Quiz 5

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11 lego blocks. k blue. $(11-k)$ red.

(a) Total number of microstates

$$\binom{11}{2} = \frac{11!}{2!\,(11-2)!} = \frac{11\cdot 10}{2} = 55$$

(b)

let $C(\mathscr{C})$ be the cardinality of the set of microstates of the macrostate $\mathscr{C}$

$$C(\text{two reds}) = \binom{11-k}{2} = \frac{(11-k)!}{2!\,(11-k-2)!}$$

$$C(\text{two blues}) = \binom{k}{2} = \frac{k!}{2!\,(k-2)!}$$

$$C\left(\begin{matrix}\text{one red}\\ \text{one blue}\end{matrix}\right) = k(11-k) = 11k - k^2$$

let $p: \mathbb{R} \to \mathbb{R}$ $\quad k \mapsto 11k - k^2$

$p': \mathbb{R} \to \mathbb{R}$ $\quad k \mapsto 11-2k$

$p'(k) = 0 \iff 11-2k = 0 \iff k = \frac{11}{2}$

it follows that. when $k=5$ or $k=6$

$C\left(\begin{matrix}\text{one red}\\ \text{one blue}\end{matrix}\right)$ reaches maximum.

if $k=5$ $\quad C\left(\begin{matrix}\text{one red}\\ \text{one blue}\end{matrix}\right) = 5(11-5) = 30$

if $k=6$ $\quad C\left(\begin{matrix}\text{one red}\\ \text{one blue}\end{matrix}\right) = 6(11-6) = 30$

So we see that, when $k=5$ or

k=6, the number of microstates for "one blue and one red" reaches maximum. It follows that the probability of getting mismatched pairs of legos is greatest at k=5 or k=6. The greates probability is given by the following:

$$P\begin{pmatrix}\text{one red}\\ \text{one blue}\end{pmatrix} = \frac{C\begin{pmatrix}\text{one red}\\ \text{one blue}\end{pmatrix}\Big|_{k=5}}{55} = \frac{30}{55} = \frac{6}{11}$$

(c) This is exactly what we expected. Intuitively, we would expect such probability is at the greatest when there is equal, or about the same, amount of the two colors in the system. So the number of red in the system should be either 5 or 6, and this is exactly what we found above

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$$\left\{\begin{matrix}\text{500 g water} & \\ & \text{at } 0^\circ C\\ \text{400 g ice} & \end{matrix}\right\} + \left\{\begin{matrix}\text{300 g water}\\ \text{at } 100^\circ C\end{matrix}\right\}$$

Latent heat for ice: $L = 334 \frac{kJ}{kg}$

water specific heat: $C_p = 4.186\ kJ/kg^\circ C$

Energy needed to melt ice: E_m

Energy released by boiling water: E_b

$$E_b = M_b C_p \Delta T = (0.3\,kg)(4.186 \times 10^3\, J/kg\cdot°C)(100\,°C) = 1.258 \times 10^5\, J$$

$$E_m = M_{ice} L = (0.4\,kg)(334 \times 10^3\, J/kg) = 1.336 \times 10^5\, J$$

Here we see that the energy released by the boiling water from 100°C to 0°C is not enough to melt the ice so we conclude that the final temperature is the same as the original temperature of the system. which is 0°C

$$T_{final} = 0\,°C$$