Constellation Rotation in Spatial Coupled Multiple Access

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***Abstract*—In this paper, we propose to apply constellation rotation in spatial coupling multiple access system, where each data stream is modulated with different rotation angle, the coupling data is the linearly superposition of certain data streams with time offsets and iterative detecting and decoding is performed. We also seek out the optimal rotation angle set using the mutual information. Simulation results show that constellation rotation contributes to improve the system performance and reduce the complexity of the system. The system with the optimal rotation angle set performs significantly better than the others.**

I. Introduction

With the growing demand of mobile communication, the fifth generation(5G) wireless communication faces many challenges including high spectral efficiency and massive connection. Therefore, conventional orthogonal multiple access(OMA)is not able to meet the demand and non-orthogonal multiple access(NOMA)[1] catch numerous attractions for its resource allocation in time/frequnecy domain. In NOMA, data streams from different user are allowed to be superimposed which makes the system achieve higher spectral efficiency. NOMA modulation[2], power allocation and user scheduling[3] are popular research direction to improve system performance.

Spatial graph coupling is a method that identical replications of a single graph are connected to produce a new graph. The technique was first applied to construct conventional low-density parity-check(LDPC) codes[4] which proved to achieve similar threshold to the maximum a posteriori probability(MAP) decoding threshold of corresponding LDPC block codes[5]. Recently, the method is also found applied in multiple access. In[6], the full capacity region of the additional white Gaussian noise(AWGN) multiple access channel has been achieved. Spatial coupling is used in code-division multiple-access(CDMA)[7] which improves the performance of iterative multiuser detection. [8] proposes a multiple access demodulation with lower complexity which obtains the same performance as the others.

Constellation rotation contributes to increase the modulation diversity and is widely used to make further distinction when data is superimposed. [9] combines constellation rotation with symbol mapping to increase spectral efficiency. The multi-dimensional SCMA codebook design with constellation rotation proposed in [10] achieves better performance.

In this paper, we focus on applying constellation rotation in spatial coupling multiple access system. Each data stream is equal-power and independent, encoded by a binary information sequence with LDPC codes[11] and transmitted via AWGN channel. During modulation, different data streams are rotated with different angles. Data stream coupling is constructed by superimposing the data streams with time offsets. At the receiver, we use iterative detection and decoding for data processing in which log likelihood ratios(LLRs) are exchanged between detector and decoder for determined times. Furthermore, we search for the optimal angle set using the method of maximizing mutual information(MI). We draw the conclusion that rotation contribute to the performance improvement using extrinsic information transfer(EXIT) charts and bit error rate(BER) curves.

The rest of this paper is organized as follows: In the section II, system model of the spatial coupling multiple access is introduced. We make elaborated theoretical analysis containing constellation rotation, rotation angles optimization and the iteration between detector and decoder in Section III. The simulation results and analysis are represented in section IV with the help of BER curves and EXIT curves. Finally, section V summarizes this paper.

II. System Model

We consider a scheme where constellation rotation is applied in spatial coupling multiuser data transmission. The system model considered in this paper is shown in Fig. 1. To simplify the description, we take $x_{l,s}$ as an example, which represents the sth data stream of user l, $l \in [1,\ldots,L]$, $s \in [1,\ldots,S]$, L and S denote the number of user and data stream per user, respectively. At the transmitter, P packages consisting of binary information sequence $x_{l,s} = \left[{x^T}_{l,s,1}, {x^T}_{l,s,2}, \ldots, {x^T}_{l,s,P}\right]^T$ are encoded by LDPC encoder with code rate $R = K/N$, where sequence $x_{l,s,p} = [x_{l,s,p,1}, x_{l,s,p,2}, \ldots, x_{l,s,p,K}]$ and $p \in [1,2,\ldots,P]$. In modulator, we propose constellation rotation scheme and the encoded data stream $\tilde{x}_{l,s}$ is modulated with $\theta_{l,s}$, $\theta_{l,s} \in [0,180)$. The output of the modulator is denoted by sequence $v_{l,s} = \left[{v^T}_{l,s,1}, {v^T}_{l,s,2}, \ldots, {v^T}_{l,s,P}\right]^T$, where $v_{l,s,p} = \left[v_{l,s,p,1}, v_{l,s,p,2}, \cdots, v_{l,s,p,N}\right]^T$. Then each package of $v_{l,s}$ is replicated M times and permuted with different interleavers, which produces the packages of $\tilde{v}_{l,s}$, $\left\{\tilde{v}^1_{l,s,1}, \ldots, \tilde{v}^M_{l,s,1}, \ldots, \tilde{v}^1_{l,s,P}, \ldots, \tilde{v}^M_{l,s,P}\right\}$. $\tilde{v}_{l,s}$ is superimposed with other data streams to generate the coupled signal c

when spatial coupling. We define the system described above as a (L, S, P, M)-multiuser spatial coupling system. Without

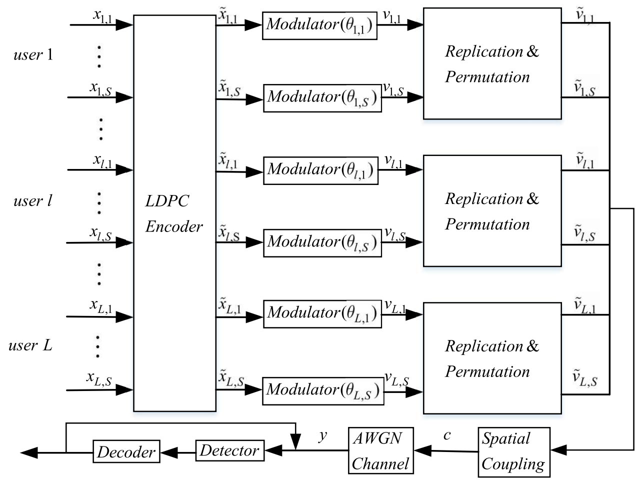

Fig. 1. System model of N user each with L streams.

loss of generality, we consider $(2, 2, 5, 3)$-multiuser spatial coupling system. The spatial coupling procedure is represented in Fig. 2, where each row is a data stream of a user and the packages are transmitted one after another. The block $\tilde{v}^{m}_{l,s,p}$ can be denoted as $\tilde{v}^{m}_{l,s,p} = \pi^{m}{}_{l,s,p} v_{l,s,p}$, where $\pi^{m}{}_{l,s,p}$ is the corresponding interleaver. At time $t = 0$, two data streams of the first user start to transmit. The signal transmitted add two data streams of one user each τ time interval in order until the number of user reaches the limit.

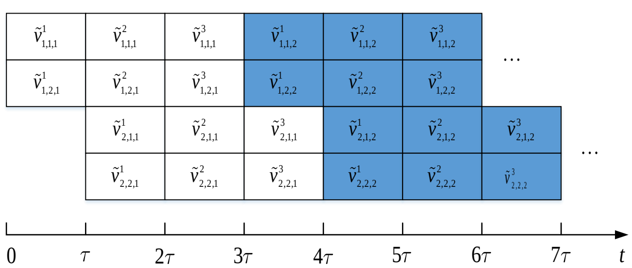

Fig. 2. Coupling of modulated data streams with time offset τ in a (L, S, P, M)-multiuser spatial coupling system.

We use spatial coupling matrix H to achieve above prodecure. The construction of H is shown in (1), where $\pi^{m}{}_{l,s,p}$ is the permutation matrix of $\pi \times \pi$ unit matrix. Therefore, the signal c can be described as $c = H^T v$,where $v = \left[v^{T}{}_{1,1,1}, \cdots, v^{T}{}_{1,1,P}, \cdots, v^{T}{}_{L,S,1}, \cdots, v^{T}{}_{L,S,P} \right]^T$.

$$H = \left[\begin{array}{cccccccccccc}
\pi^1{}_{1,1,1} & \cdots & \pi^M{}_{1,1,1} & \cdots\cdots & \pi^1{}_{1,1,P} & \cdots & \pi^M{}_{1,1,P} & & & & & \\
\pi^1{}_{1,2,1} & \cdots & \pi^M{}_{1,2,1} & \cdots\cdots & \pi^1{}_{1,2,P} & \cdots & \pi^M{}_{1,2,P} & & & & & \\
\vdots & \cdots & \vdots & \cdots\cdots & \vdots & \cdots & \vdots & & & & & \\
\pi^1{}_{1,S,1} & \cdots & \pi^M{}_{1,S,1} & \cdots\cdots & \pi^1{}_{1,S,P} & \cdots & \pi^M{}_{1,S,P} & & & & & \\
 & \pi^1{}_{2,1,1} & \cdots & \pi^M{}_{2,1,1} & \cdots\cdots & \pi^1{}_{2,1,P} & \cdots & \pi^M{}_{2,1,P} & & & & \\
 & \pi^1{}_{2,2,1} & \cdots & \pi^M{}_{2,2,1} & \cdots\cdots & \pi^1{}_{2,2,P} & \cdots & \pi^M{}_{2,2,P} & & & & \\
 & \vdots & \cdots & \vdots & \cdots\cdots & \vdots & \cdots & \vdots & & & & \\
 & \pi^1{}_{2,S,1} & \cdots & \pi^M{}_{2,S,1} & \cdots\cdots & \pi^1{}_{2,S,P} & \cdots & \pi^M{}_{2,S,P} & & & & \\
 & & \ddots & & & & & & \ddots & & & \\
 & & & \pi^1{}_{L,1,1} & \cdots & \pi^M{}_{L,2,1} & \cdots\cdots & \pi^1{}_{L,1,P} & \cdots & \pi^M{}_{L,1,P} & & \\
 & & & \pi^1{}_{L,2,1} & \cdots & \pi^M{}_{L,2,1} & \cdots\cdots & \pi^1{}_{L,2,P} & \cdots & \pi^M{}_{L,2,P} & & \\
 & & & \vdots & \cdots & \vdots & \cdots\cdots & \vdots & \cdots & \vdots & & \\
 & & & \pi^1{}_{L,S,1} & \cdots & \pi^M{}_{L,S,1} & \cdots\cdots & \pi^1{}_{L,S,P} & \cdots & \pi^M{}_{L,S,P} & &
\end{array} \right] \quad (1)$$

The total load in the system we proposed is shown in (2), which is actually the ratio between rows and columns of H.

$$Load = \frac{LSPK}{(PM + L - 1)\,K} = \frac{LSP}{(PM + L - 1)} \quad (2)$$

When the signal is passed through the AWGN channel, channel output can be calculated by (3), where λ is the power normalization coefficient, n is gaussian noise, its mean and variance are 0 and σ^2, respectively.

$$y = \lambda c + n \quad (3)$$

At the receiver, iterative detecting and decoding scheme based on message passing algorithm(MPA) is considered. Inner the detector and the decoder, the extrinsic messages are iteratively exchanged along edges between channel nodes and variable nodes. During the iteration between detector and decoder, the output of the detector is transmitted to the decoder, the output of the decoder is used as the input of the detector of next iteration, note that each package of a data stream is encoded and decoded individually. We define the maximum number of iteration as $I_{\max}$.

III. PROPOSED SCHEME

A. Constellation rotation

For each data stream, the principle of the constellation rotation we proposed is illustrated in Fig. 4, where the modulated signal is constructed by anticlockwise rotating the signal modulated by BPSK with a certain angle θ. Assuming $x'_{l,s}$ denotes the BPSK modulation signal, $x'_{l,s} = \left[x'^{T}{}_{l,s,1}, x'^{T}{}_{l,s,2}, \cdots, x'^{T}{}_{l,s,P} \right]^T$, then $v_{l,s}$ can be described as

$$v_{l,s} = e^{i\theta_{l,s}} x'_{l,s} \quad (4)$$

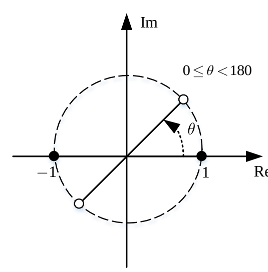

Fig. 3. The constellation rotation of each data stream.

We illustrate the scheme with the example listed in section II and focus on the constellation of the superimposed signal c. In Fig. 5, there are three constellations with different rotation angle set. As can be seen from the figure, we conclude that the constellation of c containing 16 points when there is no identical angle in the angle set. For the constellation with two or more same rotation angles, some constellation points are overlapping, which lead to the diversity reduction. We call this phenomenon constellation aliasing which make it difficult to distinguish which data streams the superimposed data come from in the detector and lead to performance degradation.

When considering constellation, we naturally come up with the minimum Euclidean distance which reflects the mutual

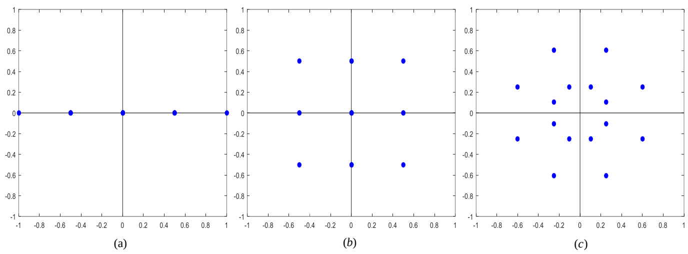

Fig. 4. The constellation of coupled signal c. The combinations of rotation angles from left to right are $(0,0,0,0)$, $(0,90,0,90)$, $(0,90,45,135)$, respectively.In case $L=2$, $S=2$, $M=3$.

interference between constellation points. The shorter the distance is, the greater the interference is. The minimum Euclidean distance decrease when the diversity of the constellation increase. Therefore, it is necessary to make a trade off between constellation aliasing and mutual interference and search for the optimal rotation angle set.

B. Rotation angles optimization

The optimization of rotation angles is equivalent to find certain angle set with maximum mutual information(MI) between the input c and the output y. The maximum MI represents the maximum amount of information about c that can be conveyed through the channel and is also called channel capacity. Assuming the constellation points set of c is denoted by $\{c_m\}, (m=1,2,\cdots,M)$,$c_m = a_m + ib_m$, $y = u + iv$. The channel transition probability is given as

$$\begin{aligned} p(y|c_m) &= \int\int \frac{1}{\pi N_0} e^{-\frac{(u-a_m)^2+(v-b_m)^2}{N_0}} dudv \\ &= \int\int \frac{1}{\pi N_0} e^{-\frac{|y-c_m|^2}{N_0}} dudv \end{aligned} \tag{5}$$

Where N_0 is the noise power spectral density. In order to facilitate the calculation, we set N_0 to 1. The computational formula of MI is derived as

$$I(c;y) = \sum_{m=1}^{M} p(c_m) \int\int p(y|c_m) \log \frac{p(y|c_m)}{\sum_{m=1}^{M} p(c_m)\, p(y|c_m)} \tag{6}$$

As c_m occurs with equal probability, we combine (5) with (6), and get the following equation:

$$\begin{aligned} &I(c;y) = \log M - \\ &\frac{1}{M}\sum_{m=1}^{M} \int\int \frac{1}{\pi N_0} e^{-\frac{|y-c_m|^2}{N_0}} \log \sum_{n=1}^{M} e^{-\frac{|y-c_n|^2+|y-c_m|^2}{N_0}} dudv \end{aligned} \tag{7}$$

In our scheme, we traverse all sets of rotation angle when the total number of data streams LS is determined and select the optimal rotation angles with the maximum MI.

C. Iterative detecting and decoding

In this section, we put the process of iteration detecting and decoding scheme in detail. During the detection, the log-likelihood ratios(LLRs) is used as the extrinsic information passed between nodes given in Fig. 5, where circles represent variable nodes corresponding to the data blocks in Fig. 2, channel nodes are coupling data and denoted by squares. The received message of one node along one edge is not allowed to update the message sent on the same edge.

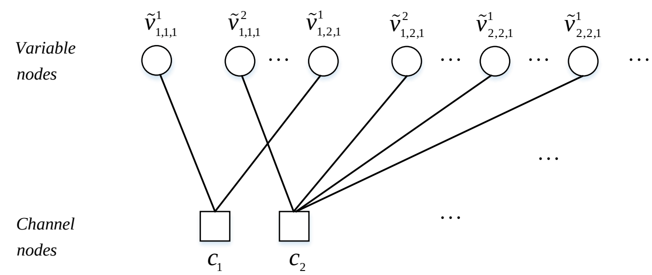

Fig. 5. Factor graph representation of spatial coupling multiple access system corresponding to the Fig .2.

Let the edge between variable node $\tilde{v}_{l,s,p}$ and channel node c_t as observation. Set $\xi(l,s,p)\backslash t$ is the index collection of channel nodes connected with variable node $\tilde{v}_{l,s,p}$ and set $\xi t\backslash(l,s,p)$ is the index collection of variable nodes connected with channel nodes c_t. Symbol $l_{\tilde{v}_{l,s,p}\to c_t}$ and $l_{c_t\to\tilde{v}_{l,s,p}}$ represents the message sent from $\tilde{v}_{l,s,p}$ and c_t, respectively. We initialize $l_{\tilde{v}_{l,s}\to c_t}$ as follows

$$l_{\tilde{v}_{l,s,p}\to c_t} = \log \frac{p(x'_{l,s,p}=+1)}{p(x'_{l,s,p}=-1)} \tag{8}$$

The message updated during iteration can be write as

$$l_{\tilde{v}_{l,s,p}\to c_t} = \sum_{t'\in\xi(l,s,p)\backslash t} l_{c_{t'}\to\tilde{v}_{l,s,p}} \tag{9}$$

$$\begin{aligned} l_{c_t\to\tilde{v}_{l,s,p}} \quad &= \quad \log \frac{p(\tilde{v}_{l,s,p}=+e^{i\theta_{l,s}}|c_t,\tilde{\mathbf{v}}^{[t]}\backslash\tilde{v}_{l,s,p})}{p(\tilde{v}_{l,s,p}=-e^{i\theta_{l,s}}|c_t,\tilde{\mathbf{v}}^{[t]}\backslash\tilde{v}_{l,s,p})} \\ = \log &\frac{p(c_t|\tilde{\mathbf{v}}^{[t]},\tilde{v}_{l,s,p}=+e^{i\theta_{l,s}})p(\tilde{\mathbf{v}}^{[t]}|\tilde{v}_{l,s,p}=+e^{i\theta_{l,s}})}{p(c_t|\tilde{\mathbf{v}}^{[t]},\tilde{v}_{l,s,p}=-e^{i\theta_{l,s}})p(\tilde{\mathbf{v}}^{[t]}|\tilde{v}_{l,s,p}=-e^{i\theta_{l,s}})} \end{aligned} \tag{10}$$

Where $\tilde{\mathbf{v}}^{[t]}$ denotes the set containing all signals superimposed on the coupled signal c_t. (10) is derived by using Bayes' rule listed below

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)} \propto p(y|x)p(x) \tag{11}$$

The conditional probability density function(pdf) of the coupled signal c_t and the set $\mathbf{v}^{[t]}$ are given separately as

$$p\left(c_t|\tilde{\mathbf{v}}^{[t]}\right) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}\left\|c_t - \mathbf{h}^{[t]T}\tilde{\mathbf{v}}^{[t]}\right\|^2\right) \tag{12}$$

$$p\left(\tilde{\mathbf{v}}^{[t]}|\tilde{v}_{l,s,p}\right) = \prod_{(l',s',p')\in\xi t\backslash(l,s,p)} p(\tilde{v}_{l',s',p'}) \tag{13}$$

where $\mathbf{h}^{[t]}$ is normalized coefficient vector of $\tilde{\mathbf{v}}^{[t]}$ and a priori probability of $v_{l',s',p'}$ is written as

$$p(\tilde{v}_{l',s',p'}) = \exp\left(\frac{\tilde{v}_{l',s',p'}}{2} l_{\tilde{v}_{l',s',p'}\to c_t}\right) \tag{14}$$

Substituting (12), (13) and (14) into (10), we get (15) shown on the top of this page, where $\max^*$ operation[12] is defined

$$l_{c_t \to \tilde{v}_{l,s,p}} = \max^{\star}_{\substack{\tilde{\mathbf{v}}^{[t]} \\ \tilde{v}_{l,s,p} = +e^{i\theta_{l,s}}}} \left(\sum_{(l',s',p') \in \xi t \setminus (l,s,p)} \frac{\tilde{v}_{l,s,p}}{2} l_{\tilde{v}_{l,s,p} \to c_t} - \frac{1}{2\sigma^2} \left\| c_t - \mathbf{h}^{[t]T} \tilde{\mathbf{v}}^{[t]} \right\|^2 \right) - \max^{\star}_{\substack{\tilde{\mathbf{v}}^{[t]} \\ \tilde{v}_{l,s,p} = -e^{i\theta_{l,s}}}} \left(\sum_{(l',s',p') \in \xi t \setminus (l,s,p)} \frac{\tilde{v}_{l,s,p}}{2} l_{\tilde{v}_{l,s,p} \to c_t} - \frac{1}{2\sigma^2} \left\| c_t - \mathbf{h}^{[t]T} \tilde{\mathbf{v}}^{[t]} \right\|^2 \right) \tag{15}$$

as follows:

$$\begin{aligned} \max^*(a,b) &= \log(\exp(a) + \exp(b)) \\ &= \max(a,b) + \log(1 + \exp(-|a-b|)) \end{aligned} \tag{16}$$

We define the iterative process from the detector to the decoder as a whole iteration. The message exchanged between detector and decoder can be described as

$$l^{DEC[i]}_{out,\tilde{v}_{l,s,p}} = l^{I_{DEC}}_{\tilde{v}_{l,s,p}} - l^{DET[i]}_{out,\tilde{v}_{l,s,p}} \tag{17}$$

where the symbol on the left of the equal sign represents the output of the decoder in ith iteration, $i \in [1, \cdots I_{\max}]$. On the right side, the subtrahend is the LLRs of the decoder after I_{DEC} inner iterations and is used for soft decision, the minuend denotes the output of the detector in ith iteration, initializes to 0 when $i = 1$ and is update as

$$l^{DET[i]}_{out,\tilde{v}_{l,s,p}} = l^{I_{DET}}_{\tilde{v}_{l,s,p}} - l^{DEC[i-1]}_{out,\tilde{v}_{l,s,p}} \tag{18}$$

where $l^{I_{DET}}_{\tilde{v}_{l,s,p}}$ is the total LLRs received from neighbors of $\tilde{v}_{l,s,p}$ after I_{DET} iterations. The symbol $l^{DEC[i-1]}_{out,\tilde{v}_{l,s,p}}$ is the $i-1$th iterative output of the detector.

IV. SIMULATION RESULTS

In this section, we analyze the system performance using EXIT charts and BER curves. We have verified that our constellation rotation is also applicable to any $(3,2,5,2)$-multiuser spatial coupling structure. To facilitate the analysis, we take $(2,2,5,2)$-multiuser spatial coupling system as observation, the encoding scheme considered is $(3,6)$-regular LDPC code with length $N = 1800$ and rate $1/2$ and the maximum inner iteration in the detector and the decoder are 2 and 10. The optimal rotation angle set we seek out is $(0, 90, 45, 135)$.

A. EXIT chart

EXIT chart is a useful tool to track the mutual information at each iteration between soft-in soft-out(SISO) constituents, and it provides an excellent prediction on the behavior of the iteration. We use EXIT charts to evaluate the performance of the detector(or the decoder) by observing whether it is conductive to increase the output mutual information I_E when the input extrinsic information I_A is given. We produce the EXIT curve with several input mutual information and the corresponding output of detector(or decoder).

There are two EXIT curves of detector with different rotation angle set over AWGN channel at $E_b/N_0 = 4dB$ shown in Fig. 6 and $E_b/N_0 = 8dB$ shown in Fig. 7, respectively. According to the figures, we reach a conclusion that constellation rotation does contribute to increase the output mutual information of the detector. The system performance is influenced by the rotation angle set, for $(0, 90, 0, 90)$, namely, two data streams of a user are perpendicular to each other, which means that there exists no interference between certain user's different data streams, but the interference between the two users can not be eliminated and results in the constellation

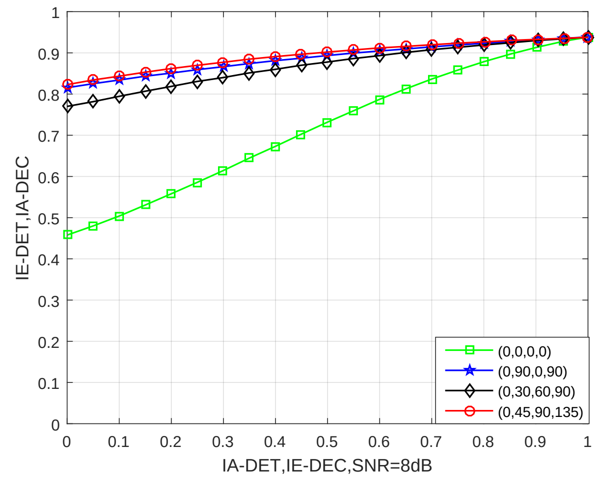

Fig. 6. The EXIT chart of the detector with 2 inner iterations for different rotation angle group in $(2,2,5,2)$-multiuser spatial coupling system with $E_b/N_0 = 3dB$.

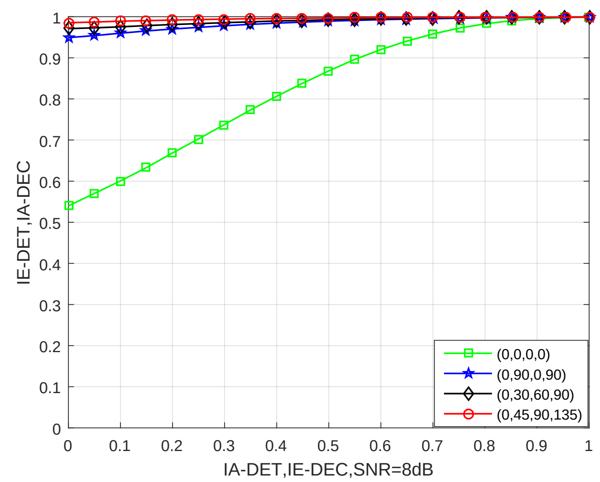

Fig. 7. The EXIT chart of the detector with 2 inner iterations for different rotation angle group in $(2,2,5,2)$-multiuser spatial coupling system with $E_b/N_0 = 8dB$.

aliasing. For the set randomly selected, such as $(0, 30, 60, 90)$, each data stream is rotated with different angles which expands diversity, but interference exists between all data streams. It can be seen that when the SNR is low at $4dB$, the performance of detector is better for $(0, 90, 0, 90)$, that is to say interference plays a dominant role in the factors affecting the performance rather than diversity. On the contrary, when the SNR is high at $8dB$, the performance of detector is better for $(0, 30, 60, 90)$, which means diversity is the main influencing factor. For the optimal angle set selected by using the method we proposed, the output mutual information of the detector is both larger than the others at $4dB$ and $8dB$.

B. BER curve

BER curve can visually describe the performance of the system. We illustrate the average BER curve of the system in Fig. 8, where dotted line and solid line represent the system with $I_{\max} = 12$ and $I_{\max} = 20$ iterations between detector and decoder respectively. When the number of iteration is fixed, it can be observed that the system with constellation rotation improves its performance indeed and rotation angle set $(0, 90, 45, 135)$ achieve the optimal BER performance, which is consistent with the analysis of EXIT charts above. Compared with the system without constellation rotation, there exists about $2.3dB$ performance improvement. The system with fixed rotation angle set achieves better performance as $I_{\max}$ increases. And the performance of the system with constellation rotation converges faster, which is beneficial to reduce the implementation complexity and latency of the system.

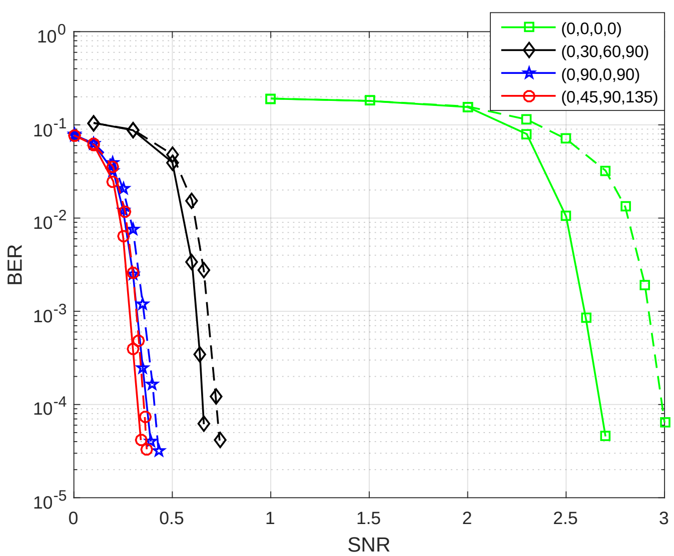

Fig. 8. The average BER curve of $(2, 2, 5, 2)$-multiuser spatial coupling system with different rotation angle groups and different iterations between detector and decoder.

CONCLUSION

In this paper, we have applied constellation rotation in spatial coupling multiple access system with LDPC codes and iterative detecting and decoding scheme, and sought out the optimal rotation angle set with the help of mutual information. The EXIT charts and BER curve simulation results verify that the constellation rotation benefits to improve the performance of the system and reduce the implementation complexity and latency, the system with the optimal rotation angle set performs significantly better than the others.

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