Class Notes

Math 591 - Differentiable Manifolds
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Multivariable Calculus

Definition 1.0.0.0.1
S^n is a $n-$sphere in $\mathbb{R}^{n+1}$ defined by:

$$S^n := \{x \in \mathbb{R}^{n+1} \mid ||x||^2 = 1\}$$

To differentiate, we just need a good local structure.

Definition 1.0.0.0.2
A topological space M is called an $n-$dimensional topological manifold provided that all $p \in M$ has a neighborhood N which is homeomorphic to $\mathbb{R}^n$.

Definition 1.0.0.0.3
Let X be a collection of subsets of a set M. X is said to be locally finite provide that all $p \in M$ has a neighborhood U such that U only intersects finitely many $C \in X$.

Definition 1.0.0.0.4
A topological space M is said to be paracompact provided that every open cover X of M admits a finite subcover.

Definition 1.0.0.0.5
A cover of a set M is a collection X of sets such that $\bigcup_{C \in X} C = M$

Definition 1.0.0.0.6
A subcover of X is a cover X^* such that every $U \in X^*$ is contained in some $U \in X$.

Definition 1.0.0.0.7
A cover X is open provided that all $C \in X$ is open

Theorem 1.1
Topological manifolds are paracompact

Definition 1.1.0.0.1
M is said to be locally compact provided that all $p \in M$ and neighborhoods U of p, there exists a neighborhood $V \subseteq U$ such that $\bar{V} \subseteq U$ is compact.

Lemma 1.1.1
Topological manifolds are locally compact

Definition 1.1.1.0.1
An exhaustion is a sequence of sets $K_n \subseteq K_{n+1}$ such that $\bigcup_n K_n = M$

Proposition 1.1.2
A secound countable locally compact Hausdorff space admits an exhaustion by compact sets.