1. For the differential equation

$$(D + 2.5)y_0(t) = 0$$

with the initial condition $y_0(0) = 3$:

(a) Find and plot the analytical (exact) solution to the differential equation for $0 \le t \le 10$.

(b) Write a program in C(++) to plot a numerical solution using (3). You may have to try several values of Δt to get a good enough approximation.

(c) Compare the exact solution with the approximate solution.

$^{1}\Rightarrow \quad \lambda + 2.5 = 0$

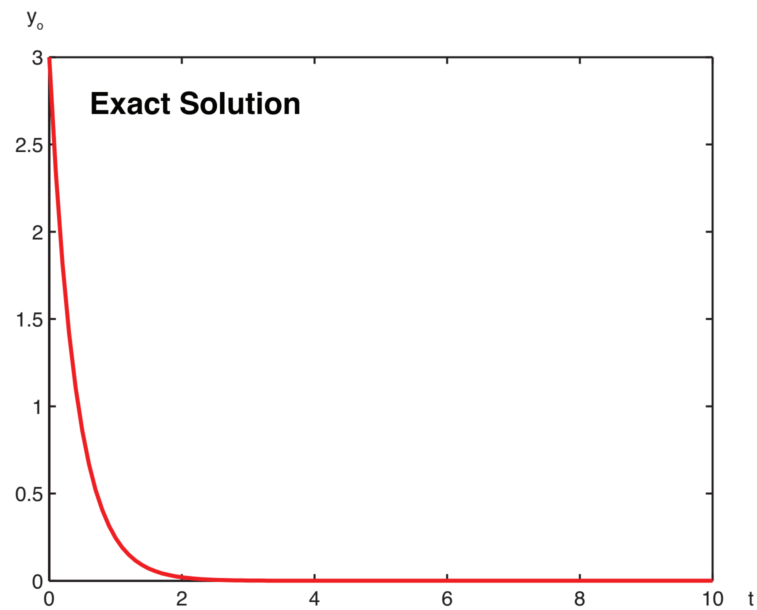
$^{2}\Rightarrow \quad \lambda = -2.5$

$^{3}\Rightarrow \quad y_o(t) = C_1 e^{-2.5t}$

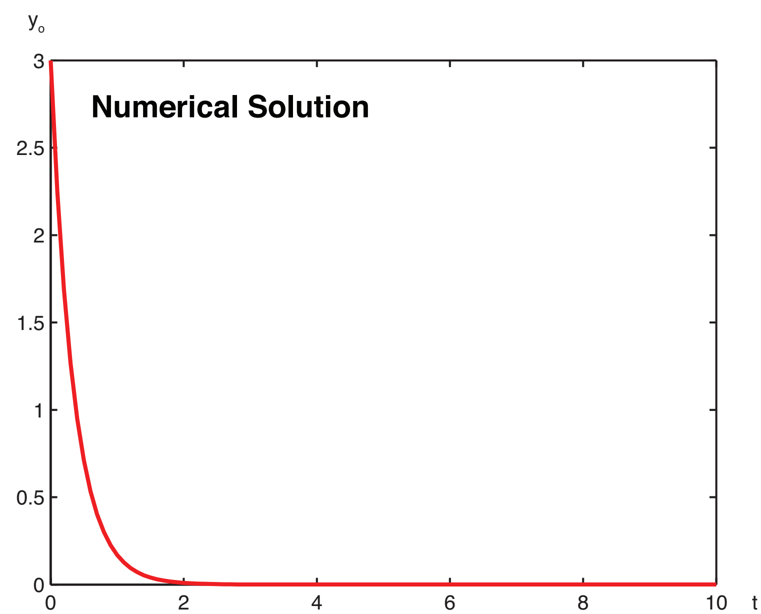
$^{4}\Rightarrow \quad 3 = C_1$

$^{5}\Rightarrow \quad y_o(t) = 3e^{-2.5t}$

t	y_o
0	3.000×10^{-0}
1	2.462×10^{-1}
2	2.021×10^{-2}
3	1.659×10^{-3}
4	1.362×10^{-4}
5	1.118×10^{-5}
6	0.918×10^{-6}
7	0.753×10^{-7}
8	0.618×10^{-8}
9	0.508×10^{-9}
10	0.412×10^{-10}




```
// 10/10/2014 - ECE 3620 - Meine, Joel
// Assignment #1 - Problem 1

#include <iostream>
#include <iomanip>
#include <math.h>
#include <vector>
#include <fstream>
using namespace std;

double a = -2.5; // Constant
double yi = 3; // Value - Initial
double dt = 0.1; // Time - Step Size
double tf = 10; // Time - Final
double Dp = 10; // Data Points, No. of

vector<double> ODE(vector<double> t)
{
        double y;
        vector<double> Y;
        y = yi;
        Y.push_back(y);
        for (int i = 0; i < t.size(); i++)
        {
```



```cpp
            y = (1 + a*dt)*y;
            Y.push_back(y);
        }
        Y.pop_back();
        return(Y);
}

int main()
{
        std::cout << "Assignment #1 - Problem 1" << std::endl;
        std::cout << "============================" << std::endl;
        std::cout << "For the differential equation..." << std::endl;
        std::cout << "(D + 2.5)yo(t) = 0" << std::endl;
        std::cout << "with initial condition yo(0) = 3:" << std::endl;
        std::cout << "Constant, a = " << a << std::endl;
        std::cout << "Value - Initial, yo(0) = " << yi << std::endl;
        std::cout << "Time - Step Size, dt = " << dt << std::endl;
        std::cout << "Time - Max, tf = " << tf << std::endl;
        std::cout << "----------------------------" << std::endl;
        std::cout << " t     y                    " << std::endl;
        std::cout << "----------------------------" << std::endl;

        double I;
        I = (tf/dt)/Dp; // Increment Length, I

        vector<double> T,sT; // Time List, T; Time List (Shortened), sT
        for (int j = 0; j <= tf/dt; j++)
        {
                T.push_back(j*dt);
        }
        for (int k = 0; k <= I; k++)
        {
                sT.push_back(T[k*I]);
        }

        vector<double> P,sP; // Print List, P; Print List (Shortened), sP
        P = ODE(T);
        for (int m = 0; m <= I; m++)
        {
                sP.push_back(P[m*I]);
        }
        for (int n = 0; n < sP.size(); n++)
        {
                printf(" %2.1f   %20.18f \n", sT[n], sP[n]);
        }
        ofstream outfile("A1_P1.txt");
        for (int o = 0; o < P.size(); o++)
        {
                outfile << T[o] << "  " << P[o] << endl;
        }

        std::cout << "----------------------------" << std::endl;
        system("pause");
        return 0;
}
```

```
Assignment #1 - Problem 1
============================
For the differential equation...
(D + 2.5)yo(t) = 0
with initial condition yo(0) = 3:
Constant, a = -2.5
Value - Initial, yo(0) = 3
Time - Step Size, dt = 0.1
Time - Max, tf = 10
----------------------------
 t     y
----------------------------
 0.0   3.000000000000000000
 1.0   0.168940544128417970
 2.0   0.009513635816801980
 3.0   0.000535746270510044
 4.0   0.000030169755484912
 5.0   0.000001698964969281
 6.0   0.000000095674688788
 7.0   0.000000005387777994
 8.0   0.000000000303404715
 9.0   0.000000000017085786
 10.0   0.000000000000962161
----------------------------
```

2. For the third-order differential equation

$$(D^3 + 0.6D^2 + 25.1125D + 2.5063)y_0(t) = 0$$

with the initial conditions $y_0(0) = 1.5$, $\dot{y}_0(0) = 2$, $\ddot{y}_0(0) = -1$:

(a) Find and plot the analytical solution to the differential equation for $0 \le t \le 10$. Identify the roots of the characteristic equation and plot them in the complex plane.

(b) Put the third-order differential equation into state-space form.

(c) Write a program in C(++) to plot an approximate solution using (8). You may have to try several values of Δt to get a reasonable approximation.

(d) Compare the exact solution with the approximate solution.

$$^{1}\Rightarrow \quad \lambda^3 + 0.6\lambda^2 + 25.1125\lambda + 2.5063 = 0$$

$$^{2}\Rightarrow \quad (\lambda + 0.1)(\lambda^2 + 0.5 + 25.0625) = 0$$

$$^{3}\Rightarrow \quad \lambda_1 = -0.25 + j5 \quad \lambda_2 = -0.25 - j5 \quad \lambda_3 = -0.1$$

$$^{4}\Rightarrow \quad y_o(t) = (C_1 \cos 5t + C_2 \sin 5t)e^{-0.25t} + C_3 e^{-0.1t}$$

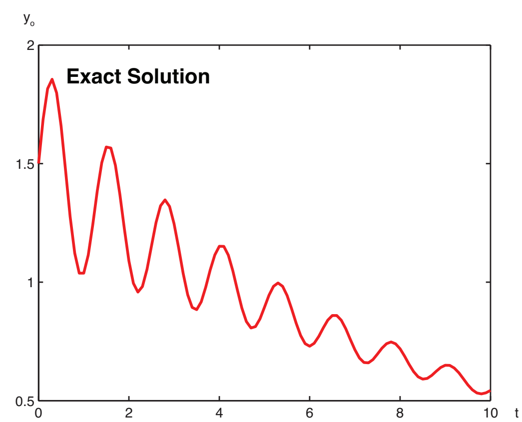
$$^{5}\Rightarrow \quad \dot{y}_o(t) = (C_1(-0.25\cos 5t - 5\sin 5t) + C_2(5\cos 5t - 0.25\sin 5t))e^{-0.25t} - 0.1C_3 e^{-0.1t}$$

$$^{6}\Rightarrow \quad \ddot{y}_o(t) = (C_1(-24.9375\cos 5t + 2.5\sin 5t) + C_2(-2.5\cos 5t - 24.9375\sin 5t))e^{-0.25t} + 0.01C_3 e^{-0.1t}$$

$$^{7}\Rightarrow \quad \begin{aligned} 1.5 &= C_1 + C_3 \\ 2 &= -0.25C_1 + 5C_2 - 0.1C_3 \\ -1 &= -24.9375C_1 - 2.5C_2 + 0.01C_3 \end{aligned}$$

$$^{8}\Rightarrow \quad C_1 = -0.002398 \quad C_2 = 0.429928 \quad C_3 = 1.5024$$

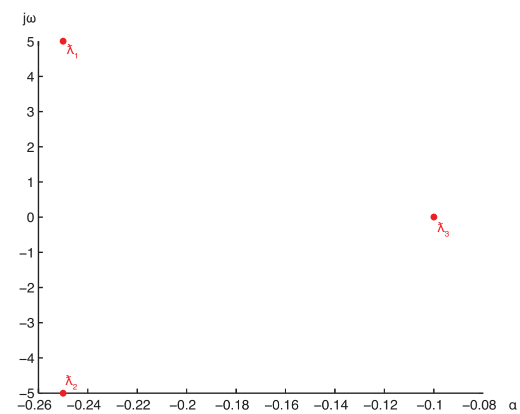
$$^{9}\Rightarrow \quad y_o(t) = (-0.002398\cos 5t + 0.429928\sin 5t)e^{-0.25t} + 1.5024e^{-0.1t}$$

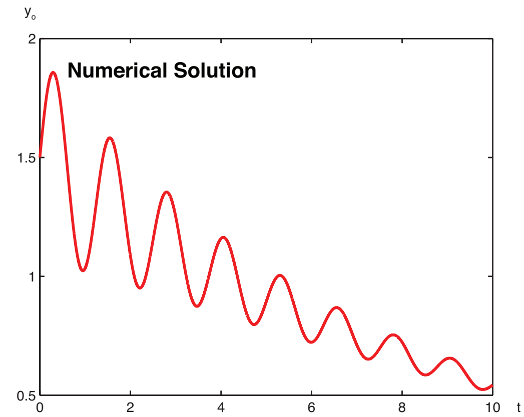


$$^{1}\Rightarrow \quad \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2.5063 & -25.1125 & -0.6 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

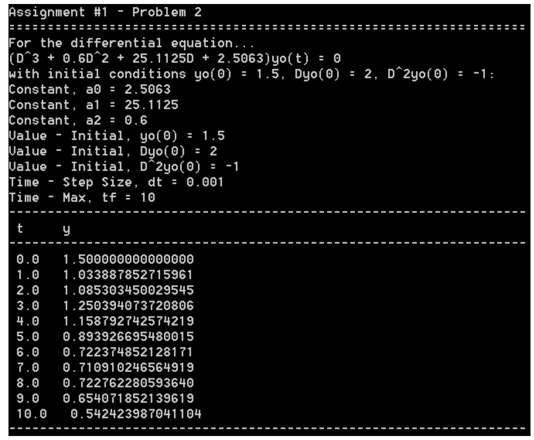
$$^{2}\Rightarrow \quad \begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= -2.5063x_1(t) - 25.1125x_2(t) - 0.6x_3(t) \end{aligned}$$

$$^{3}\Rightarrow \quad \dot{x}_3(t) + 0.6x_3(t) + 25.1125x_2(t) + 2.5063x_1(t) = 0$$






```
// 10/10/2014 - ECE 3620 - Meine, Joel
// Assignment #1 - Problem 2

#include <iostream>
#include <iomanip>
#include <math.h>
#include <vector>
#include <fstream>
using namespace std;

vector<double> a = {2.5063,25.1125,0.6}; // Constants
vector<double> yi = {1.5,2,-1}; // Values - Initial
int O = a.size(); // Order of Differential Equation
double dt = 0.001; // Time - Step Size
double tf = 10; // Time - Final
double Dp = 10; // Data Points, No. of

// NOTE: The number of items in a and yi "must" be equal.

vector<double> matrixOP_vecmul(vector<vector<double>> M, vector<double> u)
{ // Calculates v(n) = M(nxm) * u(m)
        vector<double> v;
        for (int k = 0; k < O; k++)
                v.push_back(0);
        int i; int j;
        for (i = 0; i < O; i++){
                v[i] = 0;
                for (j = 0; j < O; j++){
                        v[i] = v[i] + M[i][j] * u[j];
                }
        }
        return(v);
}

vector<vector<double>> ODE_m(vector<double> t)
{
        vector<double> y;
        vector<vector<double>> Y;
        y = yi;
        Y.push_back(y);

        // Z = w + A*dt
        vector<double> z;
        int r, s;
        for (r = 0; r < O; r++) // Column Size, Initialize
                z.push_back(0);
        vector<vector<double>> Z, w, A;
        for (s = 0; s < O; s++) // Row Size, Initialize
        {
                Z.push_back(z);
                w.push_back(z);
                A.push_back(z);
        }
        int i, j;
        for (i = 0; i < O; i++) // w
                for (j = 0; j < O; j++)
                        if (i == j)
                                w[i][j] = 1;
                        else
                                w[i][j] = 0;
        int m, n;
        for (m = 0; m < O; m++) // A
                for (n = 0; n < O; n++)
                        if (n == m + 1 && m != O - 1)
                                A[m][n] = 1;
                        else if (m == O - 1)
                                A[m][n] = -a[n];
```

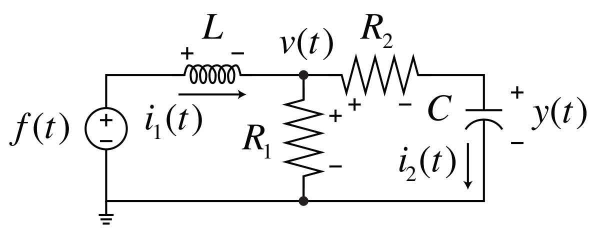


```
Assignment #1 - Problem 2
====================================================================
For the differential equation...
(D^3 + 0.6D^2 + 25.1125D + 2.5063)yo(t) = 0
with initial conditions yo(0) = 1.5, Dyo(0) = 2, D^2yo(0) = -1:
Constant, a0 = 2.5063
Constant, a1 = 25.1125
Constant, a2 = 0.6
Value - Initial, yo(0) = 1.5
Value - Initial, Dyo(0) = 2
Value - Initial, D^2yo(0) = -1
Time - Step Size, dt = 0.001
Time - Max, tf = 10
--------------------------------------------------------------------
 t     y
--------------------------------------------------------------------
 0.0   1.500000000000000
 1.0   1.033887852715961
 2.0   1.085303450029545
 3.0   1.250394073720806
 4.0   1.158792742574219
 5.0   0.893926695480015
 6.0   0.722374852128171
 7.0   0.710910246564919
 8.0   0.722762280593640
 9.0   0.654071852139619
 10.0   0.542423987041104
--------------------------------------------------------------------
```

```
                        else
                                A[m][n] = 0;
        int p, q;
        for (p = 0; p < O; p++) // Z
                for (q = 0; q < O; q++)
                        Z[p][q] = w[p][q] + A[p][q]*dt;
        // y = Z*y
        for (int r = 0; r < t.size(); r++)
        {
                y = matrixOP_vecmul(Z, y);
                Y.push_back(y);
        }
        Y.pop_back();
        return(Y);
}

int main()
{
        std::cout << "Assignment #1 - Problem 2" << std::endl;
        std::cout << "====================================================================" << std::endl;
        std::cout << "For the differential equation..." << std::endl;
        std::cout << "(D^3 + 0.6D^2 + 25.1125D + 2.5063)yo(t) = 0" << std::endl;
        std::cout << "with initial conditions yo(0) = 1.5, Dyo(0) = 2, D^2yo(0) = -1:" << std::endl;
        std::cout << "Constant, a0 = " << a[0] << std::endl;
        std::cout << "Constant, a1 = " << a[1] << std::endl;
        std::cout << "Constant, a2 = " << a[2] << std::endl;
        std::cout << "Value - Initial, yo(0) = " << yi[0] << std::endl;
        std::cout << "Value - Initial, Dyo(0) = " << yi[1] << std::endl;
        std::cout << "Value - Initial, D^2yo(0) = " << yi[2] << std::endl;
        std::cout << "Time - Step Size, dt = " << dt << std::endl;
        std::cout << "Time - Max, tf = " << tf << std::endl;
        std::cout << "--------------------------------------------------------------------" << std::endl;
        std::cout << " t     y                     " << std::endl;
        std::cout << "--------------------------------------------------------------------" << std::endl;

        double I;
        I = (tf/dt)/Dp; // Increment Length, I

        vector<double> T, sT; // Time List, T; Time List (Shortened), sT
        for (int j = 0; j <= tf / dt; j++)
        {
                T.push_back(j*dt);
        }
        for (int k = 0; k <= Dp; k++)
        {
                sT.push_back(T[k*I]);
        }

        vector<double> P, sP; // Print List, P; Print List (Shortened), sP

        vector<vector<double>> Q;
        Q = ODE_m(T);
        int f, g;
        for (f = 0; f < T.size(); f++)
                for (g = 0; g < T.size(); g++)
                        if (g == 0)
                                P.push_back(Q[f][g]); // Retrieves Only the Results of yo(t).

        for (int m = 0; m <= Dp; m++)
        {
                sP.push_back(P[m*I]);
        }
        for (int n = 0; n < sP.size(); n++)
        {
                printf(" %2.1f   %12.15f \n", sT[n], sP[n]);
        }
        ofstream outfile("A1_P2.txt");
```

```
        for (int o = 0; o < P.size(); o++)
        {
                outfile << T[o] << "  " << P[o] << endl;
        }

        std::cout << "------------------------------------------------------------------" << std::endl;
        system("pause");
        return 0;
}
```

3. For the circuit shown here:



where $R1$ = 1 kΩ, $R2$ = 22 kΩ, C = 10 μF, and L = 5 H.

(a) Determine a differential equation relating the input $f(t)$ to the output $y(t)$.

(b) Determine the initial conditions on $y(t)$ if $i_1(0) = 0.2$ A and $y(0) = 5$ V.

(c) Determine the analytical solution for the zero-input response of the system with these initial conditions.

(d) Represent the differential equation for the circuit in state variable form.

(e) Using your program, determine a numerical solution to the differential equation for the zero-input response.

(f) Plot and compare the analytical and the numerical solution. Comment on your results.

(g) Suppose that the circuit had a nonlinear element in it, such as dependent source. Describe how the analytical solution and numerical solution would change.

$$^{1}\Rightarrow \quad \frac{v}{R_1}+\frac{1}{L}\int v-f\,dt+\frac{v-y}{R_2}=0$$
$$\frac{y-v}{R_2}+C\frac{dy}{dt}=0$$

$$^{2}\Rightarrow \quad v=y+R_2C\frac{dy}{dt}$$

$$^{3}\Rightarrow \quad \frac{y}{R_1}+\frac{R_2}{R_1}C\frac{dy}{dt}+C\frac{dy}{dt}+\frac{1}{L}\int y+R_2C\frac{dy}{dt}-f\,dt=0$$

$$^{4}\Rightarrow \quad \left(1.0\times10^{-3}\right)\frac{dy}{dt}+\left(2.2\times10^{-4}\right)\frac{d^2y}{dt}+\left(10\times10^{-6}\right)\frac{d^2y}{dt}+0.2\left(y+0.22\frac{dy}{dt}-f\right)=0$$

$$^{5}\Rightarrow \quad \left(1.0\times10^{-3}\right)Dy(t)+\left(2.2\times10^{-4}\right)D^2y(t)+\left(10\times10^{-6}\right)D^2y(t)+0.2y(t)+\left(44\times10^{-3}\right)Dy(t)-0.2f(t)=0$$

$$^{6}\Rightarrow \quad \left(2.3\times10^{-4}\right)D^2y(t)+\left(45\times10^{-3}\right)Dy(t)+0.2y(t)=0.2f(t)$$

$$^{7}\Rightarrow \quad \left(D^2+195.652D+869.565\right)y(t)=869.565f(t)$$

$$^{1}\Rightarrow \quad x_1(t)=y_o(t)$$
$$x_2(t)=\dot{y}_o(t)$$

$$^{2}\Rightarrow \quad \dot{x}_1(t)=i_2(t)$$
$$\dot{x}_2(t)=-869.565i_1(t)-195.652i_2(t)$$

$$^{1}\Rightarrow \quad R_1\left(i_2(t)-i_1(t)\right)+R_2i_2+y(t)=0$$

$$^{2}\Rightarrow \quad \left(1\times10^{3}\right)\left(i_2(0)-0.2\right)+\left(22\times10^{3}\right)i_2(0)+5=0$$

$$^{3}\Rightarrow \quad i_2(0)=8.478261\text{ mA}$$

$$^{4}\Rightarrow \quad \frac{i_2(0)}{C}=\frac{dy}{dt}(0)$$

$$^{5}\Rightarrow \quad \frac{dy}{dt}(0)=847.826\text{ V/s}$$

$$^{1}\Rightarrow \quad \lambda^2+195.652\lambda+869.565=0$$

$$^{2}\Rightarrow \quad (\lambda+4.55027)(\lambda+191.102)=0$$

$$^{3}\Rightarrow \quad \lambda_1=-4.55027 \quad \lambda_2=-191.102$$

$$^{4}\Rightarrow \quad y_o(t)=C_1e^{-4.55027t}+C_2e^{-191.102t}$$

$$^{5}\Rightarrow \quad \dot{y}_o(t)=-4.55027C_1e^{-4.55027t}-191.102C_2e^{-191.102t}$$

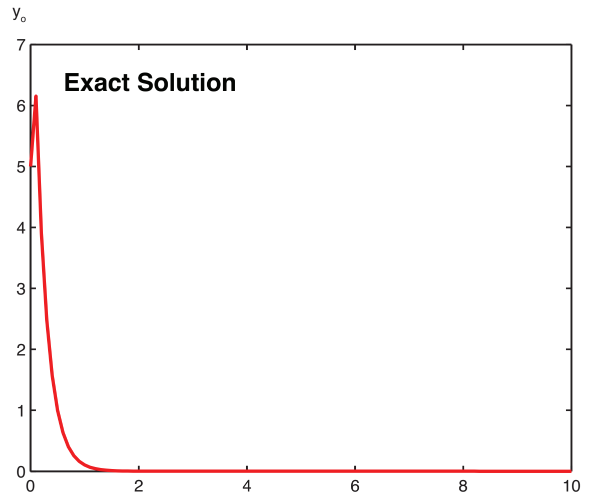
$$^{6}\Rightarrow \quad y_o(0)=C_1+C_2$$
$$\dot{y}_o(0)=-4.55027C_1-191.102C_2$$

$$^{7}\Rightarrow \quad 5=C_1+C_2$$
$$847.826=-4.55027C_1-191.102C_2$$

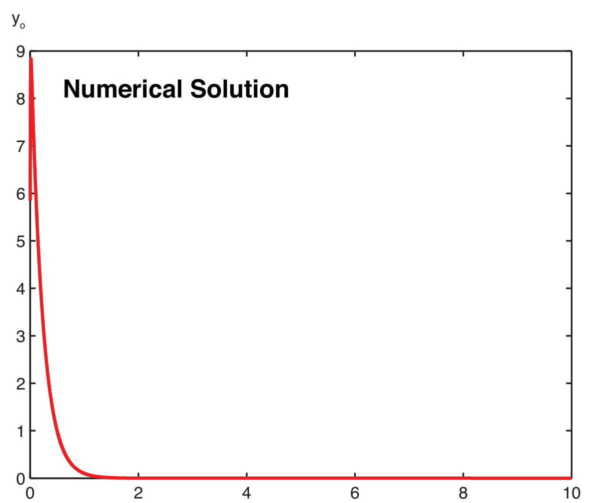
$$^{8}\Rightarrow \quad C_1=9.7 \quad C_2=-4.7$$

$$^{9}\Rightarrow \quad y_o(t)=9.7e^{-4.55027t}-4.7e^{-191.102t}$$

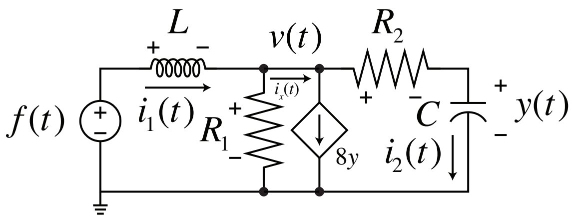
t	y_o
0	5.000×10^{-0}
1	1.025×10^{-1}
2	1.083×10^{-3}
3	1.144×10^{-5}
4	1.208×10^{-7}
5	1.276×10^{-9}
6	1.348×10^{-11}
7	1.425×10^{-13}
8	1.505×10^{-15}
9	1.590×10^{-17}
10	1.680×10^{-19}




```
Assignment #1 - Problem 3
===================================================
For the differential equation...
(D^2 + 195.652D + 869.565)yo(t) = 869.565*f(t)
with initial conditions yo(0) = 5, Dyo(0) = 847.826:
Constant, a0 = 869.565
Constant, a1 = 195.652
Value - Initial, yo(0) = 5
Value - Initial, Dyo(0) = 847.826
Time - Step Size, dt = 0.001
Time - Max, tf = 10
---------------------------------------------------
 t     y
---------------------------------------------------
 0.0   5.000000000000000
 1.0   0.101067312486863
 2.0   0.001056681139482
 3.0   0.000011047835379
 4.0   0.000000115507566
 5.0   0.000000001207657
 6.0   0.000000000012626
 7.0   0.000000000000132
 8.0   0.000000000000001
 9.0   0.000000000000000
 10.0   0.000000000000000
---------------------------------------------------
```



We see that with the increased precision of the numerical solution, it is revealed that from the starting initial condition of $y_o(0)=5$ V, the voltage rises just short of 9 V very quickly, but then decays towards stability also within a very short amount of time.



$$\overset{1}{\Rightarrow} \quad \frac{v}{R_1}+\frac{1}{L}\int v-f\,dt+\frac{v-y}{R_2}+8y=0$$

$$\frac{y-v}{R_2}+C\frac{dy}{dt}=0$$

$$\overset{2}{\Rightarrow} \quad v=y+R_2C\frac{dy}{dt}$$

$$\overset{3}{\Rightarrow} \quad \frac{y}{R_1}+\frac{R_2}{R_1}C\frac{dy}{dt}+C\frac{dy}{dt}+\frac{1}{L}\int y+R_2C\frac{dy}{dt}-f\,dt+8y=0$$

$$\overset{4}{\Rightarrow} \quad \left(1.0\times10^{-3}\right)\frac{dy}{dt}+\left(2.2\times10^{-4}\right)\frac{d^2y}{dt}+\left(10\times10^{-6}\right)\frac{d^2y}{dt}+0.2\left(y+0.22\frac{dy}{dt}-f\right)+8\frac{dy}{dt}=0$$

$$\overset{5}{\Rightarrow} \quad \left(1.0\times10^{-3}\right)Dy(t)+\left(2.2\times10^{-4}\right)D^2y(t)+\left(10\times10^{-6}\right)D^2y(t)+0.2y(t)+\left(44\times10^{-3}\right)Dy(t)-0.2f(t)+8Dy(t)=0$$

$$\overset{6}{\Rightarrow} \quad \left(2.3\times10^{-4}\right)D^2y(t)+8.045Dy(t)+0.2y(t)=0.2f(t)$$

$$\overset{7}{\Rightarrow} \quad \left(D^2+34978.3D+869.565\right)y(t)=869.565f(t)$$

$$\overset{1}{\Rightarrow} \quad i_1(0)-i_x(0)+8y(0)+i_2(0)=0 \qquad i_x=\frac{v(0)}{R_1}$$

$$R_2i_2(0)+y(0)+v(0)=0$$

$$\overset{2}{\Rightarrow} \quad v(0)=-R_2i_2(0)-y$$

$$\overset{3}{\Rightarrow} \quad i_1(0)+8y(0)+\frac{y(0)}{R_1}+\frac{R_2}{R_1}i_2(0)+i_2(0)=0$$

$$\overset{4}{\Rightarrow} \quad 23i_2(0)+40.205=0$$

$$\overset{5}{\Rightarrow} \quad i_2(0)=-1.74804\text{ A}$$

$$\overset{6}{\Rightarrow} \quad \frac{i_2(0)}{C}=\frac{dy}{dt}(0)$$

$$\overset{7}{\Rightarrow} \quad \frac{dy}{dt}(0)=-174804\text{ V/s}$$

$$\overset{1}{\Rightarrow} \quad \lambda^2+34978.3\lambda+869.565=0$$

$$\overset{2}{\Rightarrow} \quad (\lambda+34978.2)(\lambda+0.02486)=0$$

$$\overset{3}{\Rightarrow} \quad \lambda_1=-34978.2 \qquad \lambda_2=-0.02486$$

$$\overset{4}{\Rightarrow} \quad y_o(t)=C_1e^{-34978.2t}+C_2e^{-0.02486t}$$

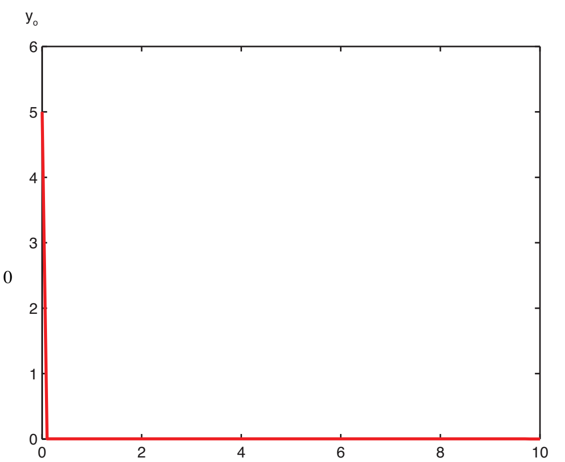
$$\overset{5}{\Rightarrow} \quad \dot{y}_o(t)=-34978.2C_1e^{-34978.2t}-0.02486C_2e^{-0.02486t}$$

$$\overset{6}{\Rightarrow} \quad \begin{aligned} y_o(0)&=C_1+C_2\\ \dot{y}_o(0)&=-34978.2C_1-0.02486C_2\end{aligned}$$

$$\overset{7}{\Rightarrow} \quad \begin{aligned} 5&=C_1+C_2\\ -174804&=-34978.2C_1-0.02486C_2\end{aligned}$$

$$\overset{8}{\Rightarrow} \quad C_1=4.99752 \qquad C_2=0.002482$$

$$\overset{9}{\Rightarrow} \quad y_o(t)=4.99752e^{-34978.2t}+0.002482e^{-0.02486t}$$



As with the earlier comparisons between the analytical solution and the numerical solution computed from the program, the differences are exceedingly marginal. In comparing the circuit against the same circuit with a dependent source in it, we see that the graph of the circuit with the dependent source shows a rate of decay that is much quicker which therefore appraoches stability faster.