P3

$$(\Delta S_x)^2 = \frac{1}{2} + \sum_{n=0}^{\infty}\left(\frac{1}{2}|C_{0,n}|^2 + C_{1,n}C_{-1,n}\right)$$

Eq. (11a) on paper

$$= \frac{1}{2} + \sum_{n=0}^{1}\left(\frac{1}{2}|C_{0,n}|^2 + C_{1,n}C_{-1,n}\right) + \sim O$$

$$(\Delta S_x)^2 - \frac{1}{2} = \frac{1}{2}|C_{0,0}|^2 + C_{1,0}C_{-1,0}$$

$$+ \frac{1}{2}|C_{0,1}|^2 + \cancel{C_{1,1}C_{-1,1}} \sim 0$$

$$= \frac{1}{2}\alpha^2\sin^2(\sqrt{2}gt) - (1-\alpha^2/2)\frac{\alpha^2}{3}(1-\cos(\sqrt{6}gt))$$

$$+ \frac{1}{2}\frac{\alpha^4}{3}\sin^2(\sqrt{6}gt)$$

(sum)

$$= \frac{1}{2}\alpha^2\sin^2(\sqrt{2}gt) - \frac{\alpha^2}{3}(1-\cos(\sqrt{6}gt)) + \frac{\alpha^4}{6}(1-\cos(\sqrt{6}gt)) + \frac{1}{2}\frac{\alpha^4}{3}\sin^2(\sqrt{6}gt)$$

$$= \frac{1}{2}\alpha^2\sin^2(\sqrt{2}gt) + \frac{\alpha^2}{3}\cos(\sqrt{6}gt) - \cancel{\frac{\alpha^4}{6}\cos(\sqrt{6}gt)} - \frac{\alpha^2}{3} + \cancel{\frac{\alpha^4}{6}} + \cancel{\frac{\alpha^4}{6}\sin^2(\sqrt{6}gt)}$$

~ 0 ~ 0 ignored term of order α^4

$$|\langle S\rangle| = \sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}$$

$$= \left(2\alpha^2\sin^2(\sqrt{2}gt) + (1-\alpha^2\sin^2(\sqrt{2}gt))^2\right)^{1/2}$$

$$= \left(\cancel{2\alpha^2\sin^2(\sqrt{2}gt)} + 1 + \alpha^4\sin^4(\sqrt{2}gt) - \cancel{2\alpha^2\sin^2(\sqrt{2}gt)}\right)^{1/2}$$

$$= \left(1 + \alpha^4\sin^4(\sqrt{2}gt)\right)^{1/2}$$

$$\sim 1 + \cancel{\frac{\alpha^4}{2}\sin^4(\sqrt{2}gt)} \sim 0$$

$$\Rightarrow |\langle S\rangle| \sim 1$$

$$\Delta S_x = \left(\frac{1}{2} + SUM\right)^{1/2} = \left(\frac{1}{2}(1+2SUM)\right)^{1/2} = \frac{1}{\sqrt{2}}(1+2SUM)^{1/2} \sim \frac{1}{\sqrt{2}}(1+SUM)$$

$\Rightarrow$ approximations leads to $\boxed{\xi_x = \sqrt{2}\,\frac{\Delta S_x}{|\langle S\rangle|} \sim 1 + SUM}$

Now employ trig-identity

from SUM. $\frac{\alpha^2}{3}\left(\cos(\sqrt{6}gt) - 1\right) = \frac{\alpha^2}{3}\left(1 - 2\sin^2(\sqrt{6}gt/2) - 1\right)$

$$= -\alpha^2\,\frac{2}{3}\sin^2(\sqrt{6}gt/2)$$

Combining all

$$\xi_x = \sqrt{2}\,\frac{\Delta S_x}{|\langle S\rangle|} \sim 1 + SUM \sim 1 + \frac{1}{2}\alpha^2\sin^2(\sqrt{2}gt) - \alpha^2\,\frac{2}{3}\sin^2(\sqrt{6}gt/2)$$

$$\Rightarrow \boxed{\xi_x \sim 1 + \alpha^2\left(\frac{1}{2}\sin^2(\sqrt{2}gt) - \frac{2}{3}\sin^2(\sqrt{6}gt/2)\right)}$$

The calculation of $\xi_{y'}$ follows similarly using Eq. (11b)

$\sqrt{2}gt$ term

$$(\Delta S_y)^2 = \langle S_y^2\rangle - \langle S_y\rangle^2 = \frac{1}{2} + \sum_{n=0}^{\infty}\left(\frac{1}{2}|c_{0,n}|^2 - c_{1,n}\,c_{-1,n}\right) - \underline{\langle S_y\rangle^2}$$

$\sqrt{6}gt/2$ term

minus

$$\Rightarrow \quad \xi_{y'} \sim 1 + \alpha^2\left(\frac{1}{2}\sin^2(\sqrt{2}gt) + \frac{2}{3}\sin^2(\sqrt{6}gt/2)\right) - \underline{\alpha^2\sin^2(\sqrt{2}gt)}$$

$$\boxed{\xi_{y'} \sim 1 + \alpha^2\left(-\frac{1}{2}\sin^2(\sqrt{2}gt) + \frac{2}{3}\sin^2(\sqrt{6}gt/2)\right)}$$