ECE 3640 - Discrete-Time Signals and Systems
Frequency Modulation

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Spring 2015

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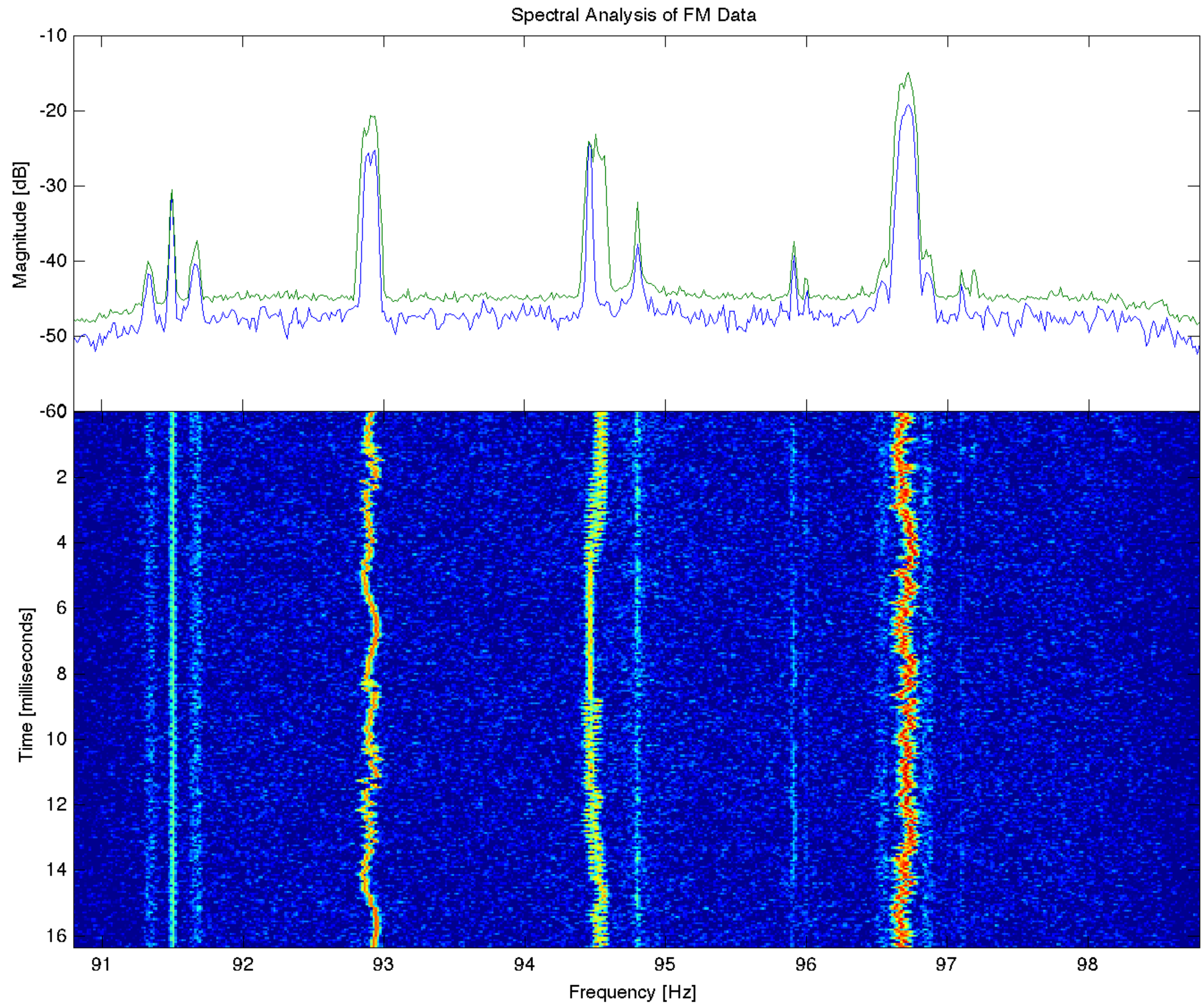
outline

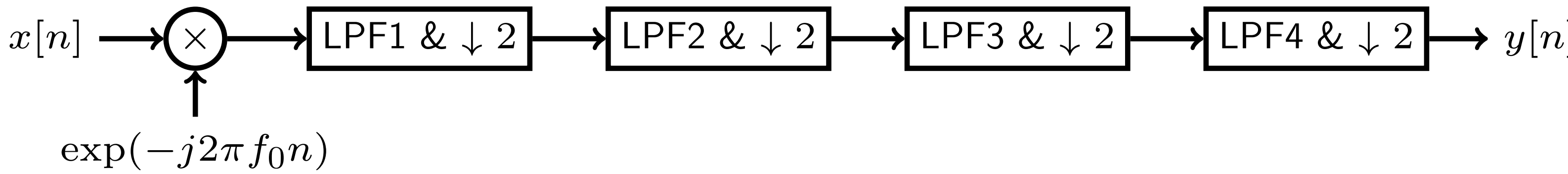
- selecting the signal of interest
- FM demodulation
- stereo decoding

data collection

- 8 MHz wide band centered at 94.8 MHz was sampled for about 10 seconds
- data saved in raw binary format with no header
- samples are complex saved with real and imaginary interleaved
- real and imaginary parts saved as 32-bit float
- sample rate is 8 MS/s
- see file `freq94_8_bw_4.bin` on the course web site
- the first 100 samples (200 floats) are garbage and can be discarded
- list of FM radio stations in Logan, Utah can be found at `http://radio-locator.com`


Spectral Analysis of FM Data
Magnitude [dB]
-10
-20
-30
-40
-50
-60
Time [milliseconds]
2
4
6
8
10
12
14
16
91
92
93
94
95
96
97
98
Frequency [Hz]

processing to select a channel and reduce the sample rate



- $f_0 = (F_t - Fc)/F_s$, where F_t is the target frequency to demodulate, F_c is the band center frequency (98.4 MHz), and F_s is the sample rate 8 MS/s

- FM signals have 200 kHz (total) bandwidth

- the basebanded signal has a 100 kHz (one-sided) bandwidth = (100 kHz)/(8 MS/s) = 1/80 cycles/sample

- we want to reduce the sample rate from 8 MS/s to a reasonable value, say, 500 kS/s

- this requires downsampling by a factor of (8 MS/s)/(500 kS/s) = 16

- this can be accomplished efficiently in 4 stages of downsampling by 2

filter design

- recall (from sample rate conversion lab) that prior to downsampling by D, a signal is low pass filtered (LPF) to prevent aliasing
- an ideal LPF would use a cutoff frequency of $1/(2D)$
- a practical filter would choose pass and stop band edge frequencies symmetric about $1/(2D)$
- if f_p is the pass band edge, then the stop band edge is placed symmetrically:

$$f_s = \frac{1}{2D} + \left(\frac{1}{2D} - f_p\right) = \frac{1}{D} - f_p$$

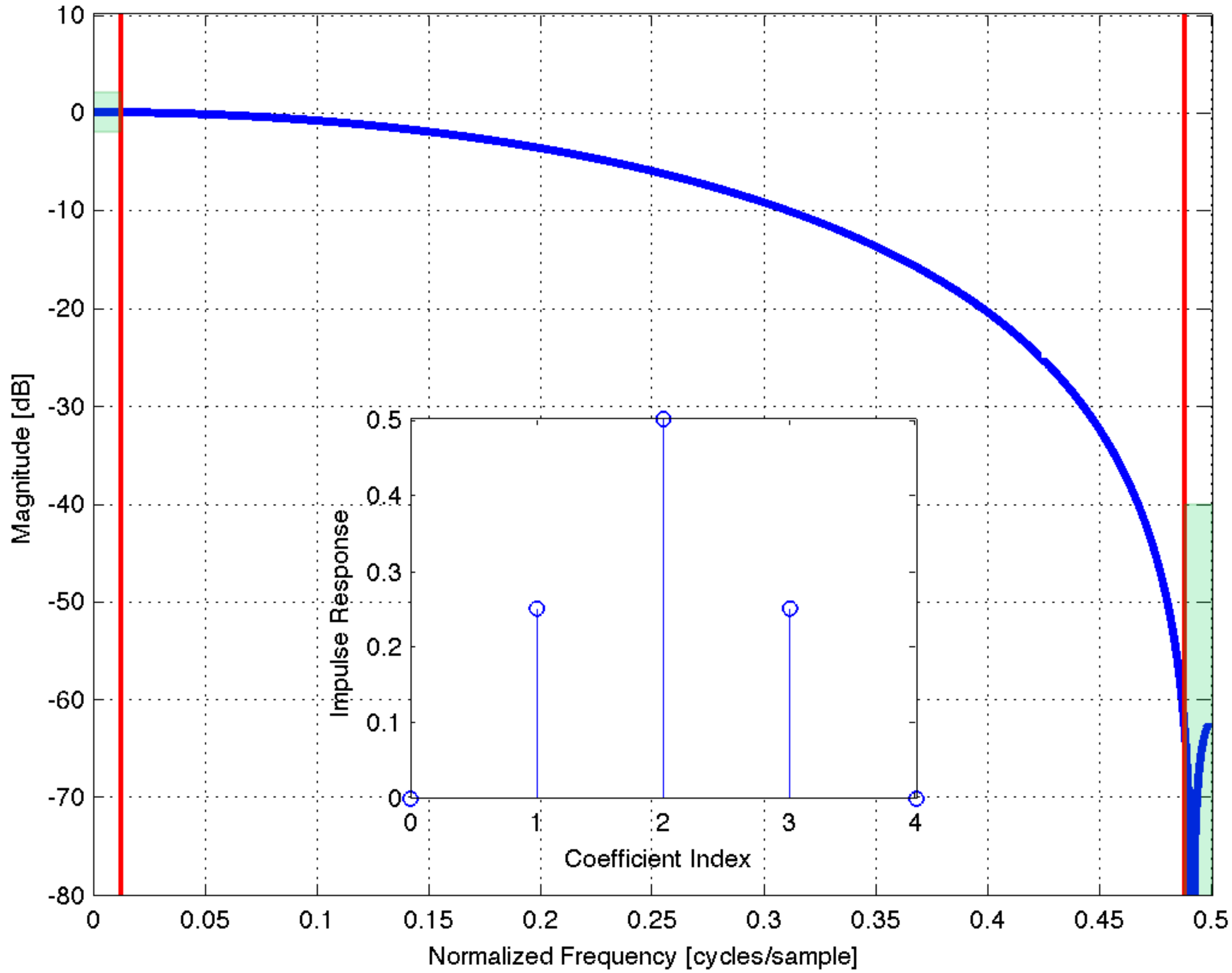
- for decimation by $D = 2$, this would be

$$f_s = \frac{1}{2} - f_p \tag{1}$$

filter design

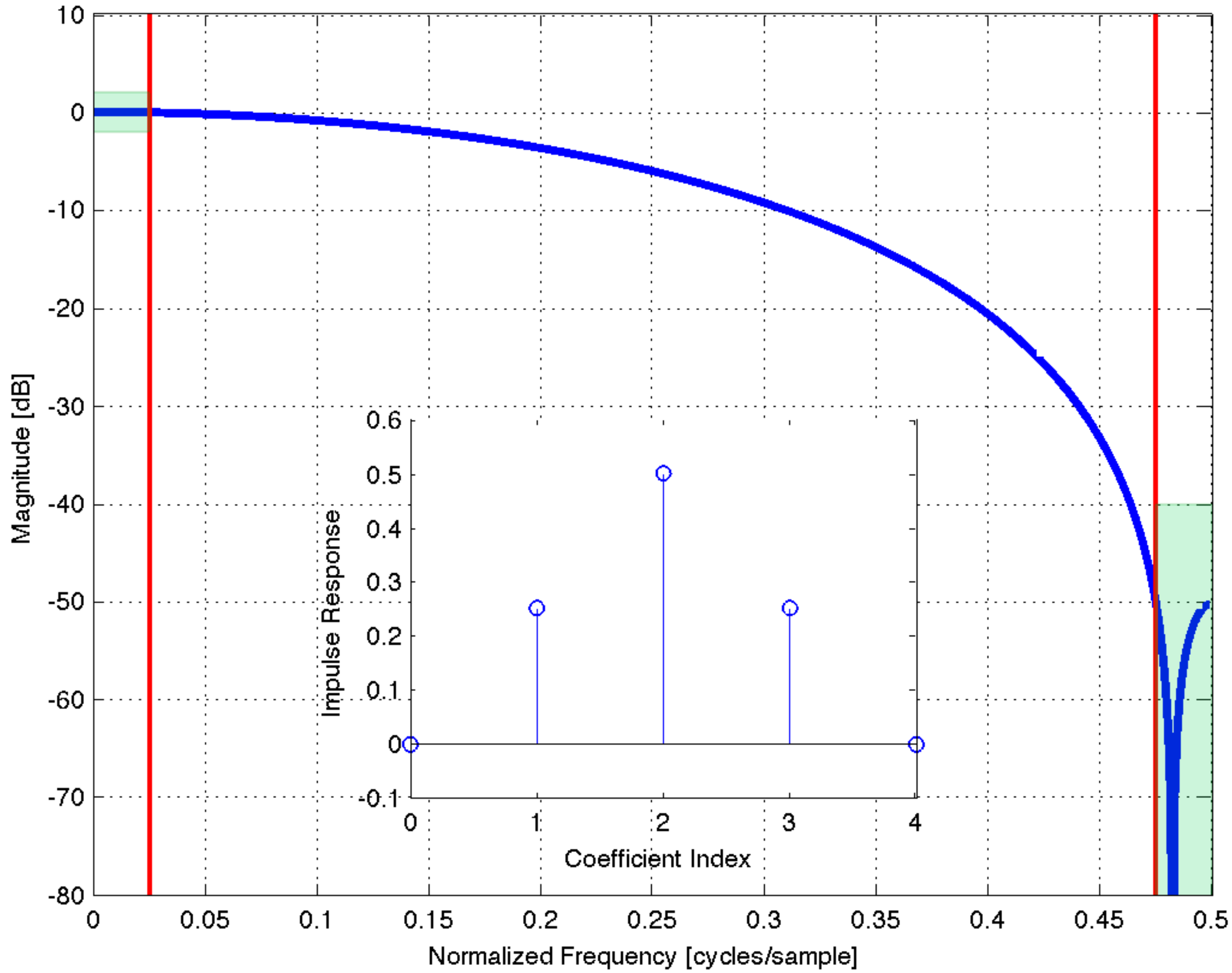
stage	$f_{p,\text{in}}$	f_s	$F_{s,\text{in}}$ [Hz]	$f_{p,\text{out}}$	$F_{s,\text{out}}$ [Hz]
1	$\frac{1}{80}$	$\frac{1}{2} - \frac{1}{80}$	8 M	$\frac{1}{40}$	4 M
2	$\frac{1}{40}$	$\frac{1}{2} - \frac{1}{40}$	4 M	$\frac{1}{20}$	2 M
3	$\frac{1}{20}$	$\frac{1}{2} - \frac{1}{20}$	2 M	$\frac{1}{10}$	1 M
4	$\frac{1}{10}$	$\frac{1}{2} - \frac{1}{10}$	1 M	$\frac{1}{5}$	500 k

LPF 1



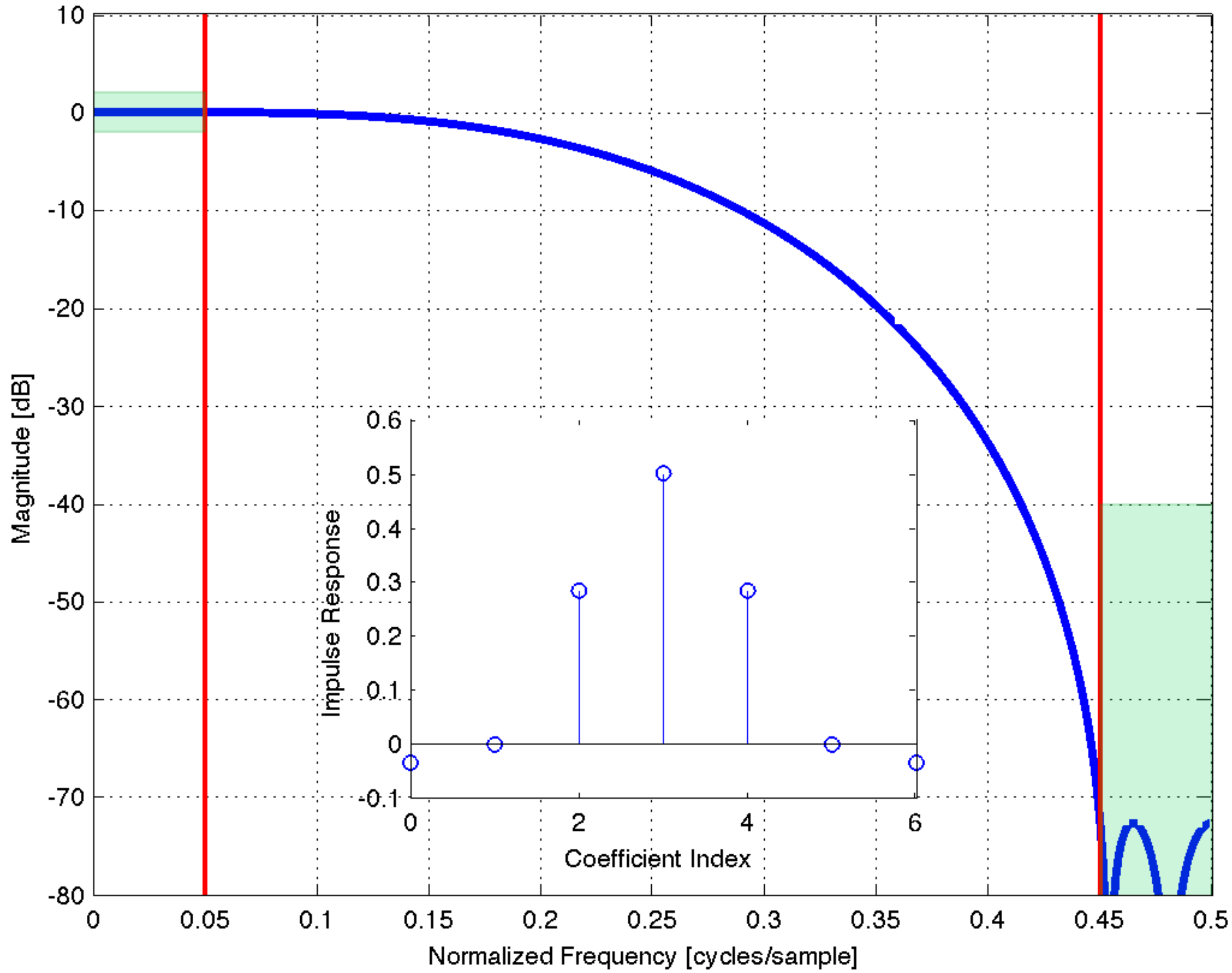
10
0
-10
-20
-30
-40
-50
-60
-70
-80
Magnitude [dB]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Normalized Frequency [cycles/sample]
0.5
0.4
0.3
0.2
0.1
0
Impulse Response
0
1
2
3
4
Coefficient Index

LPF 2

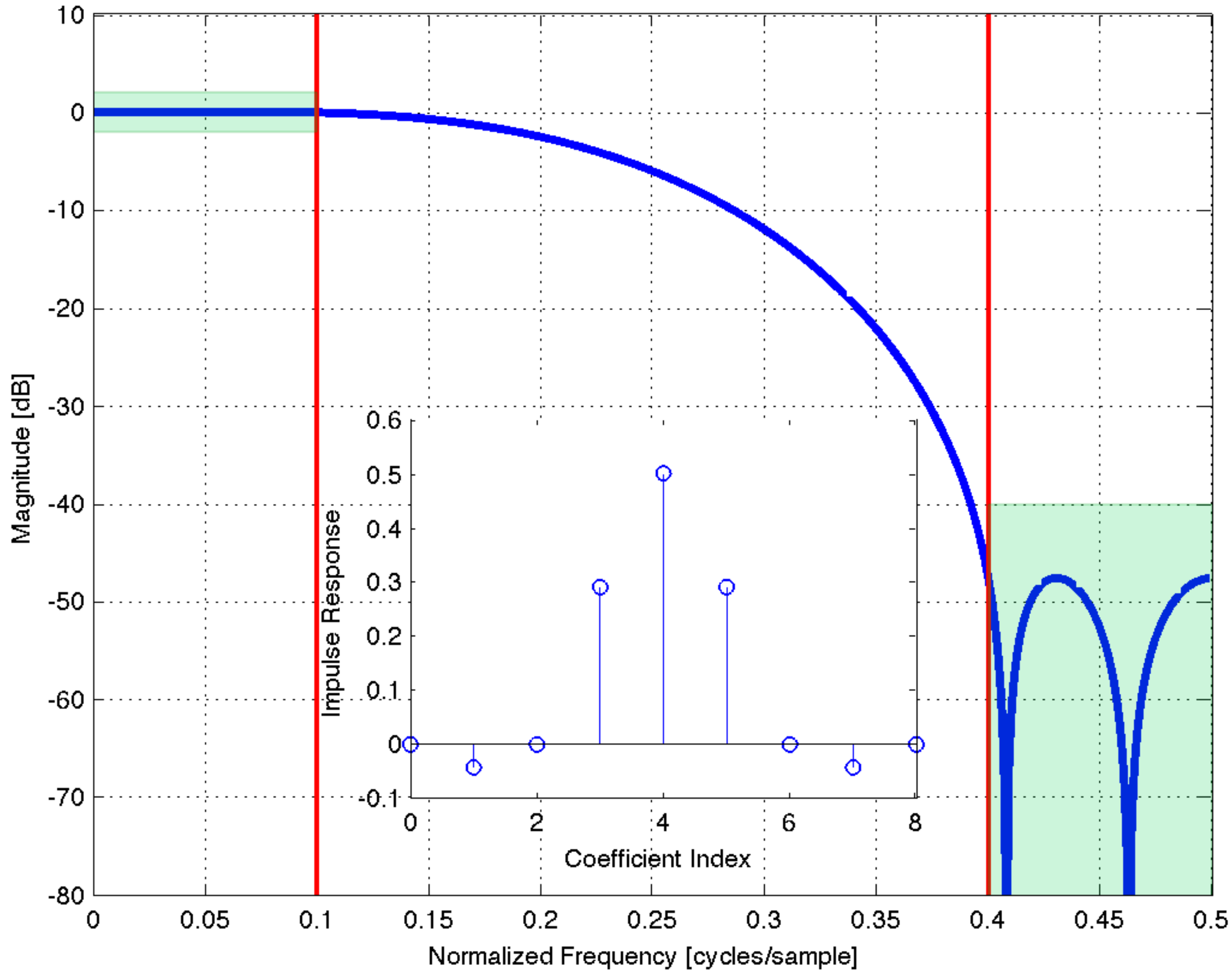


10
0
-10
-20
-30
-40
-50
-60
-70
-80
Magnitude [dB]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Normalized Frequency [cycles/sample]
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
Impulse Response
0
1
2
3
4
Coefficient Index

LPF 3

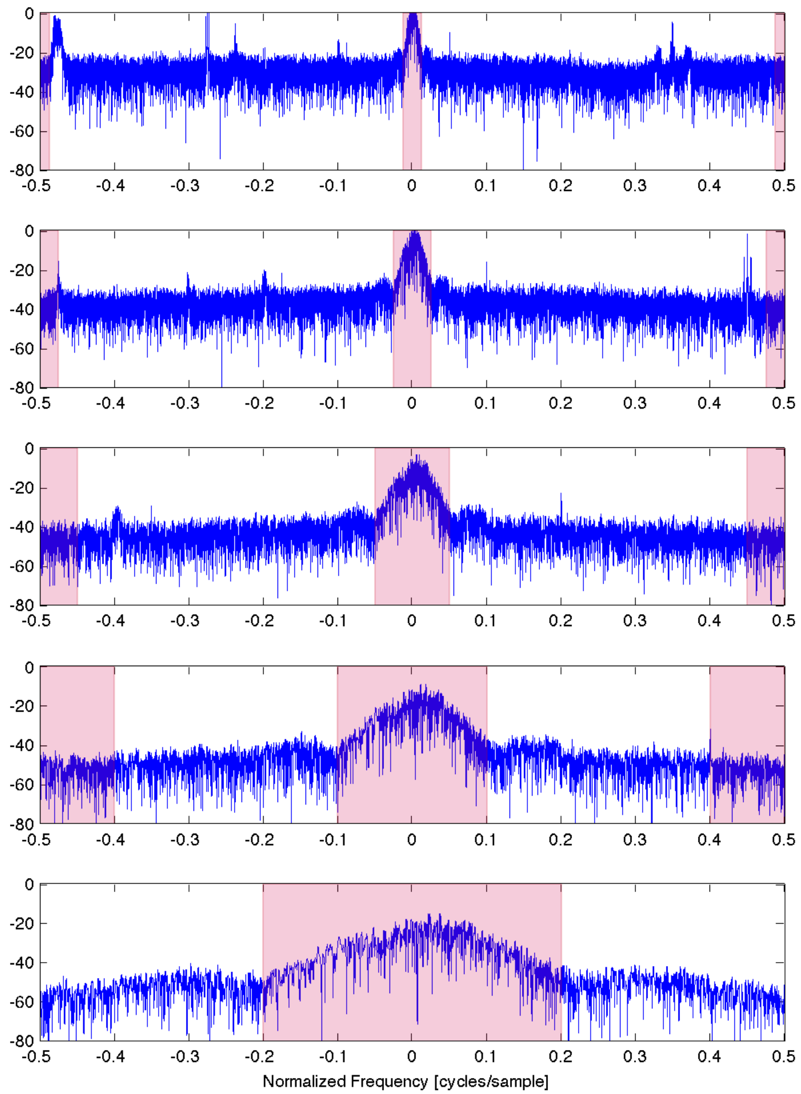

10
0
-10
-20
-30
-40
-50
-60
-70
-80
Magnitude [dB]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Normalized Frequency [cycles/sample]
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
Impulse Response
0
2
4
6
Coefficient Index

LPF 4



Magnitude [dB]
Normalized Frequency [cycles/sample]
10
0
-10
-20
-30
-40
-50
-60
-70
-80
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Impulse Response
Coefficient Index
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
2
4
6
8

filtering in action



programming assignment: part 1

- write a C program to implement the five stage processing shown previously (about page 5)
- the processing can be performed separately in several programs using files as the interface between processing stages
- design four linear-phase FIR low-pass filters for the pass and stop band edges given in the table shown previously (about page 7)
- the filters should have at least 40 dB of attenuation in the stop band
- plot the magnitude response of each filter and scale the frequency axis by input sample rate of the filter
- make a picture showing the spectrum of the original signal and the signal after each of the five processing steps (original signal, mixer, LPF1, LPF2, LFP3, LPF4) (see the previous slide for an example)
- attach a copy of your code and describe how you implemented the system

frequency modulation

- FM in continuous-time
  - $x(t)$ is the message to be transmitted
  - $y(t)$ is the frequency modulated signal
  - F_c is the carrier frequency in Hertz
  - K is the modulation index

$$y(t) = A\cos\left(2\pi\int_0^t [F_c + Kx(\tau)]d\tau\right) = A\cos\left(2\pi\left[F_c t + K\int_0^t x(\tau)d\tau\right]\right)$$

- FM in discrete-time
  - FM in discrete-time obtained by sampling at $t = n/F_s$, where F_s is the sample rate $y(n) = y(t = n/F_s)$ and $x(n) = x(t = n/F_s)$

$$y(n) \approx A\cos\left(2\pi\sum_{i=0}^{n}[f_c + kx(i)]\right) = A\cos\left(2\pi\left[f_c n + k\sum_{i=0}^{n}x(i)\right]\right)$$

$$f_c = \frac{F_c}{F_s},\ k = \frac{K}{F_s}$$

instantaneous frequency

- the instantaneous frequency $F_i(t)$ of a signal $y(t) = A\cos(\varphi(t))$ is

$$F_i(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

- FM signal

$$y(t) = A\cos\left(2\pi\left[F_c t + K\int_0^t x(\tau)d\tau\right]\right) = A\cos(\varphi(t))$$
$$\varphi(t) = 2\pi\left[F_c t + K\int_0^t x(\tau)d\tau\right]$$

- Q: why is this called frequency modulation?

- A: calculate the instantaneous frequency of $y(t)$

$$F_i(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} = F_c + Kx(t)$$

generation of FM

- continuous-time FM is generated by a voltage-controlled oscillator (VCO)

$$y(t) = A\cos\left(2\pi\left[F_c t + K\int_0^t x(\tau)d\tau\right]\right)$$

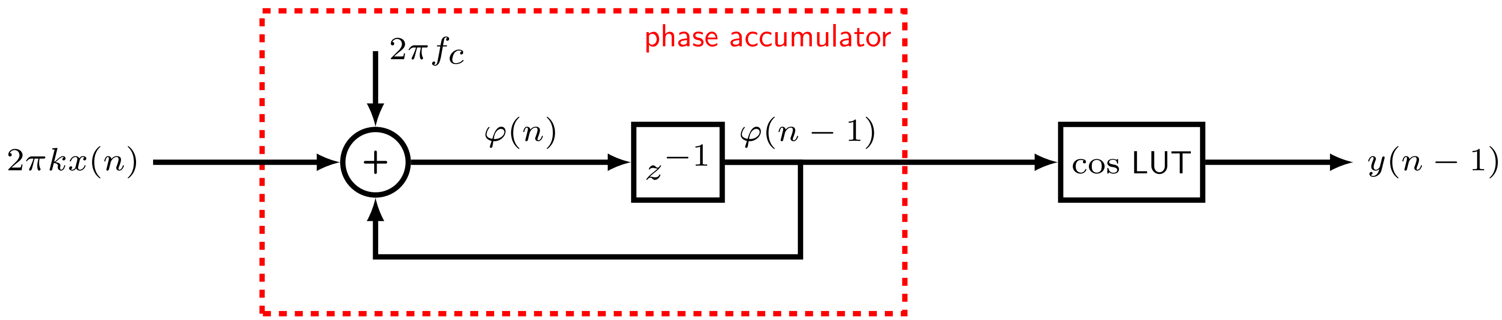
- discrete-time FM is generated by a numerically-controlled oscillator (NCO)

$$\begin{aligned} y(n) &\approx A\cos\left(2\pi\left[f_c n + k\sum_{i=0}^{n} x(i)\right]\right), \quad f_c = \frac{F_c}{F_s},\ k = \frac{K}{F_s} \\ &= A\cos(\varphi(n)) \end{aligned}$$

$$\begin{aligned} \varphi(n) &= 2\pi\left[f_c n + k\sum_{i=0}^{n} x(i)\right] && \text{(NCO phase)} \\ &= \varphi(n-1) + 2\pi[f_c + kx(n)] && \text{(recursive formula)} \end{aligned}$$

numerically controlled oscillator (NCO)

- NCO $=$ phase accumulator (PA) $+$ $\cos$ look-up table (LUT)



demodulating FM

- observe that

$$\frac{dy(t)}{dt} = 2\pi[F_c + Kx(t)]\cos\left(2\pi\left[F_c t + K\int_0^t x(\tau)d\tau\right]\right)$$

- differentiation turns FM into AM

- block diagram of FM demodulator

$$y(t) \longrightarrow \boxed{D(z)} \longrightarrow \boxed{\text{AM demod.}} \longrightarrow x(t)$$

- $D(z)$ is an FIR differentiating filter

```
d = firpm(66,[0 0.2 0.25 0.5]/0.5,[0 1 0 0],'differentiator');
```

quadrature demodulation

- modulate $y(t)$ using quadrature carriers $\cos(2\pi F_c t)$ and $-\sin(2\pi F_c t)$

$$u_0(t) = y(t)\cos(2\pi F_c t) = \frac{A}{2}\cos\left(2\pi K \int_0^t x(\tau)d\tau\right) + \text{double frequency term}$$

$$v_0(t) = -y(t)\sin(2\pi F_c t) = \frac{A}{2}\sin\left(2\pi K \int_0^t x(\tau)d\tau\right) + \text{double frequency term}$$

- now low pass filter

$$u_1(t) = \mathsf{LPF}\{u_0(t)\} = \frac{A}{2}\cos\left(2\pi K \int_0^t x(\tau)d\tau\right)$$

$$v_1(t) = \mathsf{LPF}\{v_0(t)\} = \frac{A}{2}\sin\left(2\pi K \int_0^t x(\tau)d\tau\right)$$

- now compute derivatives

$$u_2(t) = \frac{du_1(t)}{dt} = -2\pi K x(t) \frac{A}{2} \sin\left(2\pi K \int_0^t x(\tau) d\tau\right)$$

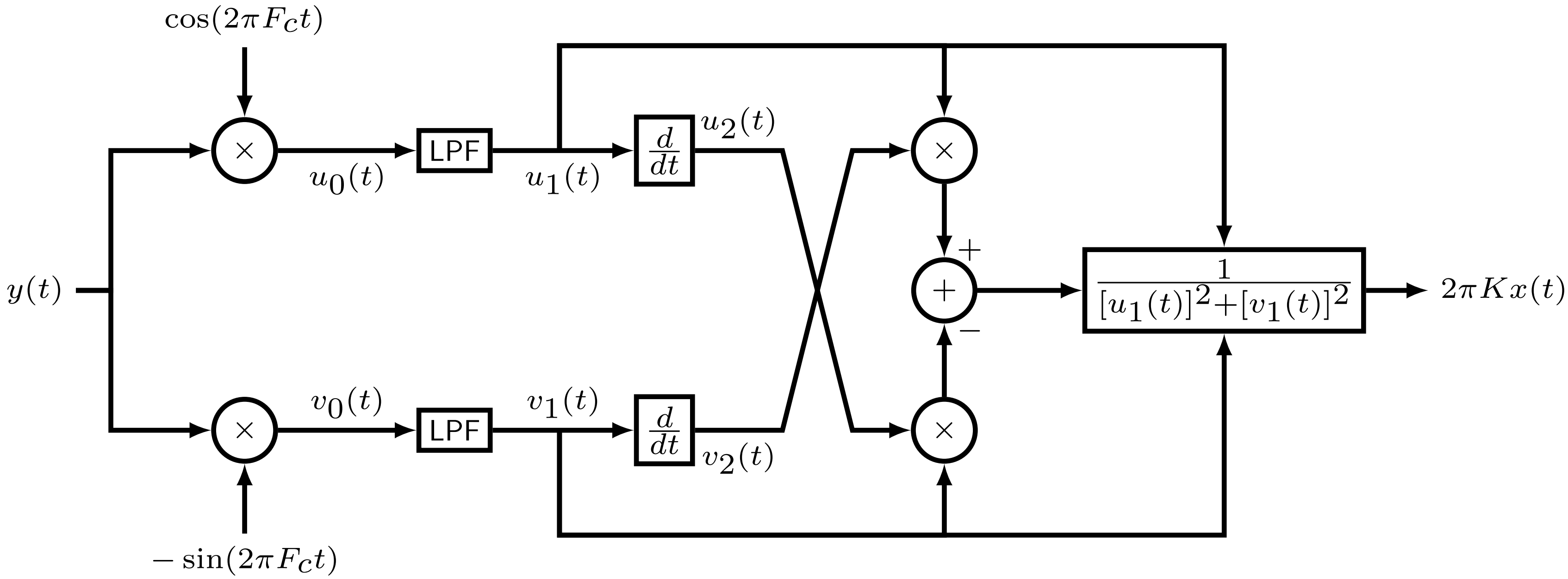
$$v_2(t) = \frac{dv_1(t)}{dt} = 2\pi K x(t) \frac{A}{2} \cos\left(2\pi K \int_0^t x(\tau) d\tau\right)$$

- now cross-multiply and normalize to solve for $x(t)$

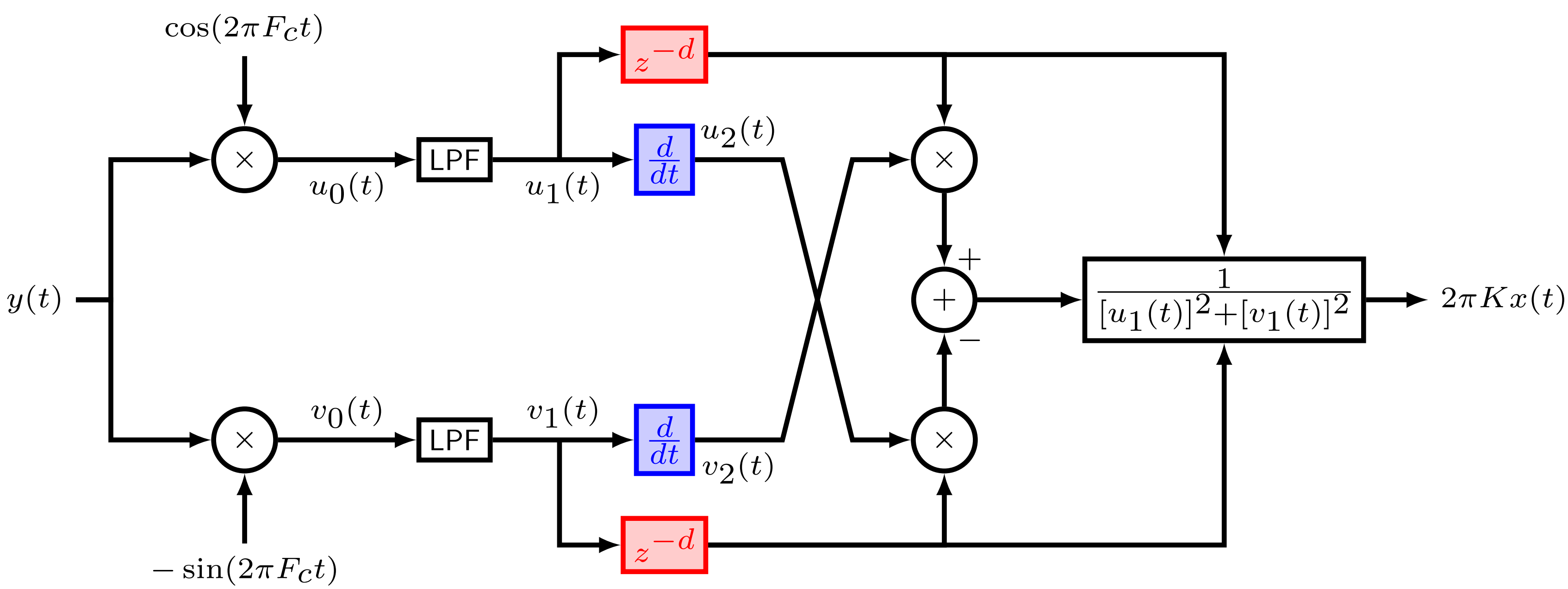
$$\frac{u_1(t)v_2(t) - v_1(t)u_2(t)}{\sqrt{[u_1(t)]^2 + [v_1(t)]^2}} = 2\pi K x(t)$$

- these operations recover the message $x(t)$

quadrature demodulation



quadrature demodulation

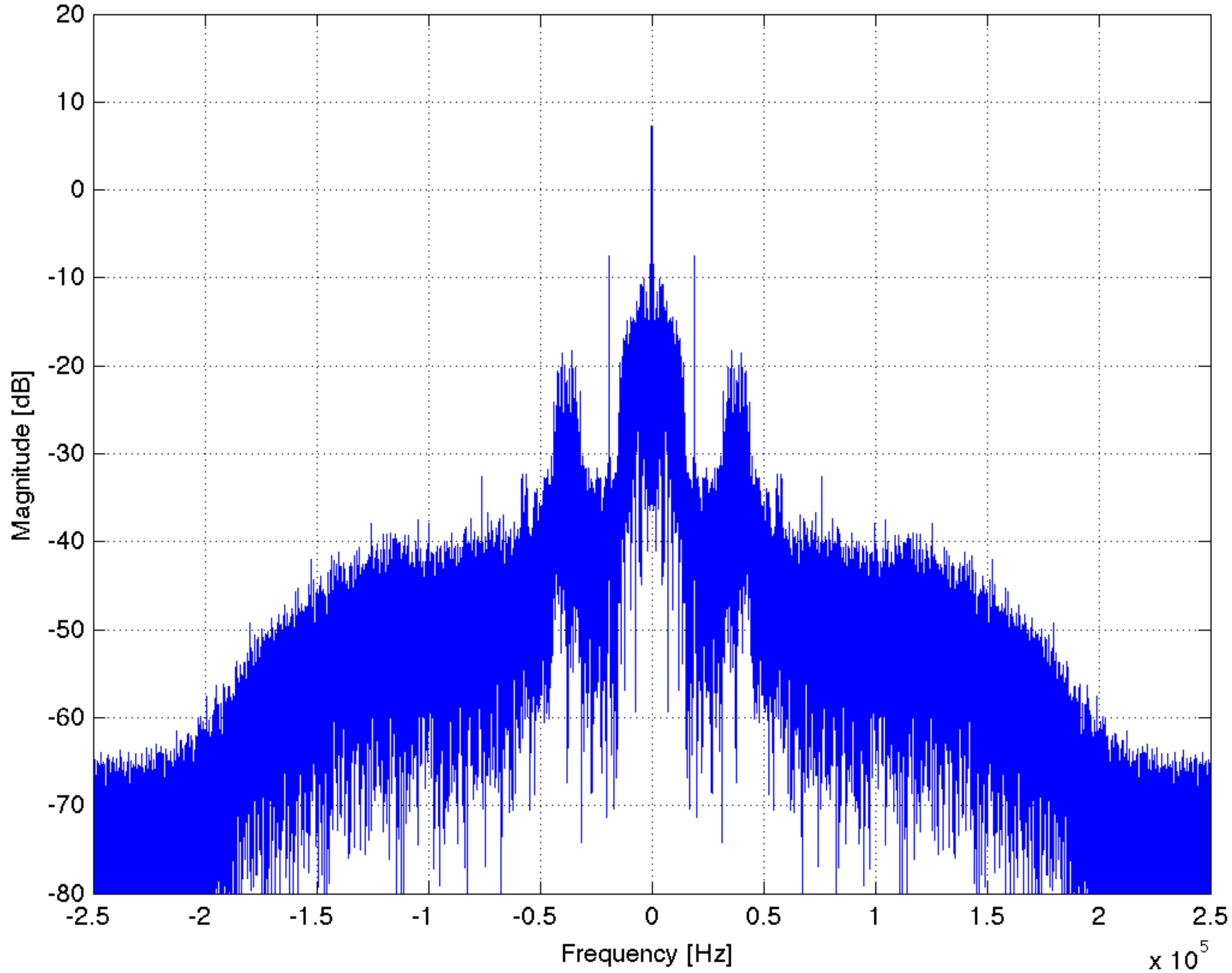


- use linear-phase type III FIR differentiator, group delay $= d$ (integer)
- insert a delay having the same group delay as the differentiator
- this keeps the signals time aligned
- we start with signals $u_1(t)$ and $v_1(t)$ from the four-stage decimator

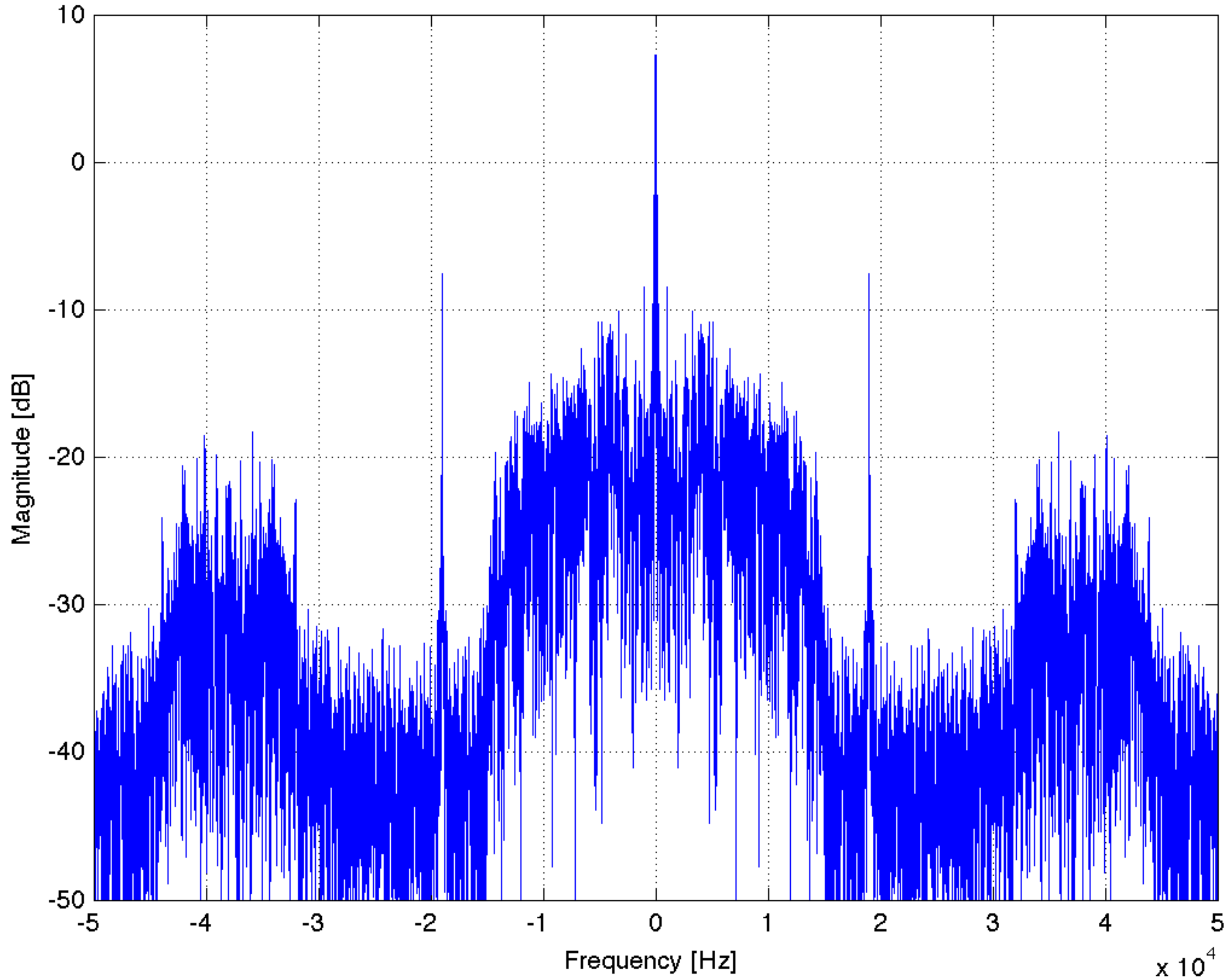
assignment

- design a differentiator and implement a delay block
- perform the FM demodulation processing shown on the previous slide
- show a plot of the spectrum of the demodulated signal and scale the frequency axis by input sample rate (see example plot on the next slide)
- attach your code and describe how you implemented your system

spectrum of demodulated signal



spectrum of demodulated signal (zoomed)

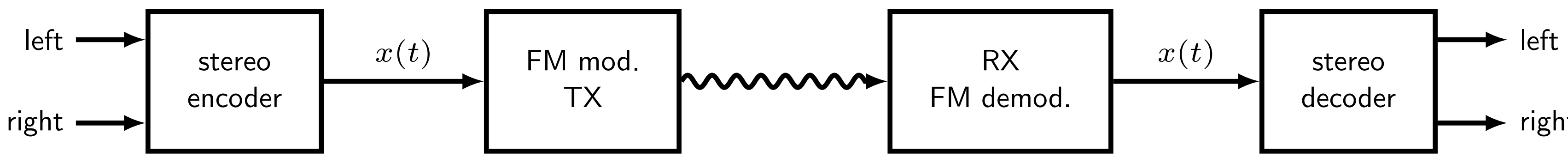


- "parts" are visible: [0-15 kHz] = L+R, 19 kHz = carrier, [23-53 kHz] = L-R

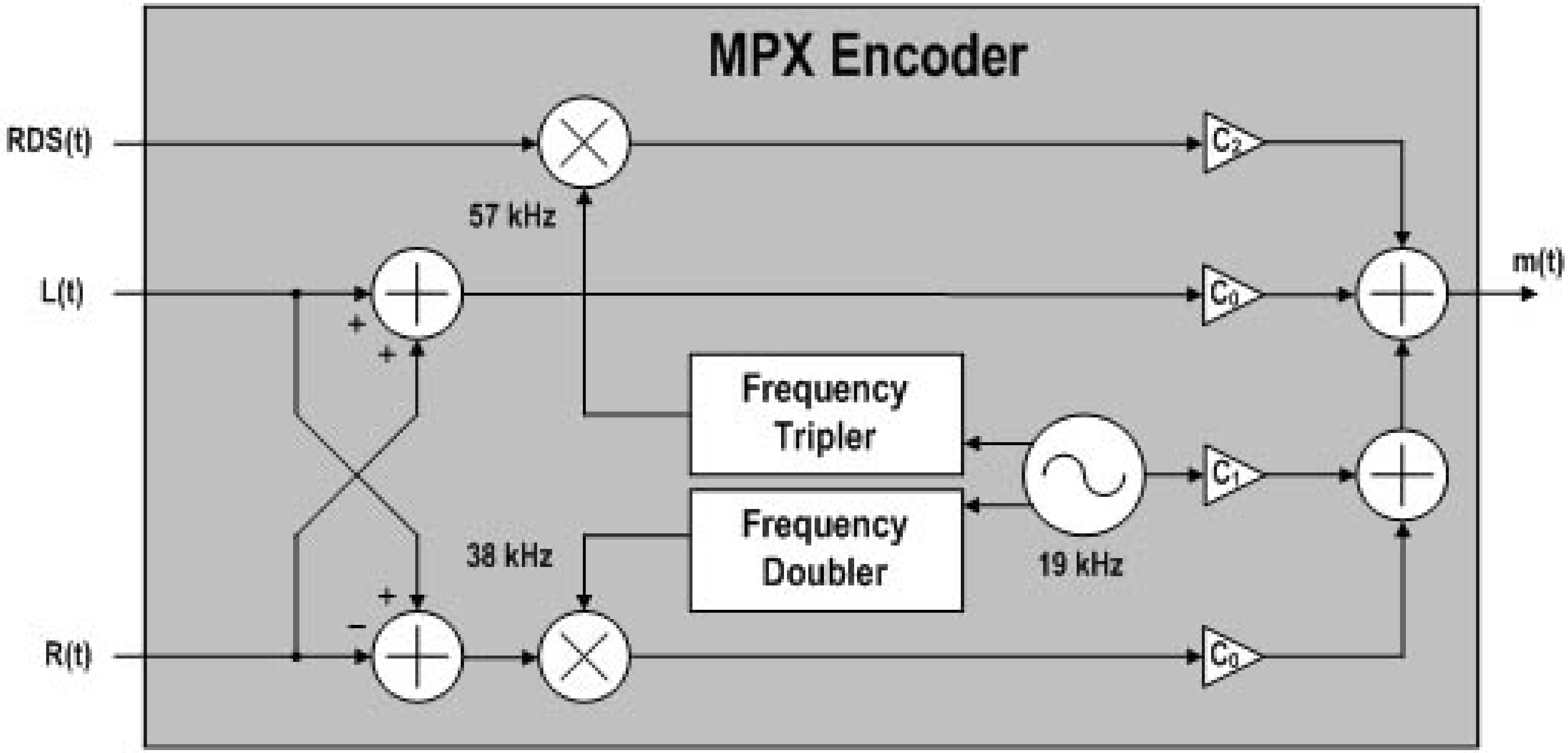
stereo encoder

- in broadcast FM, the message signal $x(t)$ is the output of a stereo encoder

- FM recievers have stereo decoders

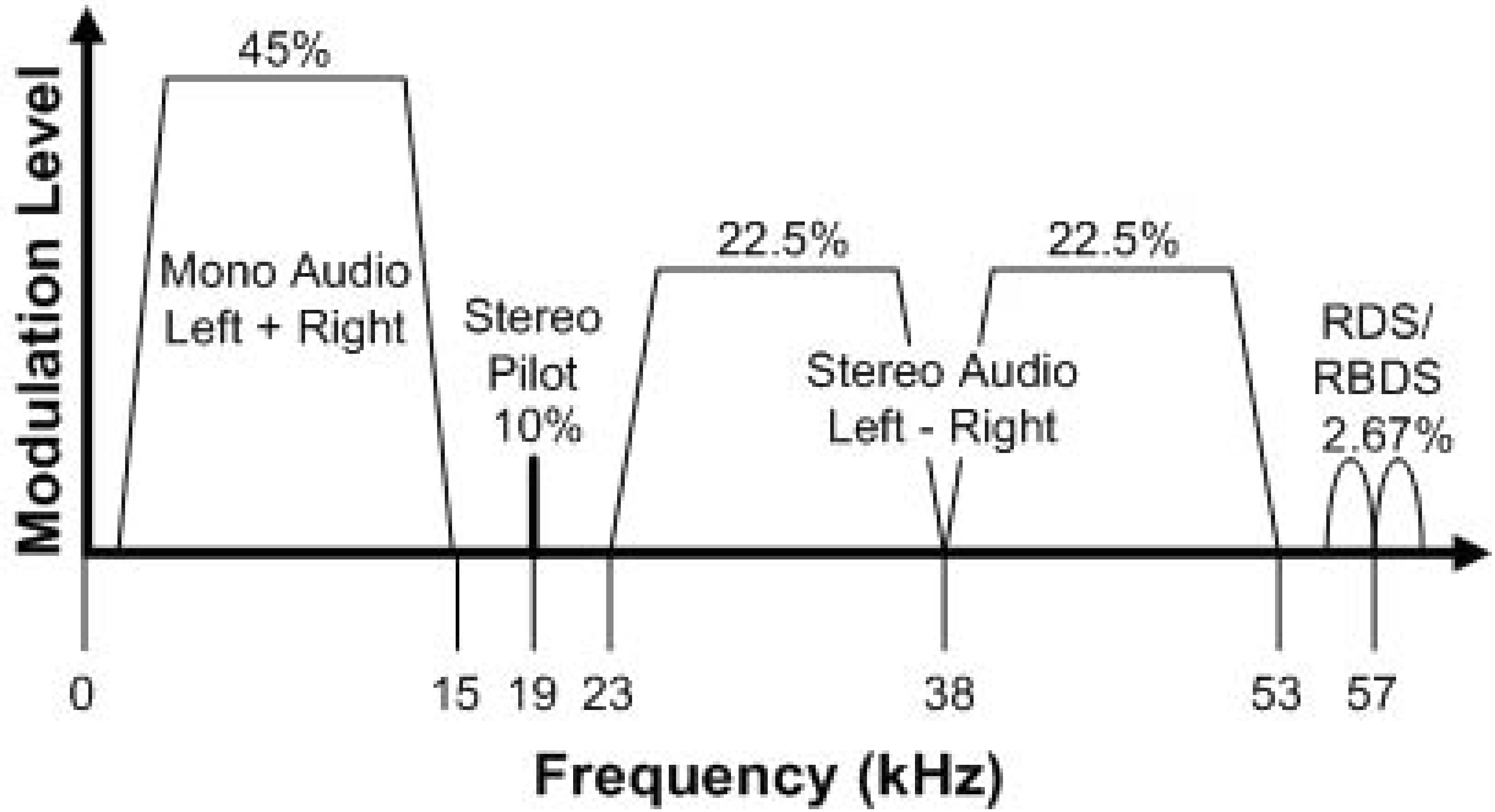


stereo encoder



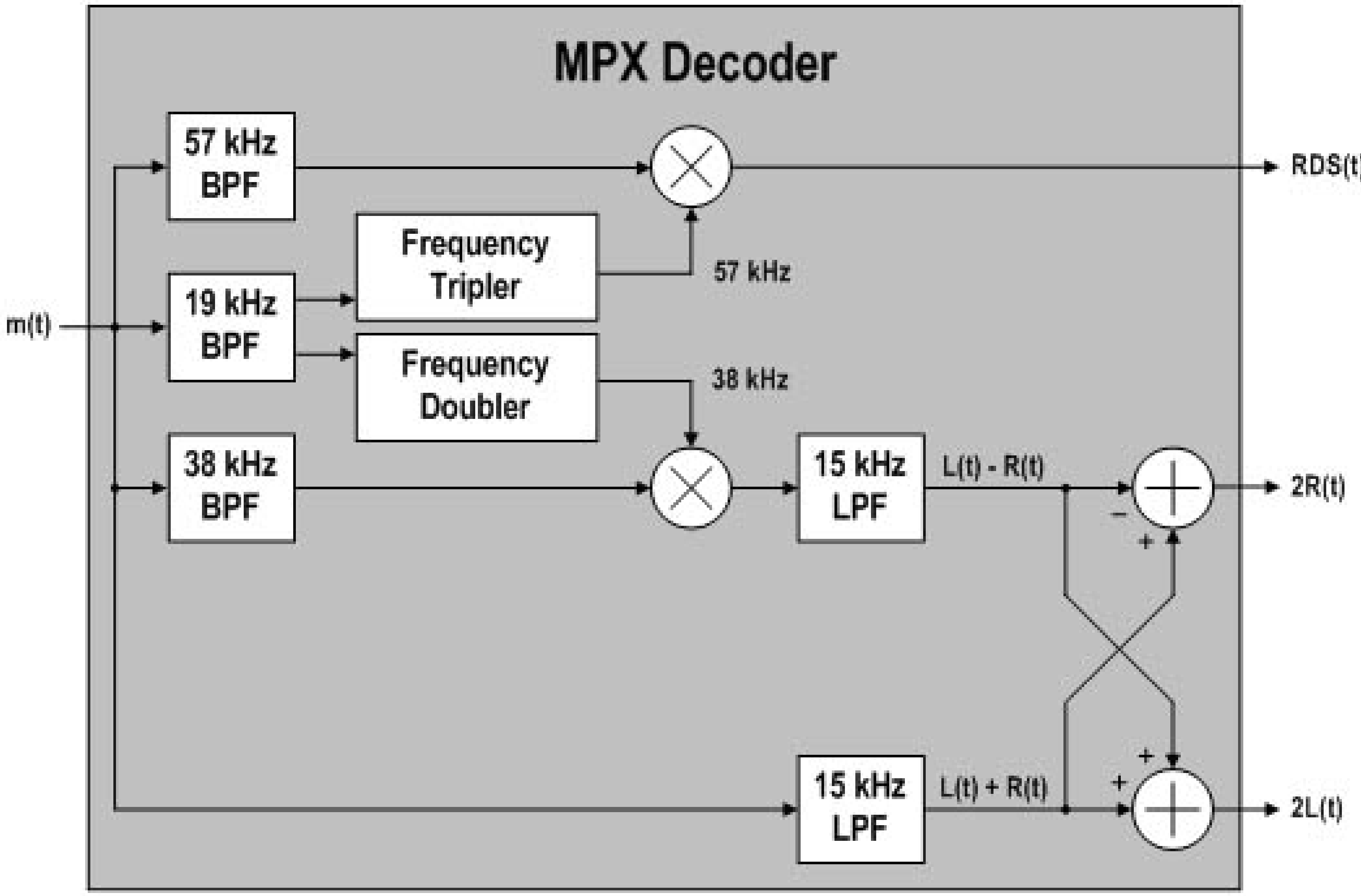
- uses principles of amplitude modulation (Fourier transform property)

spectrum of stereo signal



- for L+R audio, only need a low-pass filter

stereo decoder



- uses filters to separate the signals
- need to match delay (group delay) on all branches

extracting L+R audio

- filter and downsample to recover the L+R audio
- bandwidth of L+R audio is 15 kHz
- sample rate of 30 kS/s is needed
- input sample rate is 500 kS/s
- requires sample rate conversion by $500/30 = (2 \cdot 5 \cdot 5)/3$
- do this in multiple stages as shown below
- design linear-phase FIR filters to do this (should have > 40 dB stop band attenuation)

$$s[n] \rightarrow \boxed{\text{LPF1}} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\text{LPF2}} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\text{LPF3}} \rightarrow \boxed{\downarrow 5} \rightarrow a[n]$$

$F_s \Rightarrow$	500 kS/s	500 kS/s	250 kS/s	250 kS/s	50 kS/s	150 kS/s	150 kS/s	30 kS/s

assignment

- in a table list the input and output sample rates for each processing stage in the diagram above (previous slide)
- include in the table the input and output pass-band edge frequencies
- include in the table the stop-band edge frequencies
- show plots of the magnitude response (in dB) for each filter and scale the frequency axis by the input sample rate for the filter
- attach your code and describe how you implemented the system