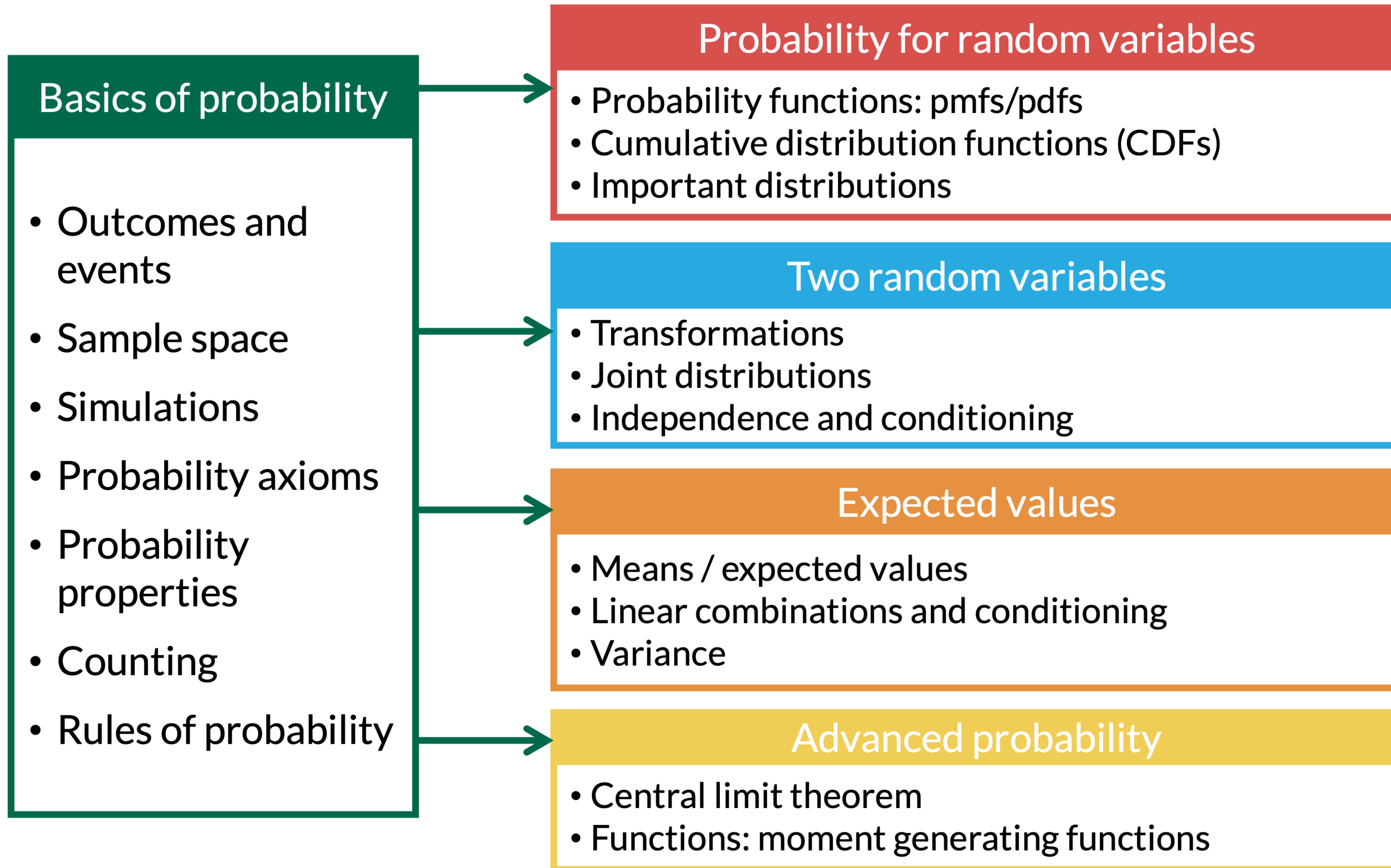# Lesson 6: Calculus Review

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# Learning Objectives

1. Find derivatives of continuous functions with one variable

2. Find antiderivatives and integrals of functions with one variable

# Where are we?

**Basics of probability**

- Outcomes and events
- Sample space
- Simulations
- Probability axioms
- Probability properties
- Counting
- Rules of probability

**Probability for random variables**

- Probability functions: pmfs/pdfs
- Cumulative distribution functions (CDFs)
- Important distributions

**Two random variables**

- Transformations
- Joint distributions
- Independence and conditioning

**Expected values**

- Means / expected values
- Linear combinations and conditioning
- Variance

**Advanced probability**

- Central limit theorem
- Functions: moment generating functions

# Differentiation

# Find the *derivative* of the following function

**Example 1.1**

$$f(x) = 2$$

**Derivative of a constant**

$$\frac{d}{dx}c = 0$$

# Find the *derivative* of the following function

**Example 1.2**

$f(x) = 2x$

# Find the *derivative* of the following function

$$f(x) = 2x + 2$$

# Find the *derivative* of the following function

**Example 1.4**

$$f(x) = x^2$$

**Derivative of $x$ to a constant**

$$\frac{d}{dx}x^n = nx^{n-1}$$

# Find the *derivative* of the following function

$$f(x) = 3\sqrt{x} + \frac{2}{x} + 5$$

# Find the *derivative* of the following function

**Example 1.6**

$$f(x) = e^x$$

**Derivative of exponential function**

$$\frac{d}{dx}e^x = e^x$$

# Find the *derivative* of the following function

**Example 1.7**

$$f(x) = \ln(x)$$

**Derivative of logarithm**

$$\frac{d}{dx}ln(x) = \frac{1}{x}$$

# Find the *derivative* of the following function

**Example 1.8**

$$f(x) = x^2 e^x$$

**Product Rule**

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

# Find the *derivative* of the following function

**Example 1.9**

$$f(x) = \frac{x^5}{2x + 7}$$

**Quotient Rule**

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

# Find the *derivative* of the following function

**Example 1.10**

$$f(x) = e^{-2x+7}$$

**Chain Rule**

$$\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big) g'(x)$$

# Find the *derivative* of the following function

Example 1.11

$$f(x) = \ln(x^2)$$

# Integration

# Find the *antiderivative* of the following function

Example 2.1

$f(x) = 2$

# Find the *antiderivative* of the following function

**Example 2.2**

$$f(x) = x$$

**Integration of x to a constant**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

# Find the *antiderivative* of the following function

$$f(x) = \frac{1}{x}$$

# Find the *antiderivative* of the following function

> **Example 2.4**
>
> $f(x) = x^{3/2}$

# Find the *antiderivative* of the following function

Example 2.5

$$f(x) = e^x$$

# Find the *antiderivative* of the following function

# Find the *antiderivative* of the following function

$$f(x) = e^{-2x}$$

# Solve the following integral

$$\int_0^1 (2x + x^5)dx$$

# Solve the following integral

## Example 3.2

$$\int_2^3 e^{-x}\,dx$$

## U-substitution

$$\int f(g(x))g'(x)\,dx = \int f(u)\,dx$$

# Solve the following integral

$$\int_2^3 xe^{x^2}\,dx$$

# Solve the following integral

## Example 3.4

$$\int_0^\infty xe^{-x}dx$$

## Integrating by Parts

$$\int f(x)g'(x)dx = f(x)g(x)-$$
$$\int f'(x)g(x)dx$$

OR

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$$

# Solve the following integral

$$\int_1^2 x^2 \ln(x) dx$$

# Solve the following integral

$$\int_1^2 \ln(x)dx$$

# Solve the following integral

$$\int_1^2 x^2 e^x \, dx$$