

MATH 420

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Team HW 1

```
In [1]: using Pkg
Pkg.activate("../p2")
Pkg.instantiate()
```

Activating project at `~/MATH420/p2`

```
In [22]: using DelimitedFiles
using LinearAlgebra
using Plots
using LaTeXStrings
using Printf

for name = ("dist", "exactdist")
    (R, num_vert_) = readdlm("../kn57Nodes1to57_" * name * ".txt", Float64, h
    num_vert = parse(Float64, num_vert_[1])
    # S = zeros(size(R))
    # rho = 0;
    # for i = 1:57, j = 1:57
    #     global S, R, rho
    #     S[i, j] = R[i, j]^2
    #     rho += S[i, j]
    # end
    S = R .^ 2
    rho = norm(S, 1)
    rho \= (2 * num_vert)
    one_col = ones(57, 1)
    v = (S - rho * I) * one_col / num_vert
    function getGram(n::Real, S::Matrix, rho::Real)::Matrix
        r = 1 / 2n * (S - rho * I) * one_col * one_col' + 1 / 2n * one_col *
        @assert issymmetric(r)
        return r
    end
    G = getGram(num_vert, S, rho)
    ev, Q = eigen(G, sortby=x -> -x)
    index = findfirst(x -> x < 0, ev)
    ev[index-1:end] .= 0
    display(scatter(ev, labels = nothing, title="Eigenvalues for G for $(na
    print("The 10 largest eigenvalues are $(first(ev, 10))")
    Λ = Diagonal(ev)
    lambda = Λ
    Q[:, 1:2]

    Y::Dict{Int,Matrix} = Dict{Int,Matrix}()
    R_hat::Dict{Int,Matrix} = Dict{Int,Matrix}()
```

```

R_norm::Dict{Int,Real} = Dict{ }
e::Dict{Int,Real} = Dict{ }
σ::Dict{Int,Real} = Dict{ }

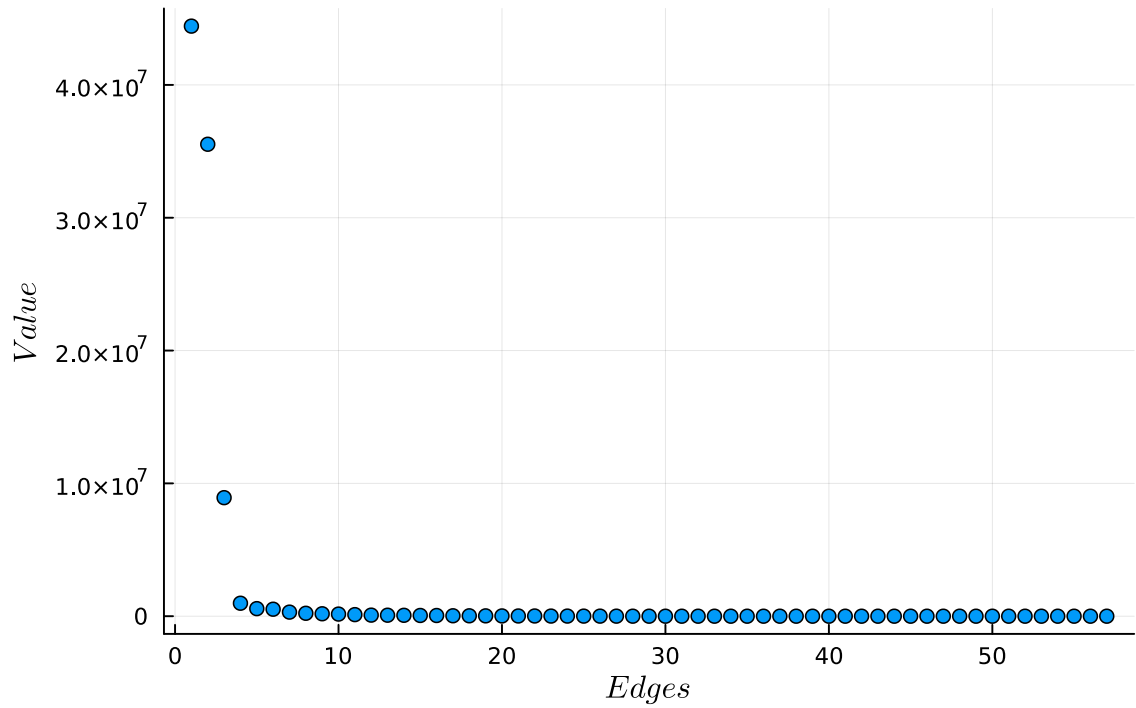
println("For data " * name)
for d = (2, 3)
    println("d = $(d)")
    Q_1 = Q[:, 1:d]
    Λ_1 = Λ[1:d, 1:d]
    Y[d] = Λ_1^1 / 2 * Q_1'
    R_hat[d] = zeros(57, 57)
    for i = 1:57, j = 1:57
        R_hat[d][i, j] = norm(Y[d][1:d, j] - Y[d][1:d, i])
    end
    R_norm[d] = norm(R - R_hat[d])
    println("Norm = $(R_norm[d])")
    e[d] = norm(G - Y[d]'Y[d])
    println("e = $(e[d])")
    σ[d] = sqrt(sum(ev[d+1:end] .^ 2))
    println("σ = $(σ[d])")
    println("Difference = $(e[d] - σ[d])")
end

function minmax(v::Vector{<:Real})
    r1 = [minimum(v), minimum(v)+(maximum(v)-minimum(v))/2, maximum(v)]
    r2 = [ @sprintf("%.1f",x) for x in r1 ]
    return (r1, r2)
end

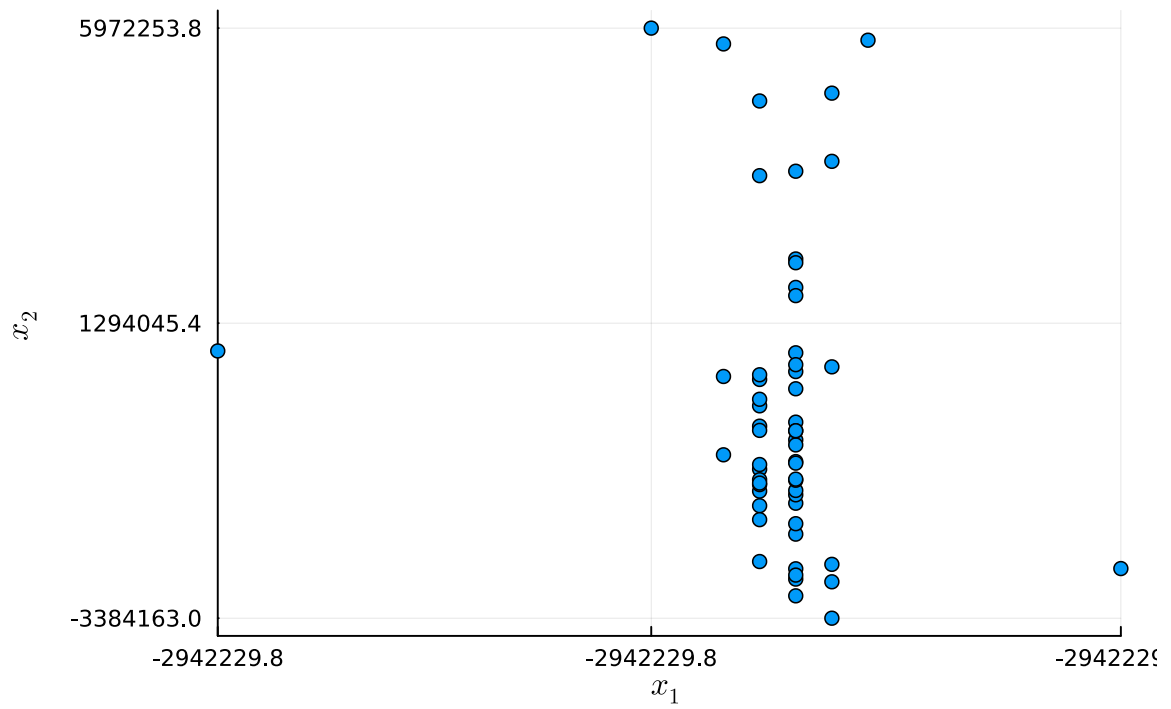
display(scatter(Y[2][1, :], Y[2][2, :], labels=nothing, title="Two dimensions",
    yticks = minmax(Y[2][2, :]), xaxis = L"x_{1}", yaxis=L"x_{2}"))
display(scatter3d(Y[3][1, :], Y[3][2, :], Y[3][3, :], title="Three dimensions",
    yticks = minmax(Y[3][2, :]), zticks = minmax(Y[3][3, :]), labels=nothing,
    yaxis=L"x_{2}", zaxis=L"x_{3}"))
# scatter3d(Y[3][1, :], Y[3][2, :], Y[3][3, :], camera=[0,0,0])
end

```

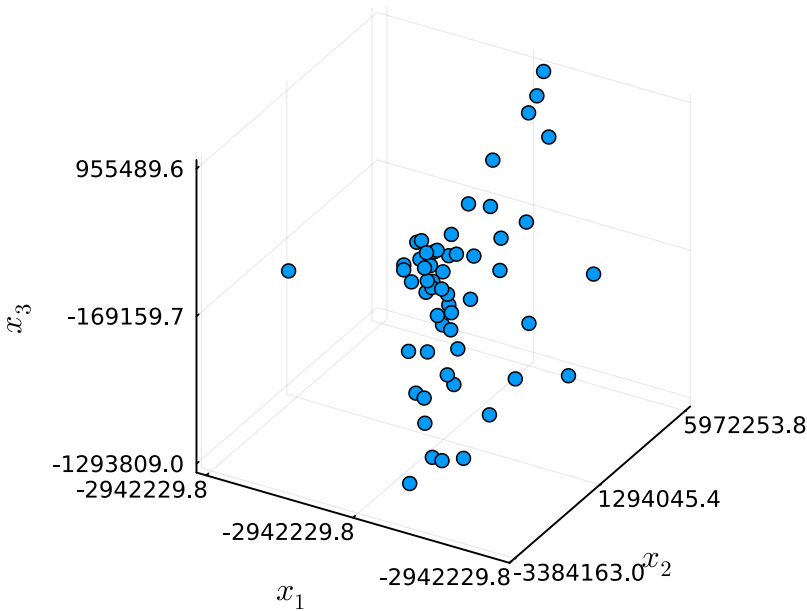
Eigenvalues for G for dist



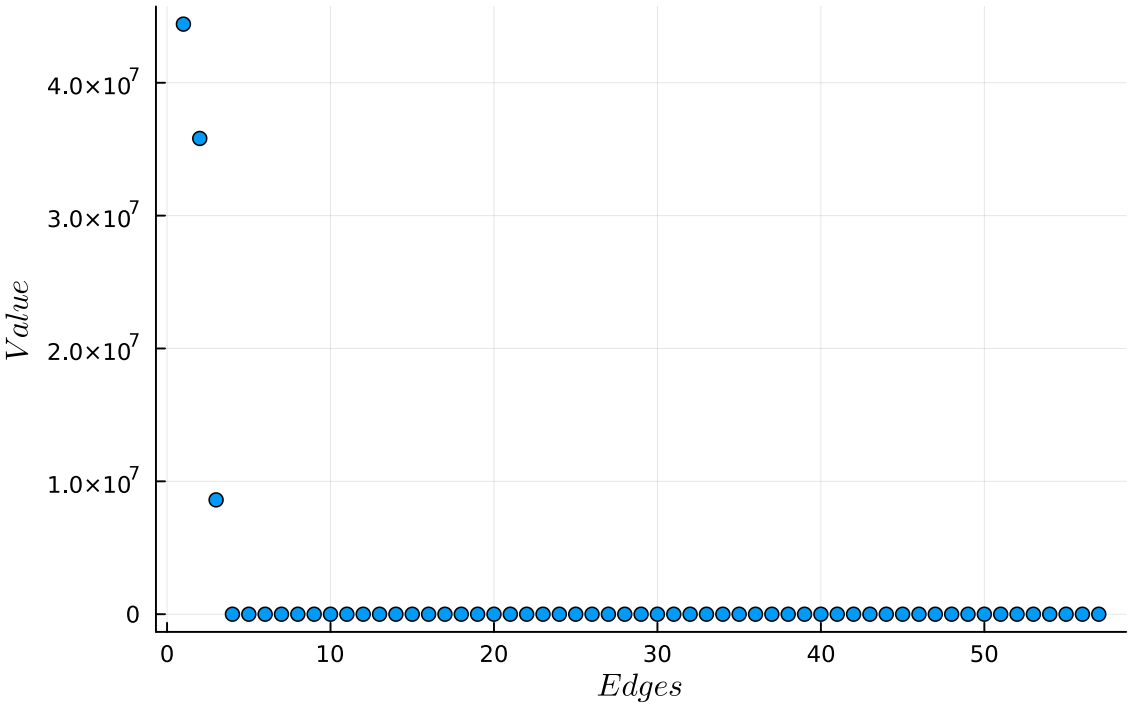
Two dimensional Embedding for dist



Three dimensional Embedding for dist



Eigenvalues for G for exactdist



The 10 largest eigenvalues are [4.4426695982456006e7, 3.553255787649086e7, 8.924037549458053e6, 975952.6526310504, 570066.7976405374, 526331.6068667904, 307163.98573833145, 220021.71828191355, 183279.62093742122, 159784.48540360644]For data dist

d = 2

Norm = 1.896225110335917e8

ϵ = 5.85751586682671e14

σ = 9.024155748387313e6

Difference = 5.857515776585152e14

d = 3

Norm = 1.9551248363265747e8

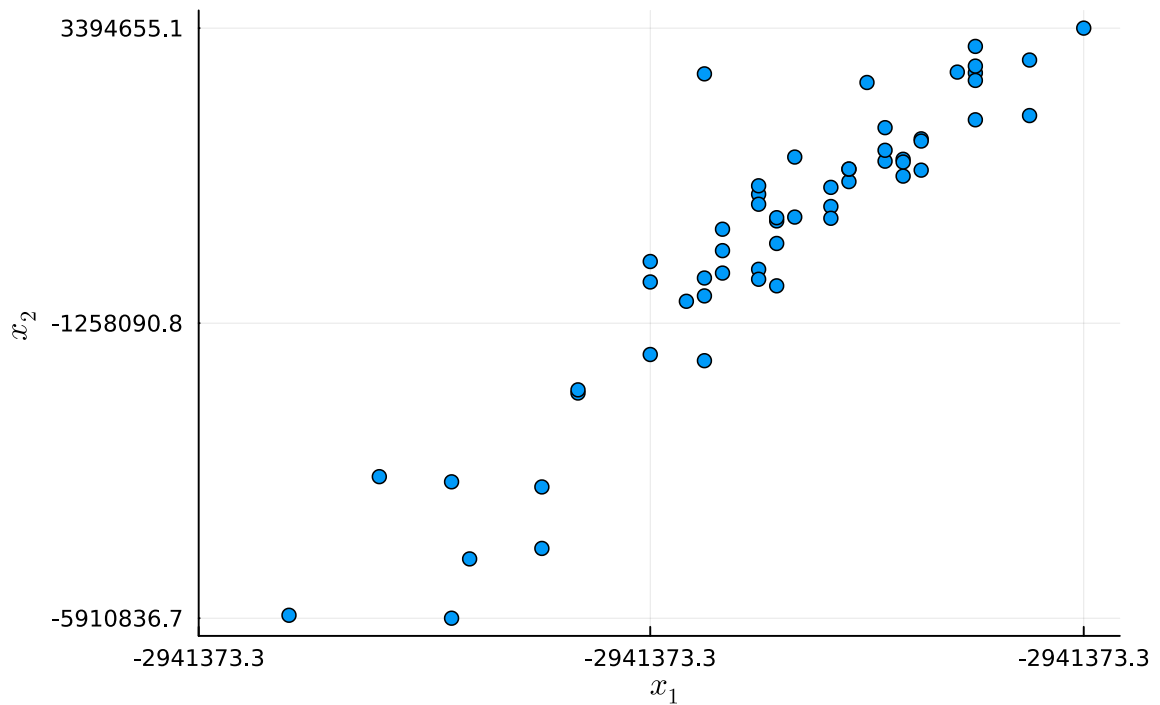
ϵ = 5.860898511131469e14

σ = 1.3405002003037904e6

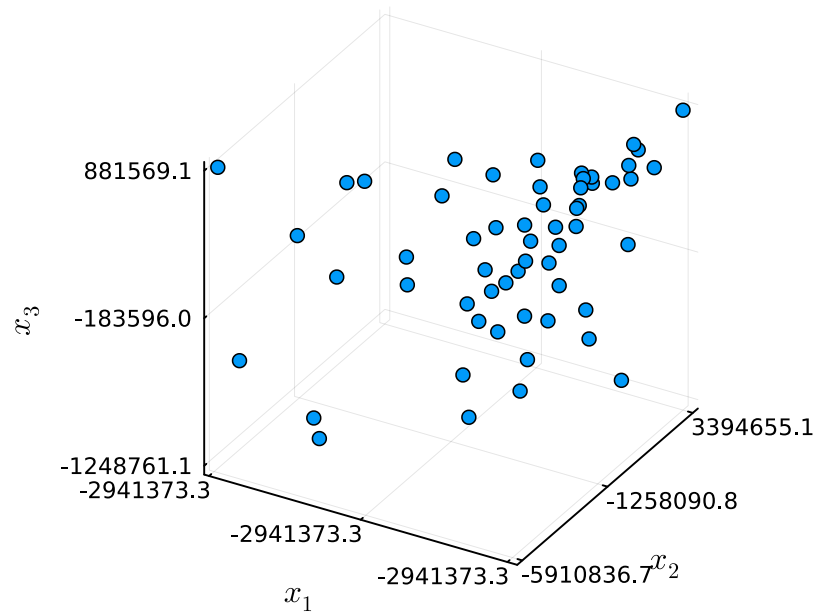
Difference = 5.860898497726466e14

The 10 largest eigenvalues are [4.4413762881676525e7, 3.580839780809973e7, 8.605365074929608e6, 0.005358653489918885, 0.004923177616982107, 0.004657428327942904, 0.003988382612363017, 0.0036938404803705353, 0.003392009912311986, 0.00270776956324139]

Two dimensional Embedding for exactdist



Three dimensional Embedding for exactdist



```
For data exactdist
d = 2
Norm = 1.9109507984747455e8
 $\epsilon$  = 5.881763597301269e14
 $\sigma$  = 8.605365074929608e6
Difference = 5.881763511247618e14
d = 3
Norm = 1.965366018568468e8
 $\epsilon$  = 5.884676404420568e14
 $\sigma$  = 0.012847672061037731
Difference = 5.884676404420568e14
```

In []: