$$\vec{p} = m\vec{v} \qquad \vec{v} = \vec{\omega} \times \vec{r} \qquad \vec{L} = I\vec{\omega} = \vec{r} \times \vec{p} \qquad \vec{N} = \frac{d\vec{l}}{dt} = \vec{r} \times \vec{F} \qquad U = \frac{1}{2}I\omega^2 \qquad I = \int_V r^2\, dm \qquad I = I_{CM} + Ml^2$$

$$dA = (r\, d\theta)(r\sin(\theta)\, d\phi) = r^2 \sin(\theta)\, d\theta\, d\phi \qquad d\Omega := \frac{dA}{r^2} = \sin(\theta)\, d\theta\, d\phi \qquad d\Omega' = 2\pi\sin(\theta)\, d\theta \qquad \frac{d\sigma}{d\Omega'} := \frac{1}{n_b}\frac{dN}{d\Omega'} = \frac{b}{\sin(\theta)}\left|\frac{db}{d\theta}\right|$$

Every beam of particle in the annulus defined by b and the $b + |db|$ area is scattered into the solid angle $d\Omega'$, define by θ and $\theta + d\theta$. Beam intensity n_b is the number of particle per unit area. N is the number of scattered particles. Note here $|\frac{db}{d\theta}|$ is absolute. Positive db may give negative $d\theta$, that is, increase in b might result in decrease in θ as force become weaker and so we have less scattering. In experiment, we often put our detector in a place with a distance d from the interaction point, and the detector detects a total area S, so the solid angle coverage of the detector is given by $\Delta\Omega = \frac{S}{d^2}$. Now we can calculate the cross section $\sigma(\theta)$ if the force law for scattering is given, and we can use $\sigma(\theta)$ to predict the event rate in experiment set up, we get $\sigma(\theta) = \frac{1}{n_b}\frac{dN}{d\Omega'}$ hence $N(\theta) = n_b \cdot \sigma(\theta) \cdot \Delta\Omega$.

In rotational frame S', $\vec{\omega}$ is the angular velocity, $\vec{v}'$ is the velocity of the object in S', $\vec{r}$ is the position of the object in S. $-2m\vec{\omega} \times \vec{v}'$ is known as the Coriolis force, and $-m\vec{\omega}$ is known as the Centrifugal force. Force acted on particle in S' reads:

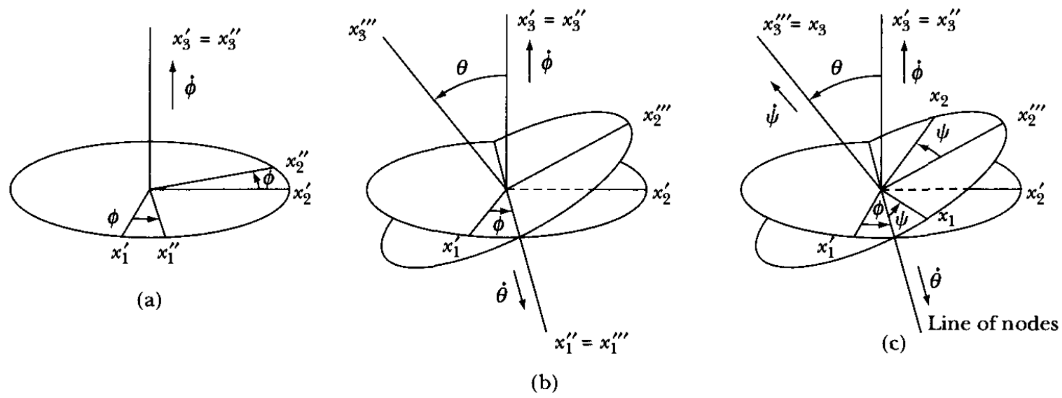
$$\vec{F}' = m\vec{a}' = \vec{F} - m\frac{d\vec{\omega}}{dt} \times \vec{r} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v}'$$

For discrete distribution, $\{a, b, c\} = \{x, y, z\}$.For continuous mass distribution, $\vec{r} = (r_1, r_2, r_3)$. Let $\{i, j, k\} = \{1, 2, 3\}$.

$$I_{aa} = \sum_i m_i(b_i^2 + c_i^2) \qquad I_{ab} = I_{ba} = -\sum_i m_i a_i b_i \qquad I_{ii} = \int (r_j^2 + r_k^2)\, dm \qquad I_{ij} = -\int r_i r_j\, dm$$

Combining, one can show that we have:

$$\vec{L} = \mathbf{I} \cdot \vec{\omega} = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \qquad \begin{cases} N_1 = I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 \\ N_2 = I_2\dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1 \\ N_3 = I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 \end{cases} \qquad \begin{cases} (I_1 - I_3)\omega_2\omega_3 - I_1\dot{\omega}_1 = 0 \\ (I_3 - I_1)\omega_3\omega_1 - I_1\dot{\omega}_2 = 0 \\ I_3\dot{\omega}_3 = 0 \end{cases}$$



$$\begin{cases} \dot{\phi}_1 = \dot{\phi}\sin(\theta)\sin(\psi) \\ \dot{\phi}_2 = \dot{\phi}\sin(\theta)\cos(\psi) \\ \dot{\phi}_3 = \dot{\phi}\cos(\theta) \end{cases} \qquad \begin{cases} \dot{\theta}_1 = \dot{\theta}\cos(\psi) \\ \dot{\theta}_2 = -\dot{\theta}\sin(\psi) \\ \dot{\theta}_3 = 0 \end{cases} \qquad \begin{cases} \dot{\psi}_1 = 0 \\ \dot{\psi}_2 = 0 \\ \dot{\psi}_3 = \dot{\psi} \end{cases} \qquad \begin{cases} \omega_1 = \dot{\phi}\sin(\theta)\sin(\psi) + \dot{\theta}\cos(\psi) \\ \omega_2 = \dot{\phi}\sin(\theta)\cos(\psi) - \dot{\theta}\sin(\psi) \\ \omega_3 = \dot{\phi}\cos(\theta) + \dot{\psi} \end{cases}$$

$$\Omega = \frac{I_3 - I_1}{I_1}\omega_3 \qquad T_{\text{rot}} = \frac{1}{2}\vec{\omega} \cdot (\mathbf{I} \cdot \vec{\omega}) = \frac{1}{2}(\omega_x L_x + \omega_y L_y + \omega_z L_z)$$

Lagrangian for coupled oscillator:

$$\mathscr{L} = T - U = \left(\frac{1}{2}\sum_{j,k} m_{jk}\dot{q}_j\dot{q}_k\right) - \left(\frac{1}{2}\sum_{j,k} A_{jk}q_j q_k\right) \qquad \frac{\partial\mathscr{L}}{\partial q_j} - \frac{d}{dt}\frac{\partial\mathscr{L}}{\partial\dot{q}_j} = 0 \quad \Rightarrow \quad \frac{\partial U}{\partial q_j} + \frac{d}{dt}\frac{\partial T}{\partial\dot{q}_j} = 0$$

$$\det\left(\mathbf{A} - \omega^2\mathbf{M}\right) = 0 \qquad q_k(t) = \sum_r c_r^+ a_{kr}e^{i\omega_r t} + c_\alpha^- a_{kr}e^{-i\omega_r t} \qquad \eta_r(t) = c_r^+ e^{i\omega_r t} + c_r^- e^{-i\omega_r t} \qquad q_k(t) = \sum_r a_{kr}\eta_r(t)$$

In summary, when dealing with general coupled oscillations, one wants to find the characteristic frequencies ω_r to describe the coordinates η_r of the normal mode motion. The actual application of the method can be summarized by the followings:

1. Choose generalized coordinates from the equilibrium of the system.
2. Find kinetic energy T and potential energy U to construct the Lagrangian of the system.
3. Find the matrix $\mathbf{A}$ and matrix $\mathbf{M}$ from the Lagrangian of the system.
4. Determine ω_r through the secular equation of the system.
5. For each ω_r, find the linear space for the coefficient $\vec{a}_r$ by equation (LSK).
6. Determine the normal coordinates η_r with linear combinations of q_j by using equation (NMS).

$$\mathscr{L} = T - U = \frac{1}{2}\sum_\alpha(\dot{\eta}_\alpha^2 - \omega_\alpha^2\eta_\alpha^2)$$

Small angle approximation:

$$\sin(\theta) \approx \theta \qquad \cos(\theta) \approx 1 - \frac{\theta^2}{2} \approx 1 \qquad \tan(\theta) \approx \theta \qquad \cos(\theta_1 - \theta_2) \approx 1$$