Homework 4

BSTA 550

Directions

Please turn in this homework on Sakai. Please submit your homework in pdf or html format.

You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. The Adobe Scan phone app is an easy way to scan photos and compile into a PDF. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

You must show all of your work to receive credit.

Questions

1. **Mystery constant.** Suppose X is a discrete random variable with a probability mass function $p_X(x) = c(4 - x)$ for x in $\{-1, 2, 3\}$ and $p_X(x) = 0$ otherwise.

    a. What is the value of c so that $p_X(x)$ is a mass?
    b. Make a plot of the probability mass function and write the piecewise pmf
    c. What is the CDF of X? Write the piecewise CDF.
    d. Make a plot of the CDF.

2. **Wastebasket basketball.** Chris tries to throw a ball of paper in the wastebasket behind his back (without looking). He estimates that his chance of success each time, regardless of the outcome of the other attempts, is 1/3. Let X be the number of attempts required. If he is not successful within the first 5 attempts, then he quits, and he lets $X = 6$ in such a case.

    a. Draw the mass of X and define the piecewise pmf.
    b. Draw the CDF of X and define the piecewise CDF.
    c. Simulate 10,000 trials of this experiment in R and plot the approximate probability distribution.

3. Suppose a density $f_X(x)$ increases linearly from $(16, 0)$ to $\left(24, \frac{1}{4}\right)$.

   a. Find the CDF of X.
   b. What is the value of a so that $P(X > a) = 0.75$?

4. For the following pdf

$$f_X(x) = \begin{cases} kx^9(1-x)^2 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   a. What is the constant k that makes the following function a valid density?
   b. What is the cumulative distribution function (cdf) $F_X(x)$?