Lab 10 Report

Math 391 - Introduction to Modern Physics Lab
Professor Wayne Lau
University of Michigan

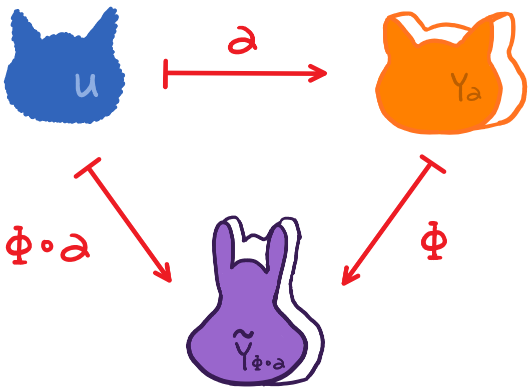

Figure 0. Integrating k-form over a manifold
is independent on the use of coordinate patches

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Lab 10 - The Hall Effect and Conductivity of Semiconductors

Introduction

The phenomenon of Hall effect was discovered by Edvin H. Hall in 1879. A charge q, moving at a velocity $\vec{v}$, perpendicular a magnetic field $\vec{B}$, with experience a force $\vec{F} = q\vec{v} \times \vec{B}$. If a conducting material in a magnetic field has physical bounds restricting such motion, the accumulated displaced charge will create a net vertical electric field, and such phenomenon is called the Hall effect, and the resulting Hall electric field is given by the following:

$$\vec{E}_H = -\frac{1}{nq}\vec{J} \times \vec{B} \qquad \text{where } \vec{J} = nq\vec{v} \tag{10.1}$$

where n is the charge carrier density.

In real materials, the translational symmetry of the crystal lattice imposes energy band gaps when the periodicity of the electronic wave function is commensurate with the lattice spacing. For semiconducting materials, the energy gap between the valence band and the conduction band, denoted as E_g, is of order $1\,eV$. Thermal excitations will promote valence electrons into the conduction band to population densities given by:

$$n_c = n_v e^{-E_g/2kT} \tag{10.2}$$

where n_v and n_c are the valence and conduction band carrier densities. However, for most applications, the doping levels of the semiconductors are high enough that at room temperature the conductivity of the material is dominated by the extrinsic carriers due to the doping process and not the thermal excitations common to intrinsic samples.[1] In Lab 10 of Physics 391, we explore the electrical properties of crystalline germanium by varying the temperature, the immersion in a magnetic field, and the applied voltage across the material.

Experimental Setup

In this lab, we explore the electronic behavior of a lightly p-doped germanium crystal by measuring the Hall voltage and the conductivity as a function of temperature. The magnetic field in this experiment is supplied by the array of permanent rare earth magnets of magnitude approximately $1\,T$. The direction of the magnetic field is determined by knowing the force experienced by a charged wire placed in the supplied field. The magnetic field in the apparatus is found to be oriented counterclockwise. We first verify that the Hall voltage is well described by (10.2). The germanium sample is inserted in the magnet ring and the temperature is assumed to be close to $20°C$. As we change the current through the sample from $-60\,mA$ to $60\,mA$, the Hall

[1] Akerlof, Carl W., *Physics 391 Experiment 10 Lab Manual*, (2018)

voltage is measured using a multimeter, and the voltage across the sample is also measured using another multimeter. The crystal cross sectional are is $1\,mm \times 10\,mm = 10^{-5}\,m^2$. The sample width is $10\,mm$.

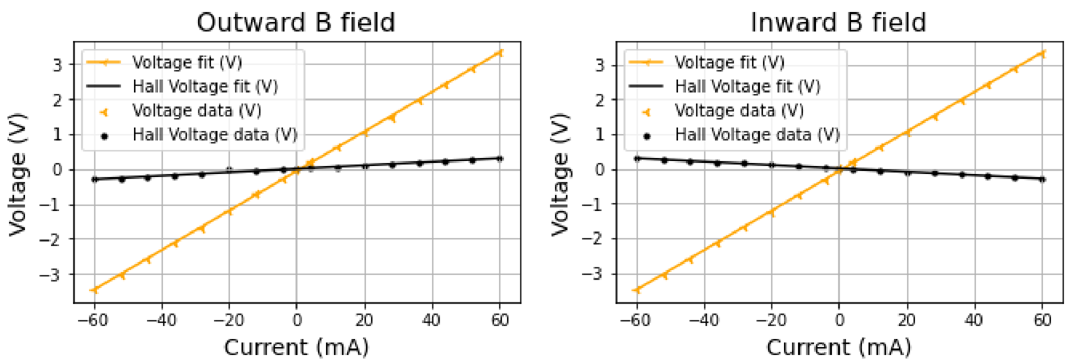
We also check the Hall voltage response over the change of magnetic field strength by lifting the germanium sample to about half inside the magnet gap, we observe that the corresponding change in the magnetic field density is about half of the original value.

Then we explore the effects on the Hall voltage when germanium crystal temperature is varied. The current through the sample in this part is fixed at $60\,mA$. The orientation of the B field is set to be inward (details about the effect of the orientation of the B field is discussed in section 10.3.1). The sample temperature is first raised up to $140°C$. Then we let the sample cool down by itself and at the same time we measure the Hall voltage and the voltage across the sample. Measurements are taken until the sample reaches around $30°C$, a bit above the room temperature. The intervals between measurements in this part is every $10°C$ drop in temperature.

Data Analysis

10.3.1 Current Dependence

In this part, we will present the result of Hall voltage's dependence on the current through the p-doped germanium crystal sample.



We have performed the measurement twice, with opposite orientation of the magnetic field, one is labeled as *Outward B field* and the other is labeled as *Inward B field*. The linear regression parameters are given by the followings:

Orientation of B field	slope k	st. dev. of k (σ_k)	y-intercept b	st. dev. of b (σ_b)
Outward	0.005023	0.000164	-0.000406	0.005884
Inward	-0.004900	0.000200	0.005065	0.000732

Table 1. Hall Voltage Linear Regression Parameters

Orientation of B field	slope k	st. dev. of k (σ_k)	y-intercept b	st. dev. of b (σ_b)
Outward	0.056681	0.000194	-0.070612	0.006933
Inward	0.057014	0.000264	-0.077259	0.009445

Table 2. Voltage Across Sample Linear Regression Parameters

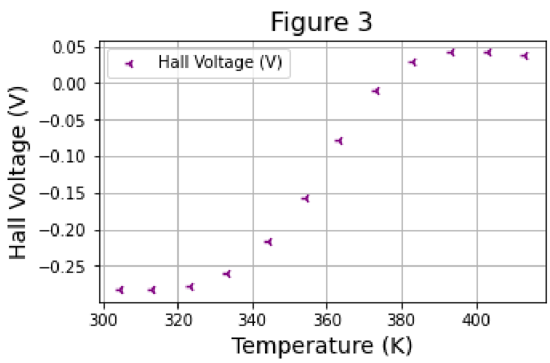
To investigate the relationship between Hall voltage and current, Table 1 is the one that we are interested in. For the outward B field orientation, the r^2-value for the linear regression is 0.9841714, and for the inward B field orientation, the r^2-value for the linear regression is 0.9997386. These results statistically indicate that we have a linear relationship between the Hall voltage and the current. This verifies the linear relationship between the Hall voltage $V_H = E_H \cdot w$ and current $|\vec{J}|$ through the sample as predicted by (10.1). Here w denotes the width of the sample. Moreover, we expect there is no difference between the absolute value of the linear dependence coefficients in the linear regression for the two orientations of the B field. That is, we should have:

$$|k_{\text{inward B}}| = |k_{\text{outward B}}|$$

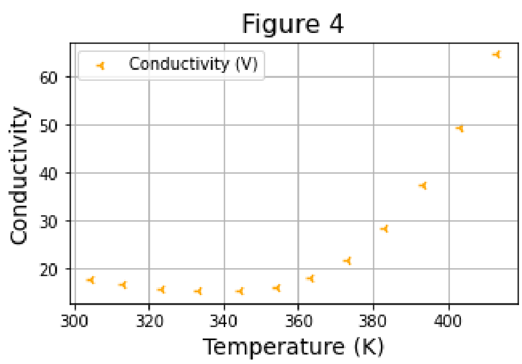
Here we simply note that $|k_{\text{outward B}}|$ is captured within 1-$\sigma_{k_{\text{inward B}}}$, and $|k_{\text{inward B}}|$ is captured within 1-$\sigma_{k_{\text{outward B}}}$. This should suffice to show statistically that the absolute value of the slopes are equal. Hence we can conclude that the orientation of the magnetic field only affects the direction of the Hall electric field, and hence the sign of the Hall voltage, and it does not affect the magnitude of the Hall electric field and Hall voltage. Moreover, based on the set up configuration and the sign of the Hall voltage V_H, we conclude here the charge carriers in the sample are of positive sign, that is, they are holes instead of electrons.

10.3.2 Temperature Dependence and Energy Gap

For this part of the experiment, the current through the sample is fixed at $60\,mA$, and the orientation of the B field is inward. First we present the data of Hall voltage over the temperature of the sample:



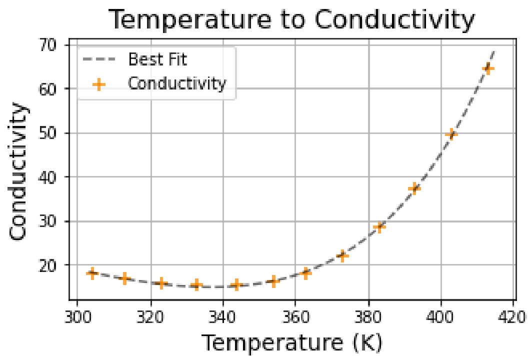
From Fig. 3, we observe that the Hall voltage changes sign as temperature increases to around $380\,K$. Here we estimate the germanium band gap potential E_g. First we compute the conductivity of the sample, and obtain the following figure:



Note that, according to the Physics 391 Lab Manual, the conductivity is defined by the ratio of the current density to the applied electric field which in turn is the longitudinal voltage applied across the sample divided by the length of 20 mm. In the high temperature regime, the conductivity is dominated by electrons promoted into the conduction band as described by (10.2), while close to room temperature, the conductivity is modulated by electron- phonon collisions that disrupt electron motion. These two effects can be modeled by the following:

$$\sigma(T) = a + \frac{b}{T} + ce^{-eE_g/2kT} \tag{10.3}$$

where a, b, c, E_g are constants. We fit (10.3) using our data and obtain the following:



The parameters of the fit is given by the followings:

Parameter	value	st. dev.
a	-49.1184	7.7010
b	20247.5686	2370.6199
c	15807680.0778	7508472.5915
E_g	0.8829	0.03567

Here we see that $E_g = 0.8829 \pm 0.03567\,V$. While the standard accepted value for E_g is $0.67\,V$, we see here our the standard value is not captured within 3-σ of our experimental value of E_g, that is we have $0.67 - 0.88 > 3\sigma = 3 \cdot 0.04 = 0.12$. Hence we conclude that the standard accepted value for E_g is not statistically well-predicted by our data.

From (10.1), we can estimate the density of carriers at room temperature in the germanium sample, where we use $V_H/I \approx 0.005 \cdot 10^3$ as we have estimated in 10.3.1.

$$n = \frac{JB}{E_H q} = \frac{IBw}{AV_H q} = \frac{Bw}{Aq}\frac{I}{V_H} = \frac{(1\,T)(10 \cdot 10^{-3}\,m)}{(10^{-5}\,m^2)(1.602 \cdot 10^{-19}\,C)} \cdot \left(\frac{1}{0.05 \cdot 10^3} A/V\right) \approx 1.25 \cdot 10^{21}\,\frac{\text{carriers}}{m^3}$$

hence the number of carriers per unit volume is around $1.25 \cdot 10^{21}$.

Finally, we make a similar calculation with copper. Assume that each copper atom contributes one electron to the conduction band. On this basis, we estimate the thickness of a copper sample that would produce a $20\,\mu V$ Hall voltage across $10 \cdot 10^{-3}\,m$ wide sample carrying a current of $60 \cdot 10^{-3}\,A$. That corresponds to a potential about 10000 times smaller than observed in this experiment with germanium.

$$20 \cdot 10^{-6}\,V = V_H = E_H w = E_H \cdot (10 \cdot 10^{-3}\,m) \qquad \Rightarrow \qquad E_H = \frac{V_H}{w} = \frac{20 \cdot 10^{-6}\,V}{10 \cdot 10^{-3}\,m} = 0.002\,V/m$$

$$n = \frac{N_A \rho}{M} = \frac{(6.022 \cdot 10^{23}\,carriers/mol)(8.94 \cdot 10^3\,kg/m^3)}{(63.55 \cdot 10^{-3}\,kg/mol)} = 8.472 \cdot 10^{28}\,\frac{\text{carriers}}{m^3}$$

$$d = \frac{IB}{wqnE_H} = \frac{(60 \cdot 10^{-3}\,A) \cdot (1\,T)}{(10 \cdot 10^{-3}\,m)(1.602 \cdot 10^{-19}\,C)(8.472 \cdot 10^{28})(0.002\,V/m)} = 2.21 \cdot 10^{-7}\,m$$

hence the width of the copper should be around $2.21 \cdot 10^{-7}\,m$, much thinner than the semiconductor sample that we use in the experiment.

Summary

In Lab 10 of Physics 391, we explored the electrical properties of crystalline germanium by varying the temperature, the immersion in a magnetic field, and the applied voltage across the material. We verified the linear relationship between V_H and current J as proposed in (10.1), and we also measured the band gap of the germanium sample to be around $0.8829 \pm 0.03567\,V$. Lastly, we calculated the width of the copper sample that is required to produce similar results.

Experiment Data

Current (mA)	Voltage (V)	Hall Voltage (V)
-60	-3.459	-0.311
-52	-3.04	-0.2712
-44	-2.588	-0.2333
-36	-2.114	-0.1912
-28	-1.6962	-0.1542
-20	-1.2061	-0.011
-12	-0.7196	-0.0664
-4	-0.2789	-0.0263
0	-0.0177	-0.0024
4	0.2067	0.0182
12	0.6087	0.0549
20	1.0622	0.0967
28	1.467	0.1337
36	1.9523	0.1779
44	2.4123	0.2189
52	2.8813	0.2605
60	3.3286	0.2993
-	-	Outward B
Current (mA)	Voltage (V)	Hall Voltage (V)
-60	-3.4852	0.2951
-52	-3.0862	0.2617
-44	-2.6153	0.2232
-36	-2.123	0.1817
-28	-1.6655	0.1431
-20	-1.2424	0.1066
-12	-0.7773	0.0664
-4	-0.326	0.0275
0	0.0204	0.0009
4	0.2223	-0.0203
12	0.6385	-0.0568
20	1.0527	-0.0932
28	1.4836	-0.1306
36	1.9861	-0.1742
44	2.4052	-0.2103
52	2.8731	-0.2493
60	3.3256	-0.2854
-	-	Inward B
temp	hall voltage	voltage
140	0.0375	0.9274
130	0.042	1.2115
120	0.0421	1.6045
110	0.0285	2.1158
100	-0.0104	2.7314
90	-0.0782	3.3215
81	-0.1568	3.7343
71	-0.2171	3.9084
60	-0.2596	3.8968
50	-0.2774	3.7603
40	-0.2831	3.5622
31	-0.283	3.369

Code

The code for computing statistics of the data sets is attached.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from google.colab import drive
drive.mount('/content/drive')

import os             ##  This module is for "operating system" interfaces
import sys            ##  This module is for functionality relevant to the
    python run time

GOOGLE_PATH_AFTER_MYDRIVE = 'Physics391/Lab10HallEffectSemiconductor'
GOOGLE_DRIVE_PATH = os.path.join('drive','My Drive', GOOGLE_PATH_AFTER_MYDRIVE)
print(os.listdir(GOOGLE_DRIVE_PATH))

# Append the directory path of this notebook to what python easily "sees"
sys.path.append(GOOGLE_DRIVE_PATH)

# Make your current working directory the directory path of this notebookand
    data
os.chdir(GOOGLE_DRIVE_PATH)

hall_head = pd.read_csv("hallEffect_head.csv")
hall_tail = pd.read_csv("hallEffect_tail.csv")
temperature = pd.read_csv("temperature.csv")

plt.figure(figsize=(5,3))
plt.scatter(hall_head["Current (mA)"], hall_head["Voltage (V)"], s=35, label="
    Voltage (V)", marker="3", color="orange")
plt.scatter(hall_head["Current (mA)"], hall_head["Hall Voltage (V)"], s=10,
    label="Hall Voltage (V)", color="black")
plt.legend()
plt.grid()
plt.title("Outward B field", fontsize=16)
plt.xlabel("Current (mA)", fontsize=14)
plt.ylabel("Voltage (V)", fontsize=14)

plt.figure(figsize=(5,3))
plt.scatter(hall_tail["Current (mA)"], hall_tail["Voltage (V)"], s=35, label="
    Voltage (V)", marker="3", color="orange")
plt.scatter(hall_tail["Current (mA)"], hall_tail["Hall Voltage (V)"], s=10,
    label="Hall Voltage (V)", color="black")
plt.legend()
plt.grid()
plt.title("Inward B field", fontsize=16)
plt.xlabel("Current (mA)", fontsize=14)
plt.ylabel("Voltage (V)", fontsize=14)


def linearFit(x, k, b):
    return k*x+b

import scipy.optimize as optm

headFit = optm.curve_fit(linearFit, hall_head["Current (mA)"], hall_head["
    Voltage (V)"])
headFit_H = optm.curve_fit(linearFit, hall_head["Current (mA)"], hall_head["
    Hall Voltage (V)"])

```

```
print("Outward magnetic field:\n\tVoltage to Current relation: V = %f.3 A + %f.3" % (headFit[0][0], headFit[0][1]))
print("\t", r"k = %f.3 +- %f.3" %(headFit[0][0], np.sqrt(headFit[1][0,0])))
print("\t", r"b = %f.3 +- %f.3" %(headFit[0][1], np.sqrt(headFit[1][1,1])))

print("\nOutward magnetic field:\n\tHall voltage to Current relation: V = %f.3 I + %f.3" % (headFit_H[0][0], headFit_H[0][1]))
print("\t", "k = %f.3 +- %f.3" %(headFit_H[0][0], np.sqrt(headFit_H[1][0,0])))
print("\t", "b = %f.3 +- %f.3" %(headFit_H[0][1], np.sqrt(headFit_H[1][1,1])))

tailFit = optm.curve_fit(linearFit, hall_tail["Current (mA)"], hall_tail["Voltage (V)"])
tailFit_H = optm.curve_fit(linearFit, hall_tail["Current (mA)"], hall_tail["Hall Voltage (V)"])

print("Inward magnetic field:\n\tVoltage to Current relation: V = %f.3 I + %f.3" % (tailFit[0][0], tailFit[0][1]))
print("\t", r"k = %f.3 $\pm$ %f.3" %(tailFit[0][0], np.sqrt(tailFit[1][0,0])))
print("\t", r"b = %f.3 $\pm$ %f.3" %(tailFit[0][1], np.sqrt(tailFit[1][1,1])))

print("\nInward magnetic field:\n\tHall voltage to Current relation: V = %f.3 I + %f.3" % (tailFit_H[0][0], tailFit_H[0][1]))
print("\t", "k = %f.3 +- %f.3" %(tailFit_H[0][0], np.sqrt(tailFit_H[1][0,0])))
print("\t", "b = %f.3 +- %f.3" %(tailFit_H[0][1], np.sqrt(tailFit_H[1][1,1])))

plt.figure(figsize=(5,3))
plt.scatter(hall_head["Current (mA)"], hall_head["Voltage (V)"], s=35, label="Voltage data (V)", marker="3", color="orange")
plt.scatter(hall_head["Current (mA)"], hall_head["Hall Voltage (V)"], s=10, label="Hall Voltage data (V)", color="black")
plt.plot(hall_head["Current (mA)"], linearFit(hall_head["Current (mA)"], headFit[0][0], headFit[0][1]), label="Voltage fit (V)", marker="3", color="orange")
plt.plot(hall_head["Current (mA)"], linearFit(hall_head["Current (mA)"], headFit_H[0][0], headFit_H[0][1]), label="Hall Voltage fit (V)", color="black")
plt.legend()
plt.grid()
plt.title("Outward B field", fontsize=16)
plt.xlabel("Current (mA)", fontsize=14)
plt.ylabel("Voltage (V)", fontsize=14)

plt.figure(figsize=(5,3))
plt.scatter(hall_tail["Current (mA)"], hall_tail["Voltage (V)"], s=35, label="Voltage data (V)", marker="3", color="orange")
plt.scatter(hall_tail["Current (mA)"], hall_tail["Hall Voltage (V)"], s=10, label="Hall Voltage data (V)", color="black")
plt.plot(hall_tail["Current (mA)"], linearFit(hall_tail["Current (mA)"], tailFit[0][0], tailFit[0][1]), label="Voltage fit (V)", marker="3", color="orange")
plt.plot(hall_tail["Current (mA)"], linearFit(hall_tail["Current (mA)"], tailFit_H[0][0], tailFit_H[0][1]), label="Hall Voltage fit (V)", color="black")
plt.legend()
plt.grid()
plt.title("Inward B field", fontsize=16)
plt.xlabel("Current (mA)", fontsize=14)
plt.ylabel("Voltage (V)", fontsize=14)

xdata = hall_head["Current (mA)"]
ydata = hall_head["Hall Voltage (V)"]
residuals = ydata - linearFit(xdata, headFit_H[0][0], headFit_H[0][1])
ss_res = np.sum(residuals**2)
```

```python
ss_tot = np.sum((ydata-np.mean(ydata))**2)
r_squared = 1 - (ss_res / ss_tot)
print(r_squared)


J = (60*(10**(-3)))/(10**(-5))
E = temperature["Voltage (V)"]/(10*10**(-3))
Cond = J/E

plt.figure(figsize=(5,3))
plt.scatter(temperature["Current (mA)"] + 273.15, Cond, s=35, label="
    Conductivity (V)", marker="3", color="orange")
plt.legend()
plt.grid()
plt.title("Figure 4", fontsize=16)
plt.xlabel("Temperature (K)", fontsize=14)
plt.ylabel("Conductivity", fontsize=14)


plt.figure(figsize=(5,3))
plt.scatter(temperature["Current (mA)"] + 273.15, temperature['Hall Voltage (V)
    '], s=35, label="Hall Voltage (V)", marker="3", color="purple")
plt.legend()
plt.grid()
plt.title("Figure 3", fontsize=16)
plt.xlabel("Temperature (K)", fontsize=14)
plt.ylabel("Hall Voltage (V)", fontsize=14)

e = 1.60217663*10**(-19)
k = 1.3807*10**(-23)
def sigma(t, a, b, c, Eg):
    res = a + b/t + c*np.exp(-e*Eg /(2*k*t))
    return res

tempFit = optm.curve_fit(sigma, temperature["Current (mA)"] + 273.15, Cond, p0
    =(-15, 10e3, 6e6, 1.0))
tempFit[0][3], np.sqrt(tempFit[1][3,3])

T_arr = np.linspace(30,142, 50) + 273.15

plt.figure(figsize=(5,3))
plt.scatter(temperature["Current (mA)"] + 273.15, Cond, s=55, label="
    Conductivity", marker="+", color="darkorange")
plt.plot(T_arr , sigma(T_arr, tempFit[0][0], tempFit[0][1], tempFit[0][2],
    tempFit[0][3]),
          label="Best Fit", color="black", ls="--", alpha=0.6)
plt.legend()
plt.grid()
plt.title("Figure 5", fontsize=16)
plt.xlabel("Temperature (K)", fontsize=14)
plt.ylabel("Conductivity", fontsize=14)
```