Aug 2020, Q 1

We know that if $P(z)$ and $Q(z)$ are poly.s and

1) $\deg(Q(z)) \geq \deg(P(z)) + 2$ ✓
2) $Q(z)$ has no zeroes on $\mathbb{R}$ ✓

then

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\,dx = 2\pi i \sum_{a \in S} \operatorname{Res}_a \frac{P}{Q}$$

zeroes of Q in UHP.

$$x^2 + 1 = 0 \implies x = \pm i$$

$$x^2 + 4 = 0 \implies x = \pm 2i$$

Let $Q(z) = (z^2+1)(z^2+4)$

$$Q'(z) = (z^2+1)(2z) + (z^2+4)(2z)$$

$$Q'(i) = (-1+4)(2i) = 6i$$

$$Q'(2i) = (-4+1)(4i) = -12i$$

$\therefore$ Answer is

$$2\pi i\left(\frac{1}{6i}-\frac{1}{12i}\right)$$

$$=2\pi\left(\frac{2-1}{12}\right)$$

$$=\frac{\pi}{6}$$