

# Homework 9

BSTA 550

## Directions

**Please turn in this homework on Sakai.** Please submit your homework in pdf or html format.

You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. The Adobe Scan phone app is an easy way to scan photos and compile into a PDF. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

*You must show all of your work to receive credit.*

Extra problems do not need to be turned in!

## Questions

1. Let  $R_X(t) = \ln(M_X(t))$ . Show that  $\text{Var}(X) = R_X''(0)$ .
2. The mgf for a Gamma distribution is  $M_X(t) = \frac{1}{(1-t/\lambda)^r}$ . Use the mgf of an Exponential distribution (from #43.9), to show that the sum of  $n$  i.i.d.  $\text{Exponential}(\lambda)$  random variables has a  $\text{Gamma}(r, \lambda)$  distribution.

## Extra problems

3. Using mgf's, show that the sum of  $n$  i.i.d. Chi Square random variables with one degree of freedom ( $\chi_{(1)}^2$ ) r.v.'s has a Chi Square with  $n$  degrees of freedom ( $\chi_{(n)}^2$ ) distribution.

*Hint:* First, look up the pdf of a  $\chi_{(n)}^2$ . This is a special case of the Gamma distribution with what parameters? Based on that and the information from # 2 above, you can determine what the mgf of a  $\chi_{(n)}^2$  is, which will help you determine whether the mgf of the sum of  $n$  i.i.d.  $\chi_{(1)}^2$  r.v.'s has a  $\chi_{(n)}^2$  distribution.

4. Use the mgf of a Poisson distribution to find the mgf of the following distributions. If the mgf is that of a common named distribution, then name the distribution and state its parameter(s).
1. The distribution of  $\sum_{i=1}^n X_i$ , if  $X_i \sim \text{Poisson}(\lambda_i)$  and are independent.
  2. The distribution of  $\sum_{i=1}^3 X_i$ , if  $X_i \sim \text{Poisson}(\lambda)$  and are independent (i.i.d. in this case).
  3. The distribution of  $3X$ , if  $X \sim \text{Poisson}(\lambda)$ .
  4. Why are the answers to (b) and (c) different?