Multi-dimensional SCMA Codebook Design Based on Constellation Rotation and Interleaving

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***Abstract*—Sparse code multiple access (SCMA) is a new non-orthogonal multiple access scheme, which effectively exploits the shaping gain of multi-dimensional codebook. In this paper, a multi-dimensional SCMA (MD-SCMA) codebook design based on constellation rotation and interleaving method is proposed for downlink SCMA systems. In particular, the first dimension of mother constellation is constructed by subset of lattice $\mathbf{Z}^2$. Then the other dimensions are obtained by rotating the first dimension. Further, the interleaving is used for even dimensions to improve the performance in fading channels. In this way, we can design different codebooks for the aim of spectral efficiency or power efficiency. And the simulation results show that the bit error rate (BER) performance of MD-SCMA codebooks outperforms that of the existing SCMA codebooks and low density signature (LDS) in downlink Rayleigh fading channels.**

I. Introduction

Sparse code multiple access (SCMA) has recently received considerable attention as a promising enabling technique in fifth generation (5G) mobile networks [1, 2]. SCMA systems can be overloaded, where the number of multiplexed users can be more than the number of resources based on the spreading factor. In addition, since the sparsity of codeword, the message passing algorithm (MPA) detection is applicable with a moderate complexity [3]. The main idea of SCMA is that the procedure of bit to quadrature amplitude modulation (QAM) symbol mapping and spreading are combined together, i.e., the incoming bits are directly mapped to multi-dimensional codewords of SCMA codebook sets. The shaping gain of multi-dimensional codebook is the main source of the performance improvement in comparison to the simple repetition of QAM symbols in low density signature (LDS).

Unfortunately, designing an optimal SCMA codebook is still an open problem. Hence a suboptimal method is used for SCMA codebook design[3,7]. However, designing a multi-dimensional constellation with good shaping gains and coding gains for SCMA codebooks is a challenging problem, even if there are many works on the constellation design [3-6]. In general, the larger the number of dimensions the worse the spectral efficiency. On the other hand, increasing the constellation size to improve the spectral efficiency while maintaining the number of dimensions unchanged. We can achieve good shaping gains by using generalized QAM constellation [4]. In [5], the authors designed good multi-dimensional compact constellation by minimizing the average symbol energy for a given minimum Euclidian distance between constellation points. Maximizing the constellation figure of merit (CFM) of multi-dimensional signal constellations is formulated as a series of optimization problems [6]. The design principles of lattice constellations are used to design the SCMA mother codebook [7], and the steps can be summarized as follow,

Step 1: For low rates, the multi-dimensional constellation can be designed by heuristic optimization [3]. For higher rates, the multi-dimensional constellation is formed by the N-fold Cartesian product of $\mathbf{Z}^2$, where $\mathbf{Z}$ is a set of integers.

Step 2: A optimal rotation matrix [8] is applied on the base constellation. In this way, the dimensional dependency, power variation, the diversity (modulation diversity or signal space diversity) order and the minimum product distance of the constellation can be control while maintaining the Euclidian distance unchanged.

Step 3: Shuffling: k-th complex dimension of complex mother constellation= (k-th real dimension of $\mathbf{A}$, k-th real dimension of $\mathbf{B}$), where $\mathbf{A}$ and $\mathbf{B}$ are the same base constellation, which are designed as the step1 and step2. It can help to reduce the decoding complexity while yet maintaining dependency among the complex dimensions of the resulted multi-dimensional constellation.

For lower rate, we have to search a good constellation for the mother constellation of SCMA codebook. For higher dimensions, the optimum rotation angle should be found first. These steps have limited constribution to implement codebook design. As so far, there is not work on designing the codebooks dependent on the systems model and the method can be easier implemented.

In this paper, we present a new method of multi-dimensional SCMA (MD-SCMA) codebook design based on constellation rotation and interleaving. Firstly, a subset of two-dimensional lattice $\mathbf{Z}^2$ is defined for the first dimension of mother constellation. And the other dimensions can be obtained by rotating the first dimenson. Then a one-to-one Gray mapping from the constellation point to the codeword of MD-SCMA mother codebook is defined. And the elements of even dimension are interleaved. On the one hand, larger minimum Euclidean distance between the codewords, a lower peak to average power ratio (PAPR) of mother constellation can be obtained. On the other hand, the interleaved codeword can get more protection against the effects of channel fade [9]. Finally, a constellation operator is defined for user with specific phase based on Latin structure and factor graph. Our simulation

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results show that the performance of MD-SCMA codebooks is better than the existing codebooks and LDS in terms of BER in downlink Rayleigh fading channels.

II. System Model

We consider a synchronous downlink SCMA system with a base station, J mobile users, K subcarriers, An SCMA encoder is defined as a mapping

$$f : \mathbb{B}^{\log_2 M} \rightarrow \mathcal{X}, i.e., \quad \mathbf{x} = f(\mathbf{b}), \tag{1}$$

where $\mathbf{b}$ is the incoming information bits and $\mathbf{x} \in \mathcal{X} \subset \mathbb{C}^K$ with cardinality $|\mathcal{X}| = M$. An SCMA encoder contains U separate layers. Without loss of generality, we assume that one user has one layer in SCMA encoder. Actually, the symbol mapping and the spreading are combined together in SCMA encoder, i.e., the input bits are directly mapped to multi-dimensional complex domain codewords selected from a predefined codebook set. However, designing an optimal multi-dimensional sparse SCMA encoder is unknown problem [3], a multi-stage approach is proposed to obtain a sub-optimal SCMA codebook. In order to describe the structure of SCMA encoder clearly, let $\mathbf{c}$ be the N-dimensional complex constellation point defined within the mother constellation set $\mathbf{C} \subset \mathbb{C}^N$ and define a mapping from $\mathbb{B}^{\log_2 M}$ to $\mathbf{C}$:

$$\mathbb{B}^{\log_2 M} \rightarrow \mathbf{C}, i.e., \mathbf{c} = g(\mathbf{b}).$$

Then the SCMA encoder in (1) can be rewritten as:

$$f :\equiv \mathbf{V}g, \ i.e., \ \mathbf{x} = \mathbf{V}g(\mathbf{b}), \tag{2}$$

where the binary mapping matrix $\mathbf{V} \in \mathbb{B}^{K \times N}$ simply maps the N dimensional complex codeword of mother constellation to a K-dimensional codeword of SCMA codebook. Here, the matrix $\mathbf{V}$ contains $K-N$ all-zero rows and all the codewords of codebook contain zero in the same $K-N$ dimensions. We can get the matrix $\mathbf{V}$ by inserting $K-N$ row zero into $\mathbf{I}_N$.

From (1) and (2), a special SCMA codebook for one user can be expressed as:

$$\mathbf{x}_j = \mathbf{V}_j(\Delta_j \circ g)(\mathbf{b}_j),$$

where Δ_j is the constellation operator for the j-th user, and $\circ$ denotes a composition operation. Note that when $\Delta_j = \mathbf{E}_j\mathbf{1}$, $\mathbf{E}_j$ is a rotation matrix defined in [7], $\mathbf{1}$ is N dimensional all one vector, $\mathbf{x}_j$ is a codeword of LDS. In other word, the LDS is a special case of SCMA.

SCMA codewords from different users are multiplexed over shared orthogonal resources, e.g. OFDMA tones. The received signal after the synchronous layer multiplexing can be expressed as

$$\mathbf{y} = \sqrt{\frac{P}{J}}\sum_{j=1}^{J} diag(\mathbf{h})\mathbf{x}_j + \mathbf{n} = \sqrt{\frac{P}{J}}\sum_{j=1}^{J} diag(\mathbf{h})\mathbf{V}_j g(\mathbf{b}_j) + \mathbf{n},$$

where $\mathbf{x}_j = (x_{1j}, x_{2j} \ldots, x_{Kj})^T$ is the SCMA codeword of user j, P is the total power at base station, $\mathbf{h} = (h_1, h_2, \cdots, h_K)^T$ is the channel vector, $h_k \sim \mathbb{CN}(0, \sigma^2)$ and $\mathbf{n} = (n_1, n_2 \ldots n_K)^T$ is the additive white Gaussian noise with zero mean and $\sigma^2\mathbf{I}_K$ variance.

III. MD-SCMA Codebook Design and Analysis

LDS is a special case of SCMA, and the codebook of LDS had obtained in [7]. SCMA outperforms LDS , mainly due to the shaping gains of multi-dimensional mother constellation. In this section, we focus on how to design the MD-SCMA mother constellation. The constellation operators based on phases of users signal is given simply.

A. Design and analysis of MD-SCMA mother codebook

As we know, the cartesian product of N m-PAM (QAM) constellations can be used to design an N dimensional costellation with $M = m^N$ points. However, we can not use this method for MD-SCMA codebook design directly, because the degree of freedom for dimensions do not be used efficiently, and the minimum Euclidean distance of codewords is smaller when each codeword energy is fixed. Thus, a MD-SCMA mother constellation can be designed by the following steps:

Step 1: A subset of lattice $\mathbf{Z}^2$ is defined as

$$\mathbf{S}_1 = \{A_m(1+i) | A_m = 2m - 1 - M, m = 1, \cdots, M\},$$

where $\mathbf{Z}$ is a set of integers. In this way, we can easily get the size of codebook for both low and high rates.

Step 2: Each point of subset $\mathbf{S}_1$ can be labeled by Gray mapping. For example, when $M = 4$, the Gray mapping is

$$\begin{array}{cccc} 00 & 01 & 11 & 10 \\ 3(1+i) & (1+i) & -(1+i) & -3(1+i) \end{array}$$

Step 3: Let $\mathbf{S}_N = \mathbf{U}_N\mathbf{S}_1, \mathbf{U}_N = diag(\mathbf{1}e^{i\theta_{l-1}}) \in \mathbf{C}^{N \times N}$, is phase rotation matrix, $\mathbf{1}$ is N-dimensional all one vector and θ_{l-1} is defined as

$$\theta_{l-1} = (l-1) \times \frac{\pi}{MN}, l = 1, \cdots, N.$$

Then a N-dimensional base constellation (mother constellation) with Gray mapping can be constructed as

$$\mathbf{M} = (\mathbf{S}_1, \cdots, \mathbf{S}_N)^T = \begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1M} \\ s_{21} & s_{22} & \cdots & s_{2M} \\ \vdots & \vdots & \ldots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NM} \end{pmatrix}.$$

N-dimensional mother constellation with M points are constructed. Since N dimensions are different, we use the degree of freedom of dimensions efficiently. On the other hand, once more users collide at a resource, the linear relations among the non-zero elements of codewords help to recover the colliding codewords from the other resources. However, the power of dimension is the same each other. And the average energy of codewords is

$$E_{avg}(\mathbf{M}) = \frac{2N}{M}\sum_{m=1}^{M} A_m^2 = \frac{2N(M^2-1)}{3},$$

the minimum square Euclidean distance of codewords can be described easily as $d_{min}^2 = 8N$, and the PAPR [10] of $\mathbf{M}$ is $10\log_{10}(\frac{\max_{1 \le m \le M}|\mathbf{s}_m|^2}{E_{avg}(\mathbf{M})})dB = 10\log_{10}(\frac{3(M-1)}{M+1})dB$.

In fact, for a fixed $E_{avg}(\mathbf{M})$, we can obtain a larger square minimum Euclidean distance of codewords. In other word, the energy efficiency of multi-dimensional constellation can be improved. On the other hand, the PAPR of $\mathbf{M}$ can be reduced by re-ordering the elements of even dimension. It is well known that the performance of interleaved codewords will be improved in fading channel as well. Therefore, we give the interleaving criteria in the following step.

Step 4: Interleaving: reorder the elements of even dimensions (rows) of base constellation $\mathbf{M}$. Since $\mathbf{S}_l$ (l is even number of N) is symmetrical, the even rows can be written as

$$\mathbf{S'}_l = \{-s_{l,M/2+1}, \cdots, -s_{l,3M/4}, s_{l,3M/4+1}, \cdots, s_{l,M}, \\ -s_{l,M}, \cdots, -s_{l,3M/4+1}, s_{l,3M/4}, \cdots, s_{l,M/2+1}\}.$$

The final base constellation is $\mathbf{M}_c = (\mathbf{S}_1, \cdots, \mathbf{S'}_l, \cdots, \mathbf{S}_N)^T$. In this way, the minimum square Euclidean distance of codewoeds of mother codebook can be rewritten as

$$d_{min}^2 = \begin{cases} 8[\widetilde{N}_o + 4\widetilde{N}_e], & M = 4 \\ 8N, & M > 4 \end{cases},$$

where $\widetilde{N}_o, \widetilde{N}_e$ are the number of odd and even dimensions, respectively. In other words, the minimum square Euclidean distance has been maximized in $\mathbf{M}_c$ for a fixed E_{avg} as $M = 4$. Moreover, the PAPR of $\mathbf{M}_c$ is

$$PAPR(\mathbf{M}_c)dB = 10\log_{10}(\frac{3[N_o(M-1)^2 + N_e]}{N(M^2-1)})dB$$

note that $PAPR(\mathbf{M}_c) = 0dB$ as $M = 4, N = 2$. Since $(M-1)^2 \sim M^2 - 1$, for $M = \infty$, $PAPR(\mathbf{M}_c) = 10\log_{10}(\frac{3N_o}{N})dB$, as M is large enough. Actually, PAPR has been reduced effectively. And we know that the elements of even dimension of $\mathbf{M}_c$ have been interleaved.

For example, $N = 3, M = 4$, from step 1 to step 3, we have

$$\theta_0 = 0, \theta_1 = \frac{\pi}{12}, \theta_2 = \frac{\pi}{6}$$

and

$$s_{1,4} = 3(1+i), s_{1,3} = 1+i, s_{2,4} = 3(1+i)e^{i\theta_1}, \\ s_{2,3} = (1+i)e^{i\theta_1}, s_{3,4} = 3(1+i)e^{i\theta_2}, s_{3,3} = (1+i)e^{i\theta_2},$$

then, $\mathbf{M} = (\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3)^T$, where

$$\mathbf{S}_l = \{s_{l,4}, s_{l,3}, s_{l,2} = -s_{l,3}, s_{l,1} = -s_{l,4}\}, l = 1, 2, 3.$$

From step 4, we re-order the points of $\mathbf{S}_2$ as follows,

$$\mathbf{S'}_2 = \{-s_{2,3}, s_{2,4}, -s_{2,4}, s_{2,3}\}$$

Finally, we obtain a MD-SCMA mother codebook $\mathbf{M}_c = (\mathbf{S}_1, \mathbf{S'}_2, \mathbf{S}_3)^T$. The MD-SCMA mother codebook design process is described in Fig.1. From Fig.1(a), we observe that basic constellation structure. $\mathbf{S}_1$ is a subset of 16QAM, $\mathbf{S_2}, \mathbf{S_3}$ and are obtained by rotating $\mathbf{S}_1$. The structure of $\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3$ is depicted in the right hand side of Fig.1(a). Then a one-to-one mapping from Fig.1(a) to Fig.1(b) is defined, i.e., one constellation corresponds to a non-zero dimension of SCMA mother codebook. Therefore, we obtain M codewords with N non-zero elements. The codewords before interleaving are described in Fig.1(c). After interleaving, the codewords of mother constellation can be described as Fig.1(d).

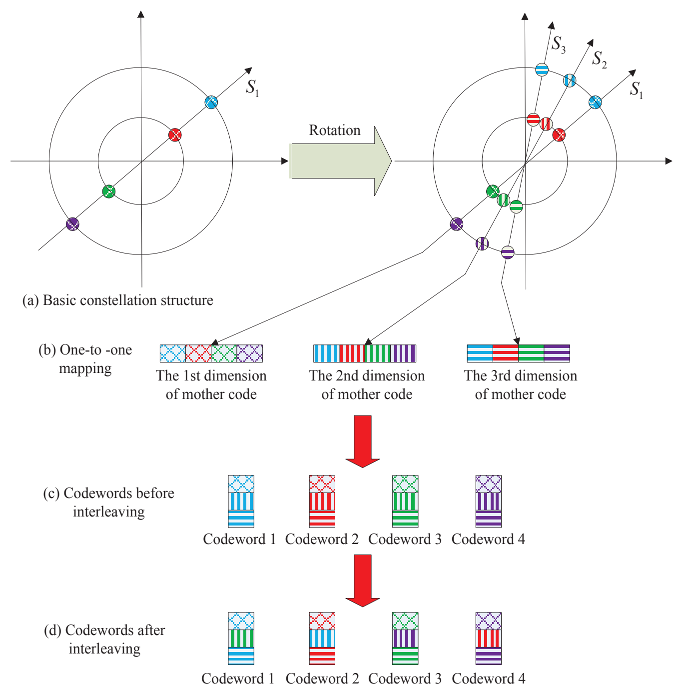

Fig. 1. Diagram of SCMA mother codebook design with $M = 4$ and $N = 3$.

B. Codebook construction for different users

Having a mother codebook $\mathbf{M}_c$, then we construct the MD-SCMA codebook for different users based on $\mathbf{M}_c$ and the constellation operator in this subsection. In order to maintain the Euclidian distance of codewords and the structure of the mother codebook unchanged, we define the constellation operators based on Latin structure [11] and factor graph.

The phase rotation angle φ_u [12] is given by

$$\varphi_u = (u-1)\frac{2\pi}{Md_f} + e_u\frac{2\pi}{M}, \ \forall u = 1, \ \cdots, d_f,$$

where e_u is any arbitrary menber of $\mathbb{Z}$, and d_f is the number of users connected to the same resource.

Then, we assign φ_u to the non-zero positions of factor graph by latin order. As an example, for a factor graph with $N = 3$, 8 users, we have

$$\mathcal{F} = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & 0 & 0 & 0 & \varphi_4 & 0 \\ \varphi_2 & 0 & 0 & \varphi_3 & \varphi_4 & 0 & \varphi_1 & 0 \\ 0 & \varphi_1 & 0 & \varphi_2 & 0 & \varphi_3 & 0 & \varphi_4 \\ \varphi_3 & 0 & \varphi_4 & 0 & \varphi_1 & 0 & 0 & \varphi_2 \\ 0 & \varphi_4 & 0 & \varphi_1 & 0 & \varphi_2 & \varphi_3 & 0 \\ 0 & 0 & \varphi_2 & 0 & \varphi_3 & \varphi_4 & 0 & \varphi_1 \end{pmatrix}, \quad (3)$$

where $\varphi_1 = 1, \varphi_2 = exp(\frac{i\pi}{6}), \varphi_3 = exp(\frac{i\pi}{3}), \varphi_4 = exp(\frac{i\pi}{2})$.

Therefore, we define operator for the j-th user as

$$\triangle_j = diag(\mathbf{f}_j) = f_j, j = 1, 2, \cdots, 8,$$

where $\mathbf{f}_j$ is the j-th column of $\mathcal{F}$ without zero elements.

Finally, we obtain the MD-SCMA codebooks for different users,

$$\mathbf{x}_j = \mathbf{V}_j \triangle_j \mathbf{M}_c, j = 1, 2, \cdots, J.$$

IV. Simulation Results and Discussions

In this section, we present simulation results for MD-SCMA and LDS in downlink Rayleigh fading channels. The BER performance of MD-SCMA codebooks is compared with SCMA codebook presented in [7] and LDS [12] under various parameter conditions. Here the LDS is considered as a special case of SCMA codebook with a multi-dimensional mother codebook constructed by the repetition of QAM constellation points over all non-zero positions of factor graph. And the message passing algorithm (MPA) detection is applied at receiver.

A. BER performance of SCMA condebooks with $N = 2$, and $M = 4$

The BER performance of MD-SCMA is compared with LDS [12] and SCMA[7] for $N = 2, M = 4$, where they both share the same factor graph as shown in Fig. 2.

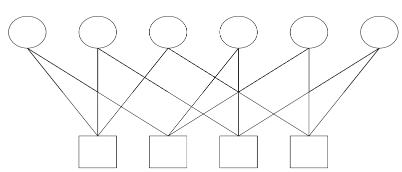

Fig. 2. Factor graph with $K = 4$, $J = 6$, and $d_f = 3$.

The main parameters of SCMA mother codebooks and LDS are listed in Table I,

TABLE I. Parameters of 2D-Mother Codebook

$\mathbf{M}_c$	d_{min}	E_{avg}	PAPR(dB)
SCMA [7]	2.4494	1	0
LDS [12]	$\sqrt{2}$	1	0
MD-SCMA	2	1	0

It can be seen from Fig. 3 that the BER performance of MD-SCMA codebooks are better than the existing SCMA and LDS in downlink Rayleigh fading channels, where 0.6 dB improvement can be achieved. From Table I, one can observe that the MD-SCMA codebooks exhibit worse energy efficiency. But the PAPR is 0 dB, which is the same as LDS and SCMA [7]. Moreover, the method presented in this paper can be easier implemented than the search method for the lower rates.

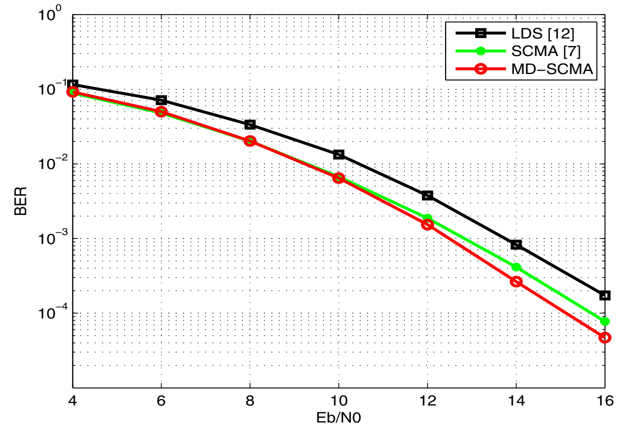

Fig. 3. Performance comparison of MD-SCMA codebooks, LDS [12], and SCMA [7] in downlink 1×2 SIMO Rayleigh fading channels.

B. BER performance of MD-SCMA codebook with different M and N

In this subsection, based on the MD-SCMA codebook parameters in Table II, the BER performance of 3-dimensional SCMA and LDS [12] with different points, and the BER performance of 4-and 5-dimensional SCMA codebooks are shown for downlink 1×2 SIMO Rayleigh fading channels in Fig. 4 and Fig. 5, respectively.

TABLE II. Parameters of MD-SCMA Codebook

N	K	U	d_f	M	$\theta_l (l = 1, \cdots, N)$	PAPR (dB)
3	6	8	4	4/8	$\theta_l = (l-1)\frac{\pi}{12/24}$	1.0266/1.9629
4	8	10	5	4	$\theta_l = (l-1)\frac{\pi}{16}$	0
5	10	12	6	4	$\theta_l = (l-1)\frac{\pi}{20}$	0.6446

It can be seen from Table II that the PAPR of MD-SCMA codebooks is increased as N becomes bigger, but increased very slowly. The sparse matrix of 3-dimensional SCMA codebook is $\mathcal{F}$ in (3). As shown in Fig. 4, the 3-dimensional SCMA have better BER performance over LDS, and the larger the mother codebook size, the higher BER gap.

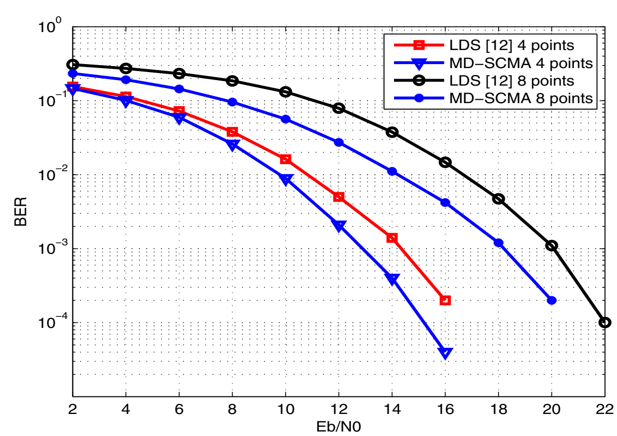

Fig. 4. BER performance between 3-dimensional SCMA and LDS [12] with different points in downlink 1×2 SIMO Rayleigh fading channels.

Fig. 5 shows the BER performance of 4- and 5-dimensional SCMA codebook with 4 points. The BER performance of MD-SCMA is better than LDS in this case as well. Moreover, the gap of BER performance will be larger as the dimension is higher.

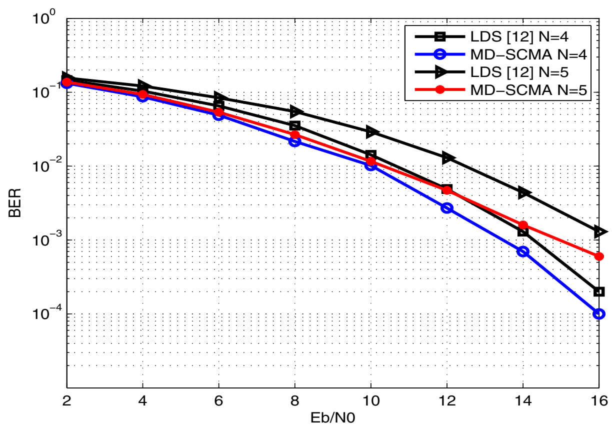

Fig. 5. BER performance between MD-SCMA and LDS [12] with different dimensions in downlink 1×2 SIMO Rayleigh fading channels.

V. Conclusion

In this paper, a new multi-dimensional SCMA codebook design method based on constellation rotation and interleaving has been presented for downlink Rayleigh fading channels. The average energy, the minimum square Euclidean distance and the PAPR of mother codebook have been analyzed. Simulation results show that the performance of MD-SCMA codebooks outperforms that of the existing SCMA codebooks and LDS in terms of BER in downlink 1×2 SIMO Rayleigh fading channels. Moreover, the method can be easier implemented for both lower and higher rates.

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