Optimal Signal Constellation Design for Nonlinear Chromatic Dispersion Optical Channel

Velimir Ilić, Tao Liu, Ivan B. Djordjević, Mo Li, Nebojša Stojanović, and Franko Küppers

Abstract— **In this paper, we propose an optimal signal constellation design algorithm suitable for the coherent optical nonlinear chromatic dispersion channels. The algorithm is a extended version of previously proposed algorithm for self phase modulation channel. Simulation results show significant improvement of bit error rate performances compared to the standard quadrature amplitude modulation format.**

Index Terms—**Signal constellation design, nonlinear fiber optics, mutual information**

I. Introduction

Fiber Kerr nonlineariities, chromatic dispersion and amplified spontaneous emission noise are the most important phenomena limiting optical fiber channel capacity [1]. Due to fiber Kerr nonlinear effects, the refractive index of optical fiber increases with optical intensity and introduces the intensity dependent nonlinear phase shift, which includes the nonlinear phase noise (NLPN) because of the noise component in optical intensity introduced by amplifier [2]. The light propagation through the fiber is further affected by chromatic dispersion which is an effect that the phase velocity and group velocity depend on the optical frequency. The interaction of these tree effects should be considered in order to increase data rates and/or extend the transmission distance of optical transmission systems.

An important task in the capacity achieving system design is finding an optimal signal constellation such that symbol error probability (SEP) or a specific criterion is minimized under an average or peak power constraint [3]. The previous results in this direction includes a maximum likelihood (ML) detector for phase-shift keying modulated signals and two-stage detector for 16-QAM constellation proposed by Lau and Kahn [4], 16-point ring constellation sets for combating the effect of NLPN [5], proposed by Beygi [5] and APSK constellations for coherent optical channels with NLPN is proposed by Hager in [6]. Although these methods can achieve better SEP performance, there is no consideration about an overall algorithm used to design the optimal signal constellation in the presence of phase noise, applicable to arbitrary signal constellation size.

Recently, a generic OSCD algorithms is proposed in [7], for ASE noise dominated scenario, and in [8] for nonlinear CD free channel. The algorithm presented in [7] creates the optimal constellation by dividing Gaussian distributed samples into clusters and the constellation points are represented as the centers of the clusters. The clustering is done by a specific method based on Euclidian distance function. The algorithm proposed in [8] is operating in a similar way, but the clustering is done at the receiver side, and is based on a log likelihood function.

In this paper, a new optimal constellation design algorithm is proposed that is applicable to coherent detection system corrupted by chromatic dispersion, Kerr nonlinearities and ASE noise. The algorithm is designed as a combination of approaches presented in [7] and [8], so that the clustering is done at the receiver side, but with a use of Euclidian distance.

Monte Carlo simulation results indicate that new constellations we obtained significantly outperform QAM constellations applied to NLCD channel.

The paper is organized as follows. In section 2, we introduce the nonlinear chromatic dispersion (NLCD) channel which is described by nonlinear Schrödinger equation. In section 3, we present the proposed OSCD algorithm for NLCD channel. Numerical results are presented in section 4.

II. Nonlinear CD Multi-span Channel Model

The sequence of input bits $b^{(1)}, \ldots, b^{(T\cdot m)}$ is divided into T subsequences and each binary subsequence of length m is converted by a natural mapping to a complex symbol from constelation set $\{c_1, \ldots, c_M\}$, where $M = 2^m$, so that the sequence of symbols $c^{(1)}, \ldots, c^{(T)}$ is formed.

The bits are transmitted with the bit rate R_b (i.e. symbol rate $R_s = R_b/m$). The signal $A(0,t)$ is formed as a superposition of Gausian pulses (GP), of width $T_s = 1/R_s$, centered the kT_s, $k = 1, \ldots, T$ with standard deviation T_0, and multiplied by $c^{(k)}$:

$$g(t) = \begin{cases} \exp\left(\frac{t^2}{2T_0^2}\right); & -\frac{T_s}{2} \leq t \leq \frac{T_s}{2} \\ 0; & \text{otherwise} \end{cases} \tag{1}$$

so that

$$A(0,t) = \sum_{k=1}^{T} c^{(k)} \cdot g(t - kT_s). \tag{2}$$

The signal is after that passed to a fiber of length L which is divided to $N = L/L_s$ spans of the length L_s.

The propagation of the signal through the spans is modeled by equation the nonlinear Schrödinger equation [9]

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} - \frac{i\beta_1}{2}\frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} + i\gamma|A|^2 A \tag{3}$$

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where β_2 is group velocity (GV) parameter and β_3 is group velocity dispersion (GVD) parameter and γ is Kerr nonlinearity parameter. The terms with parameters β_2 and β_3 describes chromatic dispersion (CD) effect (also known as dispersive pulse broadening) while the term near γ is for self-phase modulation (SPM) phase shift. In general case, the explicit solution of the equation (3) is not available, but the equation can be solved by a numerical method such as split step Fourier method [9].

After each span, the signal is amplified with an amplifier with the gain $e^{\alpha L_s}$, and each amplifier introduces the Gaussian noise with variance $\sigma^2 = N_s p h \nu \alpha \Delta\nu L_s$ (ASE noise).

In the absence of SPM and ASE noise ($\gamma = 0$, $\sigma = 0$), the signal at the end of the fiber is affected by CD only [9] and the CD can fully be compensated by all pass filter (dispersion compensation fiber) with the transfer function

$$H(\omega) = e^{i\cdot(\frac{\beta_2}{2}\omega^2 + \frac{\beta_3}{6}\omega^3)\cdot L}, \tag{4}$$

and the CD equalization is performed before the decision takes place.

All the parameters are listed in Table I.

TABLE I: Constants and parameters

Symbol	Value	Definition
T_0	7.5075ps	GP standard deviation
β_2	20.41ps^2/km	Group velocity parameter
β_3	0.1312 ps^3/km	GVD parameter
α	0.0578· 1/km	Fiber attenuation
γ	1.2· 1/W km	Nonlinearity parameter
n_{sp}	2	Spontaneous emission factor
h	$6.626 \cdot 10^{-34}$ J s	Planck's constant
ν	$1.936 \cdot 10^{14}$ Hz	Optical carrier frequence
$\Delta\nu$	$50 \cdot 10^9$ GHz	Optical bandwidth

III. OSCD Algorithm for CD Channel

The optimal signal constellation design (OSCD) algorithm for the SPM channel has been proposed in [8]. The SPM channel model is obtained from the nonlinear Schrödinger equation (3) by setting $\beta_2 = \beta_3 = 0$. In this case the nonlinear Schrödinger equation has an explicit solution given by:

$$A(z,t) = \exp(-\alpha/2) \cdot \exp(i \cdot \phi(z,T)), \tag{5}$$

where

$$\phi(z,T) = \gamma \cdot P_0 \cdot L_{eff}; \quad L_{eff} = \frac{1 - e^{(-\alpha L_s)}}{\alpha}. \tag{6}$$

In this section we propose a modification of the OSCD algorithm for nonlinear CD channel.

The first stage of the algorithm proposed in [7] is to use the conventional Arimoto-Blahut algorithm in order determine the optimum source distribution, in channel capacity sense. In the case of nonlinear CD channel this problem is still an open question, and as an alternative solution we use a Gaussian distribution, that is optimal in AWGN case, which is equivalent to multi-span system if $\gamma = \beta_2 = \beta_3 = 0$.

In the second stage, the algorithm is initialized with a set of initial constellation points, where QAM 16 constellations is used for initialization. After initialization, the training sequence is generated from the source distribution and split into clusters of points according to the Euclidian distance from constellation obtained in previous iteration.

Note that the choice of distance function $d(x_k, y_k)$ differs from OSCD algorithm [8] for SPM model, where the phase shift $\phi(z,T)$ is taken into account. The algorithm from [8] uses log-likelihood function which depends on $\phi(z,T)$ and can easily be computed. This is not the case when the CD is presented. However, as shown in Figures 1 and 2, CD pulse broadening mitigates nonlinearites introduced by Kerr nonlinear effect and Euclidian distance yields satisfactory solution. On the other hand, unlike the algorithm from [7] which also uses Euclidian distance, the proposed algorithm takes into account the channel model.

At the end of an iteration, new constellation points are obtained as the center of mass of such obtained clusters. This procedure is repeated until convergence or until a predetermined number of iterations has been reached.

The algorithm is summarized as follows [7], [8].

1) Initialization: Choose QAM constellation as initial constellation and set the size of this constellation to M.
2) Generate long training sequences $\{x_j; j = 0, \ldots, n-1\}$ from Gaussian distribution, where n denotes the length of the training sequence used for signal constellation design. Let A_0 be the initial M-level signal constellation set of subsets of constellation points.
3) Group the samples from this sequence into M clusters. The membership to the cluster is decided by Euclidian distance $d(x_k, y_k) = |y_k - x_k|$ of sample point and signal constellation points from previous iteration. Each sample point is assigned to the cluster with smallest distance squared. Given the m-th subset (cluster) with N candidate constellation points, denoted as $A_m = y_i; i = 1, \ldots, N$, find the minimum mean square error of partition $P(\hat{A}_m) = S_i; i = 1, \ldots, N$, as follows
$$D_m = D(\hat{A}_m, P(\hat{A}_m)) = \frac{1}{n}\sum_{j=0}^{n-1} min_{y \in \hat{A}_m} d(x_j, y) \tag{7}$$
where d is log-likelihood distance squared between jth training symbol and symbol y being already in the subset (cluster). With $D(\cdot)$, we denoted the distance function.
4) If the relative error $|D_{m-1} - D_m|/D_m \leq \epsilon$, where ϵ is the desired accuracy, the final constellation is described by $\{\hat{A}_m\}$. Otherwise continue.
5) Determine the new constellation points as center of mass for each cluster. With the mean square-error criterion, $x(S_i)$ is the log-likelihood center of gravity or centroid given by
$$x(S_i) = 1/(||S_i||)\sum_{j: x_j \in S_i} x_j \tag{8}$$
where $||S_i||$ denotes the number of training symbols in the region S_i. If there is no training sequence in the region, set $x(S_i) = y_i$, the old constellation point. Define

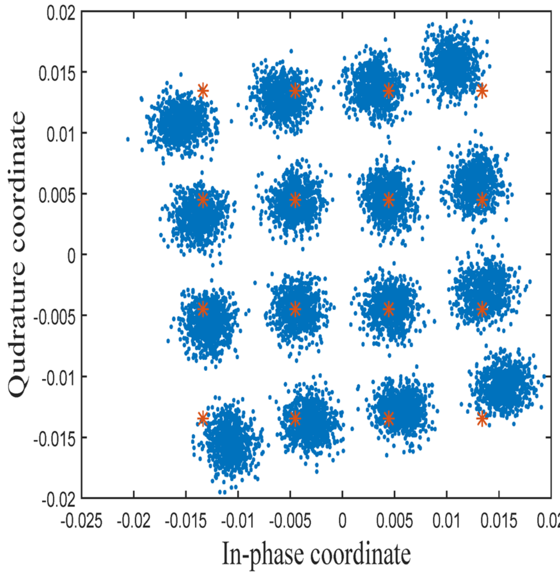

Fig. 1: Resulting signal constellation for SPM channel generated by QAM 16 for $L = 2400km, L_s = 80km$ and $P = -4dBm$

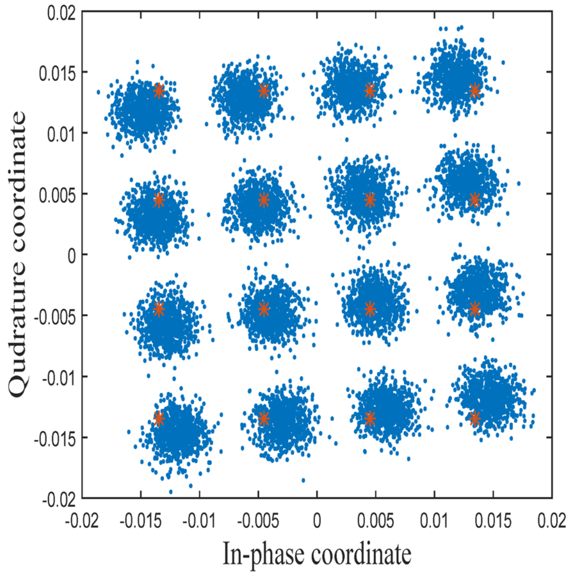

Fig. 2: Resulting signal constellation for NLCD channel generated by QAM 16 for $L = 2400km, L_s = 80km$ and $P = -4dBm$

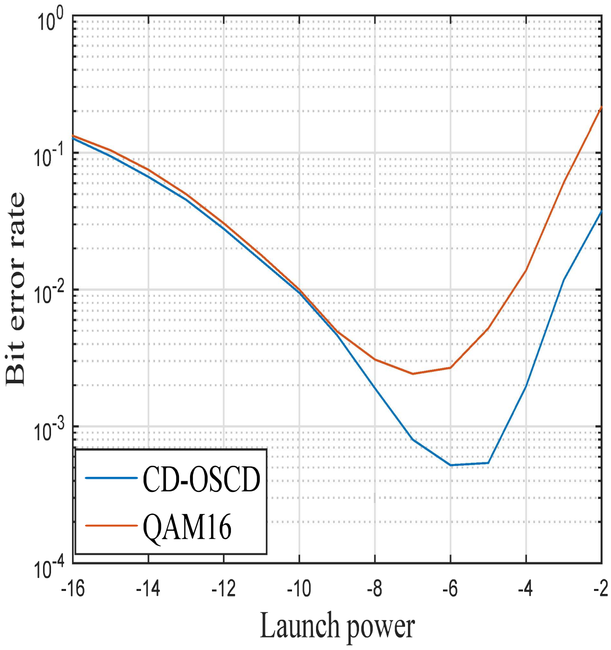

Fig. 3: (BER) as a function of P_0 for $L = 2000km$ and $L_s = 80km$

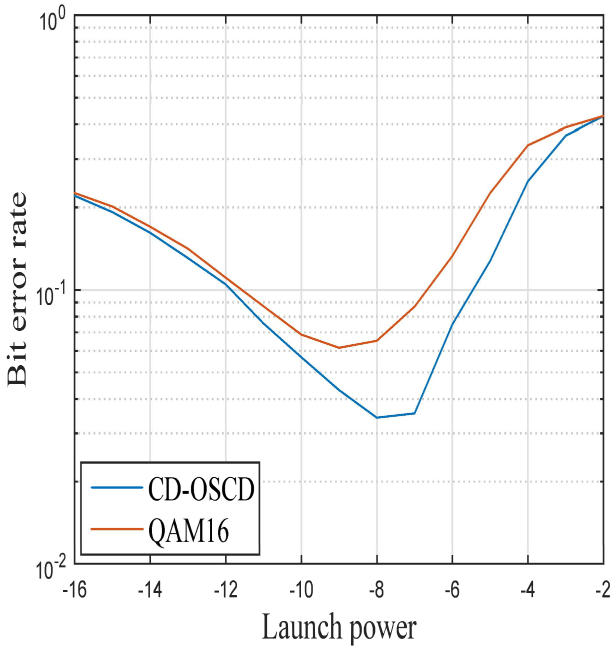

Fig. 4: (BER) as a function of P_0 for $L = 4000km$ and $L_s = 80km$

$\hat{A}_{(m+1)} = x(P(\hat{A}_m))$, replace m by $m+1$, and go to step 3). Repeat the steps 4)-6) until convergence.

IV. Numerical Results

In Figure 3 we show bit error rate (BER) as a function of launch power for the fiber of length 2000 km, divided into 25 spans of the length 80 km, for QAM 16 and proposed constellation scheme. At the optimal launch power, $P_0 = -7$ dBm, QAM 16 achieves BER of $27 \cdot 10^{-4}$, which is more than 5 times higher than the BER achieved by CD-OSCD algorithm $(5 \cdot 10^{-4})$ for the optimal launch power -6 dBm.

Note that, according to formula (6), the phase is shift is directly proportional to the launch power and that the QAM optimal power is lower than CD-OSCD one. In accordance, CD-OSCD has a higher tolerance to nonlinearities.

The resulting signal constellations at the optimal launch power are presented in Figure 5. We can observe more regular grouping of received samples around constellation points than

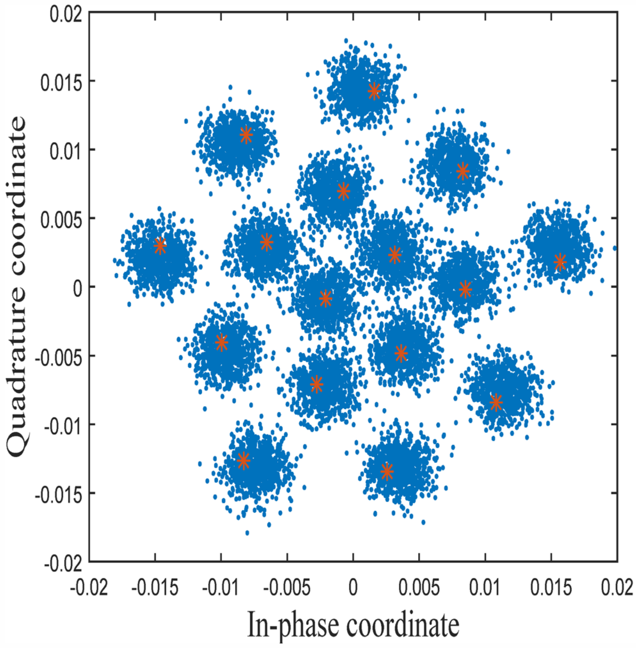

Fig. 5: Signal constellation generated by CD-OSCD algorithm and received symbols for NLCD channel: $L = 2000km, L_s = 80km$ and $P_0 = -7dBm$

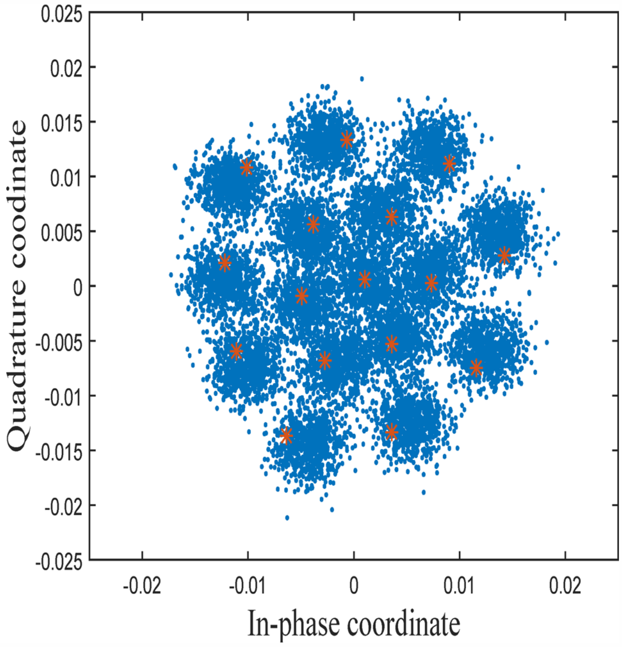

Fig. 6: Signal constellation generated by CD-OSCD algorithm and received symbols for NLCD channel: $L = 4000km, L_s = 80km$ and $P_0 = -8dBm$

in the case of QAM 16 given in Figure 2.

The similar effect can be observed in Figure 4, for the fiber of length 4000 km and span length 80 km. Note that the higher number of spans increases accumulated phase shift, which is proportional to the launch power so that the minimums of the curves are moved to the left. The resulting signal constellations for 4000 km fiber, at the optimal launch power, $P_0 = -8$ dBm, is presented in Figure 6.

V. Conclusion and Further Work

An optimal signal constellation design algorithm for the coherent optical nonlinear chromatic dispersion channels has been proposed. The algorithm is a modified version of previously proposed algorithm for self phase modulation channel. It has been shown that the OSCD CD has significantly better BER performance compared to QAM 16 constellations, for uncoded modulation case. It is expected that the coded modulation will further improve the performances which is the next step of the research in this direction.

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