5625_Project_2_sp2018_sample

May 8, 2018

Contents

Python Project II

Michael Case, Graeme Danforth, Joshua Minneti

9 May 2018

ECE 4625

```
In [72]: %pylab inline
         #%matplotlib qt
         from __future__ import division # use so 1/2 = 0.5, etc.
         import sk_dsp_comm.sigsys as ss
         import scipy.signal as signal
         from IPython.display import Audio, display
         from IPython.display import Image, SVG
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
In [73]: #%config InlineBackend.figure_formats=['svg'] # SVG inline viewing
         %config InlineBackend.figure_formats=['pdf'] # render pdf figs for LaTeX
```

Import Extra Modules

```
In [74]: import sk_dsp_comm.digitalcom as dc
         import sk_dsp_comm.synchronization as PLL # just in case we need it
```

Part I: PCM Encoding and Decoding

The functions are housed in the module scikit-dsp-comm module 'digitalcom.py.

```
def PCM_encode(x,N_bits):
    """
    x_bits = PCM_encode(x,N_bits)
    ////////////////////////////////////////////////////////////
         x = signal samples to be PCM encoded
    N_bits = bit precision of PCM samples
    x_bits = encoded serial bit stream of 0/1 values. MSB first.
    ////////////////////////////////////////////////////////////
    Mark Wickert, March 2015
    """
    xq = np.int16(np.rint(x*2**(N_bits-1)))
    x_bits = np.zeros((N_bits,len(xq)))
    for k, xk in enumerate(xq):
        x_bits[:,k] = tobin(xk,N_bits)
    # Reshape into a serial bit stream
    x_bits = np.reshape(x_bits,(1,len(x)*N_bits),'F')
    return int16(x_bits.flatten())


# A helper function for PCM_encode
def tobin(data, width):
    data_str = bin(data & (2**width-1))[2:].zfill(width)
    return map( int, tuple( data_str ) )


def PCM_decode(x_bits,N_bits):
    """
    xhat = PCM_decode(x_bits,N_bits)
    ////////////////////////////////////////////////////////////
    x_bits = serial bit stream of 0/1 values. The length of
             x_bits must be a multiple of N_bits
    N_bits = bit precision of PCM samples
      xhat = decoded PCM signal samples
    ////////////////////////////////////////////////////////////
    Mark Wickert, March 2015
    """
    N_samples = len(x_bits)//N_bits
    # Convert serial bit stream into parallel words with each
    # column holdingthe N_bits binary sample value
    xrs_bits = x_bits.copy()
    xrs_bits = np.reshape(xrs_bits,(N_bits,N_samples),'F')
    # Convert N_bits binary words into signed integer values
    xq = np.zeros(N_samples)
    w = 2**np.arange(N_bits-1,-1,-1) # binary weights for bin
```

```
                                    # to dec conversion
    for k in range(N_samples):
       xq[k] = np.dot(xrs_bits[:,k],w) - xrs_bits[0,k]*2**N_bits
    return xq/2**(N_bits-1)


def AWGN_chan(x_bits,EBN0_dB):
    """
    ////////////////////////////////////////////////////////////////////////
     x_bits = serial bit stream of 0/1 values.
    EBN0_dB = energy per bit to noise power density ratio in dB of the
              serial bit stream sent through the AWGN channel. Frequently
              we equate EBN0 to SNR in link budget calculations
     y_bits = received serial bit stream following hard decisions. This bit
              will have bit errors. To check the estimated bit error
              probability use digitalcom.BPSK_bep() or simply
              >> Pe_est = sum(xor(x_bits,y_bits))/length(x_bits);
    ////////////////////////////////////////////////////////////////////////

    Mark Wickert, March 2015
    """
    x_bits = 2*x_bits - 1 # convert from 0/1 to -1/1 signal values
    var_noise = 10**(-EBN0_dB/10)/2;
    y_bits = x_bits + np.sqrt(var_noise)*np.random.randn(size(x_bits))

    # Make hard decisions
    y_bits = np.sign(y_bits) # -1/+1 signal values
    y_bits = (y_bits+1)/2 # convert back to 0/1 binary values
    return y_bits
```

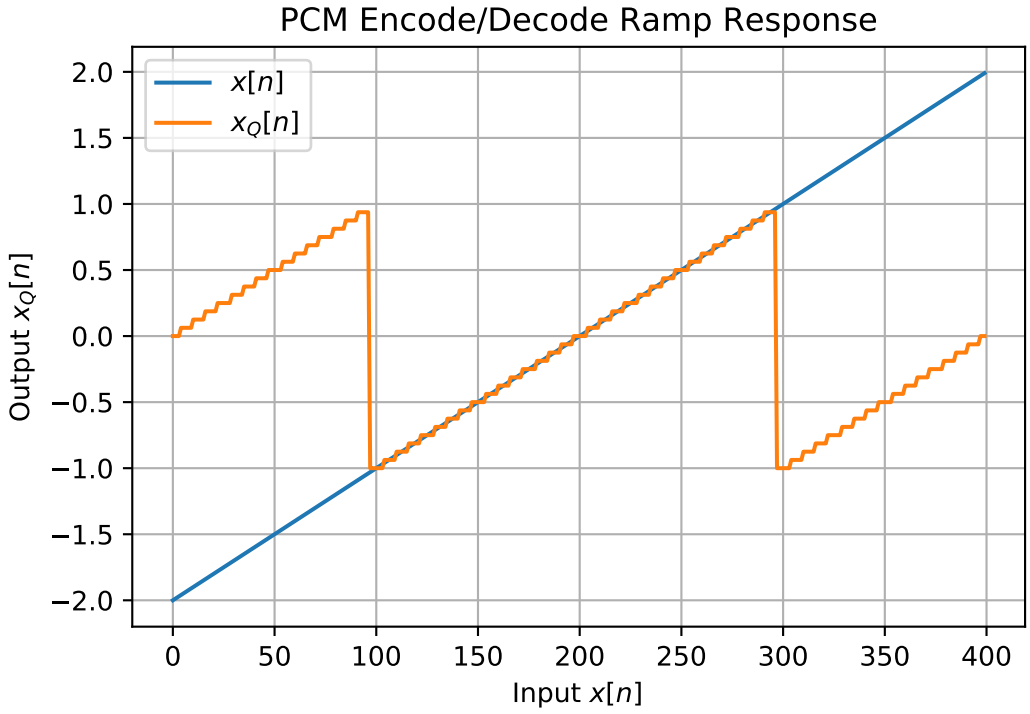
BPSK Theoretical BEP Calculation for Part I

```
In [75]: # Use the Q() function in digitalcom to find Pe_thy
         EbN0_dB = 5
         Pe_thy = dc.Q_fctn(sqrt(2*10**(EbN0_dB/10)))
         print('Pe_thy = %1.3e' % Pe_thy)

Pe_thy = 5.954e-03
```

Part I Tasks

Task 1a and 1b

```
In [76]: figure(figsize=(6,4))
         n = arange(0,400)
         # Fill in missing code:
         x = arange(-2,2,0.01);

         xbits = dc.PCM_encode(x,5)
         xq = dc.PCM_decode(xbits,5)
         plot(n,x)
         plot(n,xq)
         title(r'PCM Encode/Decode Ramp Response')
         xlabel(r'Input $x[n]$')
         ylabel(r'Output $x_Q[n]$')
         legend((r'$x[n]$',r'$x_Q[n]$'),loc='best')
         grid();
```

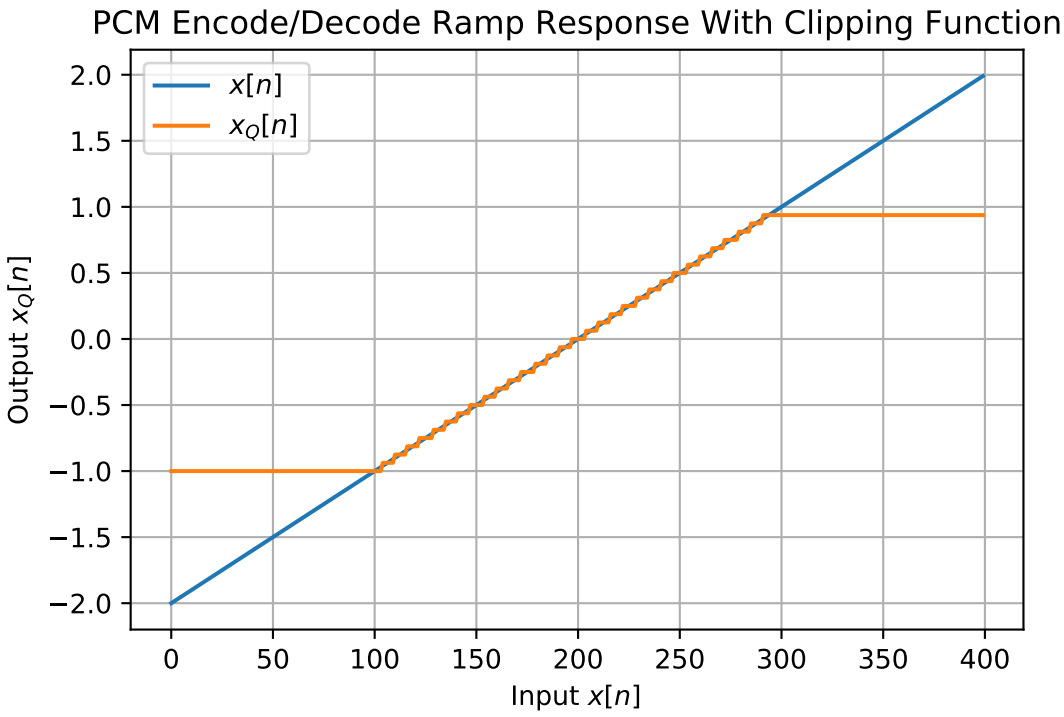


The largest quantization level for $N_B = 5$ will be $1 - 2^{-(5-1)}$, or 0.9375. It is clear on the plot that when this level is reached on the vertical axis, the decoded ramp response 'underflows' back to -1.

```
In [77]: def PCM_encode(x,N_bits, sat_mode = True):
             """
```

```
        Add saturation to dc.PCM_encode
        """

        # Write code here
        if sat_mode:
            x_clip = x.clip(-1,1-2**-(N_bits-1)) #formula given in doc
        else:
            x_clip = x

        x_bits = dc.PCM_encode(x_clip, N_bits)

        return x_bits
```

In [78]:
```
figure(figsize=(6,4))
n = arange(0,400)
x = arange(-2,2,0.01);
xbits = PCM_encode(x,5)
xq = dc.PCM_decode(xbits,5)
plot(n,x)
plot(n,xq)

# Added line:
title(r'PCM Encode/Decode Ramp Response With Clipping Function')

xlabel(r'Input $x[n]$')
ylabel(r'Output $x_Q[n]$')
legend((r'$x[n]$',r'$x_Q[n]$'),loc='best')
grid();
```

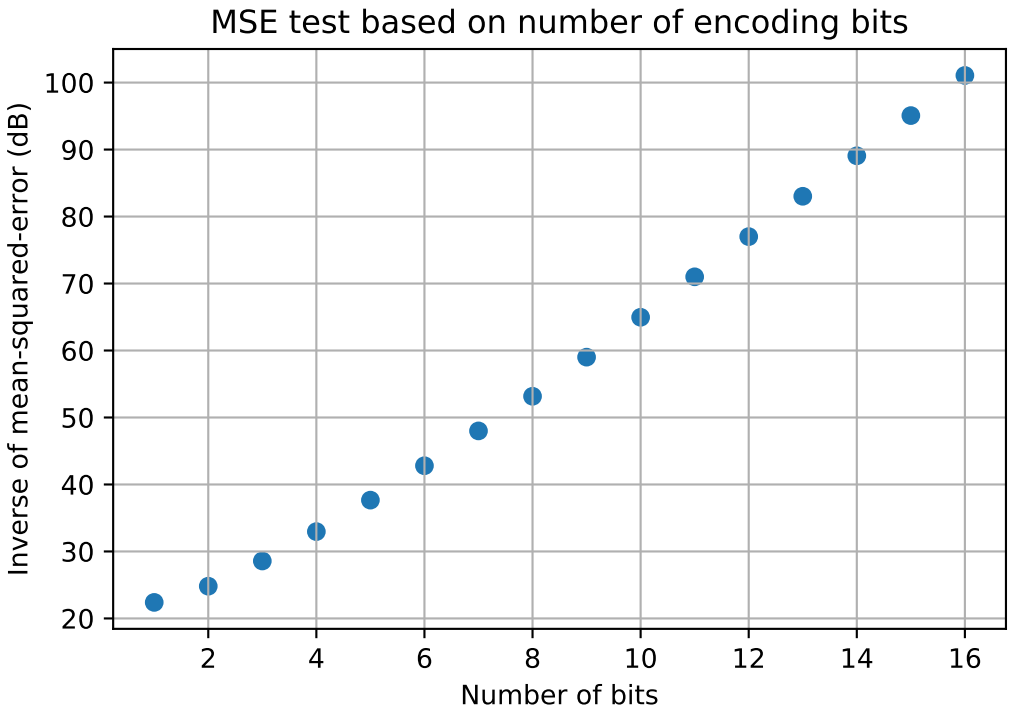


Now, instead of overflowing when the signal reaches the maximum quantization level, the output simply saturates at that level. This confirms that the clipping function is working as intended.

Task 2a and 2b: Test with Speech File

```
In [79]: N = 100000; # number of speech samples to process
         fs,m1 = ss.from_wav('OSR_uk_000_0050_8k.wav')
         m1 = m1[:N]

         # Include some scaling to be sure to fill the dynamic range, but not saturate
         m1_max = max(abs(m1))
         g = 1/m1_max    #normalize the message signal to amplitude of 1

         SNR_chan = 100 # High SNR
         MSE = ndarray(0)
         for N_B in range (1,17):

             m1_enc = PCM_encode(g*m1,N_B)
             m1_enc_hat = dc.AWGN_chan(m1_enc,SNR_chan)

             m1_hat = dc.PCM_decode(m1_enc_hat,N_B)

             fname = "NB_%d.wav"%(N_B)
```

```
        ss.to_wav(fname,8000,m1_hat)

        MSE = append(MSE,mean((m1_hat-g*m1)**2))

figure()
scatter(arange(1,17),10*log10(1/MSE))
grid()
xlabel('Number of bits')
ylabel('Inverse of mean-squared-error (dB)')
title('MSE test based on number of encoding bits');
```



Part A

The dB value of the MSE inverse increases as the number of encoding bits increases. This dB increase has a roughly linear characteristic, as seen in the plot above. Since a high inverse MSE indicates low error, it can easily be seen that increasing the number of bits in the encoder desirably reduces mean-squared-error for the received signal. Additionally, since MSE is inversely proportional, the graph also indicates that more bits in the encoder increase the quantization SNR.

```
In [80]: # 1 bits
         Audio("NB_1.wav")
```

```
Out[80]: <IPython.lib.display.Audio object>
```

```
In [81]: # 2 bits
         Audio("NB_2.wav")

Out[81]: <IPython.lib.display.Audio object>

In [82]: # 3 bits
         Audio("NB_3.wav")

Out[82]: <IPython.lib.display.Audio object>

In [83]: # 4 bits
         Audio("NB_4.wav")

Out[83]: <IPython.lib.display.Audio object>

In [84]: # 5 bits
         Audio("NB_5.wav")

Out[84]: <IPython.lib.display.Audio object>

In [85]: # 6 bits
         Audio("NB_6.wav")

Out[85]: <IPython.lib.display.Audio object>

In [86]: # 7 bits
         Audio("NB_7.wav")

Out[86]: <IPython.lib.display.Audio object>

In [87]: # 8 bits
         Audio("NB_8.wav")

Out[87]: <IPython.lib.display.Audio object>

In [88]: # 9 bits
         Audio("NB_9.wav")

Out[88]: <IPython.lib.display.Audio object>

In [89]: # 10 bits
         Audio("NB_10.wav")

Out[89]: <IPython.lib.display.Audio object>

In [90]: # 11 bits
         Audio("NB_11.wav")

Out[90]: <IPython.lib.display.Audio object>

In [91]: # 12 bits
         Audio("NB_12.wav")
```

```
Out[91]: <IPython.lib.display.Audio object>

In [92]: # 13 bits
         Audio("NB_13.wav")

Out[92]: <IPython.lib.display.Audio object>

In [93]: # 14 bits
         Audio("NB_14.wav")

Out[93]: <IPython.lib.display.Audio object>

In [94]: # 15 bits
         Audio("NB_15.wav")

Out[94]: <IPython.lib.display.Audio object>

In [95]: # 16 bits
         Audio("NB_16.wav")

Out[95]: <IPython.lib.display.Audio object>
```

Part B

The absolute minimum number of bits that was intelligible when listening to the output was somehwere around 4. However, it was extremely difficult to understand. The minimum number of encoding bits that we felt provided acceptable intelligibility was 6. This level of quantization had a low enough MSE that most offensive static noise and clipping was not noticeable in the output audio, and thus it was decided that this quantization level was the minimum acceptable resolution for this voice communication system

Tasks 3a-d: Obtain SNR Data and Comments

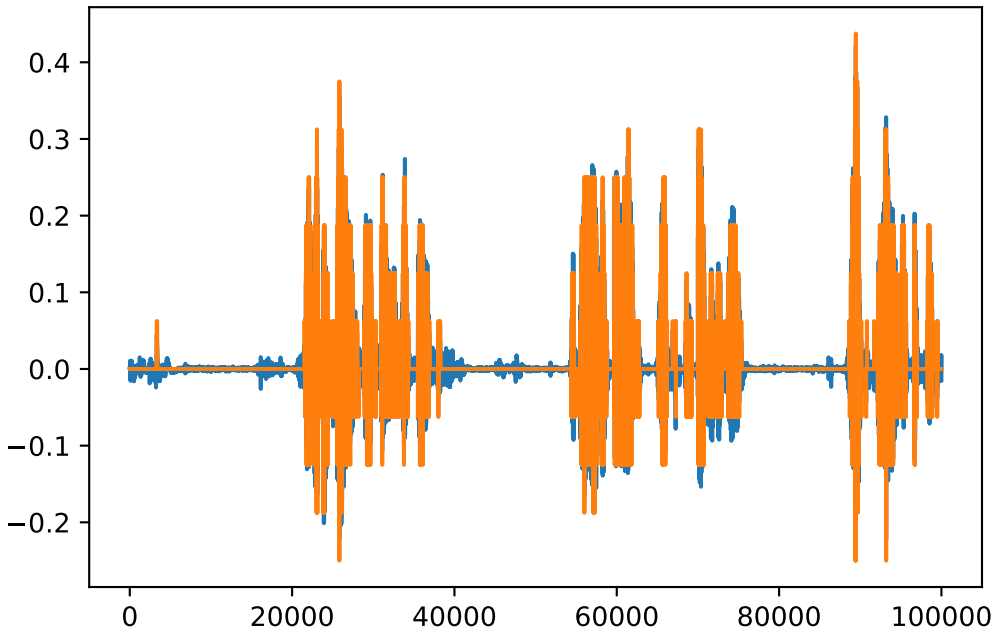
```
In [96]: N_B = 5
         SNR_chan = 100
         a = PCM_encode(m1,N_B) # use your PCM_encode with clipping
         a_hat = dc.AWGN_chan(a,SNR_chan)
         m1_hat = dc.PCM_decode(a_hat,N_B)
         # calculation to find MSE
```

Compare Visually

To gain confidence in what you are doing compare the input and output waveforms via an overlay plot or by plotting the difference.

```
In [97]: plot(m1)
         plot(m1_hat)
         # or
         #plot(m1_hat - m1)
```

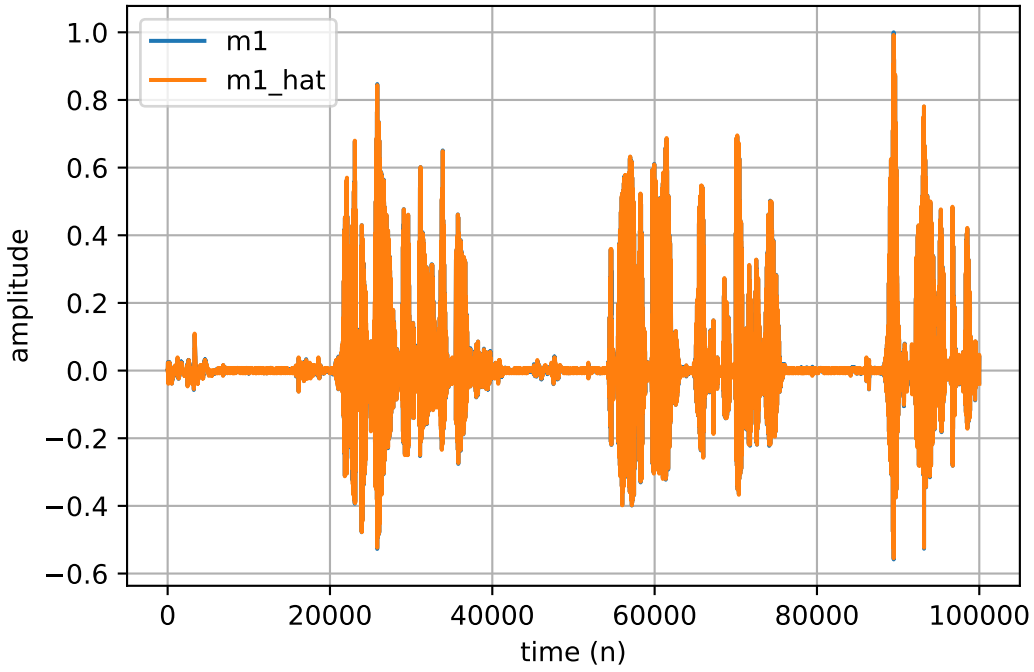
Out[97]: [<matplotlib.lines.Line2D at 0x11862978>]



Part A)

In [98]:
```
#define number of bits
N_B = 8

#define SNR vector
SNR_chan = arange(0,21,1)
MSE_8 = zeros((1,size(SNR_chan)))

#loop through and find MSE for each SNR
for ii in range (0, size(SNR_chan)):

    m1_enc = PCM_encode(g*m1,N_B)
    m1_enc_hat = dc.AWGN_chan(m1_enc,SNR_chan[ii])
    m1_hat = dc.PCM_decode(m1_enc_hat,N_B)

    MSE_8[(0,ii)] = 1/size(m1)*sum((m1_hat-g*m1)**2)

plt.plot(g*m1)
plt.plot(m1_hat)
grid()
xlabel('time (n)')
ylabel('amplitude')
legend(('m1','m1_hat'))
```

Out[98]: <matplotlib.legend.Legend at 0x54f24e0>

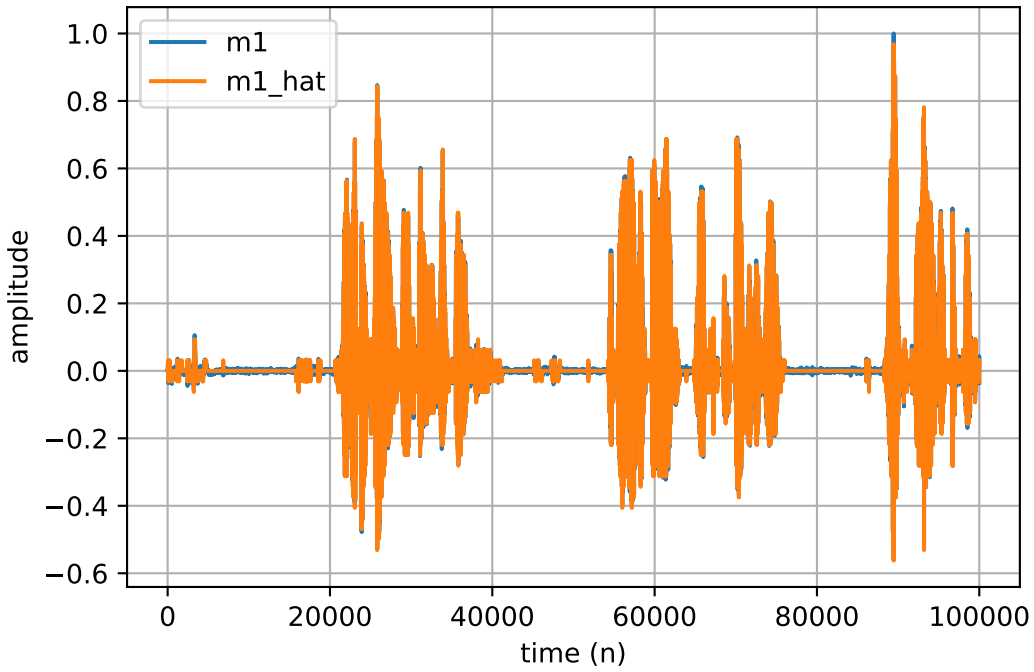


Part B)

In [99]:
```
#define number of bits
N_B = 6

#define SNR vector
SNR_chan = arange(0,21,1)
MSE_6 = zeros((1,size(SNR_chan)))

#loop through and find MSE for each SNR
for ii in range (0, size(SNR_chan)):

    m1_enc = PCM_encode(g*m1,N_B)
    m1_enc_hat = dc.AWGN_chan(m1_enc,SNR_chan[ii])
    m1_hat = dc.PCM_decode(m1_enc_hat,N_B)

    MSE_6[(0,ii)] = 1/size(m1)*sum((m1_hat-g*m1)**2)

plt.plot(g*m1)
plt.plot(m1_hat)
grid()
xlabel('time (n)')
```

```
ylabel('amplitude')
legend(('m1','m1_hat'))
```

```
Out[99]: <matplotlib.legend.Legend at 0xe613e10>
```

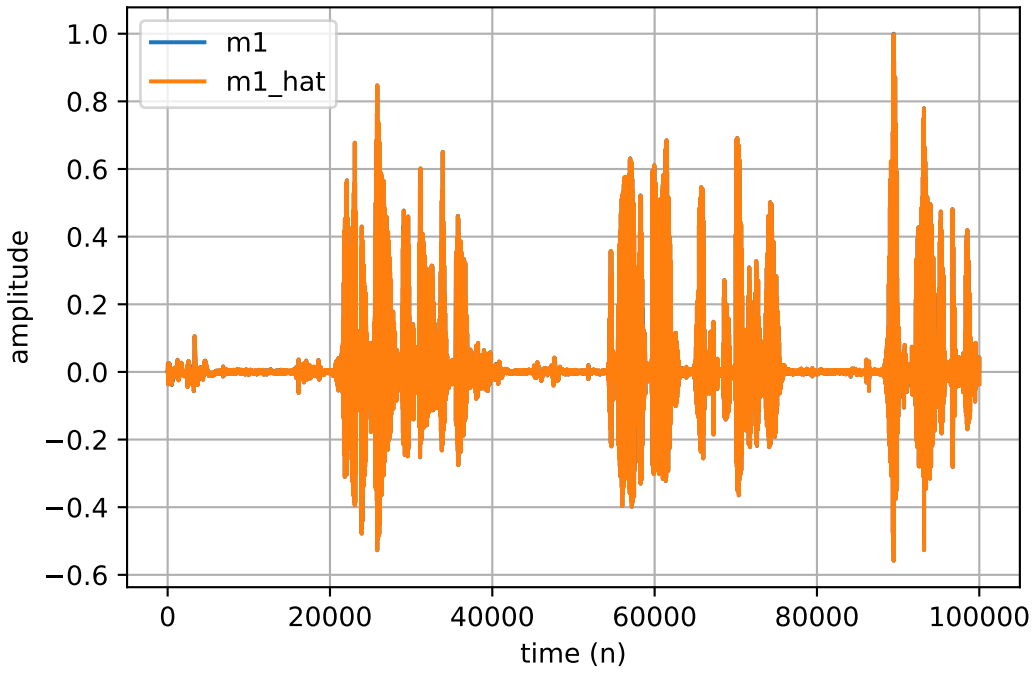


Part C)

```
In [100]: #define number of bits
          N_B = 10

          #define SNR vector
          SNR_chan = arange(0,21,1)
          MSE_10 = zeros((1,size(SNR_chan)))

          #loop through and find MSE for each SNR
          for ii in range (0, size(SNR_chan)):

              m1_enc = PCM_encode(g*m1,N_B)
              m1_enc_hat = dc.AWGN_chan(m1_enc,SNR_chan[ii])
              m1_hat = dc.PCM_decode(m1_enc_hat,N_B)

              MSE_10[(0,ii)] = 1/size(m1)*sum((m1_hat-g*m1)**2)

          plt.plot(g*m1)
          plt.plot(m1_hat)
```

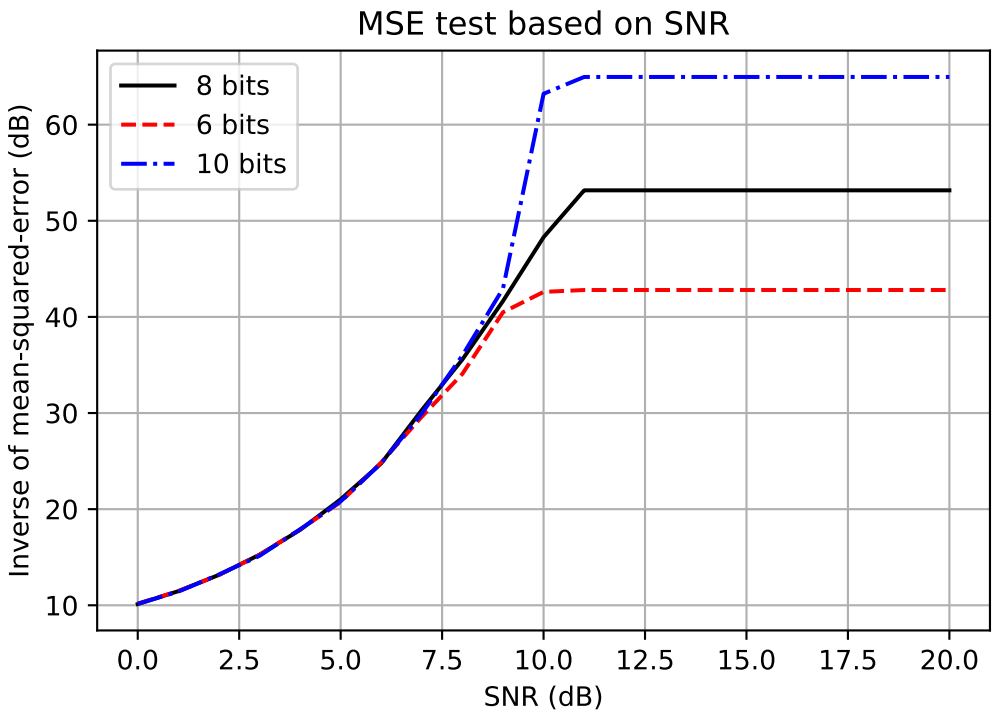
```
grid()
xlabel('time (n)')
ylabel('amplitude')
legend(('m1','m1_hat'))
```

Out[100]: <matplotlib.legend.Legend at 0xbb75a58>



Plots

In [101]:
```
#Compare SNR and MSE inverse for each N_B

figure()
plt.plot(SNR_chan,10*log10(1/MSE_8[0]), 'k-')
plt.plot(SNR_chan,10*log10(1/MSE_6[0]), 'r--')
plt.plot(SNR_chan,10*log10(1/MSE_10[0]), 'b-.')
grid()
xlabel('SNR (dB)')
ylabel('Inverse of mean-squared-error (dB)')
title('MSE test based on SNR');
legend(('8 bits','6 bits','10 bits'));
```



Part D) As expected, the SNR has a noticeable effect on the error of the recieved signal. A high SNR increases the inverse mean squared error, which indicates that error is inversely proportional with SNR, as expected. Additionally, the number of quantization bits has an effect. Generally, one would expect a higher number of bits to correspond to a lower MSE, and thus a higher *inverse* MSE. This is verified with the above plot, as the maximum inverse MSE for 10 bit encoding is over 60dB, while the same quantity for 6 bit encoding is just over 40 dB. This verifies that the channel and encoding scheme behave as expected with relation to number of quantization bits and SNR.

Part II: QAM Background Examples

Begin with some functions for working with QAM. Specifically, * `QAM_gray_encode_bb` which produces single-sample per symbol QAM waveforms and `Ns` sample per symbol waveforms with pulse shaping. Binary to Gray mapping is performed using look-up-tables (LUTs). Soon this function will be merged into `digitalcom.py` on GitHub. * `QAM_gray_decode` which takes as input single sample per QAM symbol inputs, with noise and interference, and decodes them to a serial bit stream Gray to binary LUTs. Soon this function will be merged into `digitalcom.py` on GitHub.

The QAM Encoder/Decoder with Gray Mapping Functions

Soon these functions will be merged into the `digitalcom.py` module on GitHub, for now they are also here in this notebook.

```
In [102]: def QAM_gray_encode_bb(N_symb,Ns,M=4,pulse='rect',alpha=0.35,ext_data=None):
              """
              QAM_gray_bb: A gray code mapped QAM complex baseband transmitter
              x,b,tx_data = QAM_gray_bb(K,Ns,M)

              ////////////// Inputs ////////////////////////////////////////////////
                N_symb = the number of symbols to process
                    Ns = number of samples per symbol
                     M = modulation order: 2, 4, 16, 64, 256 QAM
                         Note 2 <=> BPSK, 4 <=> QPSK
                 alpha = squareroot raised codine pulse shape bandwidth factor.
                         For DOCSIS alpha = 0.12 to 0.18. In general alpha can
                         range over 0 < alpha < 1.
                 pulse = 'rect', 'src', or 'rc'
              ////////////// Outputs ///////////////////////////////////////////////
                     x = complex baseband digital modulation
                     b = transmitter shaping filter, rectangle or SRC
               tx_data = xI+1j*xQ = inphase symbol sequence +
                         1j*quadrature symbol sequence

              Mark Wickert April 2018
              """
              # Create a random bit stream then encode using gray code mapping
              # Gray code LUTs for 4, 16, 64, and 256 QAM
              # which employs M = 2, 4, 6, and 8 bits per symbol
              bin2gray1 = [0,1]
              bin2gray2 = [0,1,3,2]
              bin2gray3 = [0,1,3,2,7,6,4,5] # arange(8)
              bin2gray4 = [0,1,3,2,7,6,4,5,15,14,12,13,8,9,11,10]
              x_m = sqrt(M)-1
              # Create the serial bit stream [Ibits,Qbits,Ibits,Qbits,...], msb to lsb
              # except for the case M = 2
              if N_symb == None:
                  # Truncate so an integer number of symbols is formed
                  N_symb = int(floor(len(ext_data)/log2(M)))
                  data = ext_data[:N_symb*int(log2(M))]
              else:
                  data = randint(0,2,size=int(log2(M))*N_symb)
              x_IQ = zeros(N_symb,dtype=complex128)
              N_word = int(log2(M)/2)
              # binary weights for converting binary to decimal using dot()
              w = 2**np.arange(N_word-1,-1,-1)
              if M == 2: # Special case of BPSK for convenience
                  x_IQ = 2*data - 1
                  x_m = 1
              elif M == 4: # total constellation points
                  for k in range(N_symb):
                      wordI = data[2*k*N_word:(2*k+1)*N_word]
```

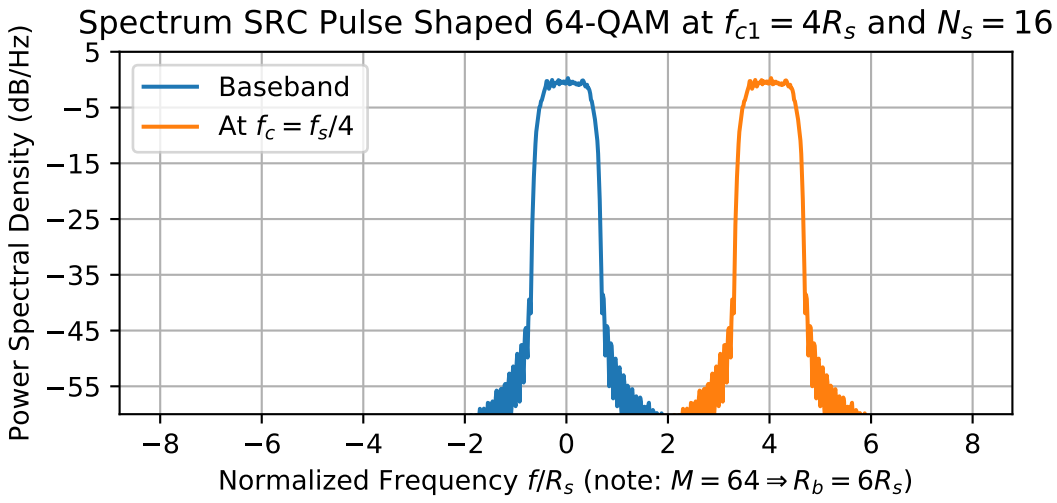
```
            wordQ = data[2*k*N_word+N_word:(2*k+1)*N_word+N_word]
            x_IQ[k] = (2*bin2gray1[dot(wordI,w)] - x_m) + \
                   1j*(2*bin2gray1[dot(wordQ,w)] - x_m)
    elif M == 16:
        for k in range(N_symb):
            wordI = data[2*k*N_word:(2*k+1)*N_word]
            wordQ = data[2*k*N_word+N_word:(2*k+1)*N_word+N_word]
            x_IQ[k] = (2*bin2gray2[dot(wordI,w)] - x_m) + \
                   1j*(2*bin2gray2[dot(wordQ,w)] - x_m)
    elif M == 64:
        for k in range(N_symb):
            wordI = data[2*k*N_word:(2*k+1)*N_word]
            wordQ = data[2*k*N_word+N_word:(2*k+1)*N_word+N_word]
            x_IQ[k] = (2*bin2gray3[dot(wordI,w)] - x_m) + \
                   1j*(2*bin2gray3[dot(wordQ,w)] - x_m)
    elif M == 256:
        for k in range(N_symb):
            wordI = data[2*k*N_word:(2*k+1)*N_word]
            wordQ = data[2*k*N_word+N_word:(2*k+1)*N_word+N_word]
            x_IQ[k] = (2*bin2gray4[dot(wordI,w)] - x_m) + \
                   1j*(2*bin2gray4[dot(wordQ,w)] - x_m)
    else:
        raise ValueError('M must be 2, 4, 16, 64, 256')

    if Ns > 1:
        # Design the pulse shaping filter to be of duration 12
        # symbols and fix the excess bandwidth factor at alpha = 0.35
        if pulse.lower() == 'src':
            b = dc.sqrt_rc_imp(Ns,alpha,6)
        elif pulse.lower() == 'rc':
            b = dc.rc_imp(Ns,alpha,6)
        elif pulse.lower() == 'rect':
            b = np.ones(int(Ns)) #alt. rect. pulse shape
        else:
            raise ValueError('pulse shape must be src, rc, or rect')
        # Filter the impulse train signal
        x = signal.lfilter(b,1,dc.upsample(x_IQ,Ns))
        # Scale shaping filter to have unity DC gain
        b = b/sum(b)
        return x/x_m, b, data
    else:
        return x_IQ/x_m, 1, data

In [103]: def QAM_gray_decode(x_hat,M = 4):
    """
    Decode MQAM IQ symbols to a serial bit stream using
    gray2bin decoding
```

```
    x_hat = symbol spaced samples of the QAM waveform taken at the maximum
            eye opening. Normally this is following the matched filter

    Mark Wickert April 2018
    """
    # Inverse Gray code LUTs for 4, 16, 64, and 256 QAM
    # which employs M = 2, 4, 6, and 8 bits per symbol
    gray2bin1 = [0,1]
    gray2bin2 = [0,1,3,2]
    gray2bin3 = [0,1,3,2,6,7,5,4] # arange(8)
    gray2bin4 = [0,1,3,2,6,7,5,4,12,13,15,14,10,11,9,8]
    x_m = sqrt(M)-1
    if M == 2: x_m = 1
    N_symb = len(x_hat)
    N_word = int(log2(M)/2)

    # Scale input up by x_m
#     x_hat = x_hat*x_m
    x_hat = x_hat/(std(x_hat) * sqrt(3/(2*(M-1))))

    k_hat_gray = (x_hat + x_m*(1+1j))/2
    # Soft IQ symbol values are converted to hard symbol decisions
    k_hat_grayI = int16(clip(rint(k_hat_gray.real),0,x_m))
    k_hat_grayQ = int16(clip(rint(k_hat_gray.imag),0,x_m))
    data_hat = zeros(2*N_word*N_symb,dtype=int)
    # Create the serial bit stream [Ibits,Qbits,Ibits,Qbits,...], msb to lsb
    for k in range(N_symb):
        if M == 2: # special case for BPSK
            data_hat = k_hat_grayI
        elif M == 4: # total points of the square constellation
            data_hat[2*k*N_word:2*(k+1)*N_word] \
              = hstack((dc.tobin(gray2bin1[k_hat_grayI[k]],N_word),
                        dc.tobin(gray2bin1[k_hat_grayQ[k]],N_word)))
        elif M == 16:
            data_hat[2*k*N_word:2*(k+1)*N_word] \
              = hstack((dc.tobin(gray2bin2[k_hat_grayI[k]],N_word),
                        dc.tobin(gray2bin2[k_hat_grayQ[k]],N_word)))
        elif M == 64:
            data_hat[2*k*N_word:2*(k+1)*N_word] \
              = hstack((dc.tobin(gray2bin3[k_hat_grayI[k]],N_word),
                        dc.tobin(gray2bin3[k_hat_grayQ[k]],N_word)))
        elif M == 256:
            data_hat[2*k*N_word:2*(k+1)*N_word] \
              = hstack((dc.tobin(gray2bin4[k_hat_grayI[k]],N_word),
                        dc.tobin(gray2bin4[k_hat_grayQ[k]],N_word)))
        else:
            raise ValueError('M must be 2, 4, 16, 64, 256')
```

```
        return data_hat
```

In [104]:
```
#QAM_gray_encode_bb(N_symb,Ns,M=4,pulse='rect',alpha=0.35,ext_data=None)
Ns = 16
M = 64
xbb1,b,data = QAM_gray_encode_bb(10000,Ns,M,'src')
n = arange(0,len(xbb1))
# Translate baseband to fc1/fs = 4.0/16
# Relative to Ns = 16 samps/bit & Rb = 1 bps
xc1 = xbb1*exp(1j*2*pi*4.0/Ns*n)
```

In [105]:
```
figure(figsize=(6,2.5))
psd(xbb1,2**10,Ns);
psd(xc1,2**10,Ns);
ylim([-60,5])
xlabel(r'Normalized Frequency $f/R_s$ (note: $M=64 \Rightarrow R_b = 6R_s$)')
legend((r'Baseband',r'At $f_c = f_s/4$'))
title(r'Spectrum SRC Pulse Shaped 64-QAM at $f_{c1} \
      = 4R_s$ and $N_s = %d$' % Ns);
```

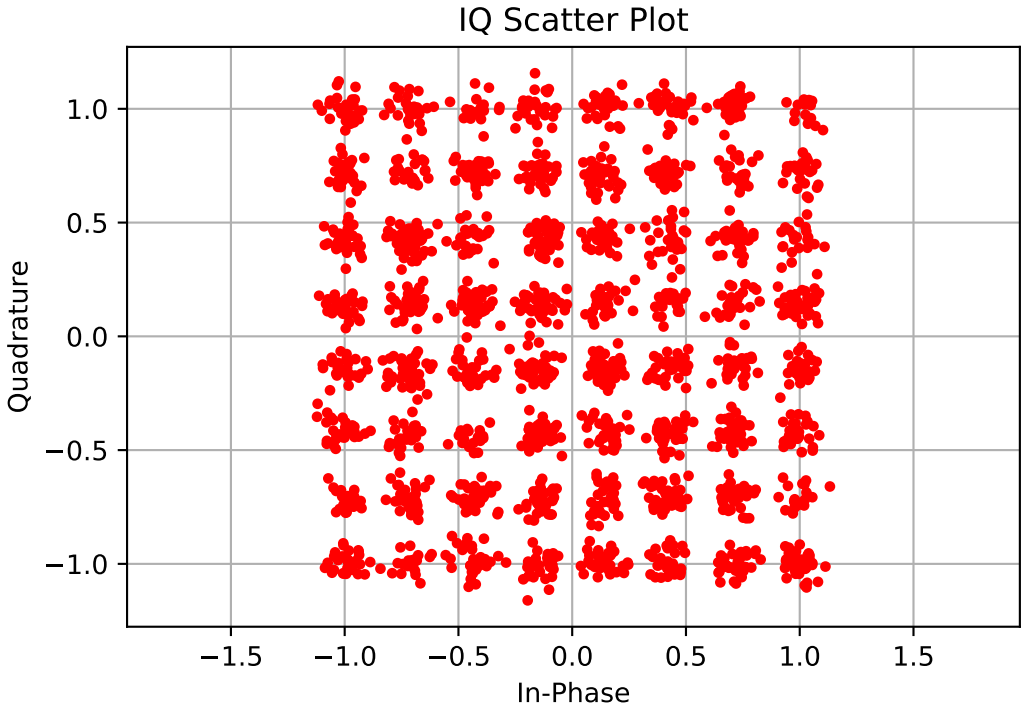


64QAM Simulation Example

In [106]:
```
M = 64
Nsymb = 100000
Ns = 1 # With Ns=1 there is no need for the matched filter
EbN0_dB = 15
EsN0_dB = 10*log10(log2(M))+ EbN0_dB
print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (EbN0_dB,EsN0_dB))
xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')
rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)
```

```
Eb/N0 = 15.00 dB and Es/N0 = 22.78 dB
```

```
In [107]: Npts = 2000
          scat_data = rbb
          plot(scat_data[:Npts].real,scat_data[:Npts].imag,'r.')
          axis('equal')
          title('IQ Scatter Plot')
          ylabel(r'Quadrature')
          xlabel(r'In-Phase')
          grid();
```




```
In [108]: data_hat = QAM_gray_decode(rbb,M)
          Nbits,Nerrors = dc.BPSK_BEP(data,data_hat)
          print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
                (Nbits, Nerrors, Nerrors/Nbits))

kmax =  0, taumax = 0
BEP: Nbits = 600000, Nerror = 459, Pe_est = 7.650e-04
```

```
In [109]: def MQAM_BEP(EbN0,M):
              """
              Approximate symbol error probability of MQAM
```

```
    Mark Wickert April 2018
    """
    if M == 2:
        PE = dc.Q_fctn(sqrt(2*EbN0))
    elif M > 2:
        EsN0 = log2(M)*EbN0 # convert Eb/N0 to Es/N0
        PE = 4*(1 - 1/sqrt(M))*dc.Q_fctn(sqrt(3/(M-1)*EsN0))/log2(M)
    return PE
```

- Let's check the simulation results above against theory:

```
In [110]: P_e_thy = MQAM_BEP(10**(EbN0_dB/10),64)
          print('P_e_bit_thy = %4.2e' % P_e_thy)
```

```
P_e_bit_thy = 7.72e-04
```

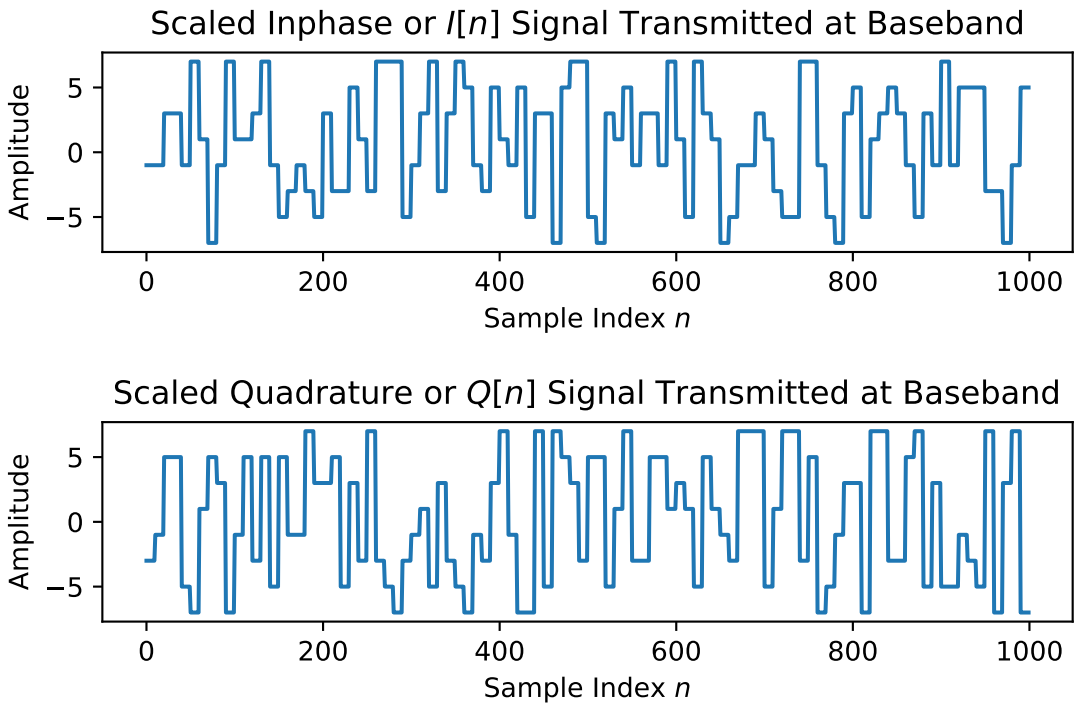
- Yes, the results are close

Waveform Level Simulation

```
In [111]: M = 64
          Nsymb = 500
          Ns = 10 # With Ns=10 we have a waveform
          EbN0_dB = 100 # Make the addive noise negligible for now
          EsN0_dB = 10*log10(log2(M)) + EbN0_dB # Convert Eb/N0 to Es/N0
          print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (EbN0_dB,EsN0_dB))
          xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'rect')
          # Enter the comm channel by adding noise
          rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)
```

```
Eb/N0 = 100.00 dB and Es/N0 = 107.78 dB
```

```
In [112]: Nplot = 100
          subplot(211)
          plot(rbb[:Nplot*Ns].real*7) # Remove the unity amplitude scaling to see the +/-1, +/
          ylabel(r'Amplitude')
          xlabel(r'Sample Index $n$')
          title(r'Scaled Inphase or $I[n]$ Signal Transmitted at Baseband')
          subplot(212)
          plot(rbb[:Nplot*Ns].imag*7)
          ylabel(r'Amplitude')
          xlabel(r'Sample Index $n$')
          title(r'Scaled Quadrature or $Q[n]$ Signal Transmitted at Baseband')
          tight_layout()
```



Including the Receiver Matched Filter to Mitigate Noise

Take the received waveform into the receiver by first passing the signal through the *matched filter* `b`, which is conveniently returned by `QAM_gray_encode_bb`.

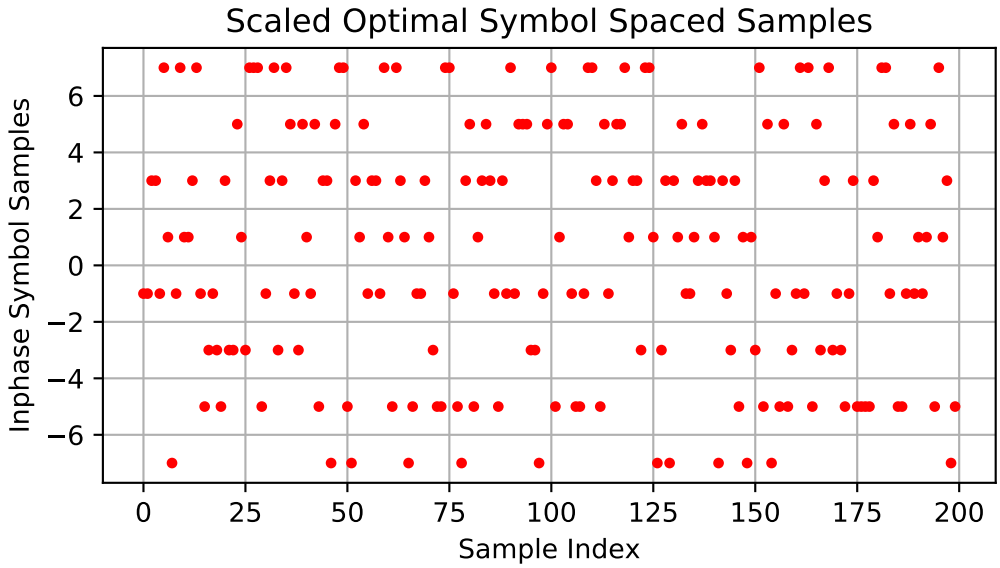
```
In [113]: # Enter the receiver by passing through the matched filter
          # Note carrier frequency ofset and phase tracking would likely be needed, but here
          # both are zero to keep things simple.
          y = signal.lfilter(b,1,rbb)
```

Choosing the Proper Once Per Symbol Sampling Instant

The symbol synchronization operation requires sampling the matched filter output onnce per symbol such that the maximum SNR is achieved, and hence the minimum BEP is obtained. Here we use `ss.downsample(y,Ns,phase)` to do this. The optional third argument `phase`, chosen from [0,Ns-1], which is exactly what we need. In practice a PLL symbol synchronizer performs this automatically.

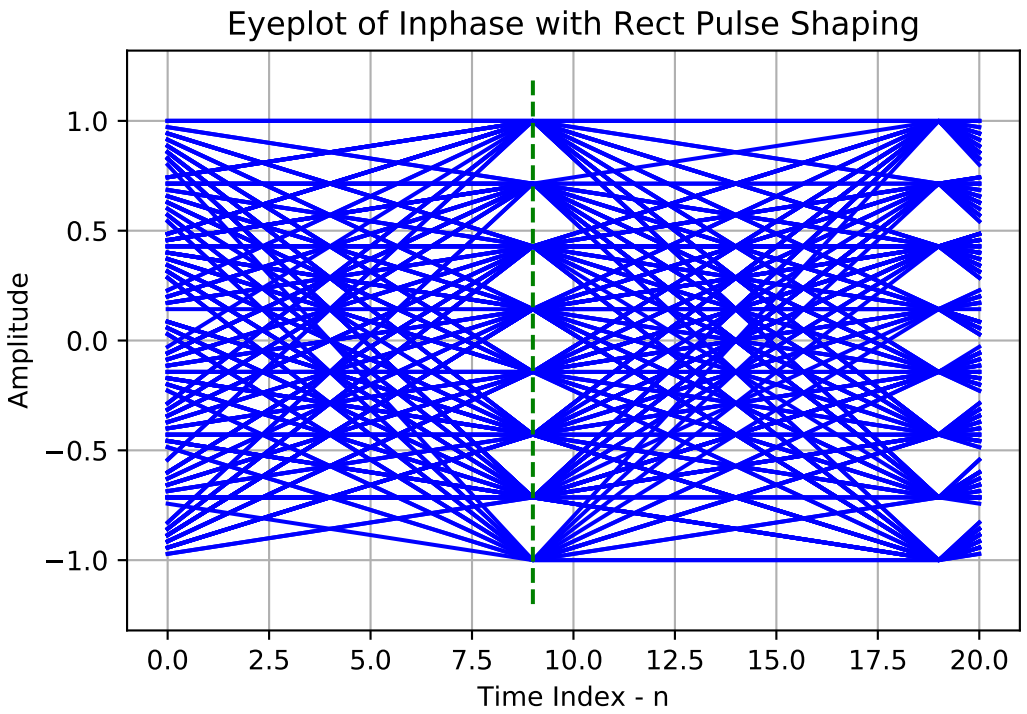
```
In [114]: # Symbol synchronize manually
          z = ss.downsample(y,Ns,9)
          figure(figsize=(6,3))
          plot(z[:2*Nplot].real*7,'r.') # Again scale to see that the sample values are aligni
                                        # with the expected unscaled tx values.
          title(r'Scaled Optimal Symbol Spaced Samples')
```

```
ylabel(r'Inphase Symbol Samples')
xlabel(r'Sample Index')
grid();
```



We can use the `eyeplot` function to get a better handle on the options.

```
In [115]: dc.eye_plot(y.real,2*Ns,12*Ns) # The 12*Ns removed the pulse shaping filter transien
          plot([9,9],[-1.2,1.2],'g--')
          title(r'Eyeplot of Inphase with Rect Pulse Shaping')

Out[115]: Text(0.5,1,'Eyeplot of Inphase with Rect Pulse Shaping')
```



Quick Look with SRC Pulse Shaping

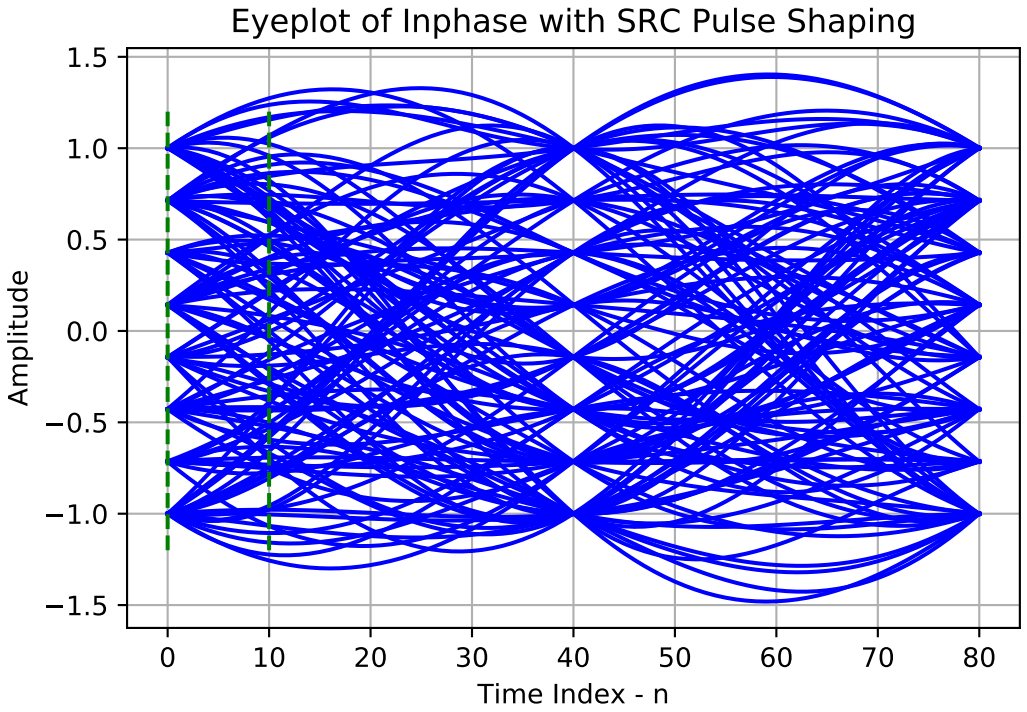
Change from the easy to visualize rectangular pulse shape to the square-root raised cosine (`src`) shape, which has a very compact spectrum. Here in particular the optimum sampling instant shifts from `phase = 9` to `phase = mod(10,Ns) = 0`.

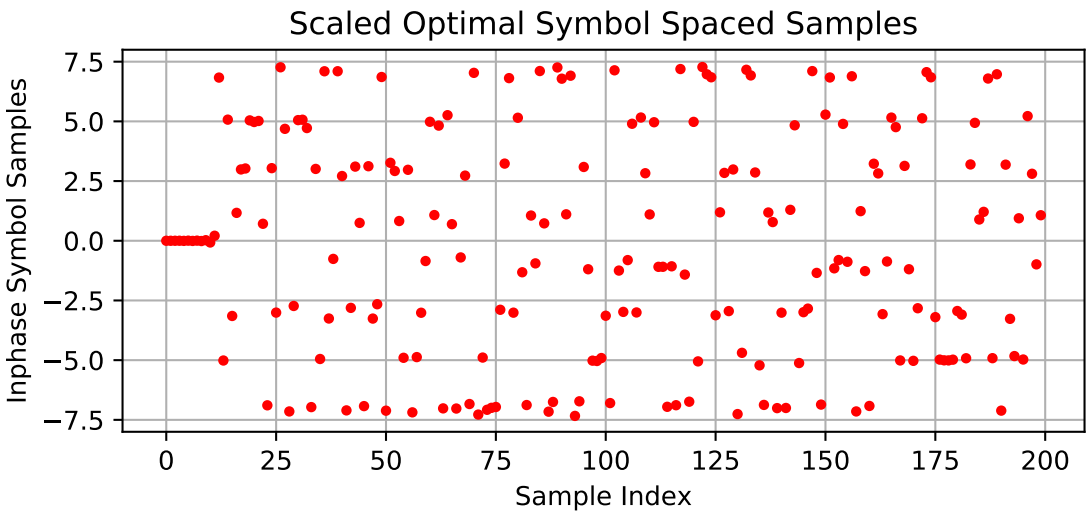
```
In [116]: M = 64
          Nsymb = 100000
          Ns = 40 # With Ns=10 we have a waveform
          EbN0_dB = 50 # Make the addive noise negligible for now
          EsN0_dB = 10*log10(log2(M)) + EbN0_dB
          print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (EbN0_dB,EsN0_dB))
          xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')
          # Enter the comm channel by adding noise
          rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)
          # Enter the receiver by passing through the matched filter
          # Note carrier frequency offset and phase tracking would likely be needed, but here
          # both are zero to keep things simple.
          y = signal.lfilter(b,1,rbb)
          # Timeout for an eye plot to check timing
          figure(figsize=(6,2))
          dc.eye_plot(y[:10000].real,2*Ns,12*Ns) # 12*Ns removes the filter transients
          plot([0,0],[-1.2,1.2],'g--')
          plot([10,10],[-1.2,1.2],'g--')
```

```
title(r'Eyeplot of Inphase with SRC Pulse Shaping')
# Symbol synchronize manually
z = ss.downsample(y,Ns,1)
figure(figsize=(6,3))
# Scale so the peak is +/- (sqrt(M) - 1).
plot(z[:2*Nplot].real/(std(z) * sqrt(3/(2*(M-1)))),'r.')
title(r'Scaled Optimal Symbol Spaced Samples')
ylabel(r'Inphase Symbol Samples')
xlabel(r'Sample Index')
ylim([-8,8])
grid();
tight_layout()
```

```
Eb/N0 = 50.00 dB and Es/N0 = 57.78 dB
```

```
<Figure size 432x144 with 0 Axes>
```





BEP Testing with Waveforms

Here we feed the reference bit stream output, `data`, from `QAM_gray_encode_bb` along with the decoded bit stream `QAM_gray_decode` into `BPSK_BEP`. For the SRC pulse shaped waveform the net filter delay from the encoder and the matched filter is 12 symbols. A cross-correlation is performed inside `BPSK_BEP` to align the two input bit streams for error detection. This of course assumes the are not too many errors.

```
In [117]: data_hat = QAM_gray_decode(z,M)
          Nbits,Nerrors = dc.BPSK_BEP(data,data_hat)
          print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
                (Nbits, Nerrors, Nerrors/Nbits))

kmax =  0, taumax = 72
BEP: Nbits = 599928, Nerror = 0, Pe_est = 0.000e+00
```

Zero bit errors at high SNR (E_b/N_0) is the expected result.

End-to-End Audio Test

The objective is to send audio through the 64QAM link using the PCM encoder/decoder pair. Some timing adjustment is needed to:

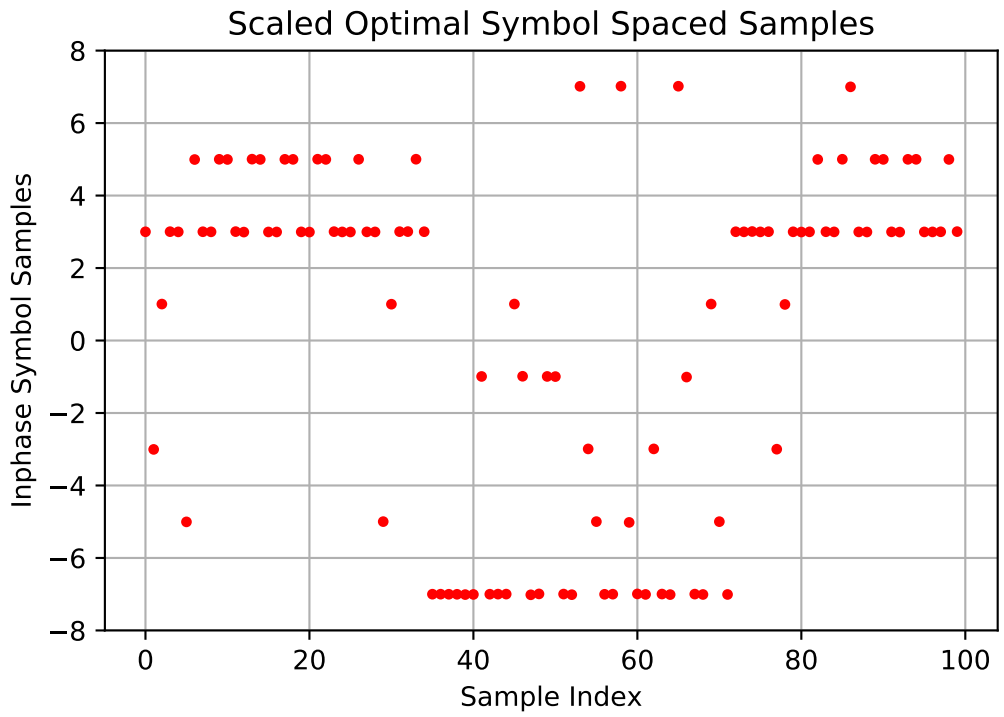
1. Make sure the filtering transient is properly removed at the start of the received serial bit stream
2. The number of bits processed bby `PCM_decode` is an integer multiple of the PCM word length in bits

We will also see how `QAM_gray_encode_bb` can work with an externally supplied bit stream, as opposed to its internally generated random bit stream.

In [118]:
```
Nstart = 20000
N = 60000; # number of speech samples to process
fs,m1 = ss.from_wav('OSR_uk_000_0050_8k.wav')
m1 = 2*m1[Nstart:Nstart+N]
N_PCM = 8 # PCM encode with 8 bits per audio sample
data_ext = dc.PCM_encode(m1,N_PCM) # Obtain external data bits for QAM_gray_encode
M = 64 # QAM modulate using a 64 point constellation (8x8 points)
Ns = 10 # With Ns=10 we have a waveform
EbN0_dB = 100 # Make the addive noise negligible for now
EsN0_dB = 10*log10(log2(M)) + EbN0_dB
print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (EbN0_dB,EsN0_dB))
xbb,b,data_ext_trim = QAM_gray_encode_bb(None,Ns,M,'src',ext_data=data_ext)
# Enter the comm channel by adding noise
rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)
# Enter the receiver by passing through the matched filter
# Note carrier frequency offset and phase tracking would likely be needed, but here
# both are zero to keep things simple.
y = signal.lfilter(b,1,rbb)
# Symbol synchronize manually
z = ss.downsample(y,Ns,0)
data_ext_hat = QAM_gray_decode(z,M)
Nbits,Nerrors = dc.BPSK_BEP(data_ext_trim,data_ext_hat)
print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
      (Nbits, Nerrors, Nerrors/Nbits))
```

```
Eb/N0 = 100.00 dB and Es/N0 = 107.78 dB
kmax =  0, taumax = 72
BEP: Nbits = 479928, Nerror = 0, Pe_est = 0.000e+00
```

In [119]:
```
plot(z[2000:2100].real*7,'r.')
ylim([-8,8])
title(r'Scaled Optimal Symbol Spaced Samples')
ylabel(r'Inphase Symbol Samples')
xlabel(r'Sample Index')
grid();
```



As we get ready to decode the recovered/demodulated bit stream `data_ext_hat` we have to bear in mind the signal imposed by the two SRC pulse shaping filters. Why? Bits that enter the PCM decoder must be properly framed, that is `N_PCM` is the word length, in this case 8 bits, so the word boundaries have to align for the decoder the do its job.

A clue that delay has occured is to look at the `taumax` output from the BEP functioin. In the above example we see `taumax = 72`, which says that the cross-correlation detection in `BPSK_BEP` found achieved its maximum by lagging the reference bits from the transmitter (`data_ext_trim`) by 72 bits. The default pulse shaping filter, `b`, introduces a delay of 6 symbols as does the matched filter in the receiver. Hence a total delay of 12 symbols is introduced. For $M = 64$ QAM we transmit 6 bits per symbol. The total delay in bits is thus $6 \times 12 = 72$.

The concern is this delay an integer multiple of $N_{\text{PCM}} = 8$? Yes, as $72/8 = 9$.

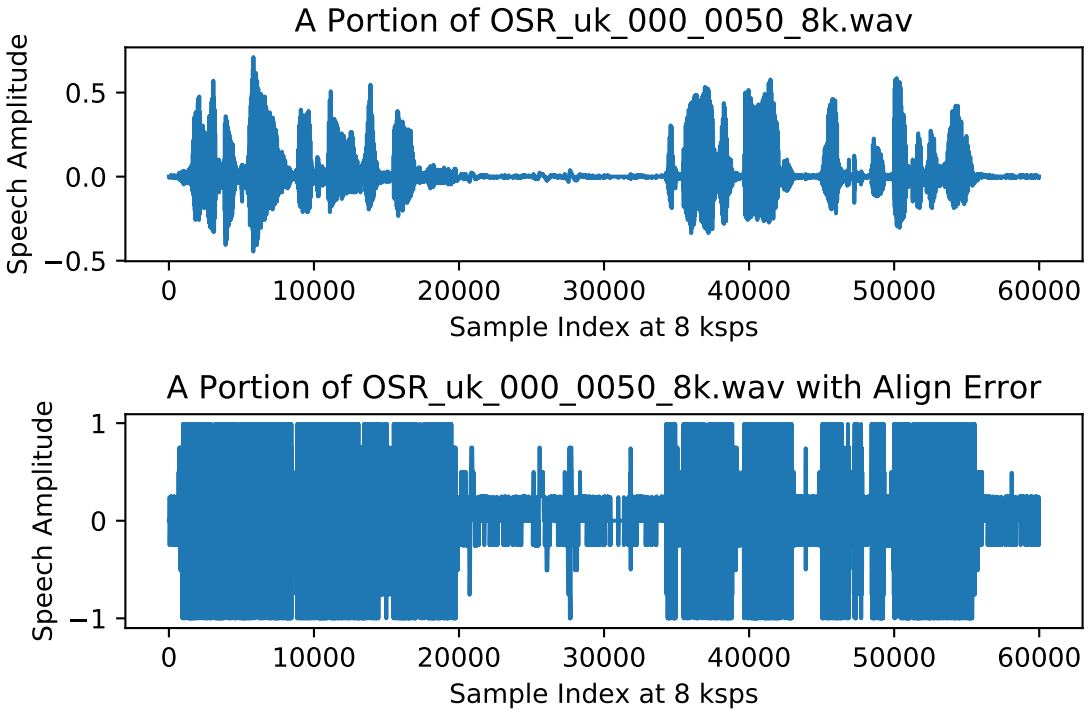
```
In [120]: 72//8
```

```
Out[120]: 9
```

Great, we do not have to do any bit trimming at the front of `data_ext_hat`. We do have to make sure that the length of `data_ext_hat` is an integer multiple of `N_PCM`. We can automate this as shown below, and while we are at it trim some of the spurious bits being pushed out of the *bit pipe* due to the delay.

```
In [121]: N_offset = 8*9
          data_ext_hat_trim = data_ext_hat[N_offset:\
                                           N_offset+N_PCM*int(len(data_ext_hat)/N_PCM)]
          m1_hat = dc.PCM_decode(data_ext_hat_trim,N_PCM)
```

```
subplot(211)
plot(m1_hat)
ylabel(r'Speech Amplitude')
xlabel(r'Sample Index at 8 ksps')
title(r'A Portion of OSR_uk_000_0050_8k.wav')
N_offset = 8*9
# Improperly trim so that the word bondary is shifted by 5 bits
m1_hat_miss = dc.PCM_decode(data_ext_hat_trim[5:-3],N_PCM)
subplot(212)
plot(m1_hat_miss)
ylabel(r'Speech Amplitude')
xlabel(r'Sample Index at 8 ksps')
title(r'A Portion of OSR_uk_000_0050_8k.wav with Align Error')
tight_layout()
```



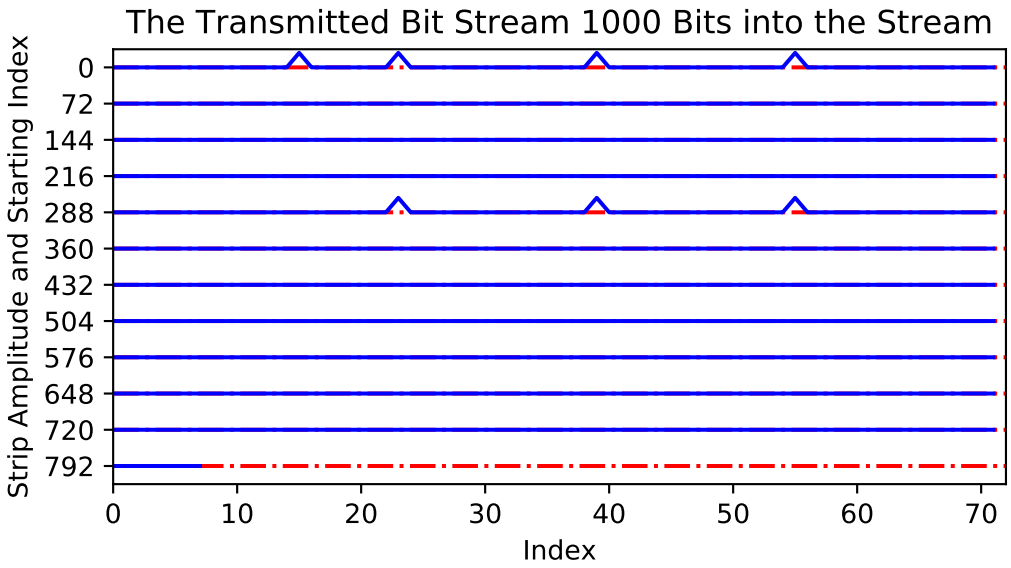

```
In [122]: ss.to_wav('QAM_PCM.wav',8000,m1_hat)
          Audio('QAM_PCM.wav')
```

```
Out[122]: <IPython.lib.display.Audio object>
```

Compare input and output bit streams by trimming off the first 72 bits using the `strips()` plot function found in `digitalcom`:

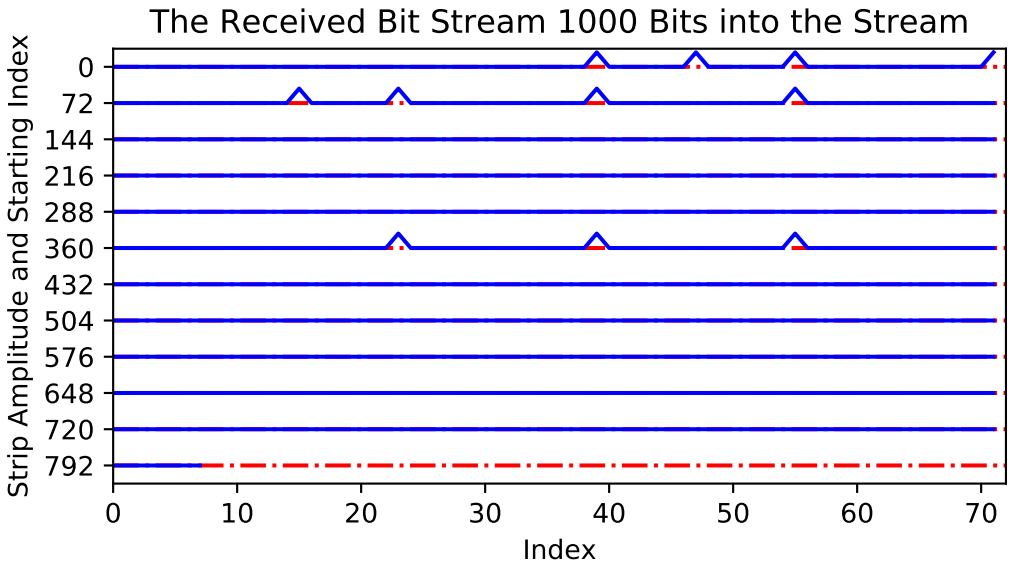
```
In [123]: dc.strips(data_ext[1000:1000+800],9*8,fig_size=(6,3)) # 9 N_PCM = 8-bit frames per s
          title(r'The Transmitted Bit Stream 1000 Bits into the Stream')
```

Out[123]: Text(0.5,1,'The Transmitted Bit Stream 1000 Bits into the Stream')



In [124]:
```
advance = 0
dc.strips(data_ext_hat[1000+advance:1000+800+advance],9*8,fig_size=(6,3))
title(r'The Received Bit Stream 1000 Bits into the Stream')
```

Out[124]: Text(0.5,1,'The Received Bit Stream 1000 Bits into the Stream')



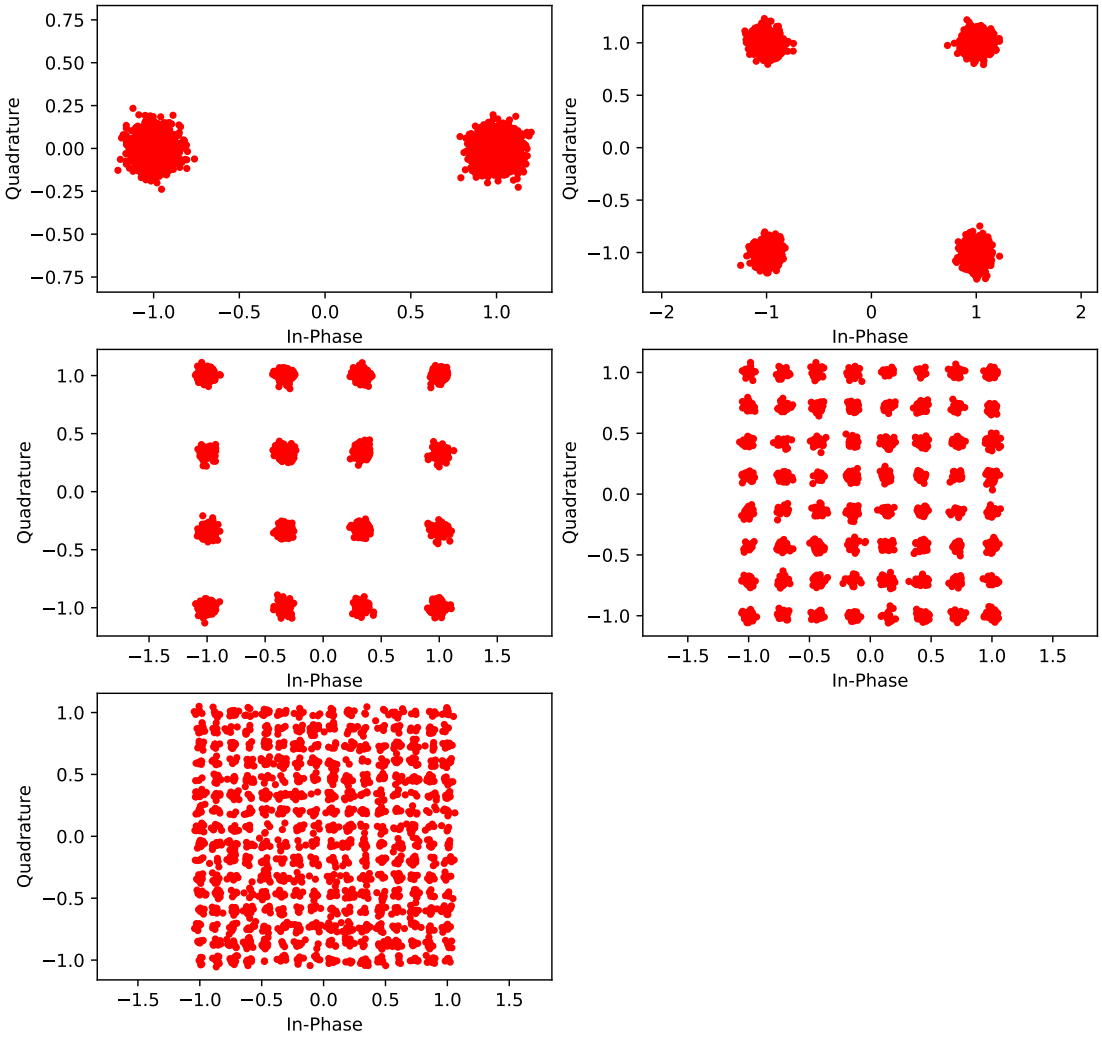
The above confirms what was suspected from the start. We see the second strip of 72 bits in the second plot matches the first strip of 72 bits from the first plot. The spurious seen in the first row of the second plot consist of all but 4 zeros. Recall a total of 12 64-QAM Symbols is equivalent to 72 bits, and this is also a multiple of `N_PCM`, so all is well on the playback of the audio too.

Part 2 Tasks

Task 4a

```
In [125]: # list of M's
          M_list = [2,4,16,64,256]

          Nsymb = 100000
          Ns = 1 # With Ns=1 there is no need for the matched filter
          EbN0_dB = 20

          figure(figsize=(10,10))

          for M, i in zip(M_list, arange(321,326,1)):
              EsN0_dB = 10*log10(log2(M))+ EbN0_dB
              print('M = %d; Eb/N0 = %4.2f dB; Es/N0 = %4.2f dB' % (M,EbN0_dB,EsN0_dB))
              xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')
              # Enter the comm channel by adding noise
              rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)
              # Plot the 64-QAM constellation points as a scatter plot
              Npts = 2000
              scat_data = rbb
              subplot(i)
              plot(scat_data[:Npts].real,scat_data[:Npts].imag,'r.')
              axis('equal')
              #title('IQ Scatter Plot')
              ylabel(r'Quadrature')
              xlabel(r'In-Phase')
              grid

              # Decode the QAM symbols back to a serial bit stream
              data_hat = QAM_gray_decode(rbb,M)
              Nbits,Nerrors = dc.BPSK_BEP(data,data_hat)
              print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e\n' % \
               (Nbits, Nerrors, Nerrors/Nbits))

M = 2; Eb/N0 = 20.00 dB; Es/N0 = 20.00 dB
kmax =  0, taumax = 0
BEP: Nbits = 100000, Nerror = 0, Pe_est = 0.000e+00

M = 4; Eb/N0 = 20.00 dB; Es/N0 = 23.01 dB
kmax =  0, taumax = 0
BEP: Nbits = 200000, Nerror = 0, Pe_est = 0.000e+00
```

```
M = 16; Eb/N0 = 20.00 dB; Es/N0 = 26.02 dB
kmax =  0, taumax = 0
BEP: Nbits = 400000, Nerror = 0, Pe_est = 0.000e+00

M = 64; Eb/N0 = 20.00 dB; Es/N0 = 27.78 dB
kmax =  0, taumax = 0
BEP: Nbits = 600000, Nerror = 0, Pe_est = 0.000e+00

M = 256; Eb/N0 = 20.00 dB; Es/N0 = 29.03 dB
kmax =  0, taumax = 0
BEP: Nbits = 800000, Nerror = 419, Pe_est = 5.238e-04
```

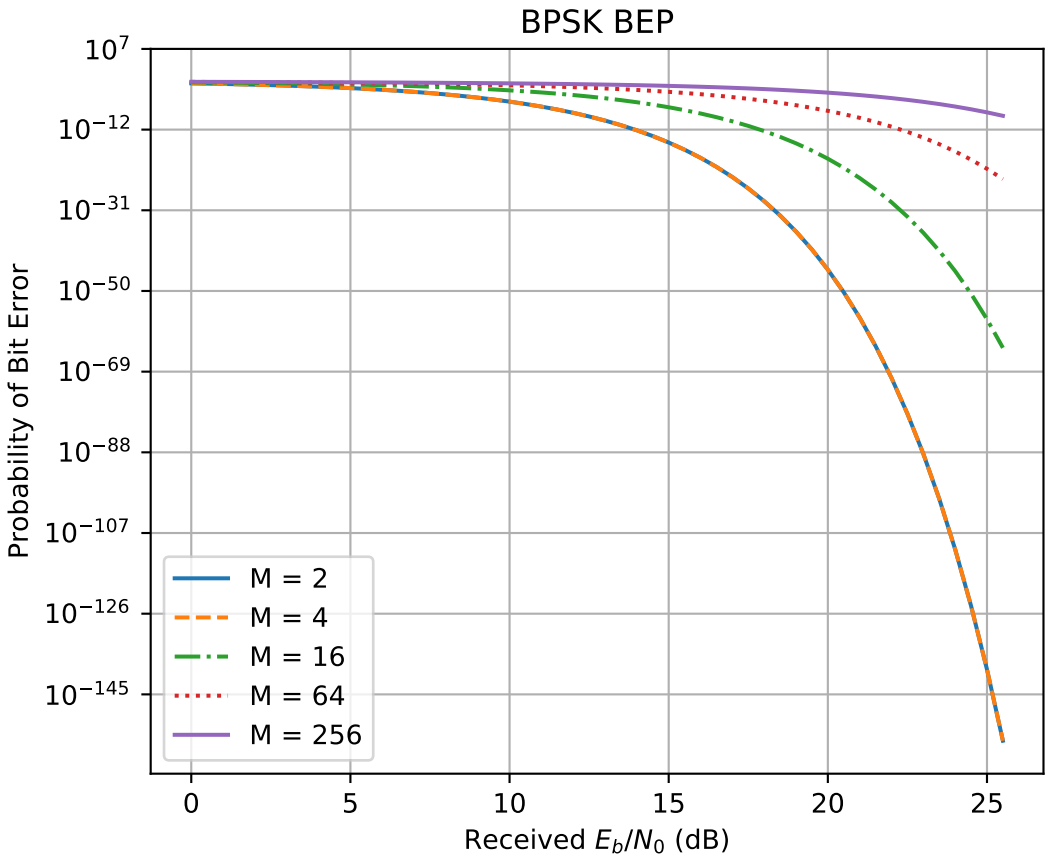


Note the noise variance is not constant as number of bits per symbol changes according to . This does however keep the cost in transmitter energy required to send one bit constant across all M values. Does it make sense that as M increases the cost in to send a bit goes up in order to maintain the same BEP? What do you get in return for this in increased cost?

Answer: As M increases, we are effectively cramming more information (ie distinct symbol locations) into the same "space" within the I/Q plot. Thus, we pay more in the form of E_b/N_0 to maintain the distict symbol locations as M increases. The upside is we have more constellation points to work with, meaning there is a greater resolution with which to send information (higher bits per symbol). This allows greater accuracy when transmitting detailed data.

Task 4b

```
In [126]: # Sample BEP Plot
          figure(figsize=(6,5))
          EbN0_dB_plot = arange(0,25.6,0.5)
          legList = list()

          linestyles = ['-', '--', '-.', ':','-']

          for M, l in zip(M_list, linestyles):
              Pe_thy_plotM2 = MQAM_BEP(10**(EbN0_dB_plot/10),M)
              semilogy(EbN0_dB_plot,Pe_thy_plotM2, linestyle=l)
              legList.append(f'M = {M}')

          ylabel(r'Probability of Bit Error')
          xlabel(r'Received $E_b/N_0$ (dB)')
          title(r'BPSK BEP')

          legend(legList,loc='best')
          grid();
```



The effect of a greater number of unique symbols (or unique points on the consetellation) is clearly seen in the plot above. As discussed previously, higher M allows more precision when transmitting information, but with a higher chance of error. This is reinforced by the BEP waterfall curves. Although all M levels have similar bit error probabilities for lower bit energy to received noise power ratios (E_b/N_0), as this ratio increases the BEP of the lower M values drops much sooner than that of the higher M values.

Task 4c

Using your limited understanding of digital comm try to explain why the a and b BEP curves overlap. You may want to refer the plots of 4a to help in your explanation.

Write your explanation here.

Based on the I/Q plots, the spacing between the constellation points for $M = 2, 4$ are identical (ie 2). Thus the their BEPs are equal for all Received E_b/N_0.

Task 5a

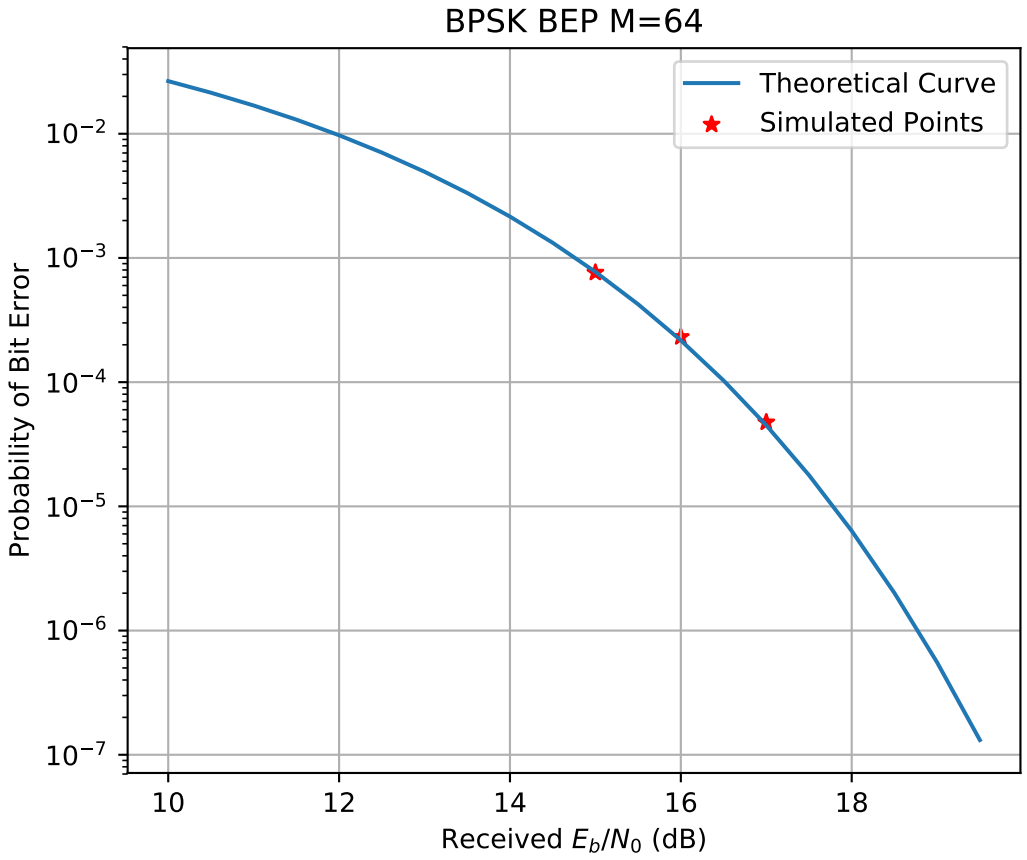
Add Some Gaussian Noise

```
In [138]: M = 64;
          EbN0_dB_plot = arange(10,20,0.5)
```

```
        EbN0_dB = arange(15,18,1)


        Nsymb = 400000
        Ns = 1 # With Ns=1 there is no need for the matched filter
        def task5a(M,EbN0_dB_plot,EbN0_dB,Nsymb,Ns):
            EsN0_dB = 10*log10(log2(M))+ EbN0_dB
            simBEP = zeros((size(EbN0_dB)))

            for ii in range(0, size(EbN0_dB)):
                xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')
                # Enter the comm channel by adding noise
                rbb = dc.cpx_AWGN(xbb,EsN0_dB[ii],Ns)
                # Decode the QAM symbols back to a serial bit stream
                data_hat = QAM_gray_decode(rbb,M)
                Nbits,Nerrors = dc.BPSK_BEP(data,data_hat)
                simBEP[ii] = Nerrors/Nbits
                print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
                 (Nbits, Nerrors, Nerrors/Nbits))


            # Sample BEP Plot
            figure(figsize=(6,5))
            legList = list()



            # Plot Theoretical
            Pe_thy_plotM2 = MQAM_BEP(10**(EbN0_dB_plot/10),M)
            plt.semilogy(EbN0_dB_plot,Pe_thy_plotM2)
            scatter
            legList.append(f'M = {M}')

            # Plot simulated
            plt.scatter(EbN0_dB,simBEP,color='r',marker='*')

            ylabel(r'Probability of Bit Error')
            xlabel(r'Received $E_b/N_0$ (dB)')
            title(r'BPSK BEP M=64')

            legend(('Theoretical Curve','Simulated Points'), loc='best')

            grid();

        task5a(M,EbN0_dB_plot,EbN0_dB,Nsymb,Ns)
```

```
kmax =  0, taumax = 0
BEP: Nbits = 2400000, Nerror = 1830, Pe_est = 7.625e-04
```

```
kmax =  0, taumax = 0
BEP: Nbits = 2400000, Nerror = 555, Pe_est = 2.313e-04
kmax =  0, taumax = 0
BEP: Nbits = 2400000, Nerror = 114, Pe_est = 4.750e-05
```



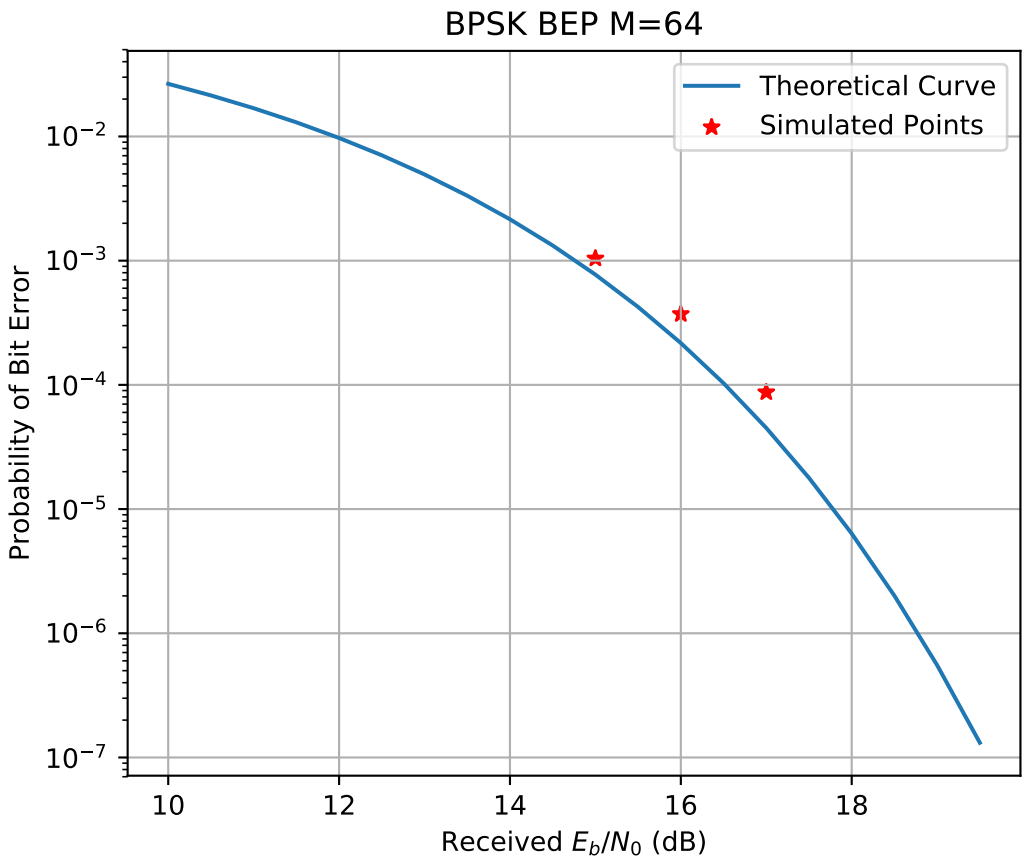
Task 5B

Add Some Phase Noise

```
In [128]: M = 64;
          EbN0_dB_plot = arange(10,20,0.5)
          EbN0_dB = arange(15,18,1)

          Nsymb = 300000
          Ns = 1 # With Ns=1 there is no need for the matched filter

          def task5b(M, EbN0_dB_plot, EbN0_dB, Nsymb, Ns):
              EsN0_dB = 10*log10(log2(M))+ EbN0_dB
              simBEP = zeros((size(EbN0_dB)))
```

```
            for ii in range(0, size(EbN0_dB)):
                xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')
                # Enter the comm channel by adding noise (gaussian/phase)
                rbb = dc.cpx_AWGN(xbb,EsN0_dB[ii],Ns)
                std_pn_deg = 1.0
                rbb_pn = rbb*exp(1j*2*pi*std_pn_deg*randn(len(xbb))/360)
                # Decode the QAM symbols back to a serial bit stream
                data_hat = QAM_gray_decode(rbb_pn,M)
                Nbits,Nerrors = dc.BPSK_BEP(data,data_hat)
                simBEP[ii] = Nerrors/Nbits
                print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
                 (Nbits, Nerrors, Nerrors/Nbits))

            disp(EbN0_dB)
            # Sample BEP Plot
            figure(figsize=(6,5))
            legList = list()



            # Plot Theoretical
            Pe_thy_plotM2 = MQAM_BEP(10**(EbN0_dB_plot/10),M)
            plt.semilogy(EbN0_dB_plot,Pe_thy_plotM2)
            scatter
            legList.append(f'M = {M}')

            # Plot simulated
            plt.scatter(EbN0_dB,simBEP,color='r',marker='*')

            ylabel(r'Probability of Bit Error')
            xlabel(r'Received $E_b/N_0$ (dB)')
            title(r'BPSK BEP M=64')

            legend(('Theoretical Curve','Simulated Points'), loc='best')

            grid();

        task5b(M, EbN0_dB_plot, EbN0_dB, Nsymb, Ns)
```

```
kmax =  0, taumax = 0
BEP: Nbits = 1800000, Nerror = 1876, Pe_est = 1.042e-03
kmax =  0, taumax = 0
BEP: Nbits = 1800000, Nerror = 669, Pe_est = 3.717e-04
kmax =  0, taumax = 0
BEP: Nbits = 1800000, Nerror = 157, Pe_est = 8.722e-05
[15 16 17]
```



The 'flare-out' for 1deg phase noise is noticeable in the plot above. The phase noise increased the BEP above the theoretical for higher bit-energy to noise-power ratios.

Task 6a

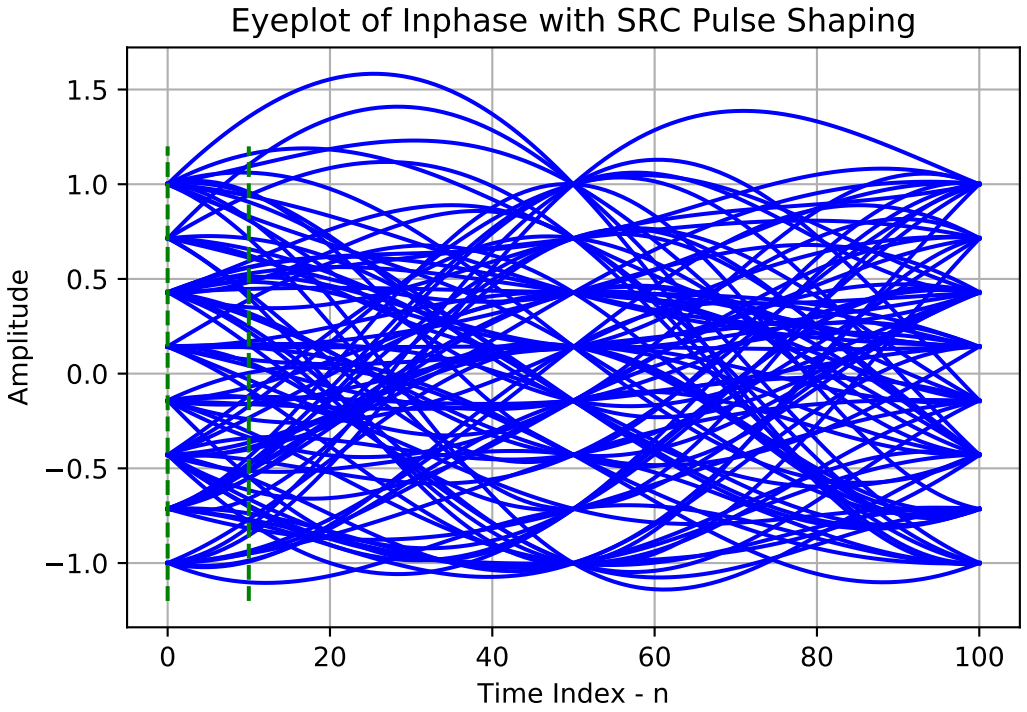
```
In [129]: M = 64
          Nsymb = 500
          Ns = 50
          EbN0_dB = 50 # Make the addive noise negligible for now
          EsN0_dB = 10*log10(log2(M)) + EbN0_dB
          print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (EbN0_dB,EsN0_dB))
          xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')

          # Enter the comm channel by adding noise
          rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)

          # Enter the receiver by passing through the matched filter
          # Note carrier frequency offset and phase tracking would likely be needed, but here
          # both are zero to keep things simple.
          y = signal.lfilter(b,1,rbb)
```

```
# Timeout for an eye plot to check timing
figure(figsize=(6,2))
dc.eye_plot(y[:10000].real,2*Ns,12*Ns) # 12*Ns removes the filter transients
plot([0,0],[-1.2,1.2],'g--')
plot([10,10],[-1.2,1.2],'g--')
title(r'Eyeplot of Inphase with SRC Pulse Shaping')
```

```
Eb/N0 = 50.00 dB and Es/N0 = 57.78 dB
```

```
Out[129]: Text(0.5,1,'Eyeplot of Inphase with SRC Pulse Shaping')
```

```
<Figure size 432x144 with 0 Axes>
```



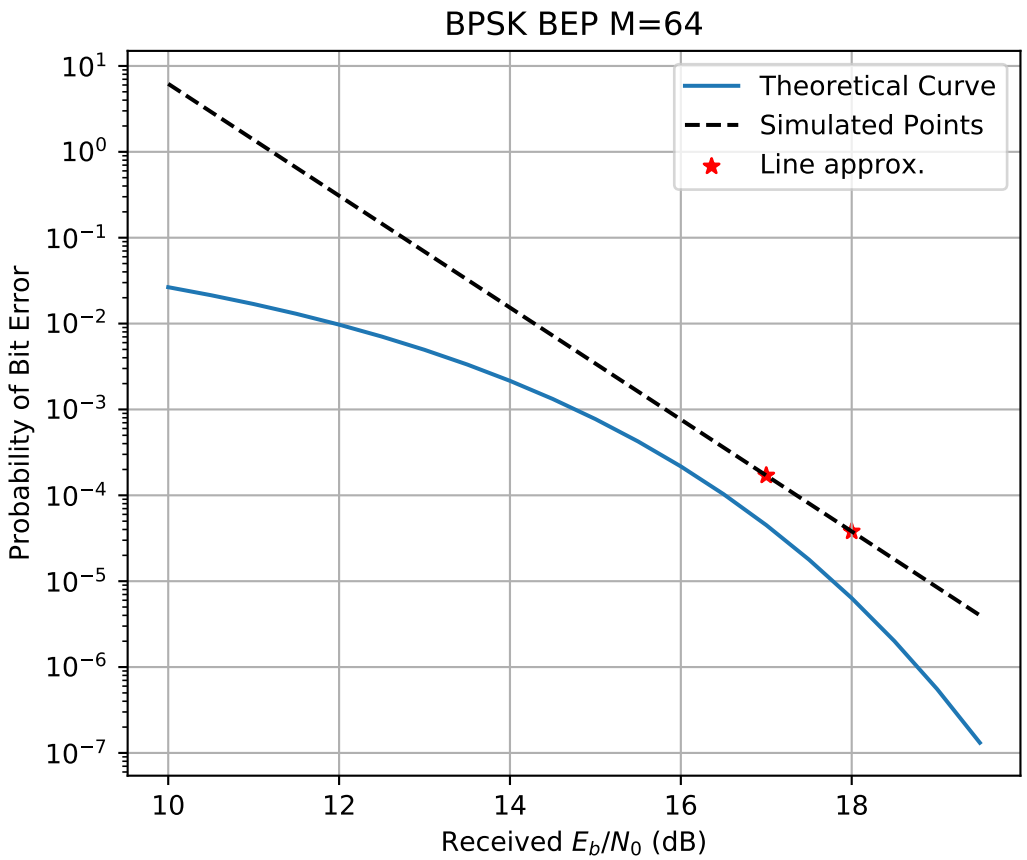
By eyeplot above, optimum is: phase = mod(50,Ns) = 0

```
In [130]: optimum = 0
          phase = optimum + 1 #cuz we're supposed to

          M = 64
          Nsymb = 500000
          Ns = 50
          EbN0_dB_plot = arange(10,20,0.5)
```

```
EbN0_dB = arange(17,19,1)

def task6a(M, Nsymp, Ns, EbN0_dB_plot, EbN0_dB, phase):
    simBEP = list()

    for i in EbN0_dB:
        EsN0_dB = 10*log10(log2(M)) + i
        print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (i,EsN0_dB))
        xbb,b,data = QAM_gray_encode_bb(Nsymb,Ns,M,'src')
        # Enter the comm channel by adding noise
        rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)

        # Enter the receiver by passing through the matched filter
        # Note carrier frequency offset and phase tracking would likely be needed, b
        # both are zero to keep things simple.
        y = signal.lfilter(b,1,rbb)

        # Get down with dat signal
        z = ss.downsample(y,Ns,phase)

        data_hat = QAM_gray_decode(z,M)
        Nbits,Nerrors = dc.BPSK_BEP(data,data_hat)
        print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
         (Nbits, Nerrors, Nerrors/Nbits))
        simBEP.append(Nerrors/Nbits)



    figure(figsize=(6,5))
    legList = list()

    # Plot Theoretical
    Pe_thy_plotM2 = MQAM_BEP(10**(EbN0_dB_plot/10),M)
    plt.semilogy(EbN0_dB_plot,Pe_thy_plotM2)
    scatter
    legList.append(f'M = {M}')

    # Plot simulated
    plt.scatter(EbN0_dB,simBEP,color='r',marker='*')


    # use y=mx+b to get line formula
    m = (log10(simBEP[1])-log10(simBEP[0]))/(EbN0_dB[1]-EbN0_dB[0])
    b = log10(simBEP[0])-m*EbN0_dB[0]
    y = 10**-4
    x = (log10(y)-b)/m
    # get degredation based on first value where theoretical curve crosses 10**-4
    x_theory = EbN0_dB_plot[where(Pe_thy_plotM2 <= 10**-4)]
```

```
            x_theory = x_theory[0]
            print('\nApprox. BEP degredation at BEP=10^-4 : %.4fdB\n'%(x-x_theory))

            x_plt = [min(EbN0_dB_plot), max(EbN0_dB_plot)]
            y_plt = [10**( m*x_plt[0]  + b ),10**( m*x_plt[1] + b )]

            plt.plot(x_plt,y_plt, 'k--')

            ylabel(r'Probability of Bit Error')
            xlabel(r'Received $E_b/N_0$ (dB)')
            title(r'BPSK BEP M=64')
            grid();

            legend(('Theoretical Curve','Simulated Points','Line approx.'), loc='best')

        task6a(M, Nsymb, Ns, EbN0_dB_plot, EbN0_dB, phase)
```

```
Eb/N0 = 17.00 dB and Es/N0 = 24.78 dB
kmax =  0, taumax = 72
BEP: Nbits = 2999928, Nerror = 511, Pe_est = 1.703e-04
Eb/N0 = 18.00 dB and Es/N0 = 25.78 dB
kmax =  0, taumax = 72
BEP: Nbits = 2999928, Nerror = 114, Pe_est = 3.800e-05

Approx. BEP degredation at BEP=10^-4 : 0.3550dB
```



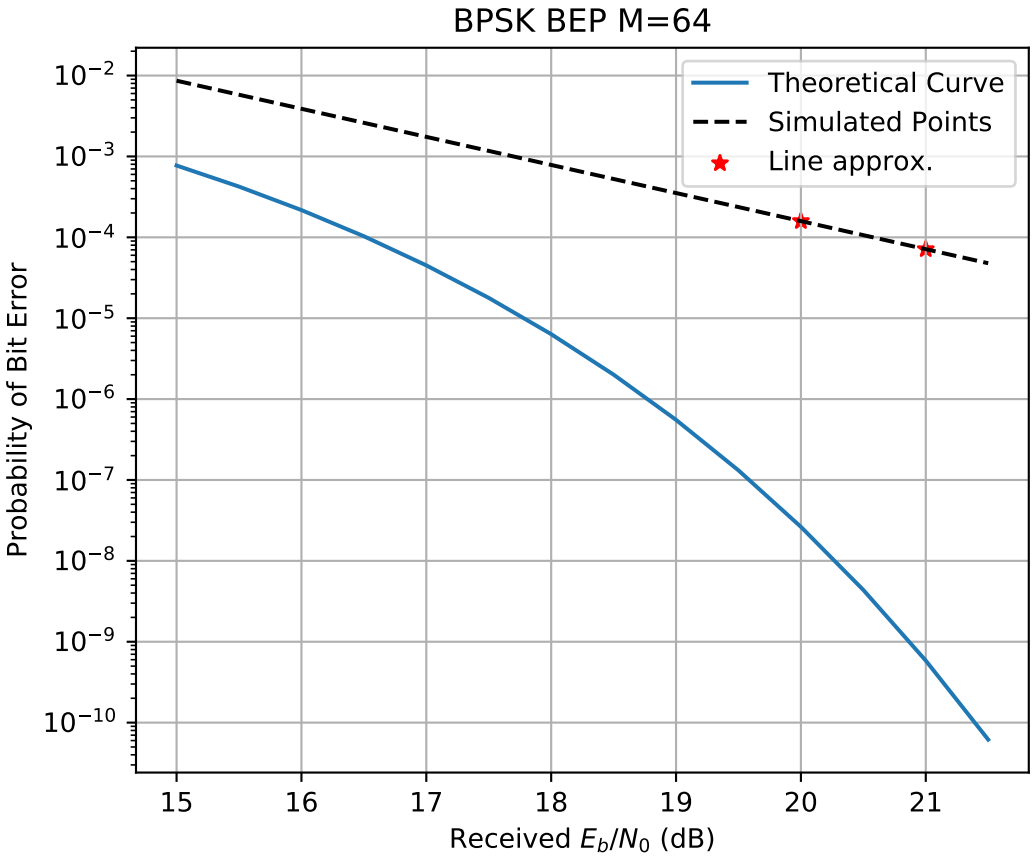
The BEP degredation is relatively low, at roughly 0.4dB

Task 6b

```
In [131]: optimum = 0
          phase = optimum + 2 #cuz we're supposed to
          M = 64
          Nsymb = 500000
          Ns = 50
          EbN0_dB_plot = arange(15,22,0.5)
          EbN0_dB = [20,21]

          task6a(M, Nsymb, Ns, EbN0_dB_plot, EbN0_dB, phase)
```

```
Eb/N0 = 20.00 dB and Es/N0 = 27.78 dB
kmax =  0, taumax = 72
BEP: Nbits = 2999928, Nerror = 476, Pe_est = 1.587e-04
Eb/N0 = 21.00 dB and Es/N0 = 28.78 dB
kmax =  0, taumax = 72
BEP: Nbits = 2999928, Nerror = 214, Pe_est = 7.134e-05
```

```
Approx. BEP degredation at BEP=10^-4 : 3.5775dB
```



The increased timing skew had the effect of increasing the BEP degredation, which is now between 3 and 4 dB.

Task 7

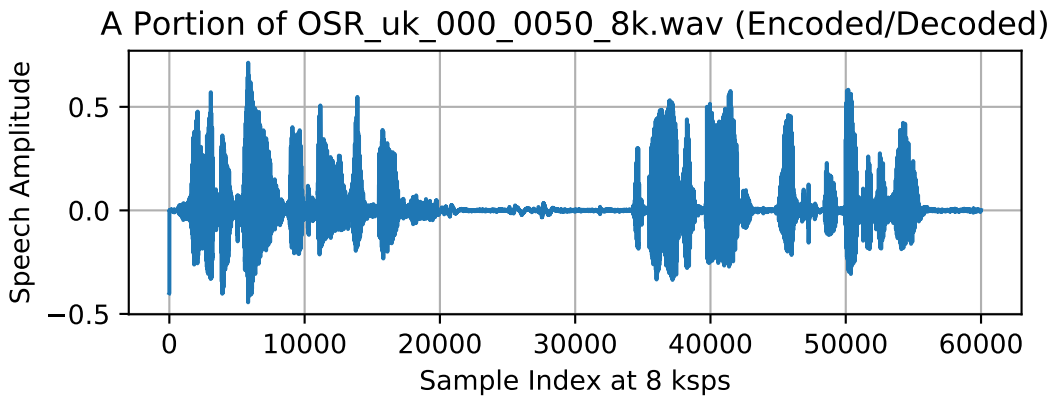
```
In [132]: Nstart = 20000
          N = 60000; # number of speech samples to process
          fs,m1 = ss.from_wav('OSR_uk_000_0050_8k.wav')
          m1 = 2*m1[Nstart:Nstart+N]
          N_PCM = 16
          data_ext = dc.PCM_encode(m1,N_PCM) # Obtain external data bits for QAM_gray_encode
          M = 256
          Ns = 8 # With Ns=10 we have a waveform
          EbN0_dB = 100 # Make the addive noise negligible for now
          EsN0_dB = 10*log10(log2(M)) + EbN0_dB
          print('Eb/N0 = %4.2f dB and Es/N0 = %4.2f dB' % (EbN0_dB,EsN0_dB))
          xbb,b,data_ext_trim = QAM_gray_encode_bb(None,Ns,M,'src',ext_data=data_ext)
```

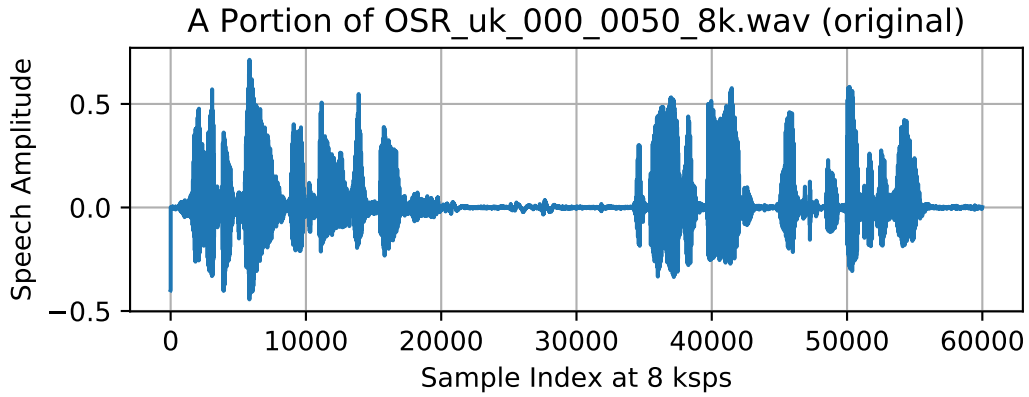
```
        # Enter the comm channel by adding noise
        rbb = dc.cpx_AWGN(xbb,EsN0_dB,Ns)

        # Enter the receiver by passing through the matched filter
        # Note carrier frequency offset and phase tracking would likely be needed, but here
        # both are zero to keep things simple.
        y = signal.lfilter(b,1,rbb)

        # Symbol synchronize manually
        z = ss.downsample(y,Ns,0)
        data_ext_hat = QAM_gray_decode(z,M)
        Nbits,Nerrors = dc.BPSK_BEP(data_ext_trim,data_ext_hat)
        print('BEP: Nbits = %d, Nerror = %d, Pe_est = %1.3e' % \
         (Nbits, Nerrors, Nerrors/Nbits))
```

```
Eb/N0 = 100.00 dB and Es/N0 = 109.03 dB
kmax =  0, taumax = 96
BEP: Nbits = 959904, Nerror = 0, Pe_est = 0.000e+00
```

Determine Offset: - Assume total delay of 12 symbols - For $M = 256 -> $ transmit 8 bits per signal - So Total Bit Delay: $8 * 12 = 96$ - Is this an integer multiple of N_PCM? - $96/16 = 6$, which is an integer - Thus N_offset = $8 * 6$

```
In [133]: N_offset = 8*6
          data_ext_hat_trim = data_ext_hat[N_offset:\
           N_offset+N_PCM*int(len(data_ext_hat)/N_PCM)]
          m1_hat = dc.PCM_decode(data_ext_hat_trim,N_PCM)
          subplot(211)
          plot(m1_hat)
          ylabel(r'Speech Amplitude')
          xlabel(r'Sample Index at 8 ksps')
          title(r'A Portion of OSR_uk_000_0050_8k.wav (Encoded/Decoded)')
          grid()

          figure()
          subplot(211)
          plot(m1_hat)
          ylabel(r'Speech Amplitude')
          xlabel(r'Sample Index at 8 ksps')
          title(r'A Portion of OSR_uk_000_0050_8k.wav (original)')
          grid();
```





The signal graph is identical to the original signal graph, indicating that the decoded speech figure matches the original sound file.

Audio Verification:

```
In [134]: ss.to_wav('Task7.wav',8000,m1_hat)
          Audio('Task7.wav')

Out[134]: <IPython.lib.display.Audio object>
```