Abstract

I present an agent-based model, inspired by the opinion dynamics (OD) literature, to explore the underlying behaviors that may induce societal polarization. My agents interact on a social network, in which adjacent nodes can influence each other, and each agent holds an array of continuous opinion values (on a 0-1 scale) on a number of separate issues. I use three measures as a proxy for the virtual society's "polarization:" the average assortativity of the graph with respect to the agents' opinions, the number of non-uniform issues, and the number of distinct opinion buckets in which agents have the same opinions after the model reaches an equilibrium.

I look at multiple model parameters that affect polarization. The first is the density of edges in the network: this corresponds to the average number of meaningful social connections that agents in a society have. First, I find that lower edge density results in higher levels of assortativity for Erdös-Rényi graphs. The second is the level of "openness" and "disgust" of agents to differing opinions; *i.e.*, how close or distant a neighboring node's opinion on an issue must be to an agent's own before the agent will adjust its opinion on a different issue. I refer to this novel mechanism as cross-issue influence. Through this mechanism, I find that when agents in the model are less open to new opinions, there will be less consensus on any given issue for all agents in the model. Additionally, I find that there will be fewer distinct opinion buckets and therefore higher polarization in models where agents follow a cross-issue influence mechanism compared to same-issue influence.

Exploring the Impact of Social Influence Mechanisms and Network Density on Societal Polarization

December 9, 2021

1 Introduction

The recent events that transpired at the U.S. Capitol on January 6th were a vivid reminder of the deep divide within the nation. There are signs that the United States is experiencing political polarization now like it has never seen before. As individuals stormed the Capitol, Americans watched in horror. Although this singular event is now in the past, the underlying tension that preceded it still remains.

Polarization – reflected in echo chambers, entrenched views, and the vilification of those whose opinion differs – can be harmful to a democratic society. It can inhibit the reaching of consensus and compromise upon which a democracy is built, and can result in even greater amounts of damage than what ensued in the U.S. on January 6th if left unchecked. Further, polarization affects not only political actors, but also the interpersonal relationships among the rank and file citizens of a country which bolster and strengthen society.

In my research, I look at multiple societal variables that I believe may significantly impact polarization in a society. The first is the density of social connections: in other words, the average number of social ties a member of that society has. The second is the degree of "openness" in the society: namely, how willing its members are to consider changing their views. The third is the degree of "disgust" in the society: namely, how easily its members are disgusted by opposing views. In addition, I look at two different influence

mechanisms. I suspect that these factors play a role in determining the aggregate polarization of a society.

In order to explore these phenomena, I created an Agent-Based Model (ABM) of heterogeneous agents in the spirit of much of the Opinion Dynamics (OD) literature. These agents interact with each other on a random, static social network and change their opinions on issues over time based on the opinions of their network neighbors. One novel feature of my model, termed cross-issue influence, is the way agents influence one another: one agent will not allow another agent to influence its opinion on an issue (the influence issue) unless the two agents already have sufficient agreement on another randomly chosen issue (the comparison issue). Additionally, the agent may potentially be repelled away from its neighbor's opinion on the influence issue if the difference in their opinions is far enough away on the comparison issue. The justification for this is related to the well-known observation of "homophily" in social psychology: people are prone to trust those who already agree with them on something, and hence are more likely to be persuaded by them on other matters.

The goal of my research is to determine what micro behaviors of individuals are sufficient to produce a change in the degree of political polarization in the society. As explained below, I measure polarization in three different ways: the average similarity of an agent to its neighbors (called "assortativity" in social network terminology), the number of non-uniform issues, and the number of distinct opinion buckets in the society at equilibrium.

2   Topic Introduction

2.1   Modeling and Simulation

First, modeling and simulation is the large field that my research falls under. Modeling and simulation is a rapidly expanding field where the goal is to model an environment to explore phenomena or effects that occur (or will occur) in the real-world. Models can be made to explain or predict real-world behavior.

In the field of modeling and simulation, professionals in many disciplines have used methods to simulate complex real-world systems. For example, many people benefit from these types of simulations every day when they check weather applications. Weather apps take in large amounts of data

from live feeds in the environment. Then, researchers use the data to create a simulated environment where they can observe and predict future weather outcomes.

Another area where this type of model is used is in the predictions of stock prices. Economists are able to input large amounts of data into a model which can predict the direction of stock prices. These types of simulations that are created to be as close to real-world systems as possible are called facsimiles. They are extremely complex and require large amounts of data to retain their predictive power. The models that I wish to do research on are different from these complex and costly models.

My goal is not to create a one-to-one model of the United States in 2010 and see if political polarization develops by 2021. This would not be feasible as I would need to model the United States in 2010 which is an impossible task. Instead, I create a simulation where agents (who represent people) interact on a social network and slowly change the views of those around them in what is called an **Agent-Based Model**.

Furthermore, I am modeling polarization in a society to discover insights about societies more generally rather than trying to predict a specific social outcome in the United States. With this in mind, I make numerous modeling decisions to model the society more generally such as having abstract issues in that model that do not represent any specific issues in today's political climate. Because my model has abstract issues, I am able to make statements about societies more generally rather than my conclusions depending on the semantics behind the issues in the model.

2.2   Modeling Techniques

Within the field of modeling and simulation, there are many techniques that have been used to model complex systems. One such technique is discrete event simulation (or DES). In DES, states are modeled as atomic representations of the environment. For example, queuing theory is often represented with DES. Using DES, we gain valuable insights such as predictions on the average length of a queue and the potential longest wait time. Another modeling technique is system dynamics (or SD). SD is a technique that involves using differential equations to model a system. This technique is widely used to model continuous relationships in studying things like epidemiology, population growth, industrial economics, and economic policy.

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2.3 Agent-Based Modeling

Although DES and SD are useful modeling techniques, they are not applicable for every complex system. These traditional modeling techniques are useful in the aggregate; however, they often fail to capture the impact of agent heterogeneity. For example, macroeconomic models assume that all agents are homogeneous which allows for a simpler model. Such models allow for conclusions about the equilibrium conditions of an economy. Although these equilibrium conclusions are helpful for policy decisions, homogeneous agents limit the complexity and authenticity of a simulation of human behavior because real people are neither carbon copies of one another, nor totally deterministic in their relationships and choices. Given my research topic of political polarization, a DES or SD model would not be the right approach. The method that I use is **agent-based modeling** which is commonly called an ABM for agent-based model. ABMs allow agents to have their own behavior according to characteristics that are unique to that particular agent which is extremely beneficial in simulating a social network.

Agent-Based modeling is a technique that is growing in popularity. Historically, agent-based modeling has not been a widely used approach in the field of modeling and simulation due to computing power restrictions. Large agent-based models generally suffer from the issues of high space and time complexity. In the past, agent-based models were even simulated by researchers calculating mathematical results by hand without the aid of computer programs. With 21st century technology, we are able to simulate large systems using ABMs of heterogeneous agents.

Some common uses of ABMs are in the study of social and cultural phenomena in economics, demography, and sociology. Additionally, ABMs are used to study natural phenomenon in fields like biology and epidemiology. With the Covid-19 pandemic, the popularity of ABMs have soared due to their ability to model disease transmission through a population where individuals respond differently to policy decisions like lock downs and mask mandates and to health offerings like vaccines.

2.3.1 The Goals of Agent-Based Models

Using an ABM, my aim is "...to 'grow' certain social structures in the computer...the aim being to discover fundamental local or micro mechanisms that are sufficient to *generate* the macroscopic social structures and collec-

tive behaviors of interest"[1]. I attempt to discover the micro mechanisms that are sufficient to produce the social and cultural phenomenon of political polarization.

A key idea of agent-based modeling defined by [1] is that "certain sets of microspecifications are *sufficient to generate* the macrophenomena of interest". The key here being the phrase "sufficient to generate". Undoubtedly, humans are complicated and complex entities, so creating a model of human behavior is a difficult task. To be clear, I am not trying to replicate every aspect of life that shapes an individual's opinion formation. Instead, I create a simple model of the mechanisms that are sufficient to generate the phenomenon of interest which is polarization. By creating a simple model that is able to generate polarization, I can discover the low-level mechanisms that are adequate in producing the phenomenon. As such, I avoid adding unnecessary complexity into my model which also aids in the implementation of the model.

2.3.2 Implementing the Agent-Based Model

In designing my model, I pull from other disciplines such as sociology and psychology for theories of human behavior that inspire the influence mechanisms of my agents. These influence mechanisms will be implemented on the micro level such that agents follow behavior specified by me when I initialize the model. I also use various computer science and statistics techniques for implementation and analysis of the macro level trends that occur in the model.

In addition to the large benefit of being able to explore macro level trends accurately, ABMs are also a practical approach to modeling due to the simple nature of their code implementation. Those that are familiar with the modern object-oriented programming approach (or OOP) may already see the connection between ABMs and OOP. In OOP, objects have instance variables and functions that are unique to that class. Objects are often contained in data structures (which may even be an instance variable of another class).

Using OOP, I can create an agent class. The agent class may contain instance variables that are useful in modeling such as the age, wealth, location, and neighbors (other connections in the social network) of that particular agent object. These agent objects represent entities that exist within the physical or abstract environment of the model. In my case, the agents will represent people, but its possible for agents to be animals, companies, or

even simple biological organisms. Oftentimes, an ABM also has a model class that follows the singleton design pattern. This model class generally contains many agents in a data structure. The encapsulation in OOP greatly aids in the development of a large ABM. With the encapsulation of agent objects in a model object, I am able to create model-level functions that provide high-level insights into the behavior of the agent objects. Using an ABM, I will study the **opinion dynamics** of an artificial society that interacts on a social network.

2.4 Opinion Dynamics

Opinion dynamics is the study of how opinions spread throughout a society. Agent-based models are especially useful for studying opinion dynamics because researchers are able to design agents and tailor their interactions. Researchers are able to test the consequences on a society of specific agent behaviors by studying how opinions flow from agent to agent. In my case, I will be designing agents to follow two different influence mechanisms. Then, I will observe how opinions flow throughout the society and the polarization that develops as a result of each respective influence mechanism.

2.5 Modeling Terminology

Parameter Suite and **Parameter Sweep**. To get robust results when investigating my hypothesis, I ran parameter suites which are batches of models that are run with the same values for each parameter in the model. With results from a parameter suite, I am able to minimize the impact of randomness and outliers which helps me determine the average result of the model with a certain set of parameters. Another useful modeling technique is the parameter sweep. Parameter sweeps allow me to vary one (or multiple) parameter(s) of the model to see how the output of the simulation varies with the parameter. Parameter sweeps consist of many parameter suites. It is helpful to think of parameter suites like playing a game against an opponent X times to find the true outcome of the game. Parameter sweeps are like playing against every opponent X times to capture the full impact of varying the opponent on your average outcome when playing the game.

Social Network. One term that I will use to describe the entirety of my model is a social network. In this instance, I do not mean a social

network like Facebook or Twitter, but rather, a social network is a group of collected nodes linked by edges that influence the opinions of one another. In my model, the nodes represent agents. Additionally, in my model the edges will be undirected. It is possible and even common for ABMs to have directed edges. In models where researchers are trying to simulate a follower-relationship like Twitter, edges are often directed. However, in my model, edges are meant to represent friendships that are mutual between two agents. If you are "friends" with another agent, you can influence them, and they can influence you. As a result, agents can influence and be influenced by their neighbors (nodes they are directly connected to on the graph).

2.6   Technologies Used

To conduct my research I used the Python programming language due to the wide array of libraries and packages that are available.

One important package that I used was Mesa. Mesa is an agent-based modeling framework for Python that allows for flexibility in modeling decisions while still providing boilerplate code and a pre-built scheduler for agents. Additionally, Mesa provides a datacollector object that allowed me to analyze the model down to the agent level and more broadly at the model level.

In addition to Mesa, I used NetworkX which is a package that allows for easy creation, manipulation, and analysis of the structure and dynamics of a graph. With NetworkX, I was able to generate different random connected graphs by using different graph generation algorithms. I used NetworkX to generate Erdös-Rényi graphs, Watts-Strogatz graphs (also known as small-world networks), and Barabási-Albert graphs (also known as preferential-attachment networks). For the purposes of this paper, I will only be presenting results that were found on Erdös-Rényi graphs.

Another package that I considered using for clustering in the model is AutoGMM. AutoGMM offers an algorithm for automatic Gaussian mixture modeling that I was contemplating using for the clustering across issues. However, estimating the optimal model and number of clusters is an NP-Hard problem, so the time complexity of using the package prohibited me from implementing it.

Other packages used were Numpy and Scipy for mathematical operations and Pandas for data analysis.

3 Related work

3.1 Opinion Dynamics

Opinion Dynamics models seek to reproduce the phenomenon of individual agents forming opinions over time via mutual influence. They allow the researcher to explore the macro-level patterns that may arise in a society from a set of simple influence rules defined on the micro-level. For instance, the Binary Voter Model (BVM), the original and perhaps most influential OD model ([2, 3]), features a set of interacting agents, each of which holds a binary opinion. The single influence rule is that agents periodically change their opinion to match one of their influencers, chosen at random. My model takes inspiration from this mechanism in that for each simulated time step of the model, every agent will take turns receiving influence from one randomly chosen neighboring agent.

Many researchers (*e.g.*, [4, 5]) have expanded this idea to model continuous, rather than discrete, opinions: these are typically expressed as real numbers between 0 and 1. I will also take inspiration from these models to include continuous opinions in my model. In addition to better capturing the nuance of real-life viewpoints (which are not usually completely black or white on any issue), continuous opinions lead naturally to incorporating a form of **homophily**[6] into the model: agents will only choose to be influenced by agents whose existing opinion is already close to their own. Termed "**bounded confidence**" (BC) by [7], this feature can result in non-convergence to uniformity depending on the value of the threshold agents use to gate influence.[7, 8, 9] The term "**clustering**" (or "opinion clustering") has been used to describe the resulting equilibrium reached by such models, in which subsets of the agents each converge on a different opinion value and are henceforth no longer persuadable by other agents.

A smaller number of studies have considered "**multidimensional opinions**," in which each agent maintains a separate opinion on each of several different "issues" rather than on just one.[1] The opinions in a multidimen-

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[1]This is to be carefully distinguished from "**opinion vectors**," which represent an agent's degree of support for each of several alternatives on the *same* issue. (See, *e.g.*, [10].) Unlike multidimensional opinions, these opinion vectors are often restricted to be members of a probability simplex.

To be concrete about the difference, an agent in a model with multidimensional opinions might ave a value of .8 for the "pro-gun control" issue, .9 for the "raise the minimum wage"

sional setting have been modeled as discrete ([8]) or even as boolean variables combined in arbitrary logic formulas ([11, 12]). Oddly, modeling multidimensional opinions as an array of continuous values is rarely seen. One purpose of using multidimensional opinions could be to see how an agent's opinions on different issues interact with one another. This is explored by the boolean expressions in [11] and [12]; in [8] the multidimensional opinion for each agent is instead used merely as an element in a vector space whose (Hamming) distance from other agents' multidimensional opinions can be computed and compared to a BC threshold.

With respect to these previous efforts, my model resembles the BVM, but gives the agents continuous, multidimensional opinions. Each agent will have one opinion on each issue in the model. If there are four issues in the model, the agents will have an opinion on a continuum from 0 to 1 for each issue.

My model also implements BC, but in a different way than models like [9] do: before accepting the influence of a fellow agent on an issue, an agent in my model must already be close in opinion to that agent on a *different* issue. Additionally, agents may be repelled away from each other on a different issue if the difference in their opinions is large enough on another separate issue. This mechanism I refer to as cross-issue influence is meant to mimic a phenomenon of human behavior: if I learn that your viewpoint on issue A is close to my own, homophily suggests that I will trust you, and I will therefore be willing to consider your viewpoint on issue B. If I think you do not have a reasonable opinion on issue A, I will move away from you on issue B. To my knowledge, this mechanism of agent influence has not been previously explored.

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issue, and .4 for the "restrict fracking" issue. By contrast, an agent in a model with opinion vectors might have a value of .2 for the "raise taxes to fund infrastructure" alternative, .7 for the "cut military spending to fund infrastructure" alternative, and .1 for the "increase IRS audits to fund infrastructure" alternative, all possible solutions to the single "how to fund infrastructure" issue. In the latter case, the options are considered mutually exclusive and must sum to 1 for any agent.

(Of course, the specific real-world examples here are only for illustration; OD models represent "issues" and the "opinions" about them completely abstractly.)

3.2 Polarization

Polarization can mean different things to different people; I therefore begin by briefly establishing a dictionary of terms that I will refer back to throughout this paper.

Arguably the most familiar manifestation of polarization – which I have termed "**diametricity**" – is when a group experiences opinions shifting away from common ground to polar sides, leaving nobody 'in the middle' on a specific issue. Although I do believe that this is a type of polarization that may be occurring, I did not study this flavor of polarization in my research.

In future research, I hope to study this type of polarization, but I wanted to focus on the types of polarization that I believe pose the greatest threat to society. In my opinion, the majority of American's views are not more diametric than they used to be. Even though certain groups of the population are extreme and pose a threat to democratic processes, the types of polarization that I feel are the most dangerous are described below.

I use the graph theory term **assortativity** to represent a second type of polarization, which is rooted in the tendency that people have to form connections with people who have similar views. This idea is supported by [13] which focuses on physical proximity breeding connections, as well as [12] which states that we are more likely to form connections with those that we already are in agreement with on another issue. The well-known concept of homophily comes into play here, as studied in [14] and [15]. Assortativity is a way to quantify the presence of "echo chambers" in a society, in which people are exposed mostly (or solely) to opinions that confirm what they already believe.[16, 17]

A third form of polarization is one that can be measured as follows: how often do opinions on issues result in **clustering**? For example, if all individuals had the same belief, there would be one opinion cluster. However, in a polarized society, there are clusters of opinions for any given issue. In this way, higher clustering in a society represents when individuals are entrenched and no longer willing to change their opinion on a given issue. The mechanism that I use in this paper to calculate the number of opinion clusters will be explained later.

Finally, the last form of polarization that I study in this paper is **issue alignment**. I define issue alignment as the tendency for people who agree on one issue to also agree on other (unrelated) issues. For example, consider the issues of vaccine mandates, abortion, and gun control. It seems clear

to me that these issues are unrelated, yet if I were to know one person's opinion on one of these issues, I believe that I could guess their opinions on the other issues to a high degree of accuracy. Simply, issues that should not be correlated with each other seem to be connected in some way.

If people generally adhere to an entire suite of opinions, I term that society "issue aligned", and claim this is an indication of polarization.

This phenomenon of issue alignment is not one I have seen studied in the opinion dynamics literature which is why I believe it is so important to understand. I have a few theories as to how issue alignment develops in a society. First, it is possible to me that there is some deep underlying principle to people's value systems that connects seemingly unconnected issues. For example, if an individual believes in personal freedom over everything else, then maybe this influences their decisions on a wide range of issues such as gun control, abortion, and vaccine mandates. However, I do not believe that this is the reason issue alignment occurs.

Another possible explanation for issue alignment is media outlets. Perhaps a small number of popular media outlets each articulates a set of opinions on various issues. If individuals only hear from their media outlet that they subscribe to, then issue alignment could occur.

A third explanation for issue alignment is that individuals following the well-proven principle of homophily are influenced by their social connections across issues. If they are friends, then homophily and common sense would suggest that agreeing on one issue makes these individuals more likely to be influenced towards each other on another issue. As a result, the individuals develop similar views on issues; thus, resulting in groups that have similar viewpoints on multiple issues that are unrelated. The societal result of this cross-issue influence mechanism is issue alignment.

4   Psychological Foundations

4.1   Openness and Disgust

Regarding the role of a society's "openness" and "disgust," one question that arises is the psychological basis for these attributes. Which personality trait plays the biggest role in an individual's likelihood to change their opinions on a particular issue?

The 'Big 5' personality trait group[18], well-researched since the 1980s,

contains Openness-to-Experience (OE) as one of its five traits. OE can be defined as "cognitive flexibility"[19], or "[openness can be] associated with having a vivid imagination and [...] receptivity to one's own and other's emotions; a willingness to try new experiences"[20].

As the research shows, openness plays a crucial role in an individual's ability to relate to others, as well as to consider adopting outside ideas as their own. On the other side of openness is disgust. More than the Big 5's Agreeableness and Conscientiousness traits, OE seems to encompass openness and disgust in my model.

4.2 Homophily

Homophily is a well-proven principle in social psychology that simply says when individuals are more similar, they are more likely to trust each other[6]. As a result, they are more likely to be influenced by those like-minded individuals because they already trust them. This principle is a large basis for both same-issue influence and cross-issue influence in my model.

5 Variables

In this section I define the four important independent variables whose effect on the model's behavior I seek to discover, and the three dependent variables I measure at simulation's end.

5.1 Independent variables

5.1.1 Openness

As mentioned earlier, research shows that **openness** plays a crucial role in an individual's ability to relate to others, as well how easily they adopt outside ideas as their own. To quantify this as a model parameter, I incorporate openness as a threshold on a continuum from 0 to 1; this threshold is used to compare agent opinions during their pairwise interactions. Low levels of openness produce models in which agents only very rarely change their opinions (namely, only when encountering neighboring agents whose opinion on another issue is very close to their own). High levels produce models in which agents eagerly incorporate the opinions of others on almost every interaction.

5.1.2 Disgust

On the other side of openness, evidence[12] shows that negative influence can happen where individuals are repulsed by each other; thus, their opinions move farther apart. I implement disgust similarly to the openness threshold. The disgust threshold is a threshold on a continuum from 0 to 1. With low levels of the disgust threshold, individuals are more likely to be pushed away from each other because the difference in two agents opinions on any given issue is more likely to greater than the disgust threshold. When the disgust threshold is high, there will be fewer interactions that result in a repulsive or negative influence. It is important to note that the disgust threshold will never be lower than the openness threshold because it would not make sense for the practicality of the influence mechanism.

5.1.3 Edge probability

Another parameter represented in my model is the *density* of social connections. To implement the concept of different degrees of social connection, I used the Erdös-Rényi graph generation algorithm to generate a random graph of connected nodes. With the Erdös-Rényi graph generation algorithm, I can specify the **edge probability** which represents the probability that there will be an edge between any two given nodes. Using the edge probability, I can control the density of the resulting graph. As a result, edge probability directly corresponds to the density of social connections in my model.

5.1.4 Cross-Issue Influence

The novel cross-issue influence mechanism that I introduced earlier is the final independent variable in the model. The cross-issue influence variable (or CI2) has a boolean value. If the value is true, all agents in the model follow the novel CI2 mechanism. If the value is false, all agents in the model follow the same-issue influence mechanism (or I2).

Same-issue influence is based on the traditional bounded-confidence mechanism mentioned earlier. Cross-issue influence is an extension of this mechanism. With same-issue influence, individuals compare and receive influence on only one issue at a time. With cross-issue influence, agents compare their opinion to their neighbors on one issue, then receive influence on another

issue that is not the same as the comparison issue. A more specific walk-through of the CI2 mechanism compared to I2 will be explained later on in the model section of this paper.

5.2 Dependent variables

5.2.1 Graph assortativity

One way I measure the simulated society's polarization is through the resulting network's "assortative mixing," or simply graph **assortativity**. This represents the degree to which an agent's opinions will have similar values to those of its network neighbors, on average.

The assortativity of a network has a value between -1 and 1, where 1 indicates "perfect assortative mixing" – *i.e.*, a situation where every agent's opinions are identical to each of its graph neighbors'. An assortativity of 0 indicates that the agents' social connections have no correlation at all with their opinion values: having a social tie with another agent does not mean an agent is any more (or less) likely to have opinions similar to that agent. This will be approximately true when the model is initialized and before the iterative process begins. (Negative assortativity values correspond to networks in which an agent is *less* likely to agree with its network neighbors than with agents in general.)

Assortativity is thus a way to measure the extent to which agents become surrounded by (only) like-minded agents, and are therefore no longer exposed to alternative points of view. Since I need to obtain the graph's assortativity with respect to *multiple* attributes (*i.e.*, the opinions an agent has on all of the issues), I simply compute the network's assortativity for each issue separately (as defined in [21], p.5) and average it over all the issues.

5.2.2 Opinion clustering

The second dependent variable of my model is opinion **clustering**. This measures how often the opinions that agents have on a given issue fail to converge to a uniform value, instead remaining bifurcated among two or more values in perpetuity. Each group of agents who, at simulation's end, have the same opinion on an issue (within some small tolerance ϵ) are termed an "opinion cluster" (a term used by [22]) on that issue.

For clarity, I refer to any issue on which all agent opinions eventually

converge to the same value as a "**uniform issue**," and any issue that instead produces opinion clusters as a "**clustered issue**."

One challenge is defining what qualifies as an clustered issue, given that agent opinions are represented as real numbers that may asymptotically converge to, but never actually reach, the same value. I use the following mechanism. To calculate the number of clusters for an issue, I add agents to a cluster after every step of the model. If the absolute value of the difference between an agent's opinion and the average opinion of a preexisting cluster is within a threshold (0.05), the agent is added to that cluster. If this is not the case, the agent is added to a new cluster in which it is the first occupant.

5.2.3   Number of Distinct Opinion Buckets

The third and final dependent variable in the model is used to quantify the issue alignment in a society. This variable I term the number of distinct opinion **buckets**. A bucket is a specific tuple of numerical opinions on the various issues. For example, a bucket could have the values (0.4, 1.0, 0.6) where the opinion values for issues one, two, and three are 0.4, 1.0, and 0.6 respectively.

It is important to note that the number of distinct opinion buckets is similar to, but not the same as, the previous dependent variable which is the number of opinion clusters. The number of distinct opinion clusters refers to how many clusters of opinions there are for a single issue. The number of distinct opinion buckets refers to the number of different tuples of opinions that exist in a society. The difference is that the number of distinct opinion clusters measures clusters on a *single issue* whereas the number of distinct opinion buckets measures the clustering of *all issues* in the model.

For example, consider a model with three issues and four agents. If two of the agents have opinion values (0.1, 0.2, 1.0) and the other two agents have opinion values (0.7, 0.6, 0.0) then there would be two distinct opinion buckets in this model. Another important note is that at any point in simulated time, each agent is only in one bucket at a time, possibly with other agents that share the same opinion values. All agents in the same bucket agree on all the issues in the model within a threshold of ϵ (0.05).

For clarity, if a pair of agents agree on *every* issue (*i.e.*, they're in the same bucket), I call them **clones** (or a "clone pair"). If a pair of agents disagree on every issue, I call them **anti-clones** (or an "anti-clone pair").

I interpret this variable differently than opinion clustering. If the number

of distinct opinion buckets in a society is high, then there are many different sets of opinions. With a healthy variety of diverse sets of opinions, there is a high number of distinct opinion buckets and low polarization.

When the number of distinct opinion buckets is low, then there is more polarization in the society because every individual in the society falls into one of the few number of buckets. It should be noted that it is possible for the model to converge to complete uniformity in which there is only one distinct opinion bucket. In this case, although the number of distinct opinion buckets is low, there is not polarization in the society. Consider an issue in which society has reached complete consensus. If this were the case, I wouldn't argue this indicates any polarization, obviously.

6 Model

The model is presented using an abbreviated version of the ODD protocol[23].

6.1 Purpose

The model simulates interactions on a random social network of agents, each with an array of continuous, numeric **opinion** attributes. Its purpose is to investigate the way in which multiple factors contribute to the emergence of polarization in the network: the **edge_probability**, a value reflecting the density of social connections in the network; the **openness** threshold, a value representing how closely one of an agent's opinions must be to that of a potential influencer in order to accept influence; the **disgust** threshold, a value representing how far away one of an agent's opinions must be to that of a potential influencer in order to be repelled away on an issue; and the presence of agents following the cross-issue influence mechanism or the same-issue influence mechanism. (See Section 6.3, below.)

Using the model, I hope to gain general insight on the emergence of this polarization within social networks and how different parameters affect this.

6.2 Entities, State Variables and Scales

The entities within the model are agents, having the following attributes:

ID A unique ID for the agent.

Opinions An array of numbers, representing opinions on issues, each having a value between 0 and 1. This represents the degree to which the agent "agrees" or "disagrees" with an issue, with 0.5 being neutral.

Neighbors A subset of the other agents in the model, to whom this Agent has a social connection. The entire set of agents and their social connections form an undirected graph (*i.e.*, all social connections are bidirectional) and the graph is fixed throughout the simulation.

6.3   Process Overview and Scheduling

After the model has been initialized, the following sequence is executed for each of a fixed number of steps in the simulation for cross-issue influence agents:

1. An agent X is chosen at random.
2. A neighbor of X (call it Y) is chosen at random.
3. An issue I_1 is chosen at random.
4. The absolute difference between X's opinion on I_1 and Y's opinion on I_1 is measured.
5. Another opinion I_2 ($\neq I_1$) is chosen at random.
6. If the difference between X's and Y's opinion on issue I_1 is less than or equal to the model's **openness** threshold, set X's opinion on I_2 to be the average of X's and Y's current I_2 opinions.
7. If the difference between X's and Y's opinion on issue I_1 is greater than or equal to the model's **disgust** threshold, calculate the difference between X's opinion and the average of X and Y's current I_2 opinions. Then, add this quantity to X's opinion on I_2. Note that for the cross-issue influence mechanism, X's opinion will move away from Y's on I_2.

For same-issue influence agents, the following sequence is executed:
1. An agent X is chosen at random.
2. A neighbor of X (call it Y) is chosen at random.
3. An issue I_1 is chosen at random.
4. The absolute difference between X's opinion on I_1 and Y's opinion on I_1 is measured.
5. If the difference between X's and Y's opinion on issue I_1 is less than or equal to the model's **openness** threshold, set X's opinion on I_1 to be the average of X's and Y's current I_1 opinions.

6. If the difference between X's and Y's opinion on issue I_1 is greater than or equal to the model's **disgust** threshold, calculate the difference between X's opinion and the average of X and Y's current I_1 opinions. Then, add this quantity to X's opinion on I_1. Note that for the same-issue influence mechanism, X's opinion will move away from Y's on I_1.

6.4   Initialization

The simulation is initialized with 100 agents, each having a variable number of opinions set to independent uniform random values between 0 and 1. The model is initialized with all agents either following the cross-issue influence mechanism or the same-issue influence mechanism. The agents are then connected to each other using a random undirected Erdös-Rényi graph[24] with parameters $N = 100, p =$ **edge_probability**. If the graph is not connected, a new random graph is generated until a connected one is obtained.

An openness threshold and disgust threshold, each having a value between 0 and 1, will be set such that the openness threshold is always less than the disgust threshold.

The model's step limit is usually set to 1000, as most change in the agent's opinions after 1000 steps is negligible.

7   Hypothesis

I form the following hypotheses about the model's behavior.

Hypothesis 1a: (H_{1a}). Mean assortativity will increase with the edge probability of an Erdös-Rényi graph.

Hypothesis 1b: (H_{1b}). Mean assortativity will increase when the openness threshold of agents in the model is lower.

Hypothesis 2a: (H_{2a}). The number of clustered issues will be negatively correlated with the edge probability of an Erdös-Rényi graph.

Hypothesis 2b: (H_{2b}). The number of clustered issues will increase when the openness threshold is lower for all agents in the model.

Hypothesis 3: (H_3). The number of distinct opinion buckets will decrease when the agents in the model follow the cross-issue influence mechanism compared to same-issue influence mechanism.

For H_{1a}, I hypothesize that increasing the connectivity of an Erdös-Rényi graph by raising the edge probability will result in higher assortativity. This hypothesis is based mainly on real-world observations: the number of social connections available to those with Internet access has increased in the past few decades (due to social media[25]), and the degree of homophily exhibited in members of a social circle has also (at least anecdotally) increased. Since both the density of connections and the homophily of those joined by such connections has increased in the real world, I presume the same effect will follow in my model.

For H_{1b}, I hypothesize that when agents in the model are less open to new opinions, there will be a higher average assortativity and therefore polarization. When all agents have a lower level of openness, they will only be interacting with agents that have opinions similar to their own; therefore, I expect to see higher levels of assortativity.

For H_{2a}, I assume that raising the connectivity of an Erdös-Rényi graph by increasing the edge probability will result in fewer clustered issues. As a graph becomes more densely connected, agents will have a wider variety of neighbors to receive influence from. As a result, agents should merge to the consensus opinion for any given issue more often in a more densely connected graph.

For H_{2b}, I believe that lowering the openness threshold of agents in the model will result in more clustered issues across the model. When agents are less open to distant opinions, there will be more variety of opinion for any given issue.

For H_3, I believe that when agents follow the cross-issue influence mechanism, there will be fewer opinion buckets because agents will converge to a few distinct sets of opinions. With the same-issue influence mechanism, there will be less convergence to sets of opinions, and therefore more distinct opinion buckets.

8   Results

8.1 H_{1a} and H_{1b}

To test H_{1a}, I first establish a model with 50 agents, 5 issues, and an openness parameter of 0.40. In order to measure the impact of varying edge probability on average assortativity across all issues, I ran each combination
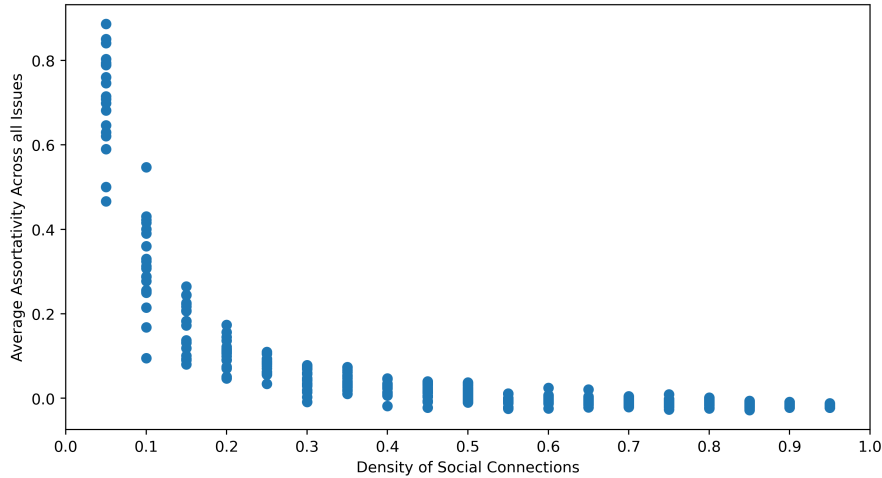
Figure 1: Average Assortativity across all Issues and Edge Probability

of parameters 20 times starting with an edge probability of 0.05 and ending with an edge probability of 0.95, in increments of .05. The results of this model run are shown in Figure 1.

From the graph, we see that as the edge probability (or density of social connections) increases, the average assortativity across all issues decreases. This is the exact opposite of my hypothesis. One possible explanation for this result is that when connections are more dense, there is a higher chance that agents will be exposed to a more diverse set of opinions. There is thus a higher chance that agents will be pulled to the 'average' opinion for a given issue, which would produce lower assortativity. With less densely connected social networks, it is easier for an individual to get stuck in an echo chamber that has a low overall diversity of opinions. In the aggregate, this would produce a social network with higher assortativity. From this result, I am able to infer that societies where individuals are more densely connected may experience less polarization than more sparsely-connected societies do.

In addition to the negative correlation between density of social connections and polarization, I also noticed that the relationship between these two variables appears negative-exponential in nature. The variance was too high, however, for me to draw a solid conclusion on whether the relationship truly conforms to a negative-exponential, a power-law, or any other standard
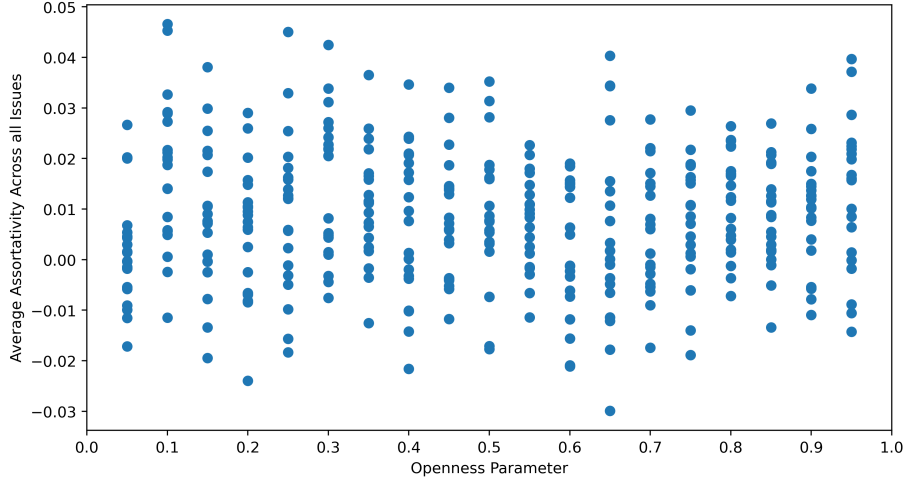
Figure 2: Average Assortativity across all Issues and Openness Threshold

distribution.

To test H_{1b}, I first establish a model with 50 agents, 5 issues, and an edge probability of 0.50. In order to measure the impact of varying the openness threshold on average assortativity across all issues, I ran each combination of inputs 20 times with an openness parameter ranging from 0.05 to 0.95 in increments of .05. The results of this model run are shown in Figure 2. As is depicted, there is no obvious relationship at all between the openness threshold and the average assortativity across all issues.

This is an interesting result. Agents in the model are influenced when they are close in opinion (within the openness threshold) to another agent on the same issue. Therefore, I believed that openness would play a role in determining the assortativity of a society. It should be noted that I tested this hypothesis with multiple different values of the edge probability (0.15, 0.40, and 0.50), to ensure that the edge probability was not having an impact on the results. Even still, I hope to investigate this hypothesis further in future research.
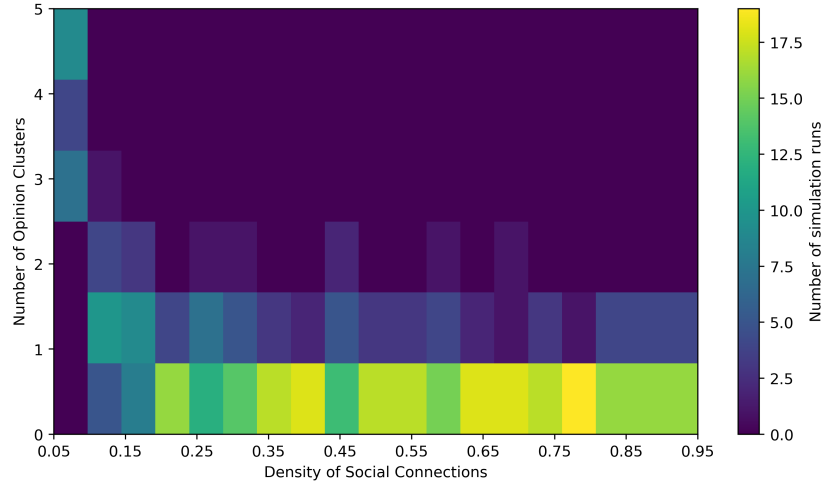
Figure 3: Number of Opinion Clusters and Edge Probability (0.05 - 0.95)

8.2 H_{2a} and H_{2b}

To test H_{2a}, I establish a model with 50 agents, 5 issues, and an openness threshold of 0.30. First, I ran a parameter sweep varying the edge probability from 0.05 to 0.95 to measure the impact of this parameter on the number of opinion clusters. The results of this parameter sweep are shown in Figure 3.

I noticed that as with H_{2a}, there appears to be a tipping point with the number of opinion clusters and the edge probability. To further explore this hypothesis, I ran another parameter sweep, this time varying the edge probability from 0.05 to 0.40 incrementing by 0.01 for each suite of 20 model runs. The results of this parameter sweep are depicted in Figure 4.

The results confirm H_{2a}; the number of opinion clusters and edge probability have a negative relationship. I believe this may be explained by the implications of a high density for a graph of nodes. For example, when a graph of 50 nodes has a density of 0.05, the average number of social connections will be 2.5. I am able to calculate the average number of social connections by multiplying the chance there will be an edge between any two nodes (edge probability) and the number of nodes. When the edge probability, or density of the graph, increases slightly to 0.2, the average number of social connections will rise to 10 connections. As a result, the geodesic
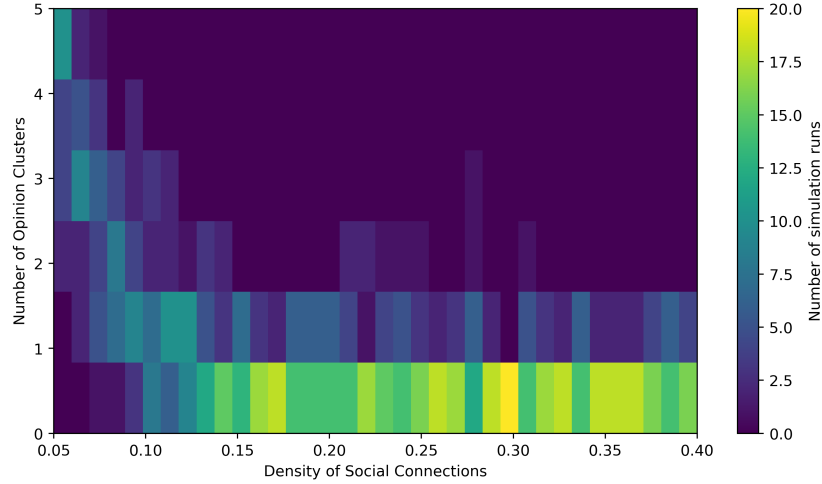
Figure 4: Number of Opinion Clusters and Edge Probability (0.05 - 0.40)

distance between two nodes decreases rapidly because each node is proportionately connected to more nodes in the graph. This may reveal why I saw that only a certain level of density is required for the number of opinion clusters to drop sharply. Undeniably, a tipping point exists with the number of opinion clusters when increasing the density of an Erdös-Rényi graph in the model.

To test H_{2b}, I establish a model with 50 agents, 5 issues, and an edge probability of 0.50. First, I ran a parameter sweep varying the openness threshold from 0.05 to 0.95 to measure the impact of varying the openness threshold on the number of opinion clusters. The results of this parameter sweep are shown in Figure 5.

I noticed that there was little to no difference between an openness threshold of 0.5 and 0.7. However, I observed that the openness threshold had more impact on the number of opinion clusters when the parameter was closer to 0.10. To further explore this result, I ran another parameter sweep with 50 agents, 5 issues, an edge probability of 0.50, and a suite size of 20. This time, I varied the openness parameter from 0.05 to 0.40. The results are depicted in Figure 6.

This graph indicates that there is a tipping point for the openness threshold. When the openness for agents in the model is very low, the agents did
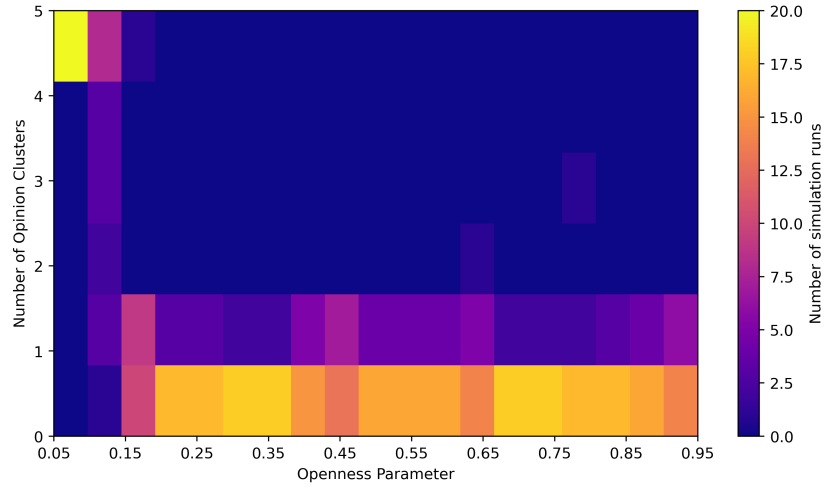
24

Figure 5: Number of Opinion Clusters and the Openness Threshold (0.05 - 0.95)
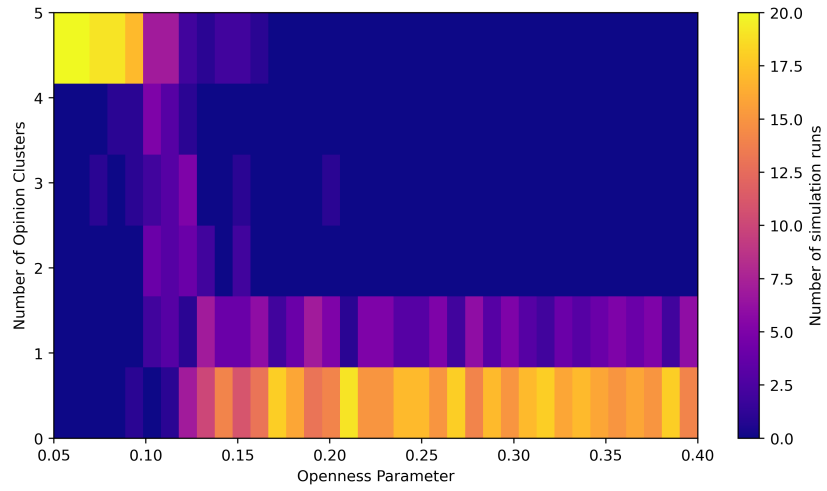

Figure 6: Number of Opinion Clusters and Openness (0.05 - 0.40)

not agree on many issues. However, as Figure 6 indicates, when I increase the openness threshold slightly, the number of opinion clusters across the model quickly drops. As a result, I can infer that low levels of openness in a society may induce more polarized societies. When agents in the model are less open to distant opinions, there are more opinion clusters for any given issue. However, the tipping point leads us to believe that slightly higher levels of openness are sufficient to reach uniformity on a given issue for all agents in the model. To conclude, when using the cross-issue influence mechanism, marginally higher levels of openness led to to less polarization in the model.

8.3 H_3

To test H_3, I establish a model with 100 agents, 3 issues, and an edge probability of 0.20. The model has an openness threshold of 0.15 and a disgust threshold of 0.55. In my explanation of this result, I analyze the plots of multiple single runs of the model.

The three plots below show what I term a census plot. The census plot shows the number of clones and anti-clones as well as the number of agent pairs that agree on one issue and two issues. The number of anti-clones represents the number of agent pairs that agree on none of the issues, and the number of clones represents the number of agent pairs that agree on every issue. This plot also has a maroon dashed line that represents the number of distinct opinion buckets. For clarity, the number of buckets is annotated on the graph once the model reaches equilibrium.

8.3.1 Same-Issue Influence (I2)

Figure 7 shows one run of the model with the combination of parameters above. The agents in this run were following the same-issue influence mechanism.

This plot shows that as the model runs, agents influence each other and get pulled towards some consensus on the issues. The number of agent pairs that agree on one issue (shown by the light grey line) increases as does the number of agent pairs that agree on two issues (shown by the darker grey line) in the model. Additionally, the number of anticlones decreases as individuals converge to some consensus. The number of clone pairs remains low because in models where I2 is the influence mechanism, there is only marginal levels of consensus across issues. Furthermore, the number of distinct opinion buckets
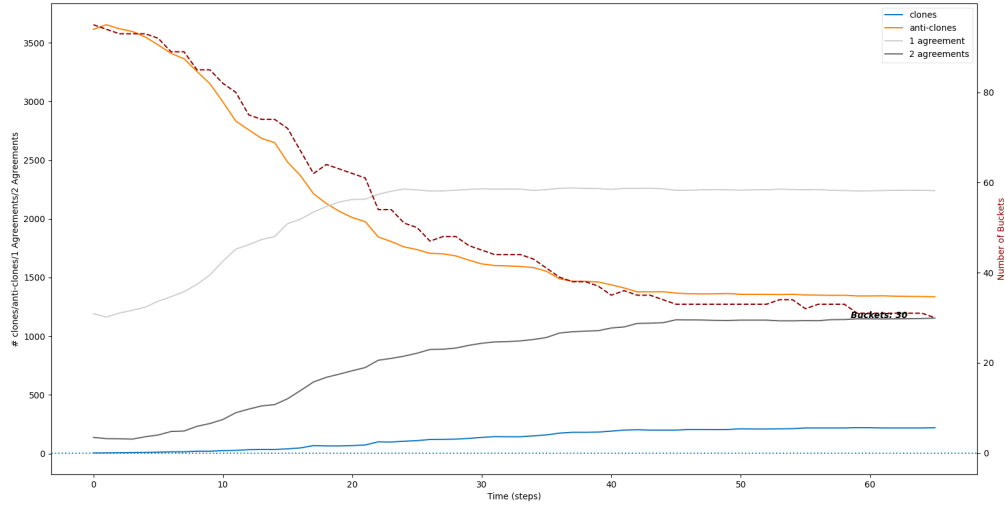
Figure 7: Number of Distinct Opinion Buckets and I2

decreases over time. When the model is initialized, each agent is given a random opinion value for each issue, so each agent is in their own bucket at simulated time step 0. As the model runs, the number of buckets decreases from around 100 (one for each agent) to 30 buckets. This represents some consensus on the issues, but I would not call the social network in this model polarized. After examining an example plot with I2, now I turned on the CI2 mechanism.

8.3.2 Cross-Issue Influence (CI2)

Figure 8 shows one run of the model with the same combination of parameters defined above. However, the agents in this run were following the cross-issue influence mechanism.

As depicted in the plot, the results of a model run with CI2 turned on are very different from the previous graph. The number of agent pairs that agree on one issue (again shown by the light grey line) increases until the number of agent pairs that agree on two issues (again shown by the darker grey line) takes over. This line increases until the number of clone pairs (represented by the blue line) rapidly begins to increase. The number of clone pairs is much higher in a model with CI2 agents compared to a model with I2 agents.

It is important to note that if a pair of agents agrees on two out of the
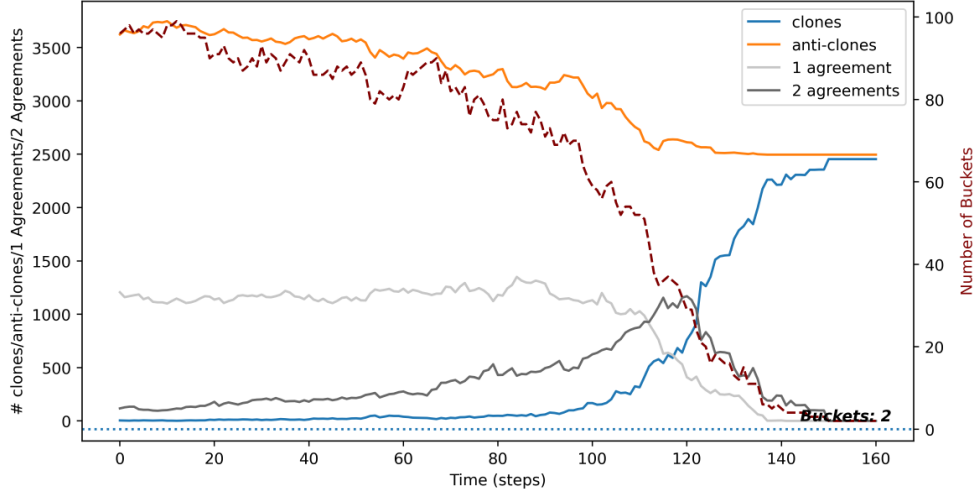
Figure 8: Number of Distinct Opinion Buckets and CI2

three issues, this pair is not counted as agreeing on one issue. This detail is a result of how the number of pairwise agreements are calculated, so the same thing happens with the number of clones taking over and the number of pairs with two agreements decreasing. Simply, if a pair agrees on two issues, they are no longer counted as agreeing on one issue.

The most interesting thing about the result shown in Figure 8 is the state of the model at equilibrium. The model starts out with each agent having randomly assigned opinion values for each issue. As a result the number of buckets is equal to the number of agents in the model because the chances of agents being in the same bucket with three randomly assigned opinion values is extremely low. Additionally, when the model is initialized, most agents are anti-clones because they are all in different buckets. However, when the model finishes running, every agent pair is either an anti-clone pair or a clone pair. Either agents are in the same bucket for each issue, or they disagree on every issue. The number of opinion buckets at equilibrium plummets to only two buckets compared to the thirty buckets at equilibrium for Figure 7. The social network depicted in Figure 8 is much more issue aligned, and therefore polarized, than the network behind the results shown in 7. With every other parameter being held constant besides the influence mechanism, I am able to attribute the increased degree of issue alignment in this society to the CI2 mechanism.

Although this is only one example of a model where agents follow the CI2 mechanism, the decreased number of buckets was consistent across many parameter suites. Regardless of the values for the openness threshold and disgust threshold, the number of distinct opinion buckets in a society with agents following the CI2 mechanism was much lower than a society where agents follow the I2 mechanism. The average number of buckets for a large sweep of the openness and disgust thresholds with CI2 turned on was 2.61, and the average number of buckets was 21.45 sweeping over the same values of the openness and disgust threshold with CI2 turned off (I2).

8.3.3 Number of Distinct Opinion Buckets with Increased Issues

One question that I had when analyzing my results was: what would the behavior of the model look like if I increased the number of issues? One advantage of the way I designed my code base is the modularity and flexibility of the model. Changing the number of issues each agent has in the model is a one line change, so I was able to easily initialize a model with 10 issues rather than only three. Then, I plotted the same census plot like Figure 7 and Figure 8.

I initialized a model with 100 agents, an edge probability of 0.20, an openness threshold of 0.10, a disgust threshold of 0.55, and 10 issues for each agent. The result of running this model to equilibrium is shown below in Figure 9.

On this plot, as the number of agreements increases, the line representing this number will get darker. The lightest grey line represents the number of agent pairs that agree on one issue, and the darkest grey line represents the number of agent pairs that agree on nine out of the ten issues. The number of anti-clone and clone pair lines remain the same color as they were on other plots.

As shown in the Figure 9, even with a large number of issues, the model still converges to a relatively low number of distinct opinion buckets. With ten issues, there are more possible sets of opinion values compared to a model with only three issues, so it is more impressive to me that the ten issue model is still able to converge to a low number of opinion buckets.

It is also interesting to look at the trend of the number of distinct opinion buckets with the number of agent pairwise agreements. For around 350 simulated time steps, the number of distinct opinion buckets does not change. However, there is a steep tipping point where the number of distinct opinion
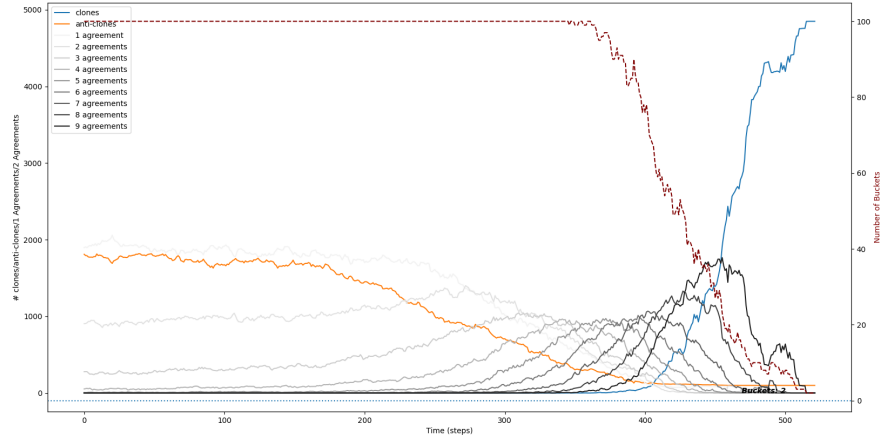
Figure 9: Number of Distinct Opinion Buckets and CI2 with 10 Issues

buckets quickly drops to only two. Before the number of buckets decreases, the agents in the society are increasingly separating and agreeing on more and more issues. This can be seen through each darker grey line peaking after the previous line. What results is a snowball-type effect on the plot where eventually the number of clones increases rapidly. Similar to the model with only three issues, at equilibrium, all agents are either anti-clones or clones. To me, this again represents a polarized society where agents have aligned into only two distinct opinion buckets.

9   Discussion and Future Work

Multiple results in this research surprised me. Firstly, the results from testing H_{1a} did not reflect my anecdotal experiences. When increasing the density of a society's connection, I instead saw *lower* assortativity. I believe this may be due to the static nature of the model's social network. In the real world, homophily not only causes existing friends to become more like each other, but also causes people to select (or reject) friends based on their similarity. In future work, I intend to add this feature to the model, producing a dynamic graph, and discover whether this addition is sufficient to produce a positive density/assortativity relationship.

The lack of a relationship for H_{1b} was another surprising result. I extensively tested this hypothesis, but the results did not indicate any statistically significant relationship. This result remains unexplained.

The tipping points observed when testing hypotheses H_{2a} and H_{2b} were compelling results. When even slightly increasing the density of a graph, the number of clustered issues can drop quickly. This would seem to indicate that the degree to which a society forms consensus can be quite sensitive to the average number of social connections people maintain, at least within a certain range. Too, the openness of a society's members – however that might be quantified in a real population – produced an even steeper tipping point. One interpretation would be that even small changes in the tolerance people have for dissenting views can produce great gains in reducing polarization. I also plan to investigate the behavior of models with agents that are heterogeneous with respect to openness, since OE and other traits are obviously not uniform across a real population.

Finally, the most exciting results in my opinion were the findings of hypothesis H_3. This result could have wide-reaching implications on how we interpret the causes of the polarization phenomenon, and what societal changes might be necessary to reduce it. First, this finding shows that neither ideological coherence nor media influence is necessary for issue alignment to develop in a society. It shows that the cross-issue influence mechanism is sufficient. Secondly, issue alignment endogenously appears through the CI2 mechanism. The model was not initialized with any interdependencies between issues, yet this phenomenon develops consistently through CI2.

Although this is an exciting result, it is hard to be joyful about replicating a negative phenomenon such as polarization. However, replicating polarization has left me with certain ideas about how to reduce polarization from the individual and societal perspective. First, relating to the finding that lower network density leads to higher polarization, I believe that a potential fix for individuals is to surround yourself with more people. By increasing your number of social connections, there is a higher chance you will be exposed to a more diverse set of opinions; thus, lowering your chance of being in an echo chamber. Second, relating to the openness findings, I believe that individuals can help to reduce societal polarization by increasing their tolerance level to dissenting opinions. Finally, relating to the issue alignment findings from H_3, I believe the solution is to allow yourself the freedom to agree with someone on one thing and disagree with them on something else. By keeping unrelated issues separate, the cross-issue influence mechanism can be reduced

which would decrease the degree of issue alignment in society. From a societal perspective, to reduce the issue alignment we need voices from more buckets. The two party system and global media outlets undoubtedly reinforce the issue alignment phenomenon. The solution on a macro level is larger variety of reasonable opinions.

10   Conclusion

To summarize, in my research I created an agent-based model that allowed me to explore the spread of political polarization in a society. Through my cross-issue influence mechanism, I find that network density is negatively correlated with assortativity. Additionally, I find that when individuals in a society are less receptive to differing opinions, there will be less consensus for any given issue in that society. Finally, and potentially most important, I conclude that the cross-issue influence mechanism is sufficient in generating issue alignment endogenously in a society.

In addition to the findings of my model, I learned many valuable skills that I will take with me beyond my time at the University of Mary Washington.

Aside from the sheer amount of python programming that this project required, I have also strengthened my data analysis skills through analyzing the results of various parameter sweeps. To ensure the robustness of my conclusions, I have explored the results graphically and through statistical methods.

Finally, this project has given me the useful ability of creating agent-based models. Increasingly, agent-based models are being used to replicate complex systems in areas where data is not readily available such as fraud detection and military simulation. The data from an agent-based model is then able to be combined with machine learning to create predictive algorithms that are profitable for modern companies.

In my professional career beyond the University of Mary Washington, I plan on continuing to utilize the skills and knowledge gained from my two semesters of research experience.

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