Homework 7
BSTA 550

Directions

Please turn in this homework on Sakai. Please submit your homework in pdf or html format.

You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. The Adobe Scan phone app is an easy way to scan photos and compile into a PDF. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

You must show all of your work to receive credit.

Extra problems do not need to be turned in!

Questions

1. Let $X_i \sim \text{Binomial}(n_i, p)$ be independent r.v.'s for $i = 1, \dots, m$.

   a. What does the r.v. $X = \sum_{i=1}^{m} X_i$ count, and what is the distribution of X? Make sure to specify the parameters of X's distribution.

   b. Find $\mathbb{E}[X]$. *Make sure to show your work for (b) and (c). However, you may use without proof what you know about the mean and variance of each X_i.*

   c. Find $Var[X]$.

2. Approximately 10% of U.S. Veterans are women. Suppose an investigator plans a study with 4500 participants that are Veterans. How many women can they expect to be included? *Your answer must be calculated by defining a random variable and showing how to calculate the expected value.*

3. Prove that for a r.v. X and constants a and b, that

$$\text{Var}[aX + b] = a^2 \text{Var}[X].$$

4. Let $\overline{X}$ be the random variable for the sample mean, $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$, where the X_i are i.i.d. random variables with common mean μ and variance σ^2.

    a. Find $\mathbb{E}[\overline{X}]$.

    b. Find $Var[\overline{X}]$.

5. Let $f_X(x) = \lambda e^{-\lambda x}$ for $x > 0$, where $\lambda > 0$.

    a. Show $Var[X] = \frac{1}{\lambda^2}$. You may use the result from class for $\mathbb{E}[X]$ without first proving it.

6. A shipping company handles containers in three different sizes: (1) 27 ft^3 (3 x 3 x 3), (2) 125 ft^3, and (3) 512 ft^3. Let X_i ($i = 1, 2, 3$) denote the number of type i containers shipped during a given week. Suppose that $\mu_1 = 200, \sigma_1 = 10, \mu_2 = 250, \sigma_2 = 12, \mu_3 = 100, \sigma_3 = 8$.

    a. Assuming that X_1, X_2, X_3 are independent, calculate the expected value and variance of the total volume shipped.

    b. Would your calculations necessarily be correct if the X_i's were not independent? Explain.

7. The automatic opening device of a military cargo parachute has been designed to open when the parachute is 200 m above the ground. Suppose opening altitude actually has a normal distribution with mean value 200 m and standard deviation 30 m. Equipment damage will occur if the parachute opens at an altitude of less than 100 m. What is the probability that there is equipment damage to the payload of at least one of the five independently dropped parachutes?

Extra problems

1. There is a bowl containing 30 cashews, 20 pecans, 25 almonds, and 25 walnuts. I am going to randomly pick and eat 3 nuts (without replacement). Find the expected value of the number of cashews by defining the number of cashews as a sum of random variables. (This one takes a little while if we don't rely on the

2. Let $\hat{p}$ be the random variable for the sample proportion, $\hat{p} = \frac{X}{n}$, where X is the number of successes in a random sample of size n. Assume the probability of success is p.

    a. Find $\mathbb{E}[\hat{p}]$.

    b. Find $Var[\hat{p}]$.