $?\{A_1, \ldots, A_n\}$

$$\frac{\Gamma \mid\!\sim A_i}{\Gamma,\, ?\{A_1, \ldots, A_n\} \;\mid\!\sim\; \bot} \qquad \text{where } 1 \leq i \leq n$$

(Negative) Partial Answer

$$\frac{\Gamma,\, B \mid\!\sim \bot}{\Gamma,\, ?\{A_1, \ldots, A_n, B\} \;\mid\!\sim\; ?\{A_1, \ldots, A_n\}}$$

We can give the semantics for a non-factive why-question, *Why B?*, as follows:

$$?\{A_1 \twoheadrightarrow B, \ldots, A_n \twoheadrightarrow B\}$$

Once $\mathcal{L}_e$ is extended to include traditional logical operators, we can give the semantics for *factive* why questions, e.g. *Why B?*, as follows:

$$?\{(A_1 \twoheadrightarrow B) \wedge B, \ldots, (A_n \twoheadrightarrow B) \wedge B\}$$

Notice that once one has an answer to this question, one is entitled to the explanans.

$$\frac{\Gamma,\ A \overset{\twoheaduparrow}{\mid\!\sim}_e B}{\Gamma\ \mid\!\sim_e\ A \twoheadrightarrow B} \qquad \text{EER}$$

$$\frac{\Gamma \mid\!\sim_e\ A \twoheadrightarrow B}{\Gamma,\ A\ \overset{\twoheaduparrow}{\mid\!\sim}_e\ B} \qquad \text{EES}$$

$$\frac{\Gamma,\ A \overset{\twoheaduparrow}{\mid\!\sim}_e B \qquad \Gamma\ \mid\!\sim_e B}{\Gamma, B, A \twoheadrightarrow B\ \mid\!\sim_e A} \qquad \text{EEL}$$

Quantified Material Consequence (QMC):

$$\Gamma, A\ \overset{\uparrow W}{\mid\!\sim} B \Longleftrightarrow_{df} \begin{cases} 1.\ W \subseteq \mathcal{P}(\mathcal{L}) & \text{and} \\ 2.\ \forall \Delta \in W(\Gamma, A, \Delta \mid\!\sim B) \end{cases}$$

Modally Robust Consequence (MRC):

$$\Gamma, A\ \overset{\Uparrow W}{\mid\!\sim} B \Longleftrightarrow_{df} \begin{cases} 1.\ \Gamma, A\ \overset{\uparrow W}{\mid\!\sim} B & \text{and} \\ 2.\ \forall W'(\Gamma, A\ \overset{\uparrow W'}{\mid\!\sim} B \Longrightarrow W' \subseteq W) & \text{and} \\ 3.\ \forall \Delta(\Gamma, A, \Delta\ \overset{\uparrow}{\mid\!\sim}\ \bot \Longrightarrow \Delta \notin W) \end{cases}$$

Sturdy Consequence (SC): [Sets]

$$\Gamma,\ \Sigma\ \overset{\twoheaduparrow}{\mid\!\sim} B \Longrightarrow \begin{cases} 1.\ \Gamma,\ \Sigma\ \overset{\Uparrow W}{\mid\!\sim} B & \text{where } B \notin \Sigma\text{, and} \\ 2.\ \forall \Theta\, (\Gamma, \Theta\ \overset{\Uparrow W'}{\mid\!\sim} B \Longrightarrow W \not\subset W') & \text{where } B \notin \Theta \end{cases}$$