Utilizing moderated nonlinear factor analysis for integrative data analysis

Joseph Kush

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Introduction

- Integrative data analysis (IDA)
  - Fitting models to data that have been pooled across multiple, independent studies

- Advantages of IDA over single study analysis
  - Increased power through larger sample sizes
  - Ability to formulate and explore previously unachievable RQs (e.g., combining multiple cross-sectional studies measured at different timepoints to allow for longitudinal analyses)
  - Improves measurement of latent constructs through robust psychometrics instruments
  - Greater heterogeneity & increased frequencies of low base-rate behaviors

Meta-analysis

- In reality, practice of combining and analyzing data is largely varied

- Meta-analysis allows for researchers to analyze results such as summary statistics or point estimates and standard errors that have been presented in different but comparable studies

Meta-analysis

- Traditionally, meta-analysis involves aggregated data (Cooper, 2009)
  - 1) researcher systematically searches and collects studies that have been conducted on the topic of interest
  - 2) extracts effect sizes based on reported summary statistics
  - 3) combines these estimates using statistically sound techniques to obtain a single average effect size and confidence interval

Integrative data analysis

- Meta-analysis based on aggregated data

- Meta-analysis based on individual participant-level data
  - often referred to as data synthesis, individual participant data meta-analysis, or integrative data analysis, involves “the central collection, checking, and re-analysis of the raw data from each study in order to obtain combined results” (Cooper & Patall, 2009)

- Allows researchers to:
  - replicate analyses performed in original studies
  - improve precision in effect estimates by increasing sample size & statistical power
  - estimate both within-study and between-study effects

Integrative data analysis

- Great deal of effort required when preparing the data

- First basic step in pre-statistical data harmonization process involved collecting datasets and obtaining codebooks from the individual studies

raceConstructs	Measure	PIRC G1	PIRC G2	P2P	Fast Track	LIFT	SAFE
Aggression	TOCA-R	X	X	X	X	X	X
Sexual orientation		X	X		X		
Race/ethnicity		X	X	X	X	X	X
Sex		X	X	X	X	X	X
Depression & Anxiety	BHIF	X	X				
	CBCL				X	X	X
	DISC	X	X		X	X	
	BSI				X	X	
	CIDI	X	X				
	CDI	X				X	
	BECK						
	CESD					X	X
Psychotic symptoms	CBCL				X	X	X
Suicide attempt & ideation	DISC	X	X		X	X	
	CBCL				X	X	X
	BSI				X	X	
Suicide death	NDI	X	X	X	X	X	X

BHIF=Baltimore How I Feel; CBCL =Child Behavior Checklist; DISC =Diagnostic Interview Schedule for Children; BSI = Brief Symptom Inventory; CDI= Child Depression Inventory; BECK=Beck Depression Inventory; CESD=Center for Epidemiologic Studies-Depression Scale; NDI=National Death Index

For LIFT

We need “w1DEMO.doc”

”There were no kid questionnaires for the 2008 to 2011 P2P trial. Just teacher ratings, classroom observations & school records. We did peer nominations for a small sub-set of 3rd to 5th graders--about 250 kids.”

NDI – We need to wait till NDI search is completed

Next task will be identifying a set of commonly observed items in BHIF, CBCL, DISC, BSI, CDI, BECK, and CESD

Note (7/13)

1. Sexual orientation variables (PIRC 1, 2, and FAST track) - Cannot be found

   “Sexuality and consequences” variable – Ask Jennifer

   Ask Rashelle and Holly about PIRC 1 and 2 – Which variable is it?

2. Alcohol-substance mis(use) measures concordance – work in progress

   Is there a codebook for SUD measures in P2P?

3. Once we identify consistent items – we will document items longitudinally (how many measures are observed when?)
4. Not sure if we can observe academic performance variables across trials

Integrative data analysis

- Next, construct concordance table, documenting which variables are collected across studies
    - confirm response scale types, identify common data domains, and examine variables over time if data were collected longitudinally

Time frame	Psychiatric disorder	Grade/Age	PIRC-G1			PIRC-G2				FastTrack		SAFE	LIFT_L	LIFT_U
			var1	var2	var3	var1	var2	var3	var4	var1	var2	var1	var1	var1
Last 12 months	GAD	G6 (11-12 y/o)	–	–	–	–	–	–	–	cdc7gdy	cds7gdy	–	–	–
Last 12 months	GAD	G9 (14-15 y/o)	–	–	–	–	–	–	–	cdc10gdy	cds10gdy	–	–	–
Last 12 months	GAD	G12 (17-18 y/o)	–	–	–	–	–	–	–	cdc13gdy	cds13gdy	–	–	–
Last 12 months	GAD	18-19 y/o								–	–	–	CFRFGA4_W15	–
Last 12 months	GAD	19-20 y/o	dsmgadyr	–	–	–	–	–	–	–	–	–	–	–
Last 12 months	GAD	20-21 y/o	–	–	–	–	–	–	–	yad15gdy	–	–	–	–
Last 12 months	GAD	22-23 y/o								–	–	–	–	CFRFGA4_W15
Last 12 months	GAD	30-32 y/o	T9_GADlf	T9_GADfstage	–	–	–	–	–	–	–	–	–	–
Time frame	Psychiatric disorder	Grade/Age	PIRC-G1			PIRC-G2				FastTrack		SAFE	LIFT	LIFT
			var1	var2	var3	var1	var2	var3	var4	var1	var2	var1	var1	var1
Lifetime	Panic disorder	G6 (11-12 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Lifetime	Panic disorder	G9 (14-15 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Lifetime	Panic disorder	G12 (17-18 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Lifetime	Panic disorder	18-19 y/o								–	–	–	CFRFPD4_W15	–
Lifetime	Panic disorder	22-23 y/o								–	–	–	–	CFRFPD4_W15
Lifetime	Panic disorder	30-32 y/o	T9_PanicLf	–	–	–	–	–	–	–	–	–	–	–
Time frame	Psychiatric disorder	Grade/Age	PIRC-G1			PIRC-G2				FastTrack		SAFE	LIFT	LIFT
			var1	var2	var3	var1	var2	var3	var4	var1	var2	var1	var1	var1
Last 12 months	Panic disorder	G6 (11-12 y/o)	–	–	–	–	–	–	–	cdc7pdy	cds7pdy	–	–	–
Last 12 months	Panic disorder	G9 (14-15 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Last 12 months	Panic disorder	G12 (17-18 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Last 12 months	Panic disorder	18-19 y/o								–	–	–	CFRFPD4_W15	–
Last 12 months	Panic disorder	22-23 y/o								–	–	–	–	CFRFPD4_W15
Last 12 months	Panic disorder	30-32 y/o	T9_PanicLf	T9_PanLstAge	–	–	–	–	–	–	–	–	–	–
Time frame	Psychiatric disorder	Grade/Age	PIRC-G1			PIRC-G2				FastTrack		SAFE	LIFT	LIFT
			var1	var2	var3	var1	var2	var3	var4	var1	var2	var1	var1	var1
Lifetime	Agoraphobia	G6 (11-12 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Lifetime	Agoraphobia	G9 (14-15 y/o)	–	–	–	–	–	–	–	–	–	–	–	–
Lifetime	Agoraphobia	G12 (17-18 y/o)	–	–	–	–	–	–	–	–	–	–	–	–

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
1	Depressed/irritable/ cranky mood	I feel sad (past 2 weeks)	Grade/Age	PIRC-G1					PIRC-G2		FastTrack					SAFE				LIFT_L						LIFT_U				
2				var1	var2	var3	var4	var5	var1	var2	var1	var2	var3	var4	var5	var1	var2	var3	var4	var1	var2	var3	var4	var5	var6	var1	var2	var3	var4	var5
3			G1 (6-7 y/o)	CDISAD11	–	–	–	–	hif31111	toc31139	–	T2RF103	P2CB103	–	–	T1CBT110	T1TOC039	P1POC043	–	CJ7101_W1	M591103_PW1	F591103_PW1	O591103_PW1	–	–	–	–	–	–	–	–	–
4			G2 (7-8 y/o)	hif02111	–	–	–	–	hif32111	toc32139	–	T3RF103	P3CB103	–	–	T2CBT110	T2TOC039	P2POC043	–	CJ7401_W4	M594103_PW4	F594103_PW4	O594103_PW4	–	–	–	–	–	–	–	–	–
5			G3 (8-9 y/o	–	–	–	–	–	hif33111	toc33139	C4GAM7	T4RF103	–	–	–	T3CBT110	T3TOC039	–	–	CJ7501_W5	–	–	–	–	–	–	–	–	–	–	–	–
6			G4 (9-10 y/o)	hif04105	B90__055	–	–	–	–	–	C5GAM7	T5RF103	P5CB103	–	–	T4CBT110	T4TOC039	P4POC043	–	CJ7601_W6	M596103_PW6	F596103_PW6	O596103_PW6	–	–	–	–	–	–	–	–	–
7			G5 (10-11 y/o)	hif05105	B91_251B	B91_084A	B91_084B	–	–	–	C6GAM7	T6RF103	P6CB103	–	–	T5CBT110	T5TOC039	P5POC043	–	–	–	–	–	–	–	CJ7101_W1	M591103_PW1	F591103_PW1	O591103_PW1	–
8			G6 (11-12 y/o)	hif06118	hip06105	B92_261B	B92_084A	B92_084B	hif36116	–	c7md001	c7md001c	p7md001e	–	–	T6CBT110	T6TOC039	P6POC043	–	CJ7801_W8	M598103_PW8	F598103_PW8	O598103_PW8	–	–	CJ7401_W4	M594103_PW4	F594103_PW4	O594103_PW4	–
9			G7 (12-13 y/o)	hif07118	toc07139	B930261B	B930084A	B930084B	hif37116	–	C8T7	–	P8CB103	–	–	T7CBT110	T7TOC039	–	–	–	–	–	–	–	–	–	CJ7501_W5	–	–	–	–
10			G8 (13-14 y/o)	hif08118	B940261B	B940084A	B940084B	–	hif38116	–	C9T7		–	–	–	C8YSD013	T8CBT110	T8TOC039	P8POC043	CJ7A01_W10	CS5A17_W10	M59A103_PW1	F59A103_PW10	O59A103_PW1	–	–	CJ7601_W6	M596103_PW6	F596103_PW6	O596103_PW6	–
11			G9 (14-15 y/o)	–	–	–	–	–	hif39116	–	c10md001	c10md001	p10md001	–	–	C9YSD013	P9POC043	–	–	–	–	–	–	–	–	–	–	–	–	–	–
12			G10 (15-16 y/o)	–	–	–	–	–	hif10116	–	C11T7	–	–	–	–	C10AYS103	P10CBP103	C10CES018	–	CJ7C01_W12	CS5C17_W12	M59C103_PW1	F59C103_PW12	O59C103_PW1	–	–	CJ7801_W8	M598103_PW8	F598103_PW8	O598103_PW8	–
13			G11 (16-17 y/o)	–	–	–	–	–	hif11116	–	C12T7	–	–	–	–	C11AYS103	P11CBP103	C11CES018	–	–	–	–	–	–	–	–	–	–	–	–	–	–
14			G12 (17-18 y/o)	–	–	–	–	–	hif1112	–	c13md001	c13md001	p13md001	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7A01_W10	CS5A17_W10	M59A103_PW1	F59A103_PW10	O59A103_PW1
15			18-19 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	C5BF103_W15	CS5F17_W15	–	–	–	–	–	–	–	–	–	–
16			19-20 y/o	cc3	–	–	–	–	–	–	c15md001	–	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7C01_W12	CS5C17_W12	M59C103_PW1	F59C103_PW12	O59C103_PW1
17			20-21 y/o	T00J0001	T00J001B	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
18			21-22 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
19			22-23 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	C5BF103_W15	CS5F17_W15	–	–	–
20			23-24 y/o	–	–	–	–	–	–	–	C19AR103	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
21			30-32 y/o	T08I0001	T08I001B	–	–	–	–	–										–	–	–	–	–	–	–	–	–	–	–
23	Depressed/irritable/ cranky mood	I feel like crying (past 2 weeks)	Grade/Age	PIRC-G1					PIRC-G2		FastTrack					SAFE				LIFT_L						LIFT_L				
24				var1	var2	var3	var4	var5	var1	var2	var1	var2	var3	var4	var5	var1	var2	var3	var4	var1	var2	var3	var4	var5	var6	var1	var2	var3	var4	var5
25			G1 (6-7 y/o)	CDICRY11	–	–	–	–	hif31103	–	–	T2RF14	P2CB14	–	–	–	–	–	–	CJ7110_W1	M59114_PW1	F59114_PW1	O59114_PW1	–	–	–	–	–	–	–	–	–
26			G2 (7-8 y/o)	hif02109	–	–	–	–	hif32103	–	–	T3RF14	P3CB14	–	–	–	–	–	–	CJ7410_W4	M59414_PW4	F59414_PW4	O59414_PW4	–	–	–	–	–	–	–	–	–
27			G3 (8-9 y/o	B89__025	–	–	–	–	hif33103	–	C4GAM8	T4RF14	–	–	–	–	–	–	–	CJ7510_W5	–	–	–	–	–	–	–	–	–	–	–
28			G4 (9-10 y/o)	hif04103	B90__059	–	–	–	–	–	C5GAM8	T5RF14	P5CB14	–	–	–	–	–	–	–	CJ7610_W6	M59614_PW6	F59614_PW6	O59614_PW6	–	–	–	–	–	–	–	–	–
29			G5 (10-11 y/o)	hif05103	B91_088A	B91_088B	–	–	–	–	C6GAM8	T6RF14	P6CB14	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7110_W1	M59114_PW1	F59114_PW1	O59114_PW1	–
30			G6 (11-12 y/o)	hif06102	hip06104	B92_088A	B92_088B	–	hif36104	–	–	–	–	–	–	–	–	–	–	CJ7810_W8	M59814_PW8	F59814_PW8	O59814_PW8	–	–	CJ7410_W4	M59414_PW4	F59414_PW4	O59414_PW4	–
31			G7 (12-13 y/o)	hif07102	B930088A	B930088B	–	–	hif37104	–	C8T8	–	P8CB14	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7510_W5	–	–	–	–
32			G8 (13-14 y/o)	hif08102	B940088A	B940088B	–	–	hif38104	–	C9T8	–	–	–	–	C8YSD002	–	–	–	CJ7A10_W10	C38A014_W10	M59A14_PW10	F59A14_PW10	O59A14_PW10	–	–	CJ7610_W6	M59614_PW6	F59614_PW6	O59614_PW6	–
33			G9 (14-15 y/o)	–	–	–	–	–	hif39104	–	–	–	–	–	–	C9YSD002	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
34			G10 (15-16 y/o)	–	–	–	–	–	hif10104	–	C11T8	–	–	–	–	C10AYS014	C10CES017	P10CBP014	–	CJ7C10_W12	C38C014_W12	M59C14_PW12	F59C14_PW12	O59C14_PW12	–	–	CJ7810_W8	M59814_PW8	F59814_PW8	O59814_PW8	–
35			G11 (16-17 y/o)	–	–	–	–	–	hif11104	–	C12T8	–	–	–	–	C11AYS014	C11CES017	P11CBP014	–	–	–	–	–	–	–	–	–	–	–	–	–	–
36			G12 (17-18 y/o)	–	–	–	–	–	hif312	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7A10_W10	C38A014_W10	M59A14_PW10	F59A14_PW10	O59A14_PW10
37			18-19 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	C5BF014_W15	–	–	–	–	–	–	–	–	–	–	–
38			19-20 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7C10_W12	C38C014_W12	M59C14_PW12	F59C14_PW12	O59C14_PW12
39			20-21 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
40			21-22 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
41			22-23 y/o	–	–	–	–	–	–	–		–								–	–	–	–	–	–	C5BF014_W15	–	–	–	–
42			23-24 y/o	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
43			24-25 y/o	–	–	–	–	–	–	–	C19AR14	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
45	Loss of interest/pleasure	Nothing makes me happy anymore (past 2 weeks)	Grade/Age	PIRC-G1					PIRC-G2		FastTrack					SAFE				LIFT_L						LIFT_U				
46				var1	var2	var3	var4	var5	var1	var2	var1	var2	var3	var4	var5	var1	var2	var3	var4	var1	var2	var3	var4	var5	var6	var1	var2	var3	var4	var5
47			G1 (6-7 y/o)	–	–	–	–	–	hif31106	–	–	–	–	–	–	–	–	–	–	CJ7104_W1	–	–	–	–	–	–	–	–	–	–	–
48			G2 (7-8 y/o)	–	–	–	–	–	hif32106	–	–	–	–	–	–	–	–	–	–	CJ7404_W4	–	–	–	–	–	–	–	–	–	–	–
49			G3 (8-9 y/o	–	–	–	–	–	hif33106	–	C4GAM29	–	–	–	–	–	–	–	–	CJ7504_W5	–	–	–	–	–	–	–	–	–	–	–
50			G4 (9-10 y/o)	hif04108	–	–	–	–	–	–	C5GAM29	–	–	–	–	–	–	–	–	CJ7604_W6	–	–	–	–	–	–	–	–	–	–	–
51			G5 (10-11 y/o)	hif05108	–	–	–	–	–	–	C6GAM29	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7104_W1	–	–	–	–
52			G6 (11-12 y/o)	hif06121	hip06106	–	–	–	hif36108	–	c7md002c	p7md002c	–	–	–	–	–	–	–	CJ7804_W8	–	–	–	–	–	–	CJ7404_W4	–	–	–	–
53			G7 (12-13 y/o)	hif07121	–	–	–	–	hif37108	–	C8T30	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	CJ7504_W5	–	–	–	–
54			G8 (13-14 y/o)	hif08121	–	–	–	–	hif38108	–	C9T30	–	–	–	–	–	–	–	–	CS5A18_W10	CJ7A04_W10	–	–	–	–	–	CJ7604_W6	–	–	–	–
55			G9 (14-15 y/o)	–	–	–	–	–	hif39108	–	c10md002	p10md002	–	–	–	–	–	–	–	CS5C18_W12	–	–	–	–	–	–	–	–	–	–	–
56			G10 (15-16 y/o)	–	–	–	–	–	hif10108	–	C11T30	–	–	–	–	C10AYS005	C10CES016	P10CBP005		CJ7C04_W12	–	–	–	–	–	–	CJ7804_W8	–	–	–	–
57			G11 (16-17 y/o)	–	–	–	–	–	hif11108	–	C12T30	–	–	–	–	C11AYS005	C11CES016	P11CBP005		–	–	–	–	–	–	–	–	–	–	–	–
58			G12 (17-18 y/o)	–	–	–	–	–	hif612	–	c13md002	p13md002	–	–	–	–	–	–	–	–	–	–	–	–	–	CS5A18_W10	CJ7A04_W10	–	–	–	–

orientation | Aggressive-oppositional+ | Impulsivity-Irritability+ | Academic engagement-perform+ | Other TOCA+ | PsychiatricDx-diagnosis-DSM | DepAnx-symptoms+ | Psychotic symptoms+ | Demographics+ | Drug-alcohol use+ | Likability,Social connected+ | +

Calculation Mode: Automatic Workbook Statistics

Integrative data analysis

- Finally, variables are ‘harmonized’ or made more homogenous
  - identification of relevant domains and instruments, developing uniform variable names and labels, creating mergeable data, etc.

- For example, consider two studies that measured aggressive behavior in first-grade students

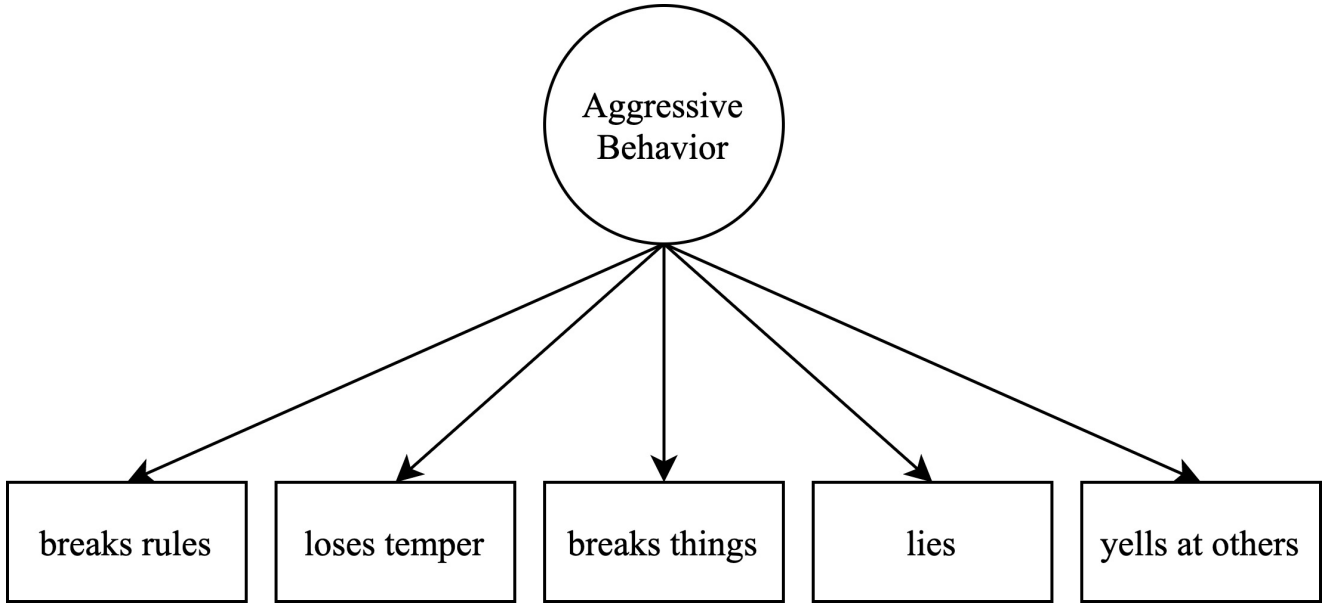
  - both studies shared the same item stem: “This student exhibits aggressive behaviors such as breaking rules, harming property, and teasing others”

Integrative data analysis

- Study A measured this item on a five-point Likert scale (0 = “Never” to 4 = “Almost always”)
- Study B measured this item as a binary variable (0 = “False”, 1 = “True”)
- May consider collapsing a sparsely distributed ordinal variable in Study A to a binary variable to ensure the two studies have equivalent response categories

Integrative data analysis

- After variables have been harmonized, we may wish to establish a theoretical construct on a common scale across studies
  - often involves estimating a latent factor from a given set of items



Integrative data analysis

- Moderated nonlinear factor analysis (MNLFA) may be useful here
  - Ensure that what we are measuring is the same in each study!

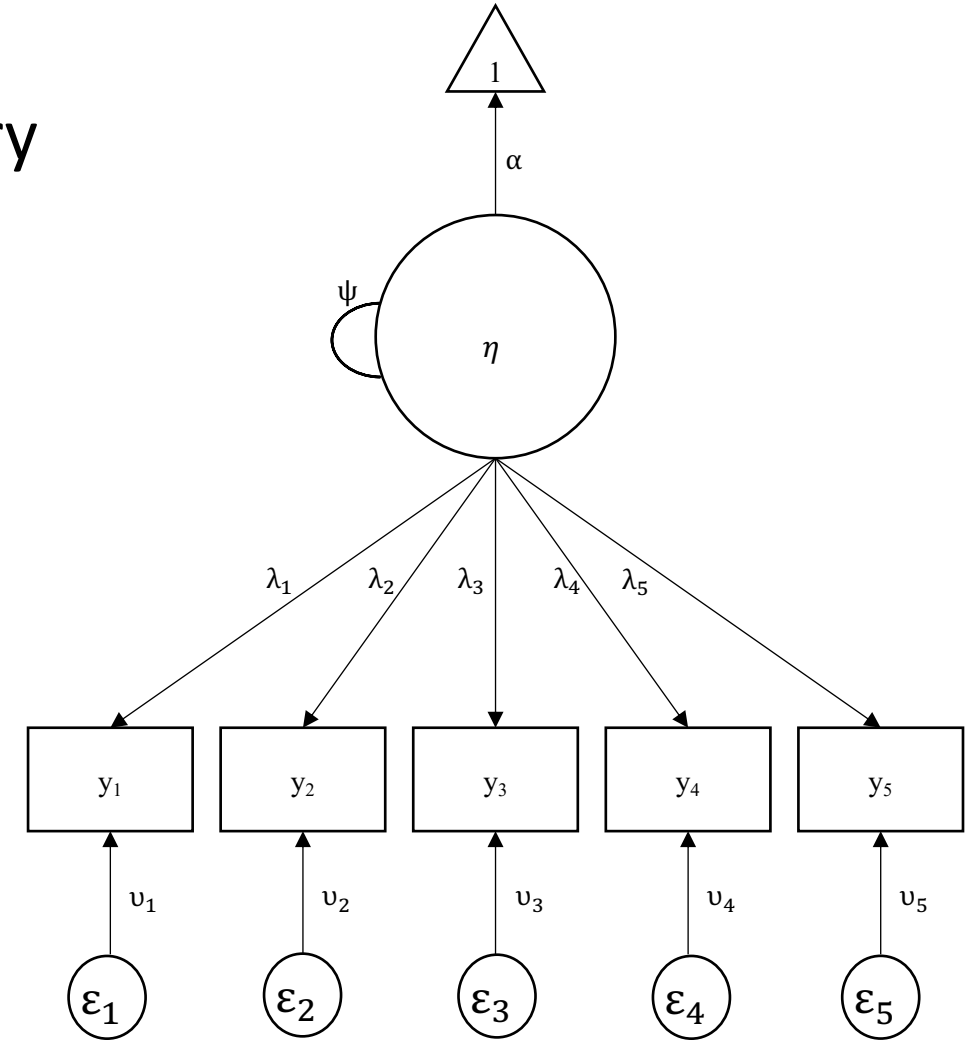
Factor analysis

- MNLFA is rooted in psychometric theory

- Linear factor model

  (continuous items!)

$$y_i = \upsilon + \Lambda\eta_i + \varepsilon_i$$



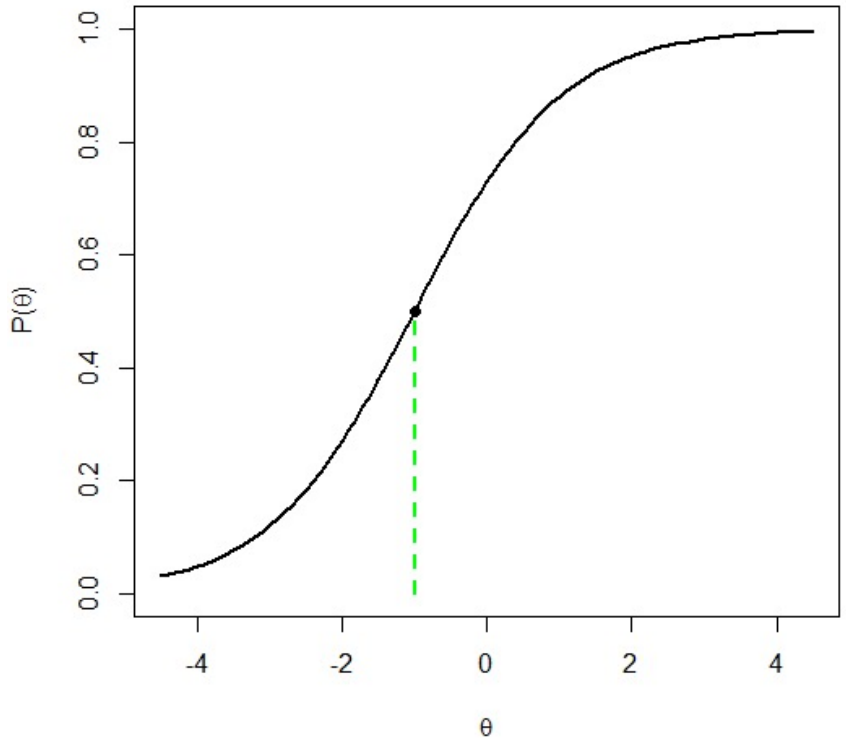
Item response theory

- 2PL (two-parameter logistic) model for binary items

$$P(y_i = 1|\theta_i) = \frac{e^{[\alpha(\theta_i - \delta)]}}{1 + e^{[\alpha(\theta_i - \delta)]}}$$

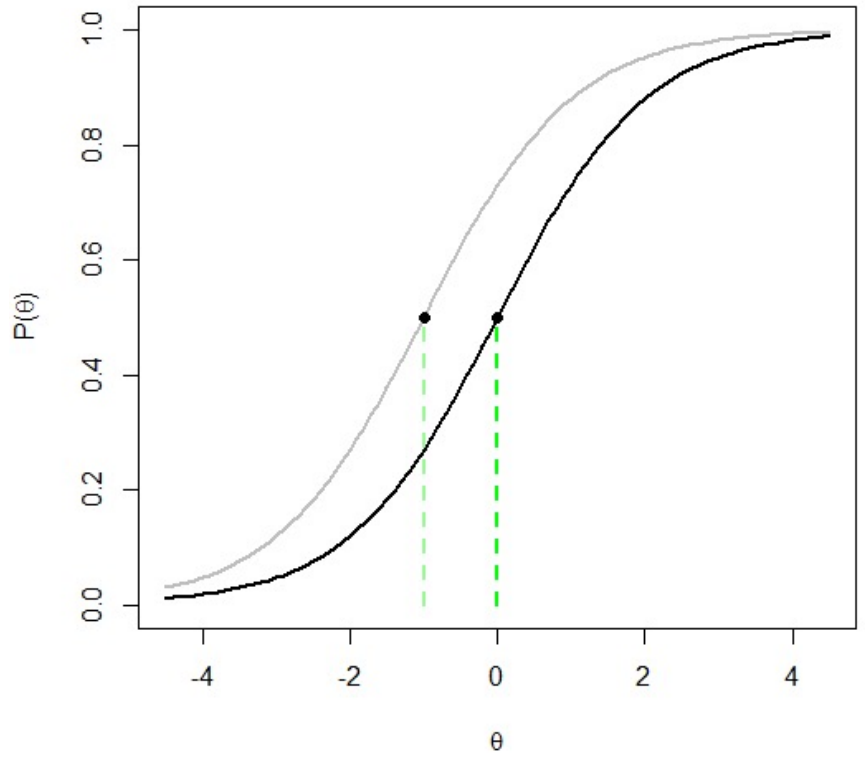
Item response theory

- Difficulty (δ = -0.5)



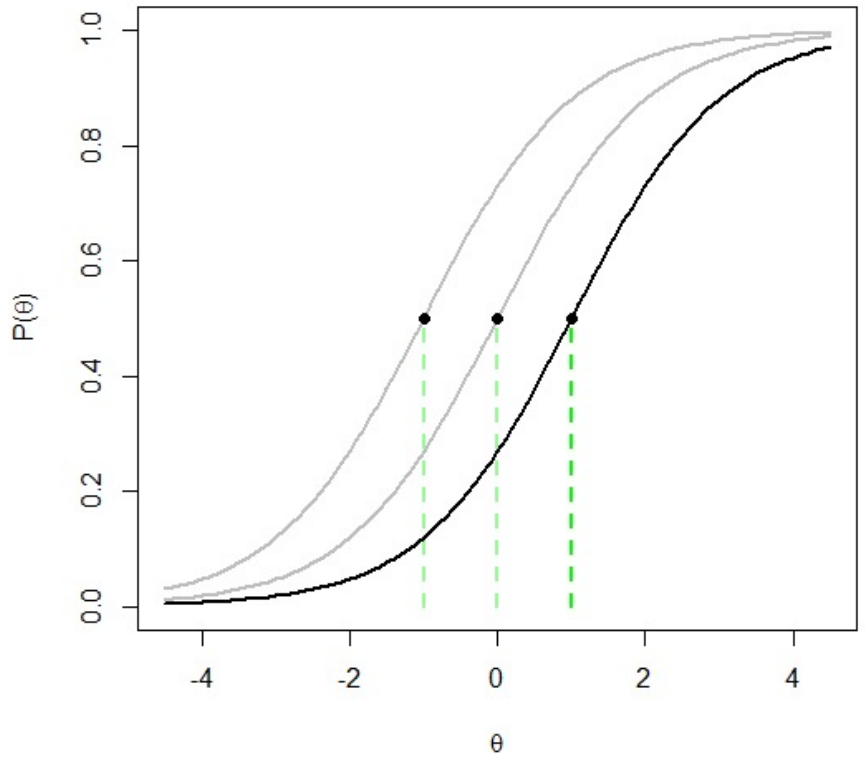
Item response theory

- Difficulty ($\delta = 0$)



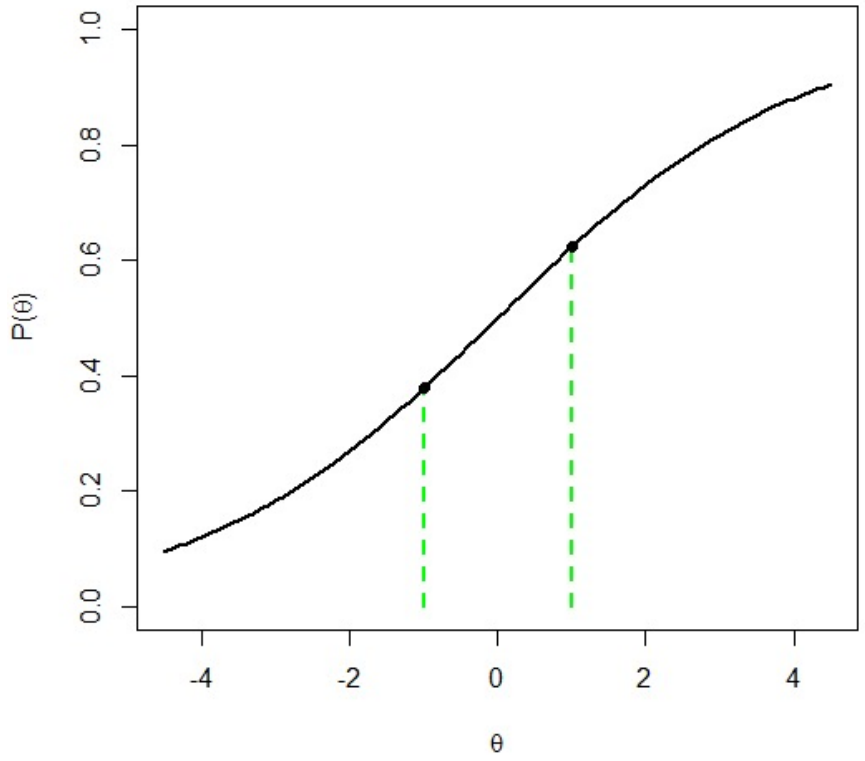
Item response theory

- Difficulty ($\delta = 1$)



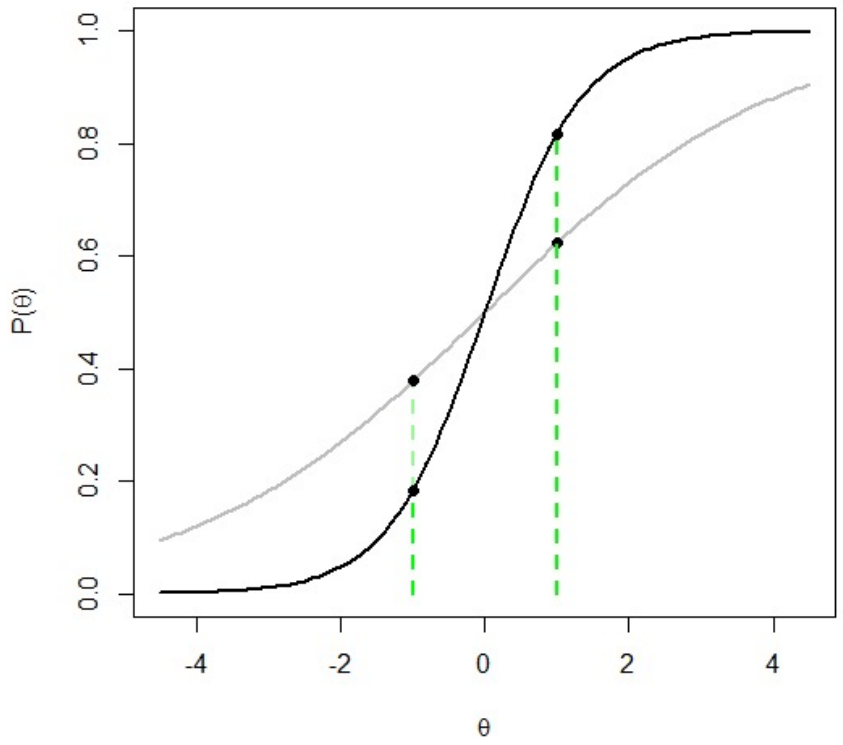
Item response theory

- Discrimination (α = 0.5)



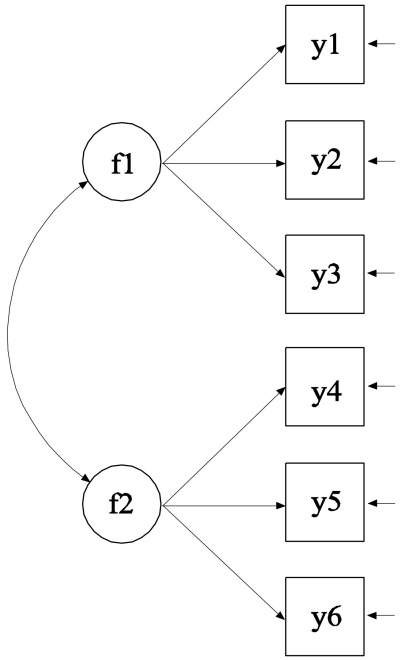
Item response theory

- Discrimination (α = 1.5)

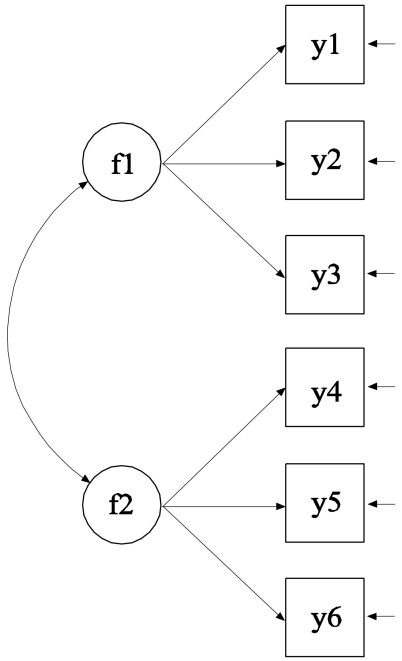


Item response theory

- Factor analysis for continuous items
- 2PL model for binary items



linear factor model



2-PL model

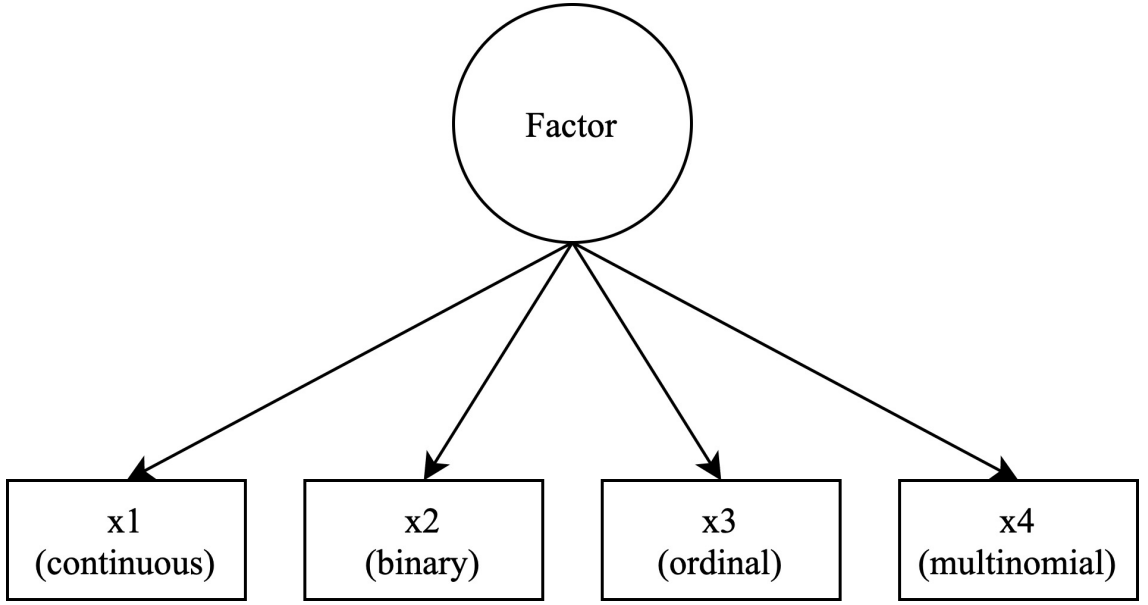
Generalized factor analysis

- Can we fit a model in which the latent factor is measured by both continuous & binary items?
    - Generalized factor analysis can accommodate items of different scale type simultaneously by specifying a response distribution and link function for a given item

Generalized factor analysis

$$g_p(\mu_{pi}) = \upsilon_p + \lambda_p \eta_i$$

- μ_{pi} = expected value of item *p* for unit *i*
- $g_p(\cdot)$= desired link function
- υ_p = item intercept
- λ_p = factor loading
- η_i = latent factor, $\eta_i \sim N(\alpha, \psi)$



Generalized factor analysis

- Continuous items

Normal response distribution:

$$y_{pi}|\eta_i \sim N(\mu_{pi}, {\sigma_p}^2)$$

Identity link function:

$$g_p(\mu_{pi}) = \mu_{pi}$$

Gives:

$$\mu_{pi} = \upsilon_p + \lambda_p \eta_i$$

Note: linear factor model from before!
(measurement error implicitly taken into account
By modeling the conditional response distribution)

Generalized factor analysis

- Binary items

Bernoulli response distribution:

$$y_{pi}|\eta_i \sim Ber(\mu_{pi})$$

Logit link function:

$$g_p(\mu_{pi}) = \ln[\mu_{pi}/(1-\mu_{pi})]$$

Gives:

$$\ln\left(\frac{\mu_{pi}}{1-\mu_{pi}}\right) = \upsilon_p + \lambda_p\eta_i$$

Or logistic link function:

$$\mu_{pi} = g_p{}^{-1}(\upsilon_{pi} + \lambda_p\eta_i)$$

Gives:

$$\frac{e^{\{\lambda_p[\eta_i-(-\upsilon_p/\lambda_p)]\}}}{1+e^{\{\lambda_p[\eta_i-(-\upsilon_p/\lambda_p)]\}}}$$

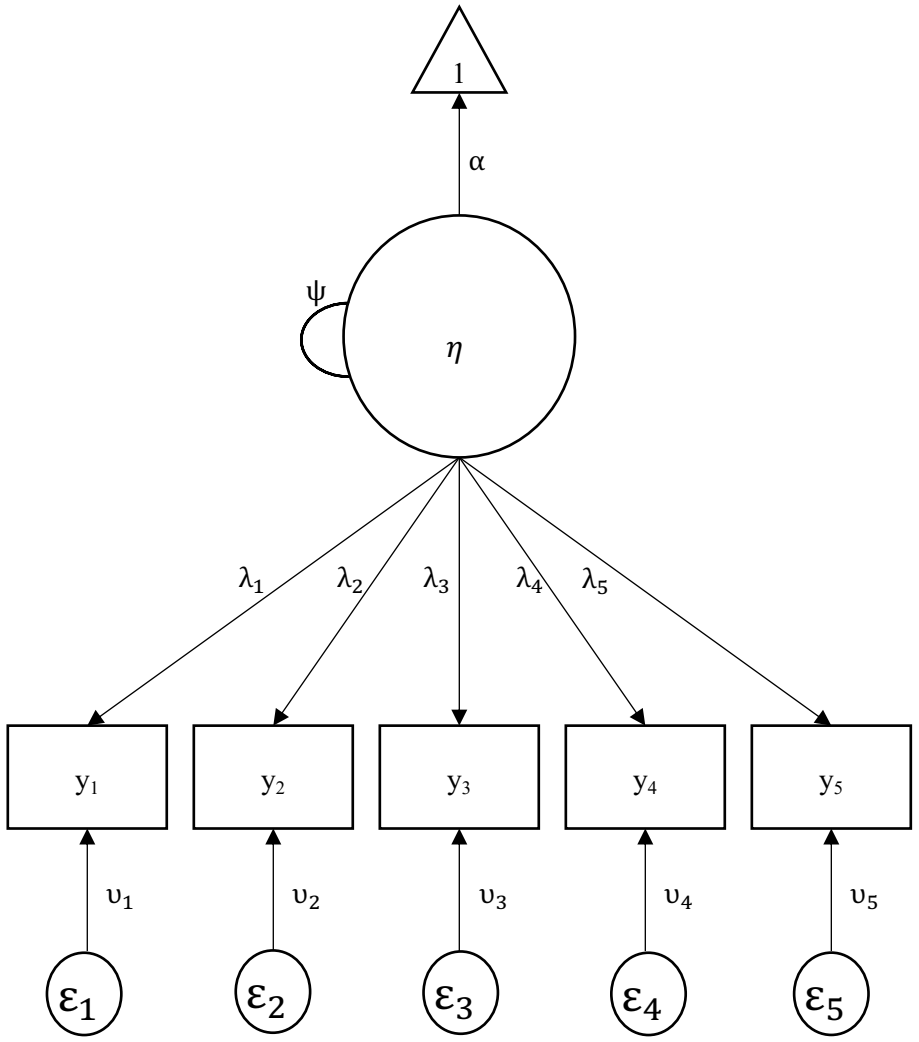
Note: reparameterization of the 2-PL model from before!

Measurement invariance

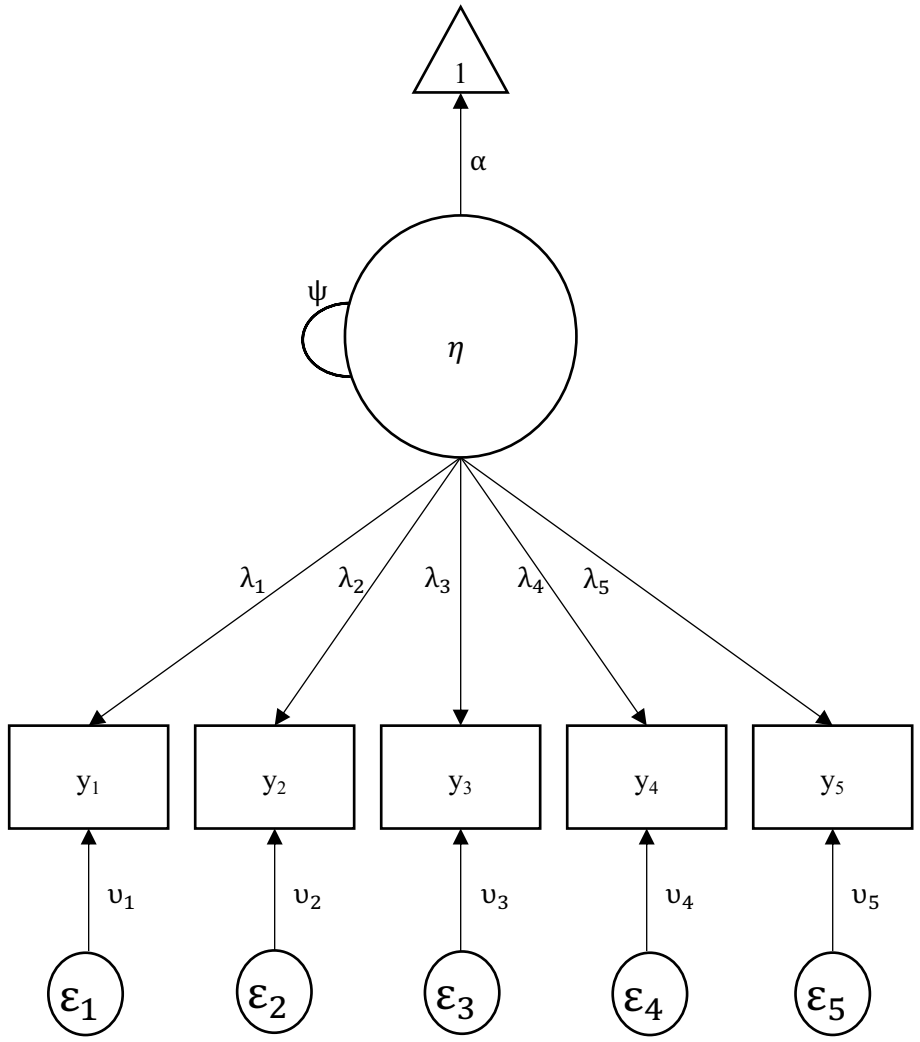
- So GFA extends FA & IRT
  - Helpful for IDA

- Still limitations regarding measurement invariance

Males



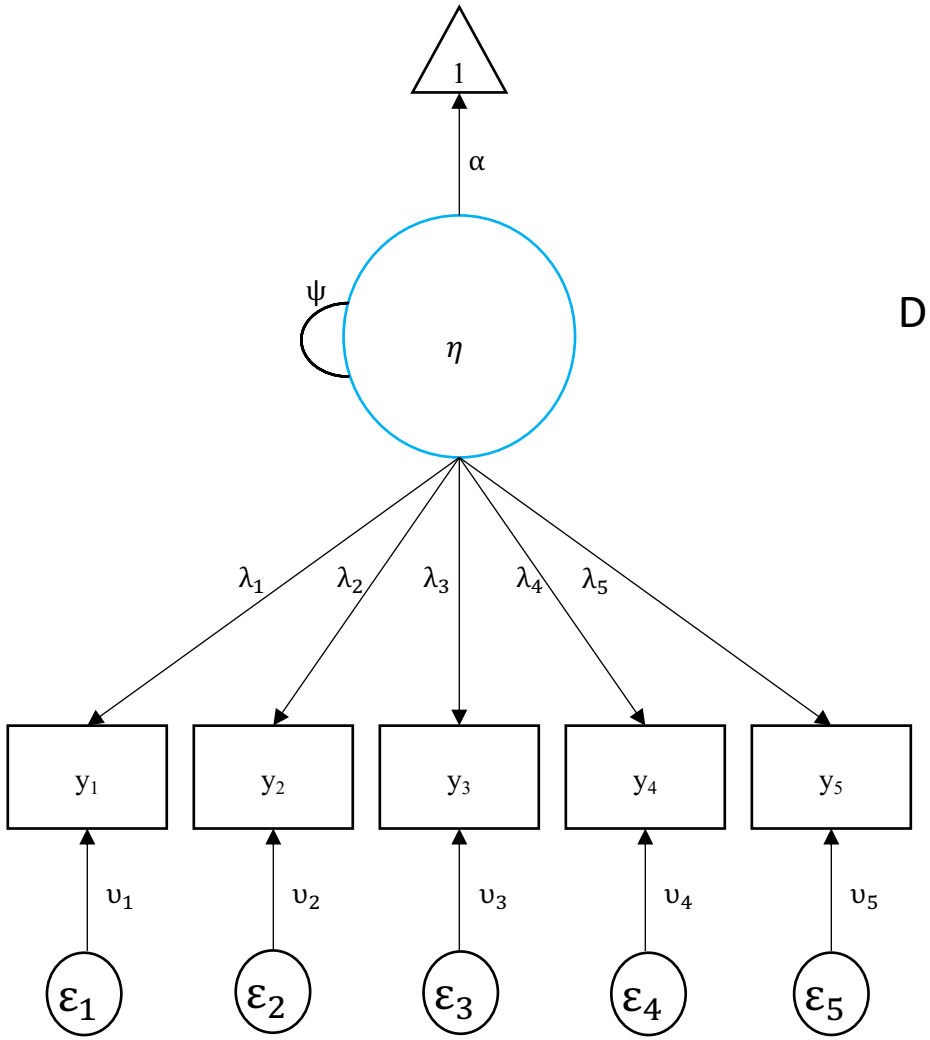
Females

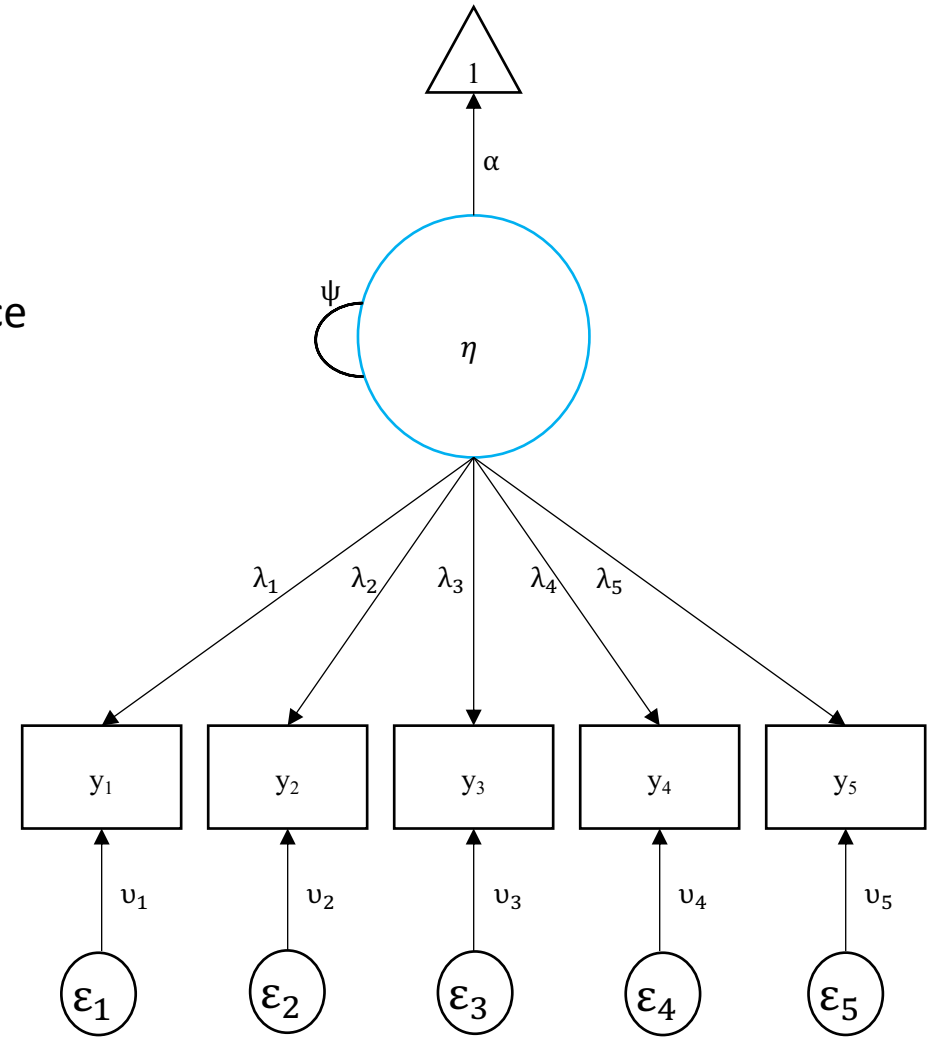


Males

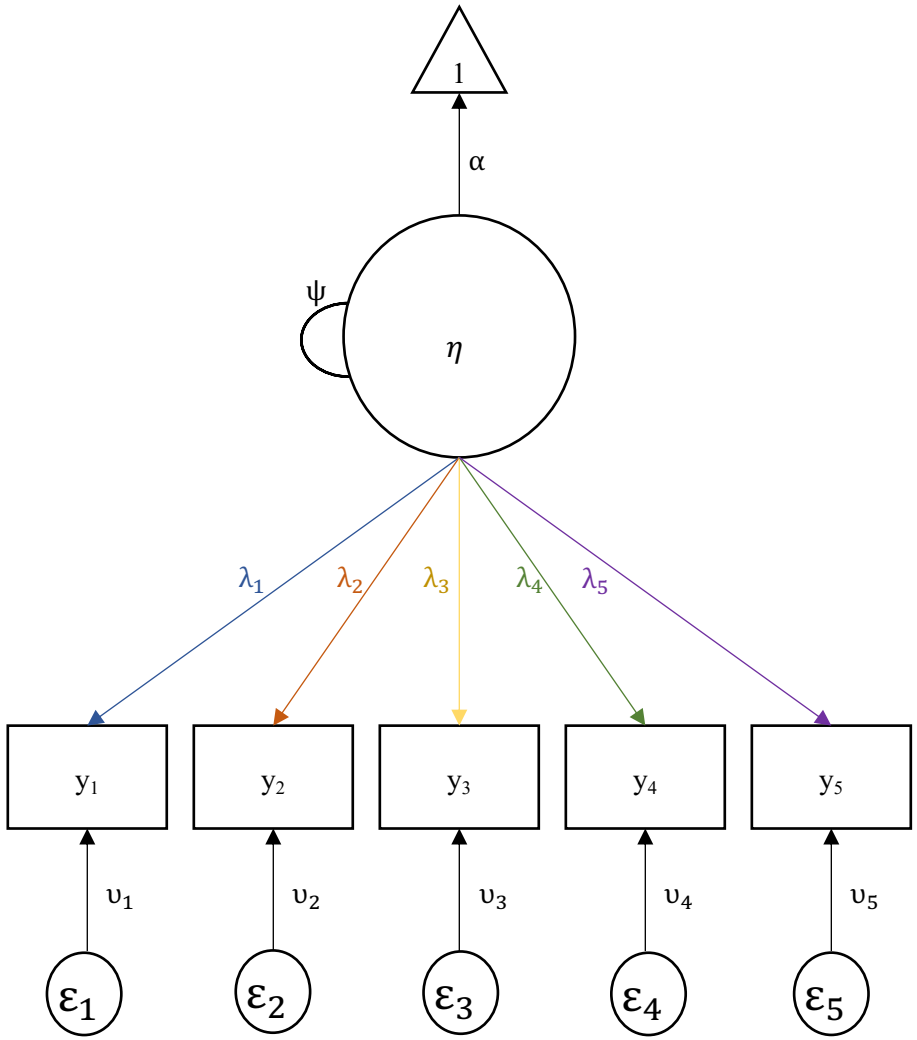
Females

Dimensional Invariance



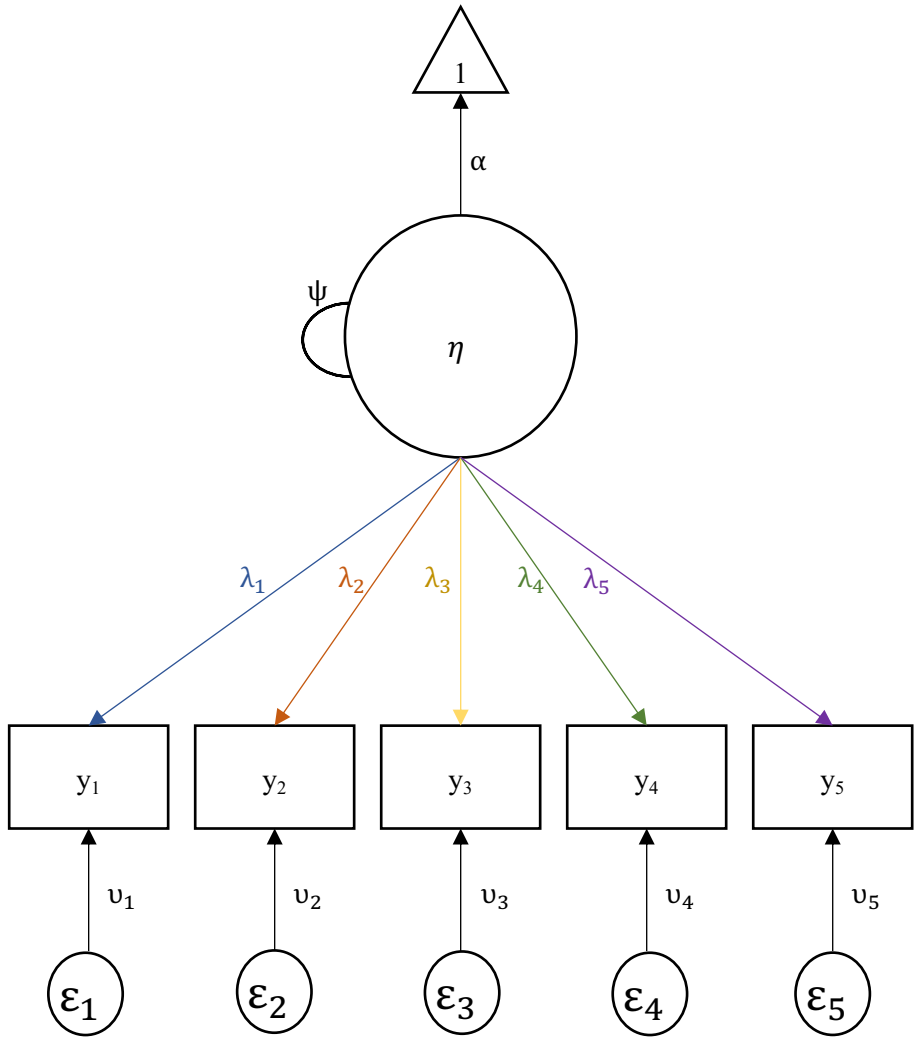


Males



Metric Invariance

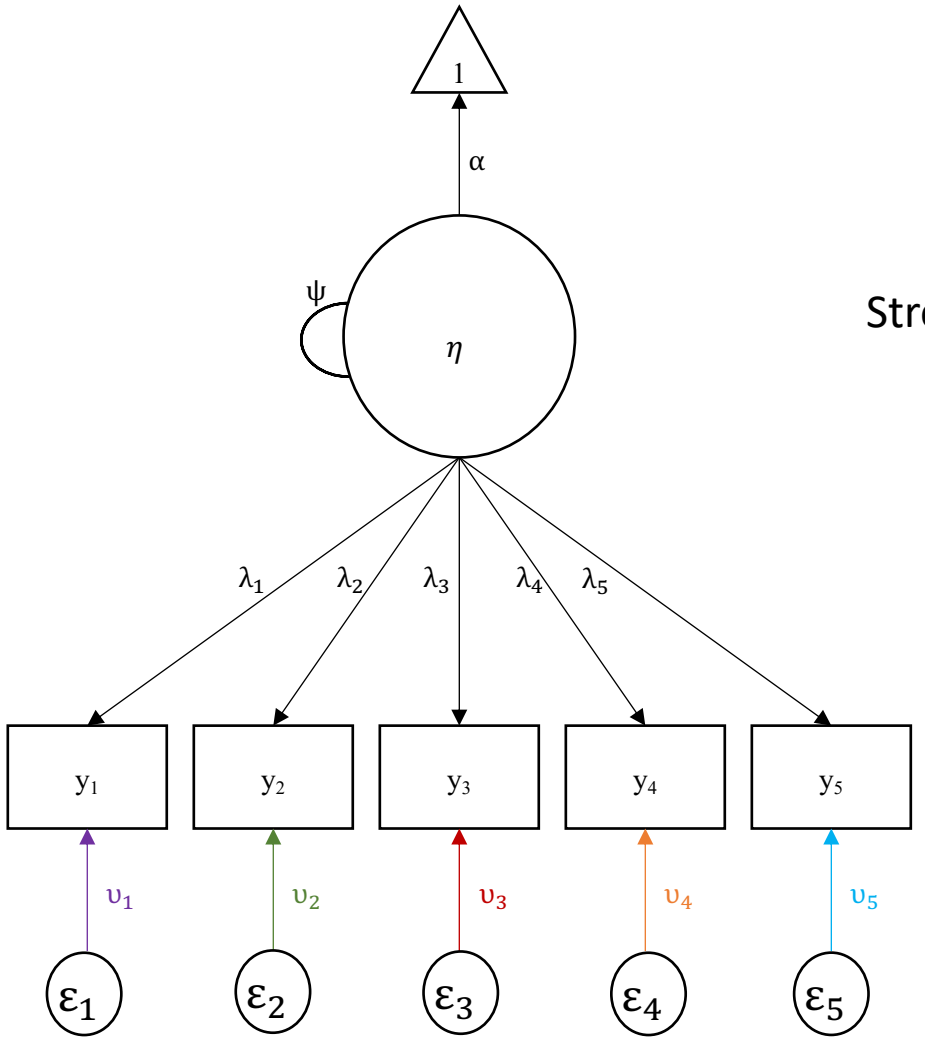
Females

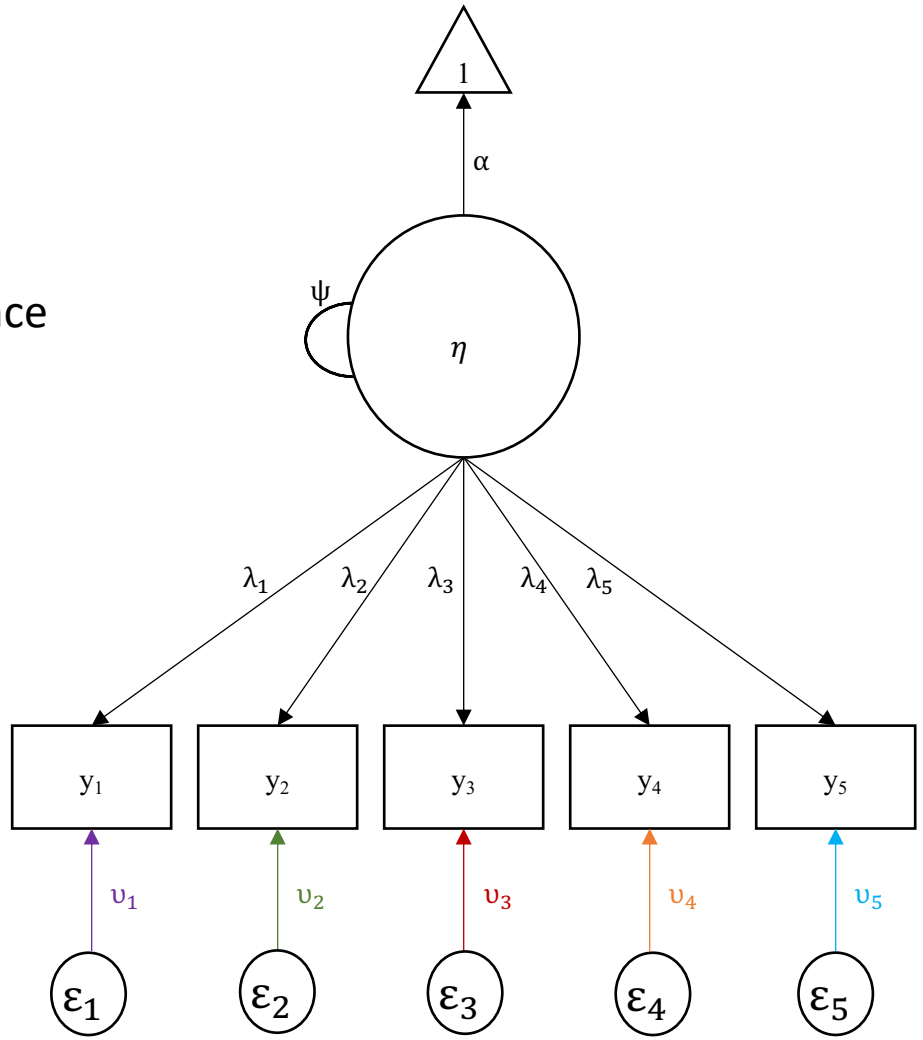


Males

Females

Strong Factorial Invariance





Measurement invariance

- Typically, when using multi-sample approaches, (a) discrete groups are (b) tested in succession

- Discrete group testing may be natural to compare group membership (treatment vs. control, old vs. young, female vs. males)
  - What about continuous covariates such as age or IQ

Measurement invariance

- Even with discrete groups, this process is typically conducted sequentially
  - Invariance between treatment vs. control is tested
  - Then, invariance between old vs. young is tested
  - Then, invariance between male vs. female is tested

- With a large number of groups, the number of tests to conduct may become unwieldy, while cell sizes of the various strata may become small
  - Treatment, old, male, vs. treatment, old, female
  - Treatment, young, male, vs. treatment, young, female

Moderated nonlinear factor analysis

- MNLFA extends the generalized factor analysis framework by allowing the four model parameters to vary as a function of covariates

α_i = latent factor mean for individual *i*

ψ_i = latent factor variance for individual *i*

υ_{pi} = intercept for item *p* for individual *i*

λ_{pi} = factor loading for item *p* for individual *i*

- In GFA, these parameters are assumed to be equal across individuals

Moderated nonlinear factor analysis

$$g_p(\mu_{pi}) = \upsilon_{pi} + \lambda_{pi}\eta_i, \text{ where } \eta_i \sim N(\alpha_i, \psi_i)$$

$$\alpha_i = \alpha_0 + \sum_{w=1}^{W} \alpha_w x_{wi}$$

$$\psi_i = \psi_0 + e^{(\sum_{w=1}^{W} \beta_w x_{wi})}$$

Where x_w denotes moderator *x* with *W* total moderators

Moderated nonlinear factor analysis

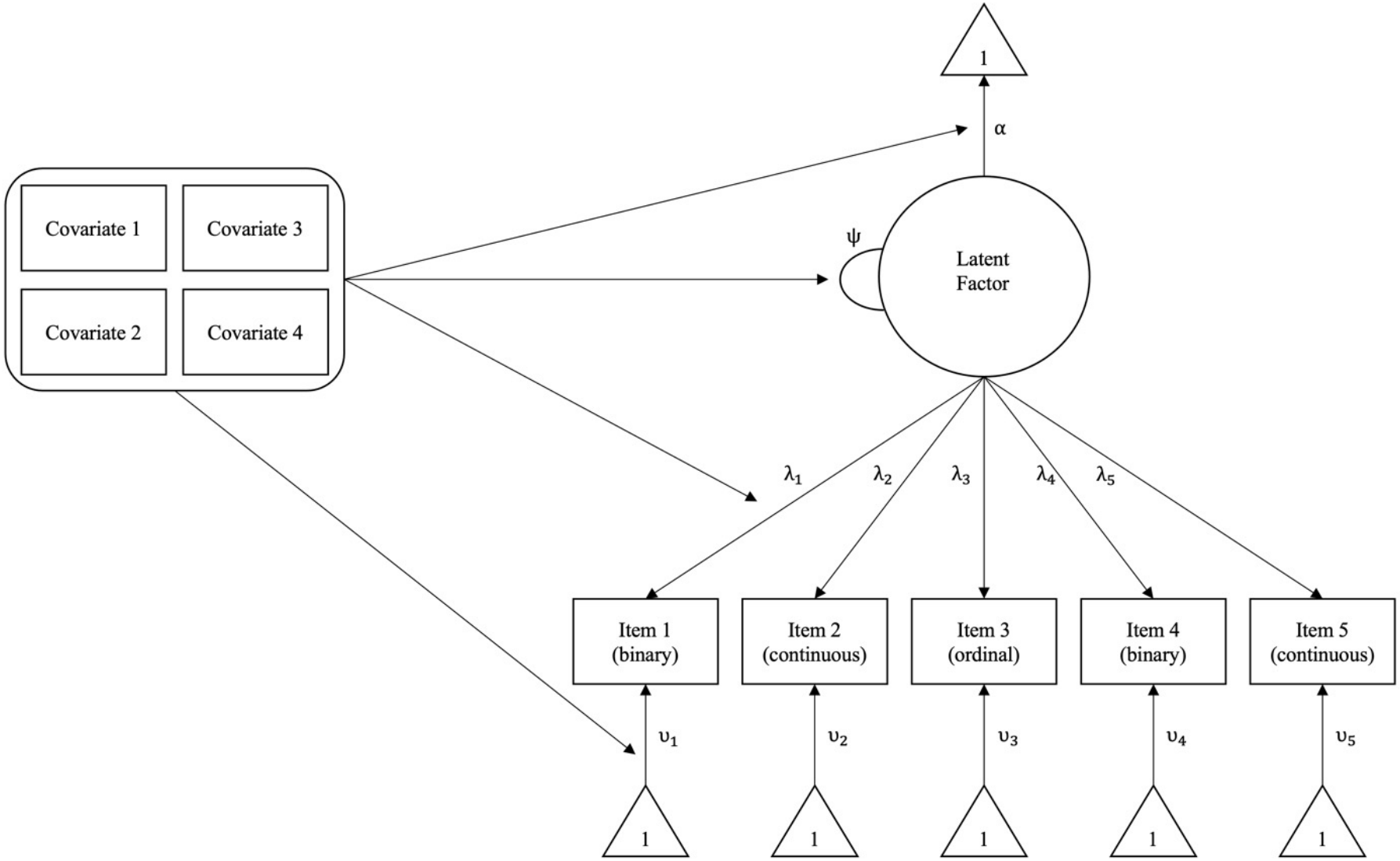
$$g_p(\mu_{pi}) = \upsilon_{pi} + \lambda_{pi}\eta_i, \text{ where } \eta_i \sim N(\alpha_i, \psi_i)$$

$$\upsilon_{pi} = \upsilon_{0p} + \sum_{w=1}^{W} \upsilon_{wp} x_{wi}$$

$$\lambda_{pi} = \lambda_{0p} + \sum_{w=1}^{W} \lambda_{wp} x_{wi}$$

υ_{wp} and λ_{wp} are effect of moderator on item intercept and factor loading

Moderated nonlinear factor analysis



Example

- Use empirical data from five independent prevention trials to estimate the effect of a latent aggressive-disruptive behavior factor on high school graduation

- 9 items total, harmonized across studies
  - Takes property
  - Teases classmates
  - Breaks rules

Sample

Table 1
Sample Demographic Characteristics

	Sample Sizes Across Study				
	Race		Sex		Total
	Black	White	Female	Male	
Study 1	432	385	254	563	817
Study 2	7	444	234	217	451
Study 3	1,322	562	1,016	868	1,884
Study 4	556	83	302	337	639
Study 5	144	13	86	71	157

	Item Endorsement Rates Across Study					
	Study					Total
	Study 1	Study 2	Study 3	Study 4	Study 5	
Breaks rules	.895	.525	.616	.521	.703	.661
Harms property	-	.175	.362	.202	.351	.307
Breaks things	.540	.175	.316	.152	.359	.331
Takes property	.668	.220	.417	.227	.487	.431
Fights	.816	.375	.360	.291	.487	.458
Lies	.738	.255	.438	.236	.583	.466
Yells at others	.816	.370	.490	.335	.506	.530
Stubborn	.876	.745	.620	.336	.551	.631
Teases others	.821	.575	.542	.382	.532	.578

Note: Item "harms property" was not measured in Study 1.

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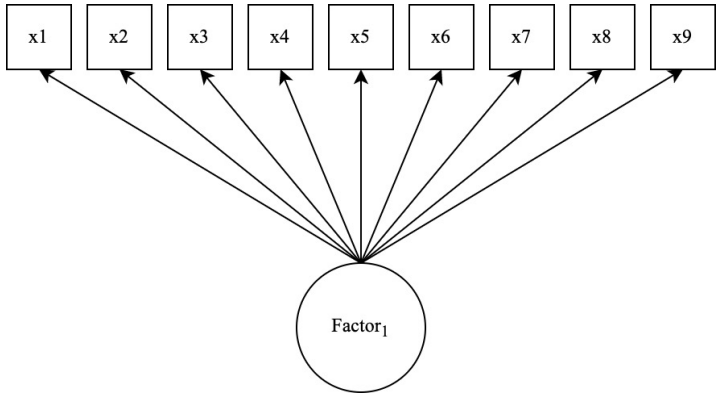
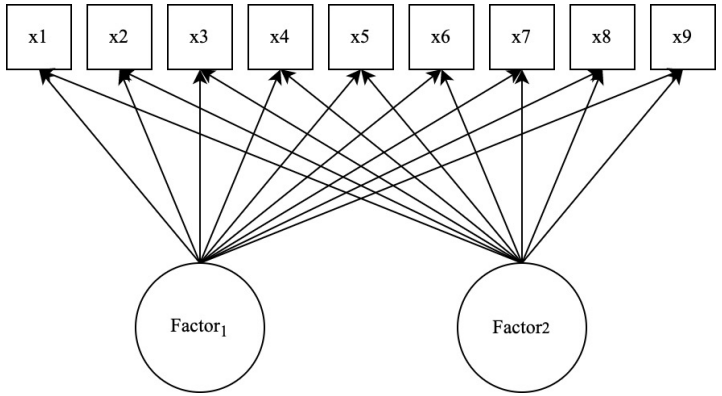
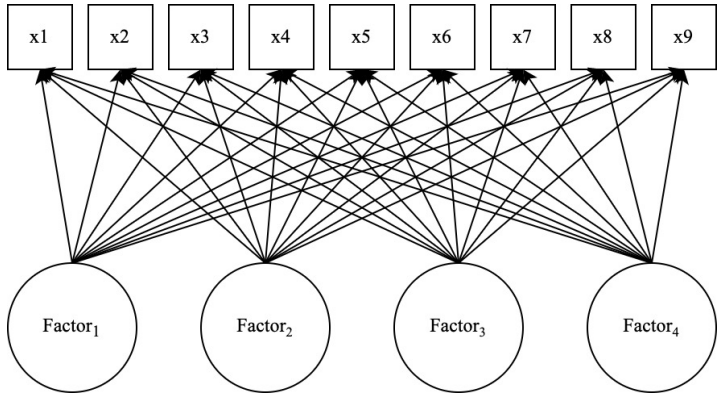
Note: Item "harms property" was not measured in Study 1.

Preliminary analyses

- Conduct CFA on the 9 items to each study independently (plus dimensionality testing)
- Final CFA model fit to all (pooled) observations simultaneously
  - Factor loadings ranging from 0.82 to 0.91 (RMSEA = .049, CFI = .996, TLI = .995, SRMR = .027)







MNLFA model building

- Consider three moderators of aggressive-disruptive behaviors:
  - Sex (0 = female, 1 = male)
  - Race (0 = Black, 1 = White)
  - Study (dummy indicators for Study 2 through Study 5)

- First, allow each of the covariates to simultaneously moderate the factor mean and factor variance
  - This establishes a baseline model, to which future models are compared

MNLFA model building

- Next, leaving each of the covariate effects on the factor mean and variance (regardless of significance), explore item-specific moderation
  - Item 1 intercept and loading moderation (in addition to factor moderation)
  - Conduct LRT comparing item 1 moderation model to baseline model (factor moderation only)
  - If LRT significant, include any item-specific covariate effects with significant *p*-values
- Continue this process for each item

Table 2

Examining DIF in Factor and Item Parameters Using a Sequential Modeling Building Approach

					Factor Mean		Factor Variance		Item Intercept		Item Loading	
Model	Covariate	Parameters	LL	SF	Est	*p*	Est	*p*	Est	*p*	Est	*p*
Baseline		29	-13603.1	1.050								
	Study 2				-1.07	< .01	0.19	.30				
	Study 3				-1.27	< .01	1.00	< .01				
	Study 4				-2.61	< .01	1.19	< .01				
	Study 5				-1.21	< .01	1.44	< .01				
	Sex				0.66	< .01	0.48	< .01				
	Race				-0.84	< .01	0.54	< .01				
Breaks rules		42	-13467.8	1.022								
	Study 2				-0.65	< .01	-0.47	.01	-0.90	.38	-0.11	.91
	Study 3				-0.85	< .01	0.07	.51	-0.70	.17	-0.44	.34
	Study 4				-1.70	< .01	0.43	< .01	-0.79	.17	**-1.24**	.01
	Study 5				-0.87	< .01	0.58	.01	-0.71	.43	-1.13	.10
	Sex				0.40	< .01	0.18	.03	-0.28	.37	**-0.61**	.01
	Race				-0.58	< .01	0.12	.17	0.67	.12	0.22	.52
Harms property		41	-13517.2	1.046								
	Study 2				-1.11	< .01	0.15	.40	**1.30**	.03	1.43	.36
	Study 3				-1.37	< .01	1.05	< .01	**1.13**	< .01	**-1.48**	< .01
	Study 4				-2.65	< .01	1.14	< .01	**1.72**	< .01	-0.37	.39
	Study 5				-1.26	< .01	1.40	< .01	**0.93**	.01	-0.20	.76
	Sex				0.65	< .01	0.48	< .01	0.04	.76	-0.20	.08
	Race				-0.84	< .01	0.54	< .01	0.08	.67	0.14	.29
Breaks things		41	-13594.4	0.980								
	Study 2				-1.05	< .01	0.23	.21	0.55	.23	-0.03	.97
	Study 3				-1.28	< .01	1.00	< .01	**1.25**	< .01	0.56	.19
	Study 4				-2.60	< .01	1.22	< .01	**0.72**	.02	0.74	.13
	Study 5				-1.20	< .01	1.47	< .01	**0.96**	.01	**0.50**	< .01
	Sex				0.65	< .01	0.48	< .01	0.16	.35	-0.31	.21
	Race				-0.84	< .01	0.52	< .01	0.23	.44	0.81	.05
Takes property		41	-13577.4	1.039								
	Study 2				-1.09	< .01	0.21	.26	-0.23	.58	-0.18	.68
	Study 3				-1.35	< .01	0.97	< .01	**0.79**	< .01	-0.16	.58
	Study 4				-2.66	< .01	1.18	< .01	0.42	.17	-0.27	.41
	Study 5				-1.29	< .01	1.39	< .01	**1.38**	.03	0.48	.55
	Sex				0.67	< .01	0.50	< .01	-0.30	.10	-0.23	.23
	Race				-0.83	< .01	0.56	< .01	-0.23	.29	-0.11	.64

Note: Table 2 continued below.

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Examining DIF in Factor and Item Parameters Using a Sequential Modeling Building Approach

					Factor Mean		Factor Variance		Item Intercept		Item Loading	
Model	Covariate	Parameters	LL	SF	Est	*p*	Est	*p*	Est	*p*	Est	*p*
Fights		41	-13520.6	1.032								
	Study 2				-1.04	< .01	0.15	.43	1.24	.22	0.83	.30
	Study 3				-1.16	< .01	1.08	< .01	**-1.59**	< .01	**-0.62**	.01
	Study 4				-2.56	< .01	1.17	< .01	0.25	.54	0.07	.84
	Study 5				-1.15	< .01	1.46	< .01	-0.69	.13	-0.40	.33
	Sex				0.62	< .01	0.48	< .01	**0.41**	.01	-0.23	.10
	Race				-0.83	< .01	0.50	< .01	0.10	.63	0.11	.54
Lies		41	-13571.0	1.025								
	Study 2				-1.03	< .01	0.25	.17	**-0.99**	.01	-0.55	.21
	Study 3				-1.30	< .01	1.05	< .01	-0.32	.15	**-0.96**	< .01
	Study 4				-2.63	< .01	1.24	< .01	-0.59	.05	**-0.89**	.01
	Study 5				-1.33	< .01	1.55	< .01	0.41	.40	**-1.31**	< .01
	Sex				0.69	< .01	0.48	< .01	**-0.40**	.03	-0.21	.16
	Race				-0.87	< .01	0.54	< .01	-0.11	.57	**-0.41**	.01
Yells at others		41	-13567.4	1.033								
	Study 2				-0.99	< .01	0.08	.67	2.35	.17	2.59	.10
	Study 3				-1.23	< .01	1.02	< .01	**-0.97**	< .01	**-0.65**	< .01
	Study 4				-2.62	< .01	1.23	< .01	**-0.75**	.03	**-0.73**	< .01
	Study 5				-1.14	< .01	1.44	< .01	**-1.17**	.01	-0.44	.26
	Sex				0.70	< .01	0.47	< .01	**-0.77**	< .01	-0.17	.19
	Race				-0.84	< .01	0.54	< .01	-0.31	.14	**-0.29**	.04
Stubborn		41	-13507.2	1.037								
	Study 2				-1.27	< .01	0.49	.01	1.06	.23	-0.44	.24
	Study 3				-1.26	< .01	1.05	< .01	**-0.90**	.01	**-0.64**	.01
	Study 4				-2.53	< .01	1.22	< .01	**-2.25**	< .01	**-0.97**	< .01
	Study 5				-1.13	< .01	1.51	< .01	**-2.22**	< .01	**-1.04**	< .01
	Sex				0.71	< .01	0.44	< .01	**-0.77**	< .01	-0.11	.28
	Race				-0.86	< .01	0.56	< .01	0.01	.97	-0.16	.18
Teases others		41	-13576.5	1.036								
	Study 2				-1.20	< .01	0.32	.08	0.69	.27	-0.40	.23
	Study 3				-1.27	< .01	1.02	< .01	**-0.60**	.02	**-0.47**	.02
	Study 4				-2.65	< .01	1.24	< .01	-0.63	.09	**-0.61**	.01
	Study 5				-1.16	< .01	1.45	< .01	**-1.43**	.00	**-0.67**	.03
	Sex				0.66	< .01	0.51	< .01	-0.36	.09	**-0.35**	.01
	Race				-0.82	< .01	0.51	< .01	**-0.58**	.01	-0.18	.19

Note: LL = log-likelihood; SF = scaling factor used in likelihood ratio test; significant item moderation effects are bolded.

Table 2

Examining DIF in Factor and Item Parameters Using a Sequential Modeling Building Approach

					Factor Mean		Factor Variance		Item Intercept		Item Loading	
Model	Covariate	Parameters	LL	SF	Est	*p*	Est	*p*	Est	*p*	Est	*p*
Fights		41	-13520.6	1.032								
	Study 2				-1.04	< .01	0.15	.43	1.24	.22	0.83	.30
	Study 3				-1.16	< .01	1.08	< .01	**-1.59**	< .01	**-0.62**	.01
	Study 4				-2.56	< .01	1.17	< .01	0.25	.54	0.07	.84
	Study 5				-1.15	< .01	1.46	< .01	-0.69	.13	-0.40	.33
	Sex				0.62	< .01	0.48	< .01	**0.41**	.01	-0.23	.10
	Race				-0.83	< .01	0.50	< .01	0.10	.63	0.11	.54
Lies		41	-13571.0	1.025								
	Study 2				-1.03	< .01	0.25	.17	**-0.99**	.01	-0.55	.21
	Study 3				-1.30	< .01	1.05	< .01	-0.32	.15	**-0.96**	< .01
	Study 4				-2.63	< .01	1.24	< .01	-0.59	.05	**-0.89**	.01
	Study 5				-1.33	< .01	1.55	< .01	0.41	.40	**-1.31**	< .01
	Sex				0.69	< .01	0.48	< .01	**-0.40**	.03	-0.21	.16
	Race				-0.87	< .01	0.54	< .01	-0.11	.57	**-0.41**	.01
Yells at others		41	-13567.4	1.033								
	Study 2				-0.99	< .01	0.08	.67	2.35	.17	2.59	.10
	Study 3				-1.23	< .01	1.02	< .01	**-0.97**	< .01	**-0.65**	< .01
	Study 4				-2.62	< .01	1.23	< .01	**-0.75**	.03	**-0.73**	< .01
	Study 5				-1.14	< .01	1.44	< .01	**-1.17**	.01	-0.44	.26
	Sex				0.70	< .01	0.47	< .01	**-0.77**	< .01	-0.17	.19
	Race				-0.84	< .01	0.54	< .01	-0.31	.14	**-0.29**	.04
Stubborn		41	-13507.2	1.037								
	Study 2				-1.27	< .01	0.49	.01	1.06	.23	-0.44	.24
	Study 3				-1.26	< .01	1.05	< .01	**-0.90**	.01	**-0.64**	.01
	Study 4				-2.53	< .01	1.22	< .01	**-2.25**	< .01	**-0.97**	< .01
	Study 5				-1.13	< .01	1.51	< .01	**-2.22**	< .01	**-1.04**	< .01
	Sex				0.71	< .01	0.44	< .01	**-0.77**	< .01	-0.11	.28
	Race				-0.86	< .01	0.56	< .01	0.01	.97	-0.16	.18
Teases others		41	-13576.5	1.036								
	Study 2				-1.20	< .01	0.32	.08	0.69	.27	-0.40	.23
	Study 3				-1.27	< .01	1.02	< .01	**-0.60**	.02	**-0.47**	.02
	Study 4				-2.65	< .01	1.24	< .01	-0.63	.09	**-0.61**	.01
	Study 5				-1.16	< .01	1.45	< .01	**-1.43**	.00	**-0.67**	.03
	Sex				0.66	< .01	0.51	< .01	-0.36	.09	**-0.35**	.01
	Race				-0.82	< .01	0.51	< .01	**-0.58**	.01	-0.18	.19

Note: LL = log-likelihood; SF = scaling factor used in likelihood ratio test; significant item moderation effects are bolded.

MNLFA model building

- Remove any covariate moderation effects if:
  - Item-specific moderation model results in non-significant LRT
  - Moderation effects are non-significant

- Estimate a penultimate model
  - Simultaneously including item-specific covariate effects previously found to be significant

- Remove any non-significant effects
  - This last pruning effort results in the final MNLFA model!

MNLFA model building

Table 3
Final MNLFA Model Examining Covariate Effects on Factor Mean and Variance

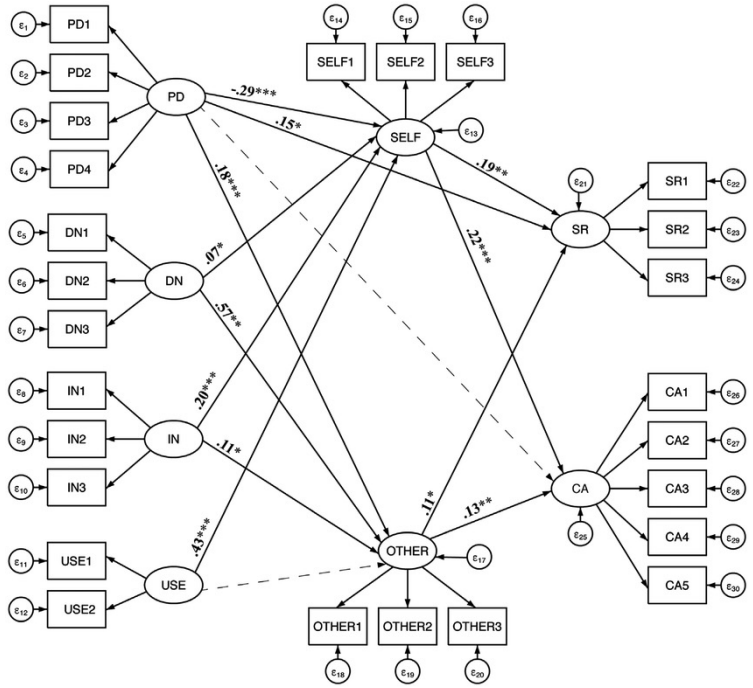
Covariate Effect	Estimate	*SE*	*p*
Factor Mean			
Study 2	-0.62	0.09	< .01
Study 3	-0.87	0.07	< .01
Study 4	-1.51	0.10	< .01
Study 5	-0.67	0.13	< .01
Sex	0.44	0.05	< .01
Race	-0.54	0.06	< .01
Factor Variance			
Study 2	-0.25	0.17	.15
Study 3	0.17	0.12	.13
Study 4	0.24	0.13	.07
Study 5	0.40	0.21	.06
Sex	0.07	0.08	.41
Race	0.12	0.09	.20

Table 4
Final MNLFA Model Examining Covariate Effects on Item Intercepts and Factor Loadings

Covariate Effect	Intercept			Loading		
	Estimate	*SE*	*p*	Estimate	*SE*	*p*
1. Breaks rules	-3.33	0.21	< .01	2.91	0.21	< .01
Study 4	-	-	-	-0.43	0.12	< .01
Sex	-	-	-	-0.34	0.10	< .01
2. Harms property	0.50	0.21	.02	3.36	0.31	< .01
Study 3	0.98	0.18	< .01	-1.73	0.30	< .01
Study 4	1.43	0.22	< .01	-	-	-
3. Breaks things	0.06	0.24	.82	3.29	0.27	< .001
Study 3	0.66	0.20	< .01	-	-	-
4. Takes property	-0.84	0.20	< .01	3.13	0.22	< .01
Study 3	0.81	0.15	< .01	-	-	-
Study 5	0.43	0.28	.12	-	-	-
5. Fights	-1.71	0.25	< .01	3.19	0.27	< .01
Study 3	-1.21	0.21	< .01	-0.79	0.27	< .01
Sex	0.20	0.12	< .01	-	-	-
6. Lies	-1.54	0.19	< .01	2.98	0.21	< .01
Study 2	-1.05	0.25	< .01	-	-	-
Study 5	-	-	-	-0.77	0.27	< .01
Race	-	-	-	-0.38	0.17	.02
7. Yells at others	-2.53	0.23	< .01	2.88	0.23	< .01
Study 3	-0.48	0.21	.02	-0.54	0.23	.02
Study 5	-0.93	0.30	< .01	-	-	-
Sex	-0.52	0.12	< .01	-	-	-
8. Stubborn	-3.04	0.18	< .01	2.09	0.15	< .01
Study 4	-1.15	0.14	< .01	-	-	-
Study 5	-1.39	0.27	< .01	-	-	-
Sex	-0.60	0.12	< .01	-	-	-
9. Teases others	-2.45	0.18	< .01	2.46	0.17	< .01
Study 5	-1.01	0.29	< .01	-	-	-
Race	-0.18	0.12	.13	-	-	-

Additional analyses

- May not be feasible to estimate MNLFA measurement model within more complex SEM



Additional analyses

- Instead, we may incorporate estimated factor scores into subsequent analyses
  - Original RQ: what is the effect of aggressive-disruptive behavior on high school graduation?

$$P(Graduate = 1) = \beta_0 + \beta_1 Factor + \beta_2 Study2 + \beta_3 Study3 + \beta_4 Study4 + \beta_5 Study5 + \beta_6 Sex + \beta_7 Race$$

- Important to include any covariates from MNLFA model
  - Think of doubly robust

Additional analyses

Table 5
Logistic Regression Results of High School Completion

	Odds Ratio	95% C.I.	*p*
Study 2	1.52	[0.97, 2.39]	.065
Study 3	0.07	[0.86, 1.32]	.546
Study 4	3.40	[2.36, 4.89]	< .001
Study 5	0.05	[0.03, 0.08]	< .001
Male	0.76	[0.63, 0.91]	.002
White	1.19	[0.97, 1.46]	.091
Factor	0.77	[0.71, 0.84]	< .001

Extensions and implications

- *Note*: this modeling approach can be used without combining data from multiple studies!
  - Development of MNLFA comes from integrative data analysis
- Intersectionality research
  - No longer discrete groups tested in succession
- Instrument development

Extensions and implications

- Useful to ensure our construct functions similarly across many covariates – simultaneously
  - Age
  - Age^2
  - Race
  - Gender
  - GPA
  - SES
  - Parent education
  - Math achievement
  - Age × Gender
  - Race × Gender
  - Age^2 × Race × Gender

Current and future work

- So far, MNLFA has only been developed for continuous latent variables
  - Factor analysis
  - Item response theory

- Extensions for mixture modeling (categorical latent variables)

- Latent classes or groups of individuals are inferred from the data

Current and future work

- Latent class growth model or growth mixture model

Discussion

- Integrative data analysis is challenging!
- Moderated nonlinear factor analysis is also challenging!

Discussion

- Curran, P. J., & Hancock, G. R. Chapter 7 Workshop

  - To summarize the model building strategy
    1. estimate the baseline unconditional CFA
    2. expand the CFA to include the covariates in the prediction of both the factor mean and factor variance
    3. add all covariate effects for the first item and conduct LRT
       - if LRT is non-significant, there is no DIF for the item
       - if LRT is significant, record significant effects at *a priori* defined α
    4. remove covariate effects, repeat for each item separately
    5. return only significant effects for items that also had a significant LRT
    6. remove any non-significant effects from the model
    7. go out for a beer to celebrate