$$\vec{p} = m\vec{v} \qquad \vec{v} = \vec{\omega} \times \vec{r} \qquad \vec{L} = I\vec{\omega} = \vec{r} \times \vec{p} \qquad \vec{N} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \qquad U = \frac{1}{2} I\omega^2 \qquad I = \int_V r^2 \, dm$$

Given $m\frac{d^2x}{dx^2} = F(x)$ with $f(x_0) = 0$ and $F'(x_0) > 0$, for small deviations from x_0:

$$x(t) = x_0 + A\cos\left(\sqrt{\frac{F'(x_0)}{m}}\; t + \phi\right)$$

with A and ϕ depending on initial conditions.

Suppose $F = -kx$, and $\omega_0^2 := \frac{k}{m}$, we get a second order ODE $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$. The solution to such ODE is given by $x = A\cos(\omega_0 t - \phi)$, where A is the maximum oscillation amplitude, and δ and ϕ are the oscillation angle offset. The period of an oscillation system is given by $\tau = 2\pi/\omega_0 = 2\pi\sqrt{m/k}$ and the frequency is then given by $\nu = 1/\tau = 1/2\pi\sqrt{k/m}$.

Assume that the the retarding force is given by $\vec{F_r} = -b\vec{v}$. The standard form of damped oscillation equation is given by $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$ with $\omega_0 = \sqrt{k/m}$ and $2\beta = b/m$
The solution to the ODE is then given by:

$$x = e^{-\beta t}(c_1 e^{\omega_1 t} + c_2 e^{-\omega_1 t}) \qquad \text{with } \omega_1 = \sqrt{\beta^2 - \omega_0^2}$$

Here $e^{-\beta t}$ is called the amplitude decay, where the unit of beta is $\frac{1}{s}$. When $(\beta^2 - \omega_0^2) < 0$, we have underdamping. When $(\beta^2 - \omega_0^2) = 0$, then we have critial damping. When we have $(\beta^2 - \omega_0^2) > 0$, we have overdamping. Note that energy of the oscillated object in damped oscillation is not a constant.

For sinusoidal driving force $F = A\cos(\omega t)$, one obtains the standard equation and its solution:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos(\omega t) \qquad \Rightarrow \qquad x = x_0 + x_p = e^{-\beta t}(c_1 e^{\omega_1 t} + c_2 e^{-\omega t}) + D\cos(\omega t - \delta)$$

$$\omega_1 = \sqrt{\beta^2 - \omega_0^2} \qquad \tan(\delta) = \frac{2\omega\beta}{\omega_0^2 - \omega^2} \qquad D = \frac{A}{(\omega_0^2 - \omega^2)\cos(\delta) + 2\omega\beta\sin(\delta)}$$

The quantity δ represents the phase difference between the driving force and the resultant motion. A real delay occurs between the action of the driving force and the response of the system.

Note that D reaches a maximum with some particular $\omega = \omega_R$. setting $\frac{dD}{d\omega} = 0$, we can solve for the amplitude resonance frequency ω_R, the result is given by $\omega_R = \sqrt{\omega_0^2 - 2\beta^2} = \omega_0\left(1 - \frac{\beta^2}{\omega_0^2}\right)$.

The Q-value of damped oscillation is $Q := (\omega_R)/(2\beta) = (\sqrt{\omega_0^2 - 2\beta^2})/(2\beta)$. If a driven oscillator is only slightly damped and driven near resonance, $Q \approx 2\pi(\text{total energy})/(\text{energy loss in one period})$. Since the oscillator is only slightly damped, then we have $\omega_R = \sqrt{\omega_0^2 - 2\beta^2} \approx \omega_0$.
We have $\omega_0 \approx \omega_R \approx \omega$ where ω is the driving frequency, and this gives $Q \approx \omega_0/(2\beta)$.

For Kinetic Energy resonance, $\dot{x} = \frac{-A\omega\sin(\omega t - \delta)}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}}$, the value of T is maximized when $\omega = \omega_0$.

Gravitational force and gravitational field are given by:

$$\vec{F} = -Gm\int_V \frac{\rho(r)\,\hat{r}}{r^2}\, d\tau = -\frac{GMm}{r^2} \qquad \vec{g} = -G\int_V \frac{\rho(r)\,\hat{r}}{r^2}\, d\tau = -\frac{GM}{r^2}\hat{r}$$

G is a constant $G = 6.673 \pm 0.010 \cdot 10^{-11}\, Nm^2/kg^2$.

For object of mass M, Clearly, we have $\nabla \times \vec{g} = 0$, thus we have $\vec{g}(r) = -\nabla\Phi(r)$, where Φ is called the gravitational potential and has dimension of (force per unit mass) $\times$ (distance), or energy per unit mass. Gravitational potential and gravitational potential energy are given by:

$$\Phi(r) = -\int_V \frac{G\,\rho(\vec{r})}{r}\, d\tau = -\int_\infty^r -\frac{GM}{(r')^2}\, dr' = -\frac{GM}{r} \qquad U = -\int \vec{F}\cdot d\vec{r} = -\frac{GmM}{r}$$

Poisson's Equation about Φ is given by the following:

$$\nabla^2\Phi(\vec{r}) = 4\pi G\rho(\vec{r}) \qquad \Rightarrow \qquad \oint_S \vec{g}\cdot d\vec{a} = \int_V \nabla \times \vec{g}\, d\tau = \int_V -\nabla^2(\Phi(\vec{r}))\, d\tau = -4\pi G\int_V \rho(\vec{r})\, d\tau$$

where V is the volume enclosed by S.

If one $\Phi(\vec{r}) = \Phi(r)$, that is, if one has mass density $\rho(\vec{r}) = \rho(r)$, then we get the following:

$$\nabla^2\Phi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{d\Phi(r)}{dr}\right) = 4\pi G\rho(r)$$