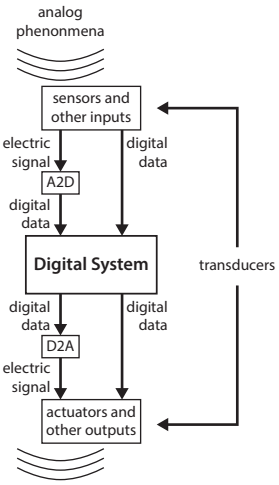
Terminology

- Digital: also known as discrete, is a signal that at any time can have one of a finite set of possible values.
- Analog: also known as continuous, is a signal that can have one of an infinite number of possible values.
- Moore's Law: the doubling of IC density every 18 months.
- Sum of Products: an equation written as an ORing of product terms such that no parentheses are present.
- Sum of Minterms: an equation in the sum-of-products form, and every product term is a minterm.
- Equivalent: two functions that return exactly the same results.
- Complement: also known as inverse, such that the outputs of two functions return exactly the opposite results.
- Optimization: a transformation that improves all criteria of interest or improves some of those criteria without worsening the others.
- Tradeoff: a transformation that improves some criteria at the expense of other criteria of interest.
- Critical Path: the longest path from any input to the circuit's output; i.e. sum the amount of processing time for each possible circuit path and whatever path returns the largest value is the critical path.



Facts

- Digital methods of recording data do not deteriorate over time.
- Digital data can be compressed.
- Custom digital circuits take less processing time than a microprocessor.

Voltage-to-Binary

t	V	B
0	0	00
1	0	00
2	1	01
3	2	10
4	3	11
5	2	10
6	1	01

V = 0012321
B = 00000110111001

Binary-to-Decimal

$$(1011)_2 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 = (11)_{10}$$

Decimal-to-Binary (Addition Method)

$(65)_{10}$

1	0	0	0	0	0	1
64	32	16	8	4	2	1

$$64 + 1 = 65 = (1000001)_2$$

Binary-to-Hexadecimal

$(11100111)_2$ NOTE: If length of binary string is not equal to a multiple of four, then append zeroes to front of string until it is.

$$(1110)_2 = 1\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 = (14)_{10} = (\mathrm{E})_{16}$$

$$(0111)_2 = 0\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0 = (7)_{10} = (7)_{16}$$

$(E7)_{16}$

Hexadecimal-to-Binary

$(B0C4)_{16}$

$$(B)_{16} = (11)_{16} = \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{array} = (1011)_2$$

$$(0)_{16} = (0)_{16} = \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 8 & 4 & 2 & 1 \end{array} = (0000)_2$$

$$(C)_{16} = (12)_{16} = \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 8 & 4 & 2 & 1 \end{array} = (1100)_2$$

$$(4)_{16} = (4)_{16} = \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 8 & 4 & 2 & 1 \end{array} = (0100)_2$$

$(1011000011000100)_2$

Boolean Algebra Properties

a + (b * c) = (a + b) * (a + c)
a + 0 = a
a * 1 = a
a + a' = 1
a * a' = 0
a + 1 = 1
a * 0 = 0
a + a = a
a * a = a
(a + b)' = a'b'
(ab)' = a' + b'

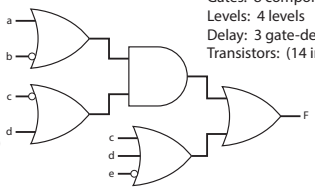
F = a+a'b+acd+c'+ab'c'd

Variables: a, b, c, d
Literals: a, a', b, a, c, d, c', a, b', c', d
Product Terms: a, a'b, acd, c', ab'c'd
Minterms: ab'c'd

Multilevel Logic Optimization

Initial	Optimized
ab + acd + ace	a(b + c(d + e))
abcd + abcef	abc(d + ef)

F = ((a+b')(c'+d))+(c+d+e')



Gates: 8 components
Levels: 4 levels
Delay: 3 gate-delays
Transistors: (14 inputs)*2 = 28

Circuit Design Metrics

- Performance: a measure of execution time for a computation on the system.
- Size: a measure of the number of transistors, or silicon area, of a digital system.
- Power: a measure of the energy consumed by the system per second, directly relating to both the heat generated by the system and to the battery energy consumed by computations.

Decimal-to-Base 5

$(128)_{10} \rightarrow (\cdots)_5$

25	5	1	0
$5\overline{)128}$	$5\overline{)25}$	$5\overline{)5}$	$5\overline{)1}$
−125	−25	−5	−0
3	0	0	1

$(1003)_5$

Binary / Decimal / Hexidecimal

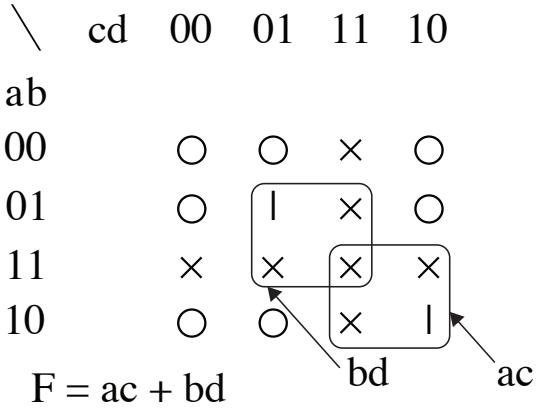
B	D	H
0 0 0 0	00	0
0 0 0 1	01	1
0 0 1 0	02	2
0 0 1 1	03	3
0 1 0 0	04	4
0 1 0 1	05	5
0 1 1 0	06	6
0 1 1 1	07	7
1 0 0 0	08	8
1 0 0 1	09	9
1 0 1 0	10	A
1 0 1 1	11	B
1 1 0 0	12	C
1 1 0 1	13	D
1 1 1 0	14	E
1 1 1 1	15	F

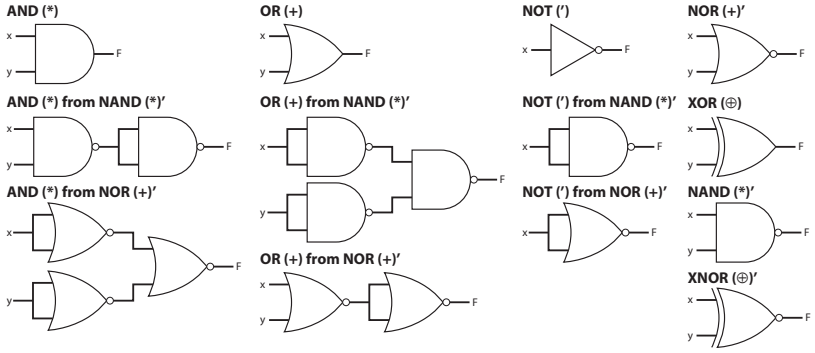
Prime Numbers

02	03	05	07	11
13	17	19	23	29
31	37	41	43	47
53	59	61	67	71
73	79	83	89	97

K-Map

- Implicants: any legal circle on the K-map.
- Prime Implicants: largest possible legal circles on the K-map.
- Essential Prime Implicants: the largest possible circle that is also the only circle that covers a particular | on the K-map.





Boolean Functions
Combinations (Rows): 2^n, n = number of variables
Functions: 2^c, c = number of combinations (rows)

a	b	a'	b'	AND ab	OR $a+b$	NOR $(a+b)'$	XOR $a \oplus b$	NAND $(ab)'$	XNOR $(a \oplus b)'$
0	0	1	1	0	0	1	0	1	1
0	1	1	0	0	1	0	1	1	0
1	0	0	1	0	1	0	1	1	0
1	1	0	0	1	1	0	0	0	1

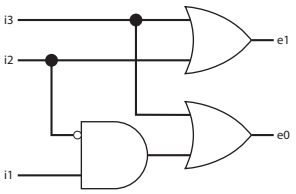
Logic Gate Operations

NOR	Detects all 0s
XOR	Detects odd number of 1s
NAND	Detects all 1s
XNOR	Detects even number of 1s

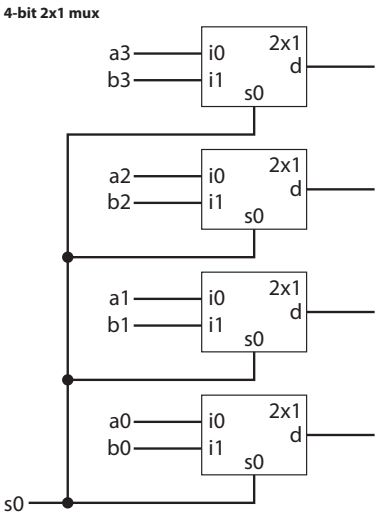
4x2 priority encoder

i3	i2	i1	i0	e1	e0
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	1
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

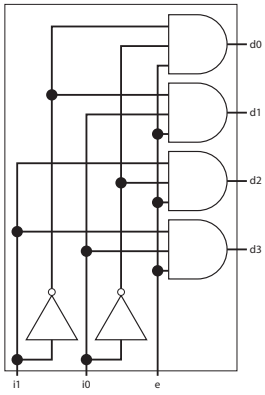
$$e1 = i3 + i2$$
$$e0 = i3 + i2'i1$$



4-bit 2x1 mux



2x4 decoder w/ enable



4x1 multiplexer