Homework 10 (Optional)

BSTA 550

Directions

Please turn in this homework on Sakai. Please submit your homework in pdf format. You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. The Adobe Scan phone app is an easy way to scan photos and compile into a PDF. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

Try to complete all of the problems listed below at some point this quarter! You may want to save some of them for studying later! Only turn in the ones listed in the "Turn In" column. Please submit problems in the order they are listed.

You must show all of your work to receive credit.

Chapter	Turn In	Extra Problems
37	TB # 24, 30	TB # 2, 4, 13, 20, 29
43	TB #9*, 10**, 11, 12**, NTB # 1, 2, 4	TB # 1-4, NTB # 3

* Include in your answer an explanation as to why we need the condition that $t < \lambda$.

** Do parts (a)-(c) below for #10 and #12:

- Answer the question using the mgf $M_X(t)$ as instructed in the book.
- Answer the question using $R_X(t)$ (as defined in class, and NTB [Ch43_R_Var] below).
- Which method did you prefer? Why?

Non-textbook problems (NTB)

1. Let $R_X(t) = \ln(M_X(t))$. Show that $\text{Var}(X) = R''_X(0)$.
2. The mgf for a Gamma distribution is $M_X(t) = \frac{1}{(1-t/\lambda)^r}$. Use the mgf of an Exponential distribution (from #43.9), to show that the sum of n i.i.d. Exponential(λ) random variables has a Gamma(r, λ) distribution.
3. Use the mgf of a Poisson distribution to find the mgf of the following distributions. If the mgf is that of a common named distribution, then name the distribution and state its parameter(s).
    1. The distribution of $\sum_{i=1}^{n} X_i$, if $X_i \sim$Poisson(λ_i) and are independent.
    2. The distribution of $\sum_{i=1}^{3} X_i$, if $X_i \sim$Poisson(λ) and are independent (i.i.d. in this case).
    3. The distribution of $3X$, if $X \sim$Poisson(λ).
    4. Why are the answers to (b) and (c) different?
4. Using mgf's, show that the sum of n i.i.d. Chi Square random variables with one degree of freedom ($\chi^2_{(1)}$) r.v.'s has a Chi Square with n degrees of freedom ($\chi^2_{(n)}$) distribution.

    Hint: First, look up the pdf of a $\chi^2_{(n)}$. This is a special case of the Gamma distribution with what parameters? Based on that and the information from # [Ch43_SumExpGamma] above, you can determine what the mgf of a $\chi^2_{(n)}$ is, which will help you determine whether the mgf of the sum of n i.i.d. $\chi^2_{(1)}$ r.v.'s has a $\chi^2_{(n)}$ distribution.