

Appendix. Calculating Standardized Factor Loadings for Generating Data

Let variable U represent a latent variable measured by p multiple indicators. A single indicator X_{pij} for item p for person i in group j may be decomposed as (Lüdtke et al., 2011):

$$X_{pij} = \lambda_{pW}x_{Wij} + \epsilon_{ij} + \lambda_{pB}x_{Bj} + \epsilon_j; p = 1, \dots, P$$

where λ_{pW} and λ_{pB} are the within- and between-factor loadings, respectively; x_{Wij} and x_{Bj} are the unobserved true scores at L1 and L2, respectively, and ϵ_{ij} and ϵ_j are the residuals at L1 and L2, respectively. Both x_{Wij} and x_{Bj} are assumed to be normally distributed with means of zero and $\text{Var}(x_{Wij}) = \sigma^2$ and $\text{Var}(x_{Bj}) = \tau^2$, respectively. For generating data, unstandardized factor loadings for each indicator at both levels were set equal to 1. Similarly, measurement error variances for each item were set to σ_e^2 and τ_e^2 at L1 and L2, respectively. As a result of parallel measures at both levels, the observed variance of a single indicator may be decomposed as:

$$\text{Var}(X_{ij}) = \sigma^2 + \sigma_e^2 + \tau^2 + \tau_e^2$$

Now, the standardized factor loading for a single indicator at L1 is:

$$\lambda_{W,std} = \frac{\sigma^2}{\sigma^2 + \sigma_e^2}$$

while the standardized factor loading for a single indicator at L2 is:

$$\lambda_{B,std} = \frac{\tau^2}{\tau^2 + \tau_e^2}$$

The standardized factor loadings were set to be equal at L1 and L2. Moreover, the total variance of a single predictor variable X was assumed to equal 1, where $\text{Var}(x_{Bj}) = \tau^2$ and $\text{Var}(x_{Wij}) = 1 - \tau^2$. Then, the item residual variance for a single item at L1 is specified as:

$$\sigma_e^2 = \frac{(1 - \tau^2)(1 - \lambda_{W,std})}{\lambda_{W,std}}$$

while the item residual variance for a single item at L2 is:

$$\tau_e^2 = \frac{\tau^2(1 - \lambda_{B,std})}{\lambda_{B,std}}$$

For example, for simulation conditions with an intraclass coefficient (ICC_0) = .25 and standardized factor loadings = 0.8, the residual variances at L1 and L2, respectively, are:

$$\sigma_e^2 = \frac{.75 * (1 - 0.8^2)}{0.8^2} = 0.42$$

and

$$\tau_e^2 = \frac{.25 * (1 - 0.8^2)}{0.8^2} = 0.14$$