

Einstein's postulates for Special Relativity is given by the following:

1. **Principle of Relativity.** The Laws of Physics are the same in every inertial frames.
2. **Speed of Light is Constant.** The speed of light in vacuum is the same in all inertial frames and it is independent of the motion of the light source.

Suppose we have inertial frame $S(x, y, z, t)$ and inertial frame $S'(x', y', z', t')$ which is moving at a speed u in the x -direction as seen from the frame S . A particle moving in S at a velocity $\vec{v} = (v_x, v_y, v_z)$.

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}, y = y', z' = z, t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - \frac{u^2}{c^2}}}. \quad v'_x = \frac{dx'}{dt'} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}, v'_y = \frac{v_y}{\gamma(1 - \frac{uv_x}{c^2})}, v'_z = \frac{v_z}{\gamma(1 - \frac{uv_x}{c^2})}$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m \vec{v} \quad \text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \vec{v} = (v_x, v_y, v_z), \quad v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \begin{cases} \gamma m \vec{a} & \vec{v} \text{ orthogonal to } \vec{F} \\ \gamma^3 m \vec{a} & \vec{v} \text{ parallel to } \vec{F} \end{cases}$$

$$KE = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) = mc^2(\gamma - 1), \quad E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

$$p'_x = \gamma \left(p_x - \frac{u}{c^2} E \right) \quad p'_y = p_y \quad p'_z = p_z \quad E' = \gamma(E - up_x)$$

$$x^2 - c^2 t^2 = x'^2 - c^2 t'^2, \quad E^2 - p^2 c^2 = m^2 c^4, \quad c^2(p_x'^2 + p_y'^2 + p_z'^2) - E'^2 = c^2(p_x^2 + p_y^2 + p_z^2) - E^2 = -m^2 c^4$$

Acceleration is also influenced under Lorentz Transformation.

An inertial frame is an object of no net force acting on it that goes in a straight line.

$T = \gamma T_0$, where $\gamma \geq 1$, **Moving clock ticks slower..** $L = \frac{L_0}{\gamma}$, **Moving object looks shorter.**

The motion of the object can be expressed as a function of x , which has a slope of $\frac{c}{v}$ and vertical axis interception $-\frac{c}{v}x_0$. A light traveling in the x direction in a space-time diagram has a slope of ± 1 .

In Special Relativity, the total relativistic energy of an object, with rest mass m and speed v , conserves.

KE gained by an electron accelerated from rest across an 1 Voltage electric potential is $1 \text{ eV} = 1.602 \cdot 10^{-19} \text{ J}$.

Photon carries energy: $E = h\nu$. Light from a source that is moving away from the observer should have less energy and lower frequency μ , so the light is redshifted. Light from a source that is moving towards the observer should have more energy and higher frequency, so the light is blueshifted.

$$E' = \gamma(E - up_x) = E \sqrt{\frac{1 - u/c}{1 + u/c}}, \quad \nu' = \nu \sqrt{\frac{1 - u/c}{1 + u/c}}$$

describes the Relativistic Doppler Formula for light source approaching the observer given u is positive. If light source is moving away, we take u be negative.

Heat is a form of energy transfer. Temperature is a measure of an object's cold or hot. More precisely, temperature is the average kinetic energy of the molecules of an object.

The Zero-th Law of Thermodynamics: For systems A, B and C. If A is in thermal equilibrium with C, and B is in thermal equilibrium with C, then A and B are in thermal equilibrium.

$$\Delta L = \alpha L_0 \Delta T \quad \Delta A = 2\alpha A_0 \Delta T \quad \Delta V = \beta V_0 \Delta T = 3\alpha V_0 \Delta T$$

If we divide the system into halves, and the variable does not change, then such variable is called Intensive variable, otherwise, if the variable does change, then such variable is called an Extensive variable.

First Order Phase Transition requires energy to take place, such energy (per unit mass) is called the Latent Heat L . Second Order Phase Transition does not require latent heat to transform from one phase to another.

$$\Delta Q = mc_p \Delta T \quad \Delta Q = mL$$

$$[L] = \text{J/kg} \\ [c_p] = \text{J/kg} \cdot \text{K}$$

$$P(\mathcal{C}) = \frac{\text{number of microstate in } \mathcal{C}}{\text{total number of microstates}}, \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The equation on the right gives the number of ways that can we pick k objects out of a set of n identical ones. without sorting or ordering, and no putting back.

$$[Q] = \text{J} \quad [H] = \text{W} = \text{J/s}$$

Conduction: $H = \frac{dQ}{dt} = kA \frac{T_H - T_C}{L} = -kA \frac{dT}{dx}$ (T_H on the left, T_C on the right) $[I] = \frac{\text{W}}{\text{m}^2}$

Radiation: $H = \frac{dQ}{dt} = eA\sigma T^4$, A is area, σ is Stefan-Boltzmann's constant, e is emissivity. Intensity of radiation is given by $I = e\sigma T^4$. Visible light is the EM wave with wavelength in the range between $\sim 380 \text{ nm}$ and $\sim 760 \text{ nm}$. The lower end is the blue light and the upper end is the red light. Below the lower end, we have ultraviolet, and beyond the upper end, we have infrared. A blackbody emits radiation at all wavelength, but it emits the most radiation at $\lambda_{MAX}(T) = \frac{2.898 \cdot 10^{-3} \text{ Km}}{T}$ known as the Wien's Displacement Law.

If $e=0$, the object is called an ideal reflector

Black body has $e=1$
 $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$

Given that $T_H > T_L$, we have $I_{T_H}(\lambda) > I_{T_L}(\lambda)$ for all λ .

$$I_T(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda kT)} - 1} \quad I_{total}(T) = \int_0^\infty I_T(\lambda) d\lambda = \sigma T^4 \quad I_T^{RJ}(\lambda) = \frac{2\pi L}{\lambda^4} kT$$

The ideal gas law states that:

$$[P] = p_0$$

$$pV = nRT = NkT$$

$$R = 8.31 \frac{J}{K \text{ mol}} \\ k = 1.38 \times 10^{-23} \frac{J}{K}$$

where n is the measure of number of molecules in gas in moles. Let N be the number of molecules, we have: $n = \frac{N}{N_A}$ where N_A is the Avogadro Number $N_A = 6.02 \cdot 10^{23} \frac{\text{molecules}}{\text{mol}}$. 1 mol is the amount of substance that contains as many molecules as there are in twelve grams of Carbon-12.

Diatomic molecules have two rotational degrees of freedom, two vibrational degrees of freedom, three translational degrees of freedom. Monatomic gas molecule only has three translational degree of freedom.

$$\bar{E}_{translational} = \frac{3}{2} kT \text{ where } k \text{ is the Boltzmann's Constant}$$

$$k = 1.38 \times 10^{-23} \frac{J}{K}$$

Boltzmann Distribution $\mathcal{P}(s) = \frac{1}{Z} e^{-\beta E_s}$, where $\beta = \frac{1}{kT}$, k is the Boltzmann's Constant, and $Z = \sum_s e^{-\beta E_s}$.

$$Z = \sum_s d(E_s) e^{-\beta E_s} = 2e^{-\beta E_1} + 8e^{-\beta E_2} + 18e^{-\beta E_3} + \dots \quad \mathcal{P}(E_s) = \frac{1}{Z} d(E_s) e^{-\beta E_s}$$

$$\frac{\mathcal{P}(s_i)}{\mathcal{P}(s_j)} = e^{-(E_i - E_j)/(kT)}$$

$$\frac{\mathcal{P}(E_i)}{\mathcal{P}(E_j)} = \frac{d(E_i) e^{-\beta E_i}}{d(E_j) e^{-\beta E_j}}$$

$$\langle f \rangle = \int_{-\infty}^{\infty} f(x) p(x) dx \quad \langle x \rangle = \int_{-\infty}^{\infty} x p(x) dx \quad \langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 p(x) dx \quad \sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

Gaussian Distribution $\mathcal{P}(x) = c \cdot e^{-\frac{(x-a)^2}{2\sigma^2}}$

Maxwell-Boltzmann Distribution $\mathcal{P}(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} \cdot 4\pi v^2 \cdot e^{-\frac{mv^2}{2kT}}$. $\mathcal{P}(v)dv$ finds the the probability of particle at speed v given T . $\mathcal{P}(v)$ is also temperature dependent, when T is higher, $\mathcal{P}(v)$ peaks at a greater v , but the peak is lower, and the shape of the curve is more normal. Here the the speed v of the peak of the distribution is called the most probable speed at given temperature T .

$$\text{Average Speed} = \langle v \rangle = \int_0^\infty v \cdot \mathcal{P}(v) dv$$

$$\text{Root mean square speed} = v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\int_0^\infty v^2 \cdot \mathcal{P}(v) dv} = \sqrt{\frac{3kT}{m}}$$

$$\langle KE \rangle = \left\langle \frac{1}{2} mv^2 \right\rangle = \int_0^\infty \frac{1}{2} mv^2 \mathcal{P}(v) dv = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$W = \int_{V_i}^{V_f} p dV \quad \Delta U = Q - W$$

Work is path dependent, while internal energy U is path independent.

Isochoric Process - Constant volume, no work done by the system, $\Delta U = Q$.

Isobaric Process - Constant pressure. $W = p(V_f - V_i)$.

Isothermal Process - Constant temperature. $p_i V_i = p_f V_f$ for ideal gas.

Adiabatic Process - No heat exchange during the process, that is, $Q = 0$, so $\Delta U = -W$.

Cyclic Process - Initial state is equal to the final state.

Isolated System - $Q = W = 0$, and we have $\Delta U = 0$ by the First Law of Thermodynamics.

In general, isochoric process and isothermal process have non zero Q , W , and ΔU . For ideal gas in isothermal process, pV is a constant as temperature of the system is a constant, and we have:

$$W = nRT \ln(V_f/V_i) = nRT \ln(p_f/p_i)$$

For non-ideal gas, T decreases as average kinetic energy decreases.

$$C_v = \frac{1}{n} \left(\frac{dQ}{dT} \right)_{\text{fixed volume}} \quad C_p = \frac{1}{n} \left(\frac{dQ}{dT} \right)_{\text{fixed pressure}}$$

For monatomic gas, we have $C_v = \frac{3}{2} R$, and for diatomic gas, we have $C_v = \frac{5}{2} R$ where $R = 8.31 J/(mol K)$. For all ideal gas, we have:

$$C_p = C_v + R$$

