Approaching the Shannon Limit Through Constellation Modulation

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Abstract: We show a spectral efficiency increase by combining constellation modulation and symbol mapping. The SE increases by 12.5% in theory and 6% in experiments for QPSK at a negligible SNR penalty for a BER=1×10^{-3}.

OCIS codes: (060.2330) Fiber optics communications; (060.1660) Coherent communications; (060.4080) Modulation

1. Introduction

Increasing the spectral efficiency (SE) in communications is a key challenge in both wired and wireless communications. As state-of-the art transmission systems are improving and getting closer to the Shannon limit, major efforts are needed to further improve the SE [1].

Large numbers of advancements have been obtained in wireless by introducing e.g. Trellis coding [2], MIMO coding [3], or time-space coding [4]. In recent years, several of these techniques have also been applied to increase the SE of optical transmission systems. Spectral shaping, orthogonal frequency division multiplexing and higher-order modulation [1, 5, 6] have all led to significant improvements. Advanced modulation formats in combination with coherent receiver technology and advanced digital signal processing (DSP) have made it possible to get closer to the Shannon limit. However, there is still a measurable gap [7]. In 2011, constellation switching was proposed as an efficient modulation format in wireless communications [8]. Unfortunately, this has not been investigated further.

In this paper, we propose to encode information both by symbol and constellation modulation to increase the SE. Theory, simulation and experiments demonstrate SE improvements at hardly any cost on the SNR. The technique is of particular interest when the SNR is low.

2. Constellation Modulation

Information can be encoded as symbols within a constellation. Yet the choice of a particular constellation from a set of possible constellation diagrams is information as well. The choice of the constellation is thus a way to encode additional information. In order to allow the detection of the constellation, the same constellation needs to be transmitted for a certain duration (meaning two or more symbols), see Fig. 1.

The encoding technique is shown in the block diagram in Fig. 1. An input bit stream is de-multiplexed in two streams of k and $L \times m$ bits. The first stream selects a constellation from a set of $K = 2^k$ constellations. Here, all K constellations can encode m bits. The second stream comprises the bits to be mapped on L symbols, each encoding m bits in the constellation selected above. Therefore, for every k bits of the first stream, $L \times m$ bits of the second stream will be encoded, see Fig. 1. The effective increase in data rate is therefore given by $\eta = k/(mL)$.

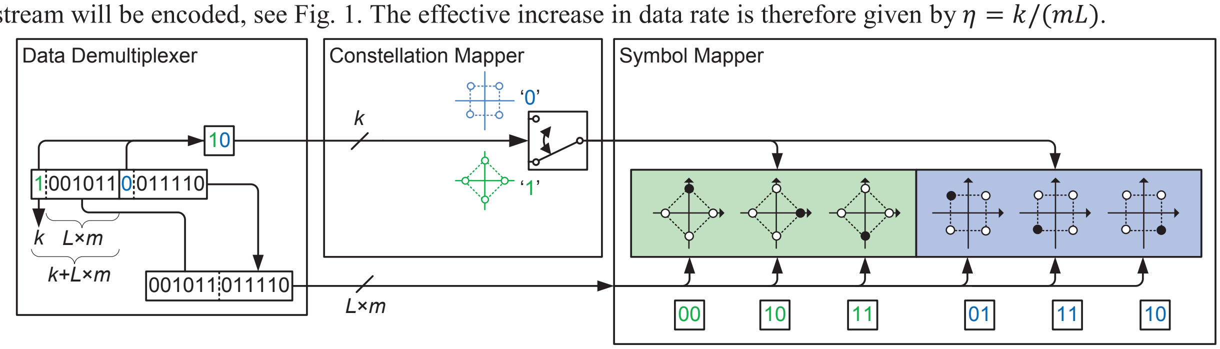

Fig. 1: This Figure shows a scenario with two constellations ($K = 2$): QPSK ('0'), and π/4 rotated QPSK ('1'). Each QPSK constellation encodes $m = 2$ bits. In this example, a block of $L = 3$ consecutive symbols are encoded in the same constellation. This way, each such block carries $k = 1$ additional bit. The effective increase in data rate is 17% in this example.

2. Theoretical Calculations and Simulations

To demonstrate the potential of this technique, we perform a theoretical analysis and perform numerical simulations. In these calculations, we start from the symbol error probability SEP_s of QPSK, which is for realistic SNR equal to

$$\mathrm{SEP}_s = \mathrm{erfc}\left(\frac{1}{\sqrt{2}}\frac{r}{\sigma}\sin\left(\frac{\pi}{4}\right)\right) = \mathrm{erfc}\left(\frac{r}{2\sigma}\right)$$

with the signal amplitude r and the noise variance σ^2.[9]

For a block of $L \geq 2$ QPSK symbols that carries $(2L+1)$ bits, the errors can be attributed to:

a) A wrong decoding of an individual symbol. This event appears with a probability of $L \times \mathrm{SEP}_s$. It leads in each case to a 1 bit error if Gray coding for QPSK is used.

b) A wrong identification of the constellation. The probability SEP_c of a wrong constellation detection is similar to the symbol error probability of a symbol in an 8-PSK constellation. Yet the noise variance is being reduced by averaging over L symbols, and thus effectively increasing the SNR by L

$$\mathrm{SEP}_\mathrm{c} = \mathrm{erfc}\left(\sqrt{L/2}\,\frac{r}{\sigma}\sin\left(\frac{\pi}{8}\right)\right)$$

Each error in the constellation detection leads to a wrong constellation bit. Additionally, the contained QPSK symbols may also be detected wrongly. As the QPSK symbols are now rotated by $\pi/2$, the decision thresholds are placed exactly on the symbols. This leads to an error probability of only 50%. Assuming Gray coding, the number of wrong bits is thus: $L/2$. The total number of bit errors caused by a wrong constellation detection is therefore in average: $L/2+1$. Finally the total error probability (BEP) for our constellation modulation example is given by

$$\mathrm{BEP}_\mathrm{CM} = \frac{(1-\mathrm{SEP}_\mathrm{c})(L\,\mathrm{SEP}_\mathrm{s}) + \mathrm{SEP}_\mathrm{c}(L/2+1)}{2L+1}.$$

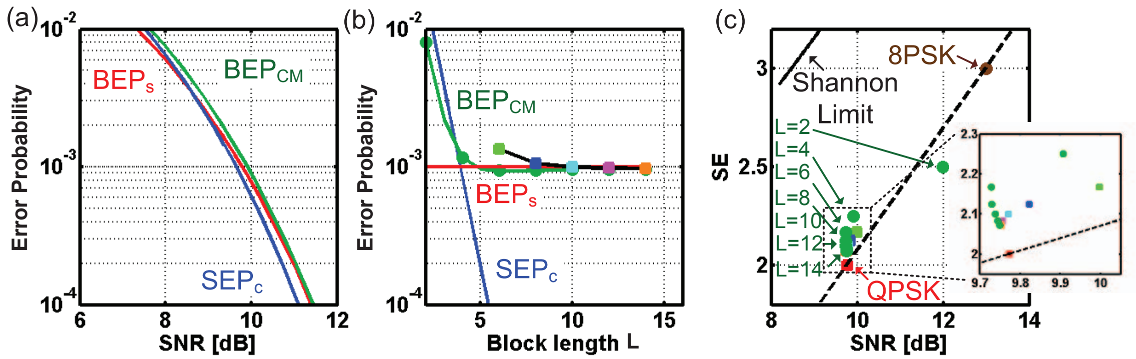

Fig. 2: Theoretical and simulation results. (a) The bit error probability of constellation modulation ($\mathrm{BEP}_{\mathrm{CM}}$–) for a block of $L = 4$ derived from the error probabilities of the standard QPSK BEP_s (–) and the constellation error detection $\mathrm{BEP}_\mathrm{m} = \mathrm{SEP}_\mathrm{m}$ (–). (b) Evolution of error probabilities with increasing block lengths L at an SNR = 9.8 dB (as required for a QPSK signal to reach a BER of 10^{-3}). The square symbols are results obtained by simulations. (c) SE gain at a BER of 10^{-3} vs. SNR for an encoding with and without constellation modulation. The dashed line is plotted as a reference. It comprises the SE versus SNR of QPSK and 8PSK at BER = 10^{-3} (solid circles). SE gains obtained with the novel constellation modulation techniques are plotted as open circles for different block lengths. Gains of up to 12.5 % are expected for block lengths of L = 4 at a BER = 10^{-3}. Simulation supports SE of 8.33% at a BER = 10^{-3} with block length L=6 (■).

The benefit of this scheme is illustrated in Fig. 2(a). Here we show the total bit error probability for constellation modulation $\mathrm{BEP}_{\mathrm{CM}}$, the bit error probability for QPSK with gray coding $\mathrm{BEP}_\mathrm{s} = \mathrm{SEP}_\mathrm{s}/2$, and the probability for a constellation identification error SEP_c as a function of the $\mathrm{SNR} = r^2/\sigma^2$. While SEP_c is large for low SNRs, it decreases quickly with increasing $\mathrm{SNR} = r^2/\sigma^2$ and increasing block length L. For $L > 4$, there is no penalty for constellation modulation over simple QPSK modulation.

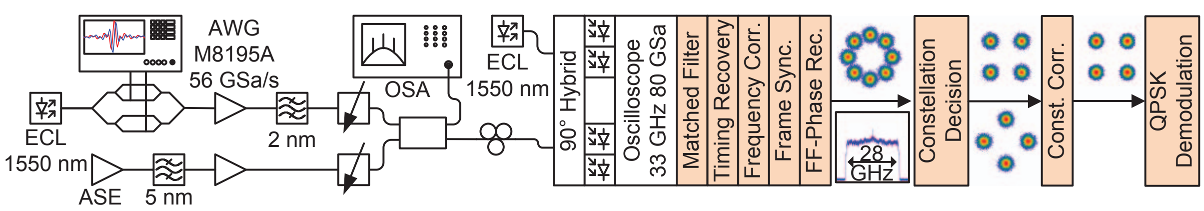

Fig. 3: Experimental setup: An external cavity laser (ECL, linewidth < 100 kHz) provides the optical carrier to the IQ-modulator that encodes the 28 GBd signal generated by the arbitrary waveform generator (AWG). The signal is then noise-loaded with an amplified spontaneous emission source (ASE). The OSNR is measured by the optical spectrum analyzer (OSA). The signal is received with a coherent receiver and offline DSP (orange boxes). The received spectrum is the same, with and without constellation modulation. Here, after matched filtering, timing recovery, and frequency offset correction, we perform data aided frame synchronization through pilot symbols and a feed forward phase recovery. The insets show the constellation similar to 8-PSK. Subsequent constellation decision with the minimum-mean-square error criterion separates the meta-symbols in two streams (see insets). The constellation is corrected and the resulting QPSK signal demodulated. Finally, bit errors are counted for the CM and QPSK data and the BER is calculated.

To verify this behavior and test our algorithm chain presented in Fig. 3, we have performed numerical simulations for constellation modulation encoded signals with different blocks lengths at an SNR of 9.8 dB. The SNR has been simulated as additive white Gaussian noise (AWGN). The simulation results are plotted as square symbols in Fig. 2(b) and (c), theory and simulations agree for $L \geq 8$. The employed algorithms [11, 12] allowed us to estimate the phase within six symbols, at the minimum length $L = 6$, an implementation penalty is observed. Improvements to these algorithms would allow for shorter block lengths, leading to a higher the SE gain.

3. Experiment

In our experiment shown in Fig. 3, we study the resilience of the constellation modulation to additive Gaussian noise for blocks of length L = 6; 8; 10; 12; 14. For each experimental point, 10^7 symbols are processed. The offline digital signal processing (DSP) is described in Fig. 3 and Refs. [10-12]. The overhead for pilot symbols is <1%.

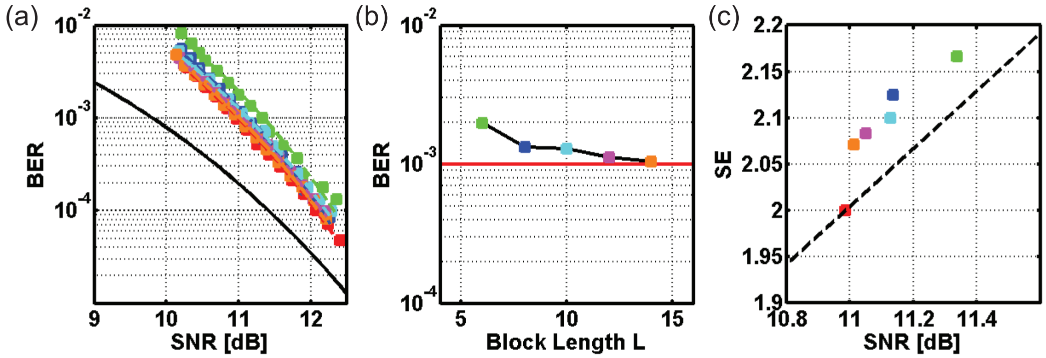

Fig. 4: Experimental results for constellation modulation with different block lengths L of 6 (■), 8 (■), 10 (■), 12 (■), and 14 (■) in comparison to QPSK (■,–). The theoretical limit is plotted as a solid line (–). (a) BER measurements for different SNRs and the various block lengths L. Theoretical limit (–) and experimental results (■,–) for QPSK serve as a reference. (b) BER for different L at the SNR required for a QPSK signal to reach a BER of 10^{-3}. (c) Spectral efficiency (SE) gain at a BER = 10^{-3} and SNR penalties for blocks of different length. For a block length of 8 (■), a spectral efficiency increase of 6.25% has been found for a minimal SNR penalty. This results in a SE versus SNR value that lies closer to the Shannon limit and above the SNR versus SE curve from conventional modulation (dashed line, introduced in Fig. 2(c)).

The experimental results are shown in Fig. 4. The measurements of the bit error ratio (BER) for different SNR show behaviour similar to simulations and the theoretical study. Fig. 4(b) shows how increasing the block length L reduces the penalty for constellation modulation in comparison with the QPSK BER (–). Fig. 4(c) shows how constellation modulation leads to a SE gain while the SNR penalty remains small. For constellation modulation with lengths $L \geq 8$ (■), there is a SE gain while there is hardly any penalty in comparison to QPSK modulation (■,–). SE gains of 8% and 6% are found for block lengths of L=6 and 8, respectively.

5. Conclusion

We have investigated and demonstrated the potential of constellation modulation to increase the spectral efficiency at negligible SNR penalties. The method promises in SE gains of up to 12.5% for signals with a low SNR, when optimum phase estimation techniques are employed. When compared to QPSK with a BER of 10^{-3}, experiments verify a spectral efficiency increase of 8.33% and 6.25% with a small BER penalty from 10^{-3} to 2×10^{-3} and 1.3×10^{-3}, respectively.

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