1. Write a subroutine (function) that will convolve two sequences using (2).

$$(2) \quad y[n] = \sum_{m=0}^{n} f[m]h[n-m] = f[n]*h[n]$$

   You might want to use a function description like

```
leny = conv(double *f1, int len1, double *f2, int len2, double *y)
```

   which would convolve the sequence in the array `f1` having `len1` points with the sequence in the array `f2` having `len2` points. The result of the convolution is returned in the array `y`, and the number of points in the convolution is returned in `leny`.

2. Test your program by convolving the following functions:

   (a) $f_1 * f_1$
   (b) $f_1 * f_2$
   (c) $f_1 * f_3$
   (d) $f_2 * f_3$
   (e) $f_1 * f_4$

   where the sequences are described by the C/C++ arrays

```
f1[] = {0,1,2,3,2,1};
len1 = 6;
f2[] = {-2,-2,-2,-2,-2,-2,-2};
len2 = 7;
f3[] = {1,-1,1,-1};
len3 = 4;
f4[] = {0,0,0,-3,-3};
len4 = 5;
```

   Remember that C/C++ arrays start with an index of 0.

   Plot these functions. Verify that the convolution is working as it should.

   Do the convolutions by hand. Also, compute the same convolutions using the `Matlab` function `conv`. Compare the results from the three methods (they better all be the same!).

```
For the given functions...
f1 = {0,1,2,3,2,1}
f2 = {-2,-2,-2,-2,-2,-2,-2}
f3 = {1,-1,1,-1}
f4 = {0,0,0,-3,-3}

the resulting convolutions are...
(a) conv(f1,f1) = {0,0,1,4,10,16,19,16,10,4,1}
(b) conv(f1,f2) = {0,-2,-6,-12,-16,-18,-18,-18,-16,-12,-6,-2}
(c) conv(f1,f3) = {0,1,1,2,0,0,-2,-1,-1}
(d) conv(f2,f3) = {-2,0,-2,0,0,0,0,2,0,2}
(e) conv(f1,f4) = {0,0,0,0,-3,-9,-15,-15,-9,-3}
```

(a) $f_1 * f_1 \quad m = \{0,1,2,3,4,5\}$

$$y[0] = \sum_{m=0}^{0} f[m]h[n-m] = (0\bullet 0)+0+0+0+0+0=0$$
$$y[1] = \sum_{m=0}^{1} f[m]h[n-m] = (0\bullet 1)+(1\bullet 0)+0+0+0+0=0+0=0$$
$$y[2] = \sum_{m=0}^{2} f[m]h[n-m] = (0\bullet 2)+(1\bullet 1)+(2\bullet 0)+0+0+0=0+1+0=1$$
$$y[3] = \sum_{m=0}^{3} f[m]h[n-m] = (0\bullet 3)+(1\bullet 2)+(2\bullet 1)+(3\bullet 0)+0+0=0+2+2+0=4$$
$$y[4] = \sum_{m=0}^{4} f[m]h[n-m] = (0\bullet 2)+(1\bullet 3)+(2\bullet 2)+(3\bullet 1)+(2\bullet 0)+0=0+3+4+3+0=10$$
$$y[5] = \sum_{m=0}^{5} f[m]h[n-m] = (0\bullet 1)+(1\bullet 2)+(2\bullet 3)+(3\bullet 2)+(2\bullet 1)+(1\bullet 0)=0+2+6+6+2+0=16$$
$$y[6] = \sum_{m=0}^{6} f[m]h[n-m] = 0+(1\bullet 1)+(2\bullet 2)+(3\bullet 3)+(2\bullet 2)+(1\bullet 1)=1+4+9+4+1=19$$
$$y[7] = \sum_{m=0}^{7} f[m]h[n-m] = 0+0+(2\bullet 1)+(3\bullet 2)+(2\bullet 3)+(1\bullet 2)=2+6+6+2=16$$
$$y[8] = \sum_{m=0}^{8} f[m]h[n-m] = 0+0+0+(3\bullet 1)+(2\bullet 2)+(1\bullet 3)=3+4+3=10$$
$$y[9] = \sum_{m=0}^{9} f[m]h[n-m] = 0+0+0+0+(2\bullet 1)+(1\bullet 2)=2+2=4$$
$$y[10] = \sum_{m=0}^{10} f[m]h[n-m] = 0+0+0+0+0+(1\bullet 1)=1$$
$$y = \{0,0,1,4,10,16,19,16,10,4,1\}$$

(b) $f_1 * f_2 \quad m = \{0,1,2,3,4,5,6\}$

$$y[0] = \sum_{m=0}^{0} f[m]h[n-m] = (0\bullet -2)+0+0+0+0+0+0=0$$
$$y[1] = \sum_{m=0}^{1} f[m]h[n-m] = (0\bullet -2)+(1\bullet -2)+0+0+0+0+0=0-2=-2$$
$$y[2] = \sum_{m=0}^{2} f[m]h[n-m] = (0\bullet -2)+(1\bullet -2)+(2\bullet -2)+0+0+0+0=0-2-4=-6$$
$$y[3] = \sum_{m=0}^{3} f[m]h[n-m] = (0\bullet -2)+(1\bullet -2)+(2\bullet -2)+(3\bullet -2)+0+0+0=0-2-4-6=-12$$
$$y[4] = \sum_{m=0}^{4} f[m]h[n-m] = (0\bullet -2)+(1\bullet -2)+(2\bullet -2)+(3\bullet -2)+(2\bullet -2)+0+0=0-2-4-6-4=-16$$
$$y[5] = \sum_{m=0}^{5} f[m]h[n-m] = (0\bullet -2)+(1\bullet -2)+(2\bullet -2)+(3\bullet -2)+(2\bullet -2)+(1\bullet -2)+0=0-2-4-6-4-2=-18$$
$$y[6] = \sum_{m=0}^{6} f[m]h[n-m] = (0\bullet -2)+(1\bullet -2)+(2\bullet -2)+(3\bullet -2)+(2\bullet -2)+(1\bullet -2)+(0\bullet -2)=0-2-4-6-4-2+0=-18$$
$$y[7] = \sum_{m=0}^{7} f[m]h[n-m] = 0+(1\bullet -2)+(2\bullet -2)+(3\bullet -2)+(2\bullet -2)+(1\bullet -2)+(0\bullet -2)=-2-4-6-4-2+0=-18$$
$$y[8] = \sum_{m=0}^{8} f[m]h[n-m] = 0+0+(2\bullet -2)+(3\bullet -2)+(2\bullet -2)+(1\bullet -2)+(0\bullet -2)=-4-6-4-2+0=-16$$
$$y[9] = \sum_{m=0}^{9} f[m]h[n-m] = 0+0+0+(3\bullet -2)+(2\bullet -2)+(1\bullet -2)+(0\bullet -2)=-6-4-2+0=-12$$
$$y[10] = \sum_{m=0}^{10} f[m]h[n-m] = 0+0+0+0+(2\bullet -2)+(1\bullet -2)+(0\bullet -2)=-4-2+0=-6$$
$$y[11] = \sum_{m=0}^{11} f[m]h[n-m] = 0+0+0+0+0+(1\bullet -2)+(0\bullet -2)=-2+0=-2$$
$$y[12] = \sum_{m=0}^{12} f[m]h[n-m] = 0+0+0+0+0+0+(0\bullet -2)=0$$
$$y = \{0,-2,-6,-12,-16,-18,-18,-18,-16,-12,-6,-2,0\}$$

(c) $f_1 * f_3 \quad m = \{0,1,2,3,4,5\}$

$$y[0] = \sum_{m=0}^{0} f[m]h[n-m] = (0\bullet 1)+0+0+0+0+0=0$$
$$y[1] = \sum_{m=0}^{1} f[m]h[n-m] = (0\bullet -1)+(1\bullet 1)+0+0+0+0=0+1=1$$
$$y[2] = \sum_{m=0}^{2} f[m]h[n-m] = (0\bullet 1)+(1\bullet -1)+(2\bullet 1)+0+0+0=0-1+2=1$$
$$y[3] = \sum_{m=0}^{3} f[m]h[n-m] = (0\bullet -1)+(1\bullet 1)+(2\bullet -1)+(3\bullet 1)+0+0=0+1-2+3=2$$
$$y[4] = \sum_{m=0}^{4} f[m]h[n-m] = (0\bullet 0)+(1\bullet -1)+(2\bullet 1)+(3\bullet -1)+(2\bullet 1)+0=0-1+2-3+2=0$$
$$y[5] = \sum_{m=0}^{5} f[m]h[n-m] = (0\bullet 0)+(1\bullet 0)+(2\bullet -1)+(3\bullet 1)+(2\bullet -1)+(1\bullet 1)=0+0-2+3-2+1=0$$
$$y[6] = \sum_{m=0}^{6} f[m]h[n-m] = 0+(1\bullet 0)+(2\bullet 0)+(3\bullet -1)+(2\bullet 1)+(1\bullet -1)=0+0-3+2-1=-2$$
$$y[7] = \sum_{m=0}^{7} f[m]h[n-m] = 0+0+(2\bullet 0)+(3\bullet 0)+(2\bullet -1)+(1\bullet 1)=0+0-2+1=-1$$
$$y[8] = \sum_{m=0}^{8} f[m]h[n-m] = 0+0+0+(3\bullet 0)+(2\bullet 0)+(1\bullet -1)=0+0-1=-1$$
$$y[9] = \sum_{m=0}^{9} f[m]h[n-m] = 0+0+0+0+(2\bullet 0)+(1\bullet 0)=0+0=0$$
$$y[10] = \sum_{m=0}^{10} f[m]h[n-m] = 0+0+0+0+0+(1\bullet 0)=0$$
$$y = \{0,1,1,2,0,0,-2,-1,-1,0,0\}$$

(d) $f_2 * f_3 \quad m = \{0,1,2,3,4,5,6\}$

$$y[0] = \sum_{m=0}^{0} f[m]h[n-m] = (-2\bullet 1)+0+0+0+0+0+0=-2$$
$$y[1] = \sum_{m=0}^{1} f[m]h[n-m] = (-2\bullet -1)+(-2\bullet 1)+0+0+0+0+0=2-2=0$$
$$y[2] = \sum_{m=0}^{2} f[m]h[n-m] = (-2\bullet 1)+(-2\bullet -1)+(-2\bullet 1)+0+0+0+0=-2+2-2=-2$$
$$y[3] = \sum_{m=0}^{3} f[m]h[n-m] = (-2\bullet -1)+(-2\bullet 1)+(-2\bullet -1)+(-2\bullet 1)+0+0+0=2-2+2-2=0$$
$$y[4] = \sum_{m=0}^{4} f[m]h[n-m] = (-2\bullet 0)+(-2\bullet -1)+(-2\bullet 1)+(-2\bullet -1)+(-2\bullet 1)+0+0=0+2-2+2-2=0$$
$$y[5] = \sum_{m=0}^{5} f[m]h[n-m] = (-2\bullet 0)+(-2\bullet 0)+(-2\bullet -1)+(-2\bullet 1)+(-2\bullet -1)+(-2\bullet 1)+0=0+0+2-2+2-2=0$$
$$y[6] = \sum_{m=0}^{6} f[m]h[n-m] = (-2\bullet 0)+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet -1)+(-2\bullet 1)+(-2\bullet -1)+(-2\bullet 1)=0+0+0+2-2+2-2=0$$
$$y[7] = \sum_{m=0}^{7} f[m]h[n-m] = 0+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet -1)+(-2\bullet 1)+(-2\bullet -1)=0+0+0+2-2+2=2$$
$$y[8] = \sum_{m=0}^{8} f[m]h[n-m] = 0+0+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet -1)+(-2\bullet 1)=0+0+0+2-2=0$$
$$y[9] = \sum_{m=0}^{9} f[m]h[n-m] = 0+0+0+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet -1)=0+0+0+2=2$$
$$y[10] = \sum_{m=0}^{10} f[m]h[n-m] = 0+0+0+0+(-2\bullet 0)+(-2\bullet 0)+(-2\bullet 0)=0+0+0=0$$
$$y[11] = \sum_{m=0}^{11} f[m]h[n-m] = 0+0+0+0+0+(-2\bullet 0)+(-2\bullet 0)=0+0=0$$
$$y[12] = \sum_{m=0}^{12} f[m]h[n-m] = 0+0+0+0+0+0+(-2\bullet 0)=0$$
$$y = \{-2,0,-2,0,0,0,0,2,0,2,0,0,0\}$$

(e) $f_1 * f_4 \quad m = \{0,1,2,3,4,5\}$

$$y[0] = \sum_{m=0}^{0} f[m]h[n-m] = (0\bullet 0)+0+0+0+0+0=0$$
$$y[1] = \sum_{m=0}^{1} f[m]h[n-m] = (0\bullet 0)+(1\bullet 0)+0+0+0+0=0+0=0$$
$$y[2] = \sum_{m=0}^{2} f[m]h[n-m] = (0\bullet 0)+(1\bullet 0)+(2\bullet 0)+0+0+0=0+0+0=0$$
$$y[3] = \sum_{m=0}^{3} f[m]h[n-m] = (0\bullet -3)+(1\bullet 0)+(2\bullet 0)+(3\bullet 0)+0+0=0+0+0+0=0$$
$$y[4] = \sum_{m=0}^{4} f[m]h[n-m] = (0\bullet -3)+(1\bullet -3)+(2\bullet 0)+(3\bullet 0)+(2\bullet 0)+0=0-3+0+0+0=-3$$
$$y[5] = \sum_{m=0}^{5} f[m]h[n-m] = (0\bullet 0)+(1\bullet -3)+(2\bullet -3)+(3\bullet 0)+(2\bullet 0)+(1\bullet 0)=0-3-6+0+0+0=-9$$
$$y[6] = \sum_{m=0}^{6} f[m]h[n-m] = 0+(1\bullet 0)+(2\bullet -3)+(3\bullet -3)+(2\bullet 0)+(1\bullet 0)=0-6-9+0+0=-15$$
$$y[7] = \sum_{m=0}^{7} f[m]h[n-m] = 0+0+(2\bullet 0)+(3\bullet -3)+(2\bullet -3)+(1\bullet 0)=0-9-6+0=-15$$
$$y[8] = \sum_{m=0}^{8} f[m]h[n-m] = 0+0+0+(3\bullet 0)+(2\bullet -3)+(1\bullet -3)=0-6-3=-9$$
$$y[9] = \sum_{m=0}^{9} f[m]h[n-m] = 0+0+0+0+(2\bullet 0)+(1\bullet -3)=0-3=-3$$
$$y[10] = \sum_{m=0}^{10} f[m]h[n-m] = 0+0+0+0+0+(1\bullet 0)=0$$
$$y = \{0,0,0,0,-3,-9,-15,-15,-9,-3,0\}$$

```
>> f1 = [0 1 2 3 2 1]; f2 = [-2 -2 -2 -2 -2 -2 -2]; f3 = [1 -1 1 -1]; f4 = [0 0 0 -3 -3];
>> conv(f1,f1)

ans =

   0   0   1   4   10   16   19   16   10   4   1

>> conv(f1,f2)

ans =

   0  -2  -6  -12  -16  -18  -18  -18  -16  -12  -6  -2

>> conv(f1,f3)

ans =

   0   1   1   2   0   0   -2   -1   -1

>> conv(f2,f3)

ans =

   -2   0   -2   0   0   0   0   2   0   2

>> conv(f1,f4)

ans =

   0   0   0   0   -3   -9   -15   -15   -9   -3
```

3. Using the results from Program #1 (which computed the zero-input response), find the **total solution** to the differential equation

$$(D^3 + 5D^2 + 12D + 15)y_o(t) = (D + 0.5)f(t)$$

with initial conditions $y_o(0) = -3$, $\dot{y}_o(0) = 2$, $\ddot{y}_o(0) = 1$ and $f(t) = \sin(3\pi t)u(t)$. Use $T = 0.001$, and let $0 \le t \le 10$. You will need to incorporate part of Program #1 into your new program to complete this part.

   (a) Using paper-and-pencil analysis, find the impulse response of the system $h(t)$. Then compute and plot its sampled values $h[k] = h(kT)$. You may use Laplace transform methods if you want.

   (b) Find the sampled values of the input function $f[k] = f(kT)$. Plot these values.

   (c) The zero-state solution is the scaled convolution $T(f[k] * h[k])$.

   (d) The zero-input solution is found using Program #1.

   (e) The total solution is the sum of the zero-state solution and the zero-input solution. (You will somehow need to incorporate the data from Program #1 in with the data from this program to get the total solution.)

   (f) Compute numerically and plot the total solution.

   (g) Find an analytical solution to the DE and plot it.

   (h) Compare the analytical and the numerical solution. (Comment)

$$\left(D^3+5D^2+12D+15\right)y(t)=(D+0.5)f(t)$$

$$y_o(0)=-3 \quad \dot{y}_o(0)=2 \quad \ddot{y}_o(0)=1 \quad f(t)=\sin(3\pi t)u(t)$$

$^{1}\Rightarrow \quad \lambda^3+5\lambda^2+12\lambda+15=0 \quad \lambda_1=-1.198 \quad \lambda_2=-2.604 \quad \omega=2.080\text{ rad/s}$

$^{2}\Rightarrow \quad y_o(t)=\left(C_1\cos 2.080t+C_2\sin 2.080t\right)e^{-1.198t}+C_3e^{-2.604t}$

$^{3}\Rightarrow \quad \dot{y}_o(t)=\left(C_1\left(-1.198\cos 2.080t-2.080\sin 2.080t\right)+C_2\left(2.080\cos 2.080t-1.198\sin 2.080t\right)\right)e^{-1.198t}-2.604C_3e^{-2.604t}$

$^{4}\Rightarrow \quad \ddot{y}_o(t)=\left(C_1\left(-2.890\cos 2.080t+4.983\sin 2.080t\right)+C_2\left(-4.983\cos 2.080t-2.890\sin 2.080t\right)\right)e^{-1.198t}+6.780C_3e^{-2.604t}$

$^{5}\Rightarrow \quad -3=C_1+C_3 \quad 2=-1.198C_1+2.080C_2-2.604C_3 \quad 1=-2.890C_1-4.983C_2+6.780C_3$

$^{6}\Rightarrow \quad C_1=-1.177 \quad C_2=-1.999 \quad C_3=-1.823$

$^{7}\Rightarrow \quad y_o(t)=\left(-1.177\cos 2.080t-1.999\sin 2.080t\right)e^{-1.198t}-1.823e^{-2.604t}$

$^{8}\Rightarrow \quad y_n(t)=\left(-0.159\cos 2.080t+0.107\sin 2.080t\right)e^{-1.198t}+0.159e^{-2.604t}$

$\dot{y}_n(t)=\left(0.413\cos 2.080t+0.202\sin 2.080t\right)e^{-1.198t}-0.413e^{-2.604t}$

$^{9}\Rightarrow \quad h(t)=P(D)y_n(t)u(t)+b_n\delta(t)$

$^{10}\Rightarrow \quad P(D)=D+0.5 \quad n=3 \quad b_3=0$

$^{11}\Rightarrow \quad h(t)=(D+0.5)y_n(t)u(t)$

$^{12}\Rightarrow \quad h(t)=\left(Dy_n(t)+0.5y_n(t)\right)u(t)$

$^{13}\Rightarrow \quad h(t)=\left[\left(0.334\cos 2.080t+0.255\sin 2.080t\right)e^{-1.198t}-0.334e^{-2.604t}\right]u(t)$

$^{14}\Rightarrow \quad y(t)=h(t)*f(t) \quad Y(s)=H(s)F(s)$

$^{15}\Rightarrow \quad H(s)=\dfrac{0.334(s+1.198)}{(s+1.198)^2+(2.080)^2}+\dfrac{0.255(2.080)}{(s+1.198)^2+(2.080)^2}-\dfrac{0.334}{s+2.604} \quad F(s)=\dfrac{3\pi}{s^2+(3\pi)^2}$

$^{16}\Rightarrow \quad Y(s)=\dfrac{\left(3.682\times10^{-12}\right)\left(s^2+\left(2.560\times10^{12}\right)s+\left(1.280\times10^{12}\right)\right)}{(s+2.604)\left(s^2+88.826\right)\left(s^2+2.396s+5.761\right)}$

$^{17}\Rightarrow \quad y(t)=\left(-5.198\times10^{-3}\right)\cos(3\pi t)-\left(9.936\times10^{-3}\right)\sin(3\pi t)+\left(0.019\cos\omega t-0.013\sin\omega t\right)e^{\lambda_1 t}-0.033e^{\lambda_2 t}$

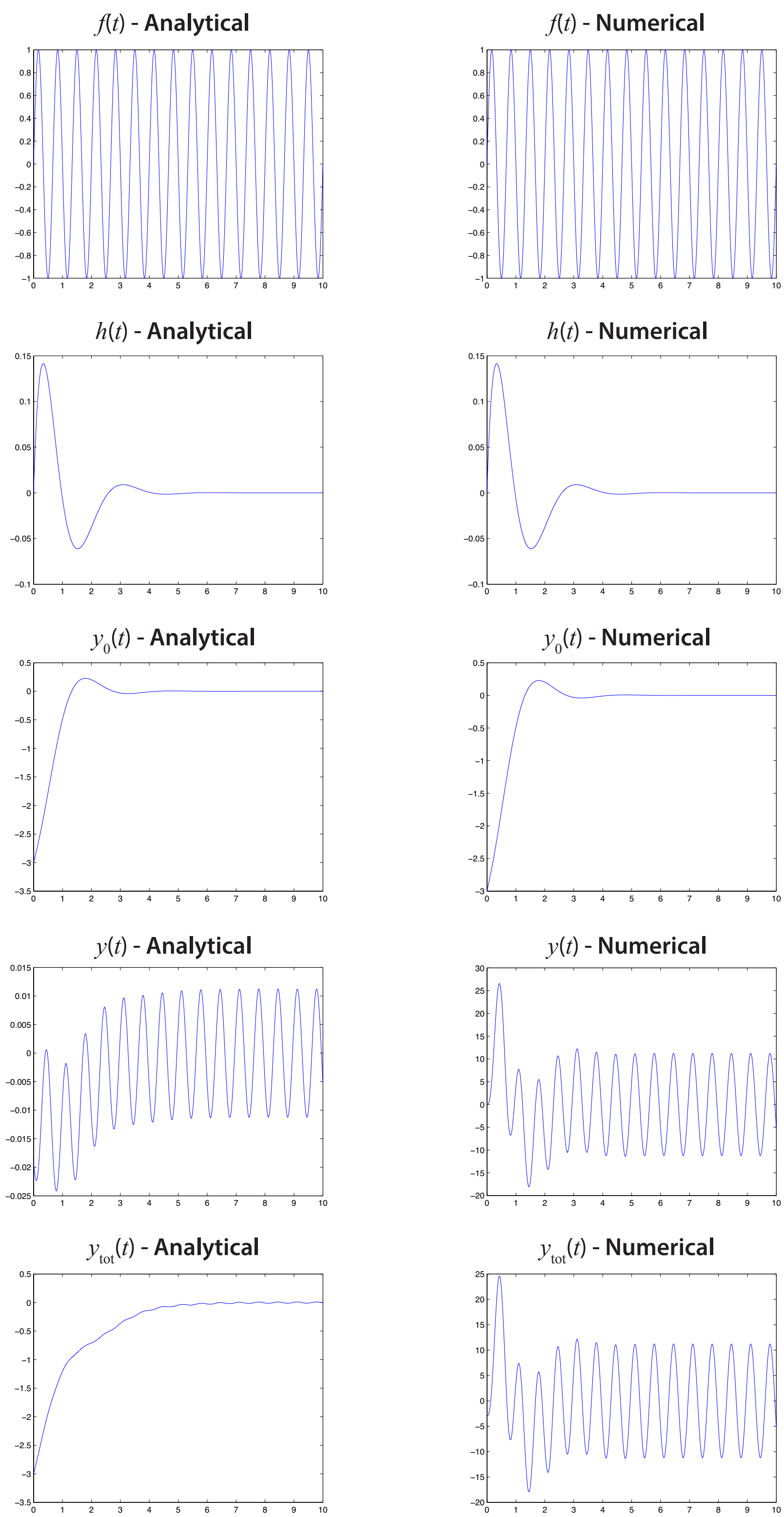
$^{18}\Rightarrow \quad y_{tot}(t)=y_o(t)+y(t)$

$y_{tot}(t)=\left(-5.198\times10^{-3}\right)\cos(3\pi t)-\left(9.936\times10^{-3}\right)\sin(3\pi t)+\left(-1.138\cos\omega t-1.973\sin\omega t\right)e^{\lambda_1 t}-1.856e^{\lambda_2 t}$

```
For the differential equation...
(D^3 + 5D^2 + 12D + 15)y0(t) = (D + 0.5)*f(t)
with initial conditions y0(0) = -3, Dy0(0) = 2, D^2y0(0) = 1 and
f(t) = sin(3*PI*t)*u(t)

h(t) = [(0.334*cos(2.080*t)+0.255*sin(2.080*t))*exp(-1.198*t)...
      ...-0.334*exp(-2.604*t)]*u(t)

Time - Step Size, dt = 0.001
Time - Initial, ti = 0
Time - Final, tf = 10
----------------------------------------------------------------
   t            y0                 y                y[tot]
----------------------------------------------------------------
      0     -3.00000000000     0.00000000000    -3.00000000000
      1     -0.48723964351     3.95118381756     3.46394417406
      2      0.20190431499    -9.19810056591    -8.99619625092
      3     -0.03117264688     6.19981412255     6.16864147567
      4     -0.01039388522    -5.14965639940    -5.16005028461
      5      0.00583501701     5.09097038663     5.09680540363
      6     -0.00078325717    -5.17103803791    -5.17182129508
      7     -0.00030167336     5.19980335377     5.19950168041
      8      0.00016074114    -5.20098063653    -5.20081989539
      9     -0.00002000183     5.19870666216     5.19868666033
     10     -0.00000876355    -5.19793077016    -5.19793953371
----------------------------------------------------------------
```


$f(t)$ - Analytical
$f(t)$ - Numerical
$h(t)$ - Analytical
$h(t)$ - Numerical
$y_0(t)$ - Analytical
$y_0(t)$ - Numerical
$y(t)$ - Analytical
$y(t)$ - Numerical
$y_{tot}(t)$ - Analytical
$y_{tot}(t)$ - Numerical

```
>> t = [0:0.001:10];

LAMBDA1 = -1.1980924212362;
LAMBDA2 = -2.6038151575276;
OMEGA = 2.079748094628;
OMEGAf = 3*pi;
C1 = -1.176604010431;
C2 = -1.99902405212;
C3 = -1.82339598957;
Ch1 = 0.33386426881482;
Ch2 = 0.25516523268429;
Ch3 = -0.33386426881443;
CyA = -0.005197992950;
CyB = 0.009935683856;
Cy1 = 0.019055008;
Cy2 = 0.012887164;
Cy3 = -0.032912023931;
CyTOT1 = -1.138493994431;
CyTOT2 = 1.97324972412;
CyTOT3 = -1.856308013501;

f = sin(3*pi*t);
h = (Ch1.*cos(OMEGA*t)+Ch2.*sin(OMEGA*t)).*exp(LAMBDA1*t)+Ch3.*exp(LAMBDA2*t);
y0 = (C1.*cos(OMEGA*t)+C2.*sin(OMEGA*t)).*exp(LAMBDA1*t)+C3.*exp(LAMBDA2*t);
y = CyA.*cos(OMEGAf*t)-CyB.*sin(OMEGAf*t)+(Cy1.*cos(OMEGA*t)-Cy2.*(OMEGA*t)).*exp(LAMBDA1*t)+Cy3.*exp(LAMBDA2*t);
yTOT = CyA.*cos(OMEGAf*t)-CyB.*sin(OMEGAf*t)+(CyTOT1.*cos(OMEGA*t)-CyTOT2.*(OMEGA*t)).*exp(LAMBDA1*t)+CyTOT3.*exp(LAMBDA2*t);

plot(t,f); plot(t,f); plot(t,y0); plot(t,y); plot(t,yTOT);
```

In comparing the plots of the analytical and numerical solutions, we see that the plots correctly match each other for the input signal $f(t)$, the impulse response $h(t)$, and the zero-input response $y_0(t)$. However, the plots for the zero-state response $y(t)$ and the total response $y_{tot}(t)$ do not match and thus indicate an error somewhere either in the analytical or the numerical method.

Intuitively I believe the analytical solution is correct and the numerical solution plots for the affected zero-state and total response are in error. Curiously, the convolution function does work correctly and the test of the function in problem 2 shows that.

4. Note that the solution to the system output in 3 above settles to a "steady-state" solution after a few seconds, where the signal form continues unchanged. Your plots should indicate this. What is the steady-state output amplitude? Why does this happen?

From the plot of the total response via the numerical method, we see that the steady-state output amplitude is 10. This happens because the roots of the differential equation are both negative and therefore indicate an asymptotically stable system.