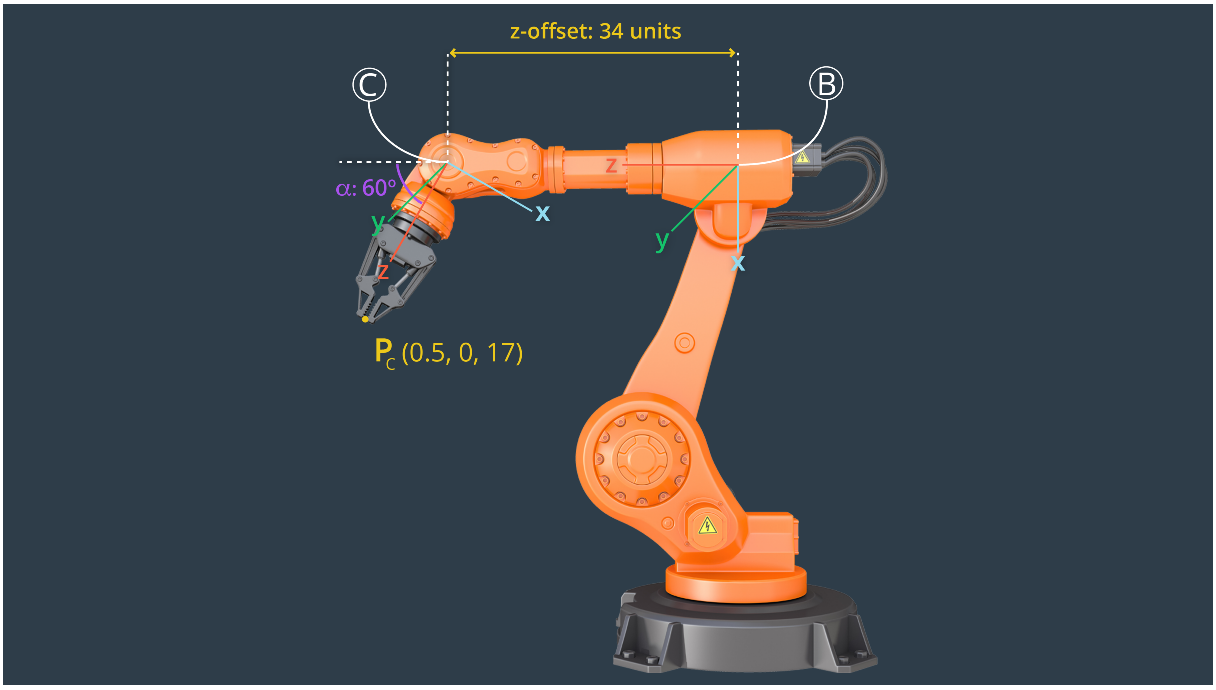
Project 2 of Robotics Nanodegree
Pick and Place,
Kuka KR210: a 6DOF manipulator



September 19, 2017

0.1 Manipulator: plans and actions

Fig. 1 shows the different steps performed by the simulator for the pick and place of each cylinder.

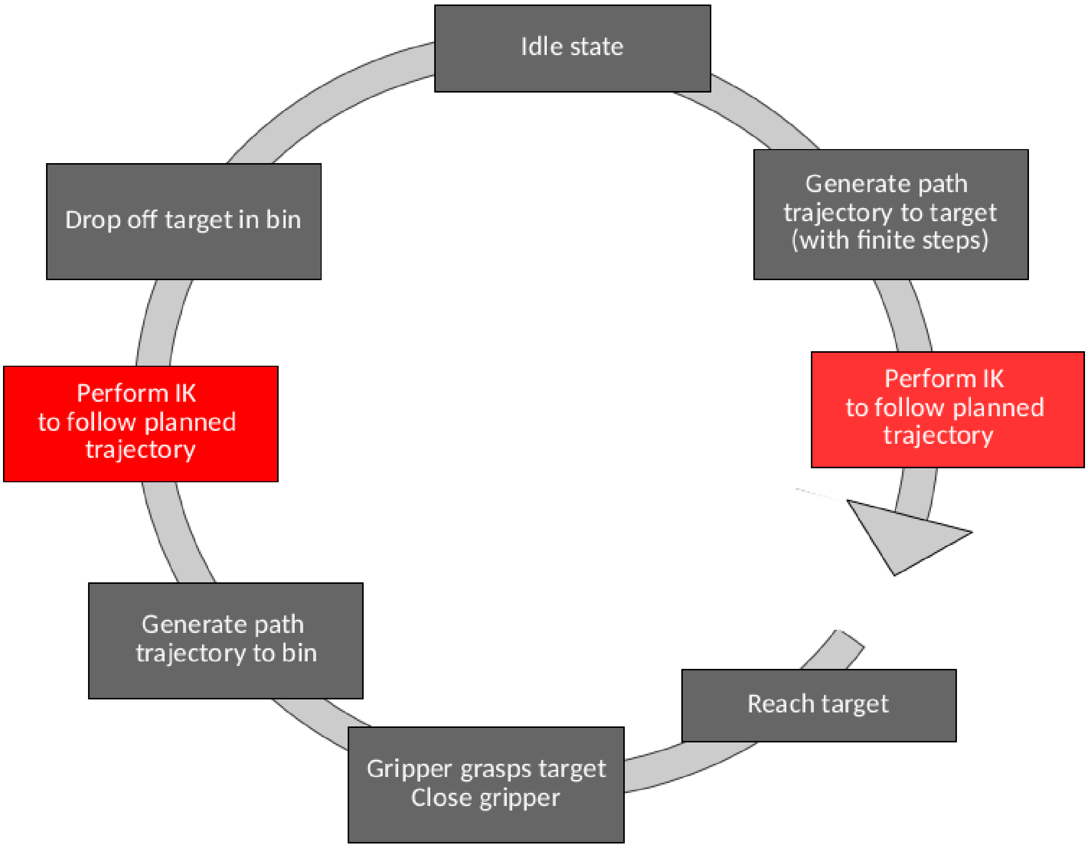


Figure 1: Steps performed by the simulator

When the ROS simulator generates the path trajectory, it provides a set of poses for the Gripper. Our goal is to compute the Inverse Kinematics so that the gripper position and orientation match the position and orientation computed by the planner.

0.2 Kinematics Analysis

0.3 Manipulator geometry: reference frames

In order to determine the modified DH parameter, we need to assign reference frames to each joint according to the DH convention. Fig. 2 shows the joint configuration with the manipulator in the *Idle state* (*i.e* all joint variables are set to 0).

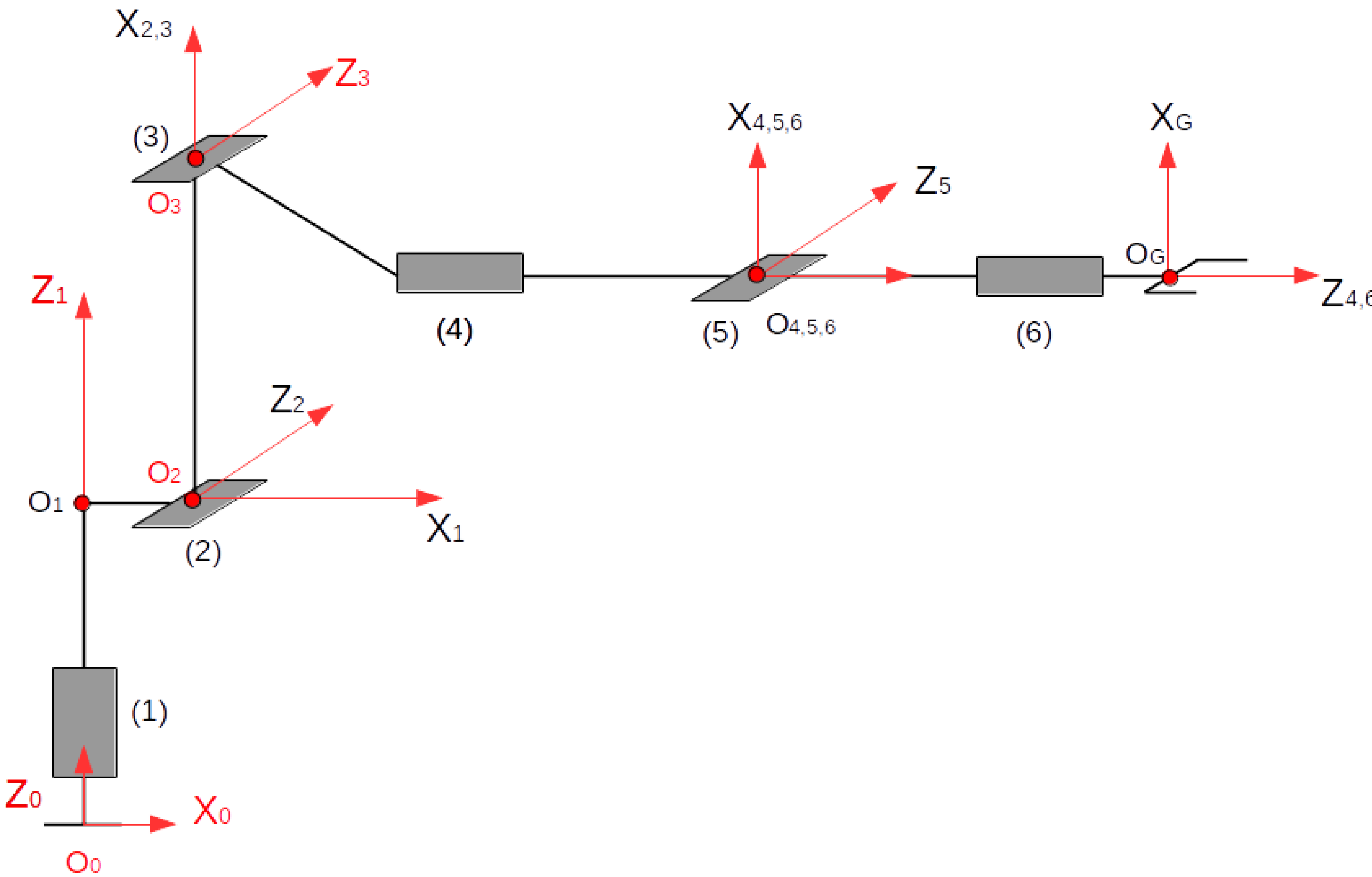


Figure 2: Initial pose of the manipulator. The joint variables $\theta_i = 0\ \forall i \in [1, 6]$. The base frame is (O_0, X_0, Y_0, Z_0)

0.4 Modified DH Parameter tables

The 4 DH parameters are defined as follows:

- link length: $a_i = [\hat{z}_{i-1}, \hat{z}_i]_{\hat{x}_i}$
- link twist: $\alpha_i = \langle \hat{z}_{i-1}, \hat{z}_i \rangle_{\hat{x}_i}$
- link offset: $d_i = [\hat{x}_{i-1}, \hat{x}_i]_{\hat{z}_{i-1}}$
- joint offset: $\theta_i = \langle \hat{x}_{i-1}, \hat{x}_i \rangle_{\hat{z}_{i-1}}$

The notations [ ] and $\langle\ \rangle$ refer respectively to a distance and an angle, along or about the axis given by the subscript.

The DH parameters are determined from the *URDF* (***kr210.urdf.xacro***) file and the frame axis attributed according to the DH convention.

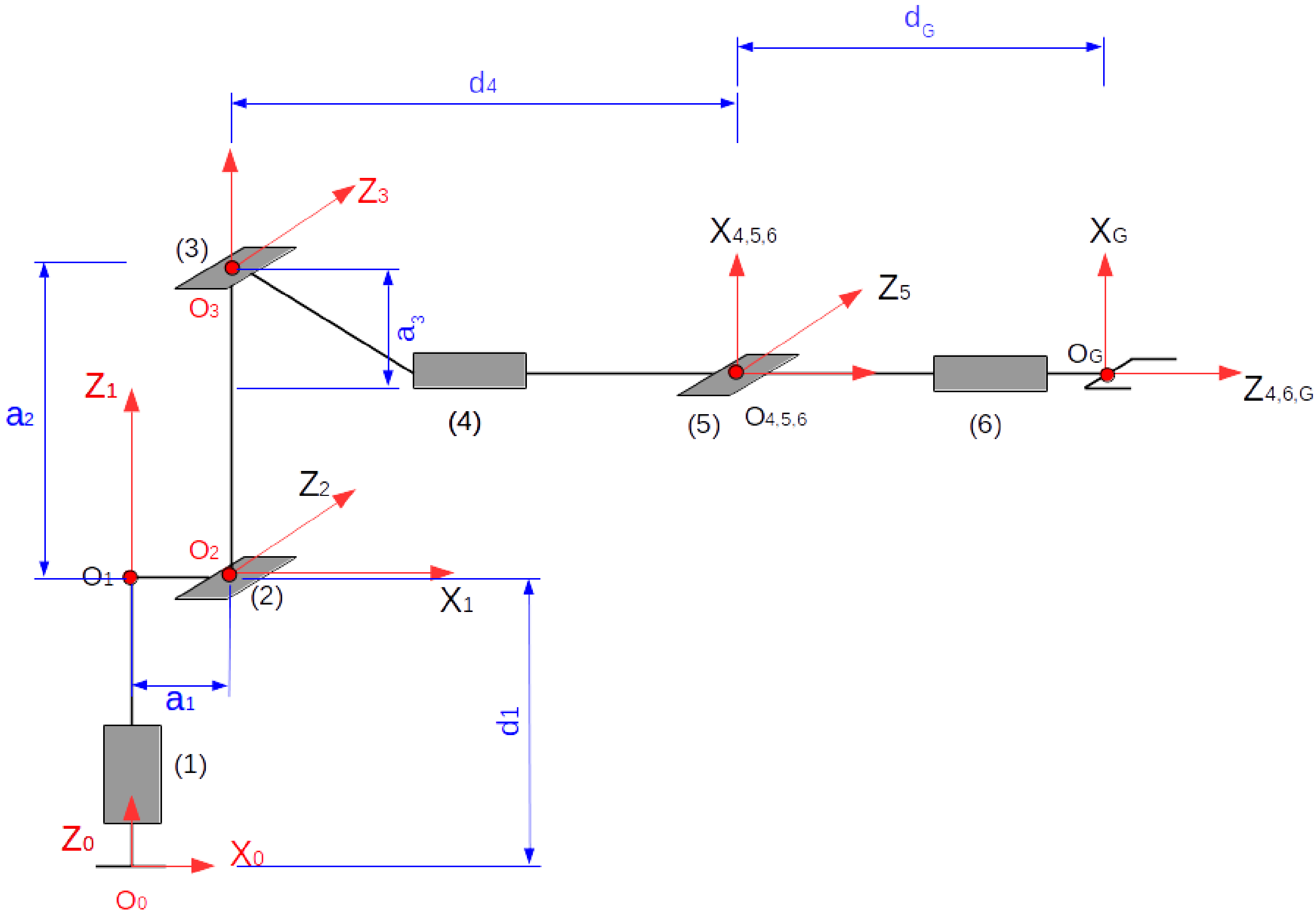


Figure 3: The non-zero DH parameters.

Table 1 shows the position and orientation of the joint $\{i\}$ wrt the joint $\{i-1\}$, as extracted from the *URDF* file.

joint	Δx	Δy	Δz	roll	pitch	yaw
1	0	0	0.33	0	0	0
2	0.35	0	0.42	0	0	0
3	0	0	1.25	0	0	0
4	0.96	0	-0.054	0	0	0
5	0.54	0	0	0	0	0
6	0.193	0	0	0	0	0
Gripper	0.11	0	0	0	0	0

Table 1: Relative distances extracted from the file ***kr210.urdf.xacro***.

In the reference frame, the position of joint $\{2\}$ relative to joint $\{1\}$ is 0.35m along $\hat{x}$ direction and 0.42 along $\hat{z}$ direction. The later distance corresponds to the distance $[O_1, O_2]$, i.e the link distance d_1. Similarly, d_G is the distance $[O_6, O_G] = 0.193+0.11$, *i.e* the x displacement along $\hat{x}$ from joint $\{5\}$ to joint $\{6\}$ added together with the displacement from joint $\{6\}$ to joint $\{G\}$ along $\hat{x}$. Similarly, we can use the *URDF* file to determine the non-zero DH parameters a_i and d_i (Table 2).

link i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0.75	$q1$
2	$-\pi/2$	0.35	0	$q2-\frac{\pi}{2}$
3	0	1.25	0	$q3$
4	$-\pi/2$	-0.054	1.50	$q4$
5	$\pi/2$	0	0	$q5$
6	$-\pi/2$	0	0	$q6$
G	0	0	$dG = 0.303$	0

Table 2: Modified DH Parameter table

0.5 Homogeneous Transforms

0.5.1 General form of the homogeneous Transform

The homogeneous transform T_{i-1}^{i} is an expression of the orientation and position of joint $\{i\}$ with respect to joint $\{i-1\}$. The homogeneous transform is composed of 2 rotations(α and θ) and 2 translations (a and d):

$$ {}_{i-1}^{\,i}T = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\ \cos\alpha_{i-1} & \cos\theta_i\ \cos\alpha_{i-1} & -\ \sin\alpha_{i-1} & -d_i\ \sin\alpha_{i-1} \\ \sin\theta_i\ \sin\alpha_{i-1} & \cos\theta_i\ \sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i\ \cos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1} $$

In the code *IK_server.py*, we create a function that generates the general form of the Homogeneous transform (Line 23-30)

0.5.2 Homogenemous Transform for each frame transition

Using the modified DH parameter table (Table 2) and Eq. 1, we determine the Homogeneous transform for each transition:

- ${}_1^0T = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- ${}_2^1T = \begin{pmatrix} \sin(q_2) & -\cos(q_2) & 0 & 0.35 \\ 0 & 0 & 1 & 0 \\ \cos(q_2) & -\sin(q_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- ${}_3^2T = \begin{pmatrix} \cos(q_3) & -\sin(q_3) & 0 & 1.25 \\ \sin(q_3) & \cos(q_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- ${}_1^0T = \begin{pmatrix} \cos(q_4) & -\sin(q_4) & 0 & -0.054 \\ 0 & 0 & 1 & 1.5 \\ -\sin(q_4) & -\cos(q_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- ${}^{4}_{5}T = \begin{pmatrix} \cos(q_5) & -\sin(q_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(q_5) & \cos(q_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- ${}^{5}_{6}T = \begin{pmatrix} \cos(q_6) & -\sin(q_6) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(q6) & \cos(q_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- ${}^{6}_{G}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.303 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Note that the last homogeneous transform (${}^{6}_{G}T$) is only a translation of the frame $\{6\}$ (no rotation).

The Homogeneous Transform matrix from base link to gripper link is a post-multiplication of the individual homogeneous transformation matrix:

$$ {}^{0}_{G}T = {}^{1}_{0}T \, {}^{2}_{1}T \, {}^{3}_{2}T \, {}^{4}_{3}T \, {}^{5}_{4}T \, {}^{6}_{5}T \, {}^{G}_{6}T \tag{2} $$

In IK_server.py, those matrices are defined line 99-126.

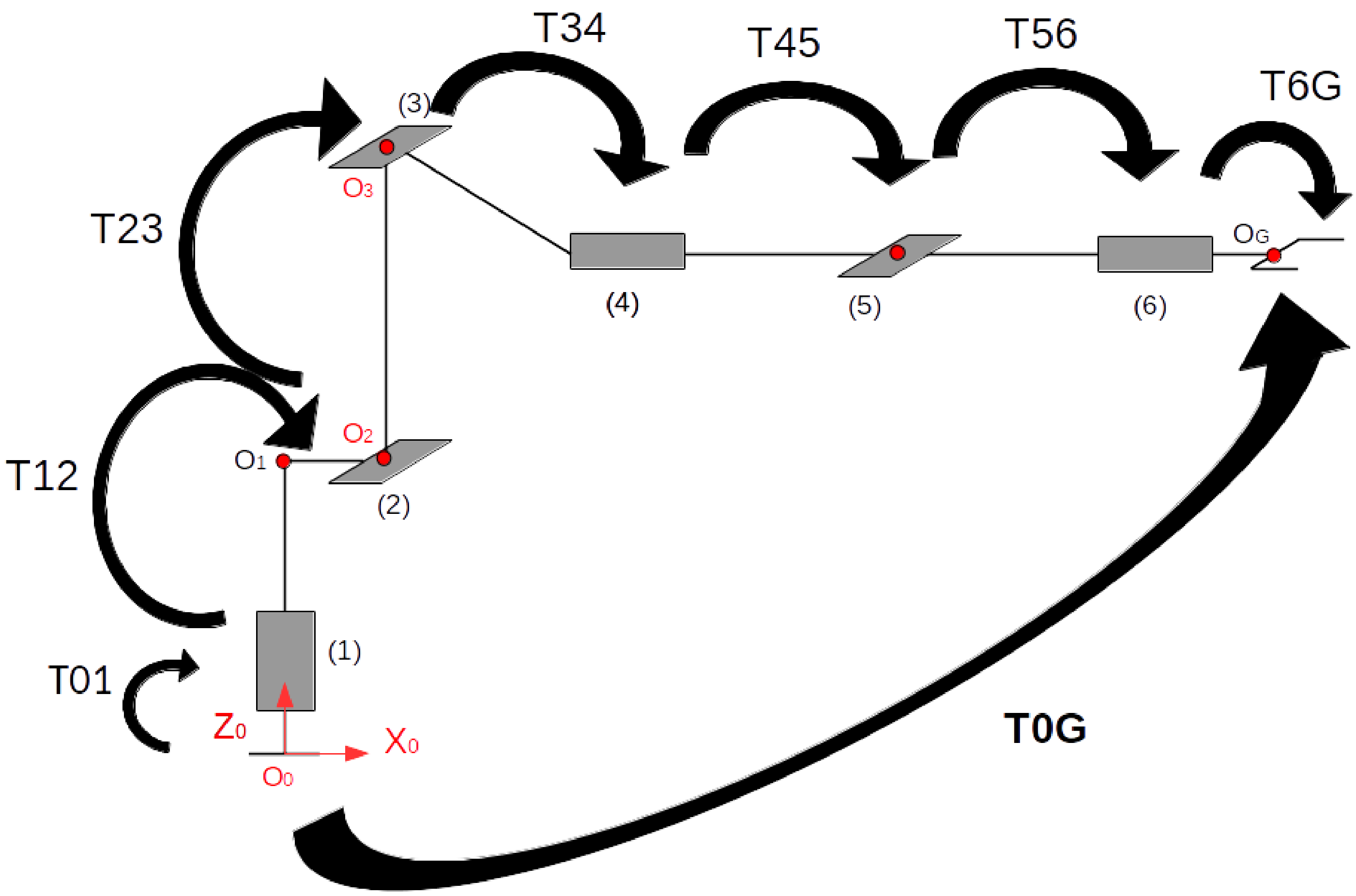


Figure 4: The individual homogeneous transforms and the complete homogeneous transform between the base frame and the gripper frame.

0.6 Inverse Kinematics

The last 3 joints (namely joint 4, 5 and 6) defines a spherical wrist: they are revolute joints with their axis intersecting at a single point (origin O_5). This point is the wrist center WC. Using the position of the wrist center, the Inverse Kinematics can be decoupled into a position and orientation problem.

In the next sections, we calculate the cartesian coordinates of WC wrt the base frame, and determine the joint variables.

0.6.1 Position of the wrist center

From Fig. 5, we have:

$$^{0}r_C \equiv^{0} r_G - d_G \ \hat{z}_G \tag{3}$$

where $^{0}r_C$ gives the position of the wrist center wrt the base frame, $^{0}r_{0G}$ is the vector from the origin to the wrist center wrt to the base frame, and d_G is the distance from the origin of the Gripper frame to the wrist center along $\hat{z}_G$. $\hat{z}_G$ can be expressed in the base reference frame by computing its projection onto the 3 axis (X_0, Y_0, Z_0), which is in fact the last column of the rotation matrix $^{0}_{G}R$. Hence:

$$^{0}r_C = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}^{0} r_G - d_G \ ^{0}_{G}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

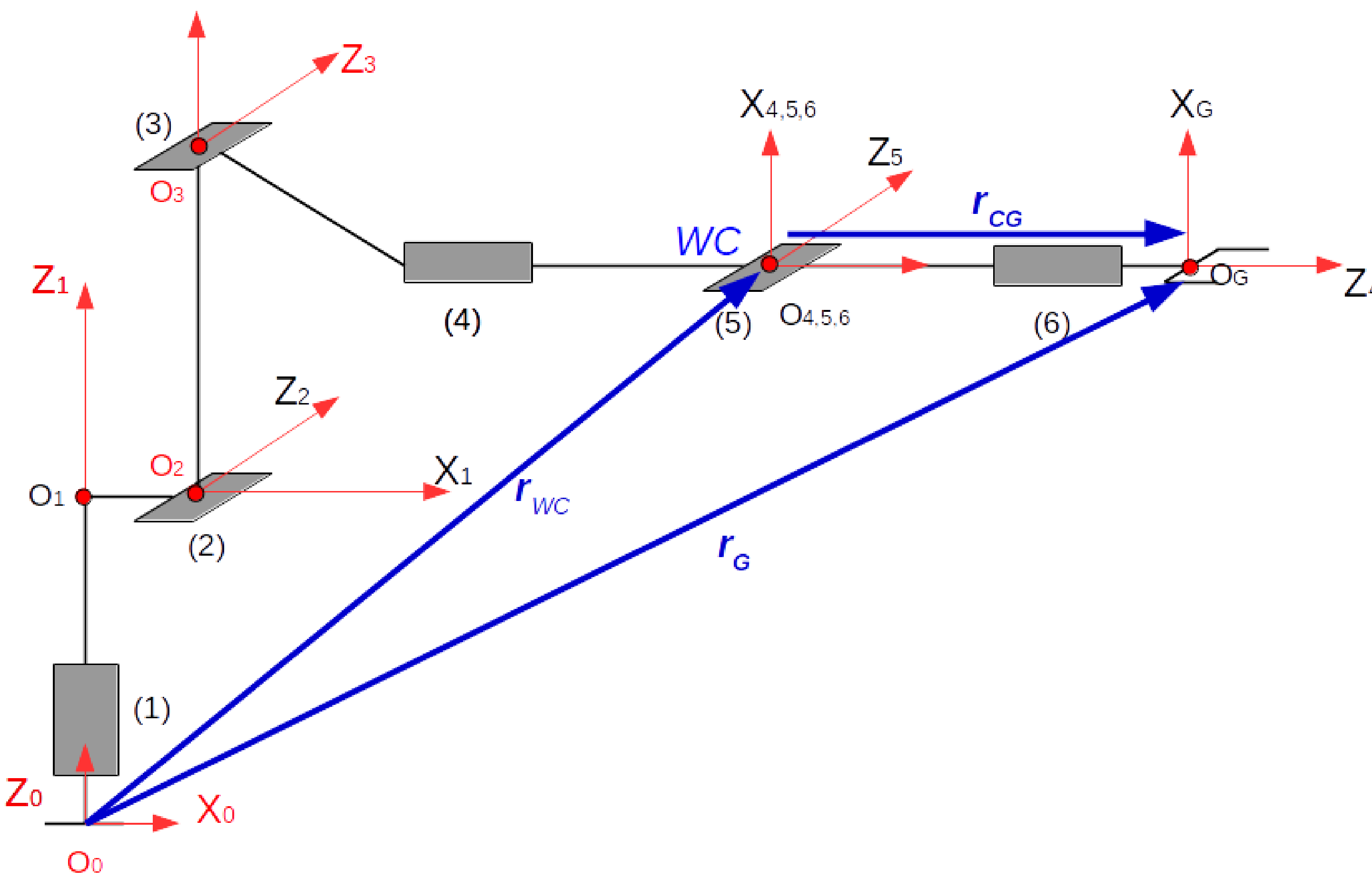


Figure 5: Wrist center position with the manipulator in Idle mode configuration.

0.6.2 Transformation Matrix between the gripper frame and the base frame

The path planner provides 6 parameters: the position and the orientation of the gripper wrt to the base frame: $(p_x, p_y, p_z, roll, pitch, yaw)$. We can build the transformation matrix (in the DH frame): ${}^0_GT$, by determining the rotation matrix ${}^0_GR$ from the roll, yaw and pitch. In addition, we need to apply a correction to account for the misalignment between the gripper frame in the urdf file and the gripper frame according to the DH convention.

1. Compute the total rotation matrix ${}^{urdf}R_G$, in the *"urdf frame"* using the individual rotation matrices with roll (r), pitch (p), yaw y angles:

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(r) & -\sin(r) \\ 0 & \sin(r) & \cos(r) \end{pmatrix} R_y = \begin{pmatrix} \cos(p) & 0 & \sin(p) \\ 0 & 1 & 0 \\ -\sin(p) & 0 & \cos(p) \end{pmatrix} \quad R_z = \begin{pmatrix} \cos(y) & -\sin(y) & 0 \\ \sin(y) & \cos(y) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

   Hence:

$$^{urdf}R_G = R_z\ R_y\ R_x \tag{6}$$

2. Apply rotation correction. From Fig. 6, the gripper axis according to the urdf file can be aligned with the gripper axis from the DH convention by applying 2 intrinsic rotations: a rotation of π about $\hat{z}_u$ axis, and a rotation of $-\pi/2$ about $\hat{y}_u$ $R_{z_u}(\pi) R_{y_u}(-\pi/2)$. Hence the orientation of the gripper wrt to the base frame and within the DH framework is:

$${}^0_GR = \left({}^{urdf}R_G\right) * R_z(\pi) * R_y(-\pi/2) \tag{7}$$

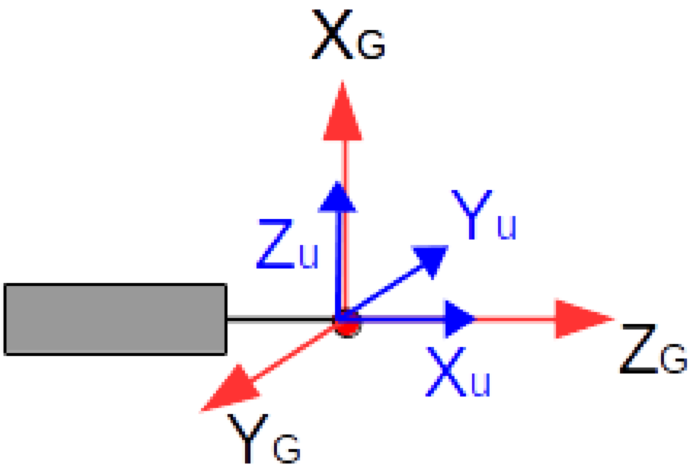


Figure 6: (X_G, Y_G, Z_G) are the axis of the gripper frame according to the DH convention (red), and (X_u, Y_u, Z_u) are the axis of the gripper frame as in the urdf file (blue).

3. Homogeneous transform matrix:

$${}^0_GT = \left[\begin{array}{c:c} {}^0_GR & p \\ \hdashline 000 & 1 \end{array}\right] \qquad \text{where } p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \tag{8}$$

In *IK_server.py*, the orientation matrix and position vector of the Gripper is computed from line 148-166.

0.6.3 Determine Joint angles

Joint angle 1: $q1 = \langle \hat{x}_0, \hat{x}_1 \rangle_{z_0}$

We can calculate $q1$ from the coordinates of the wrist center (From Fig 9.

$$q1 = atan2\left(\frac{w_y}{w_x}\right) \tag{9}$$

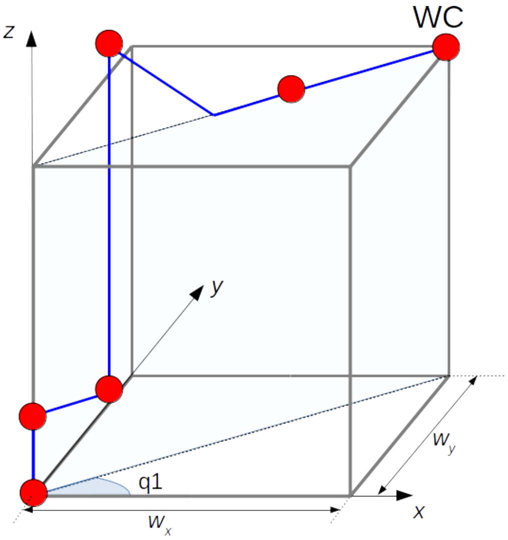


Figure 7: Joint 1 is rotated by angle $q1$. (w_x, w_y) are the coordinates of the wrist center wrt to the base reference frame.

According to the $URDF$ file, the joint 1 angles are limited to the range $-185 < q1 < 185$.

Joint angle 2: $q2 = \langle \hat{x}_1, \hat{x}_2 \rangle_{z_1}$

$q2$ is given by:

$$q2 = \pi/2 - a - a' \tag{10}$$

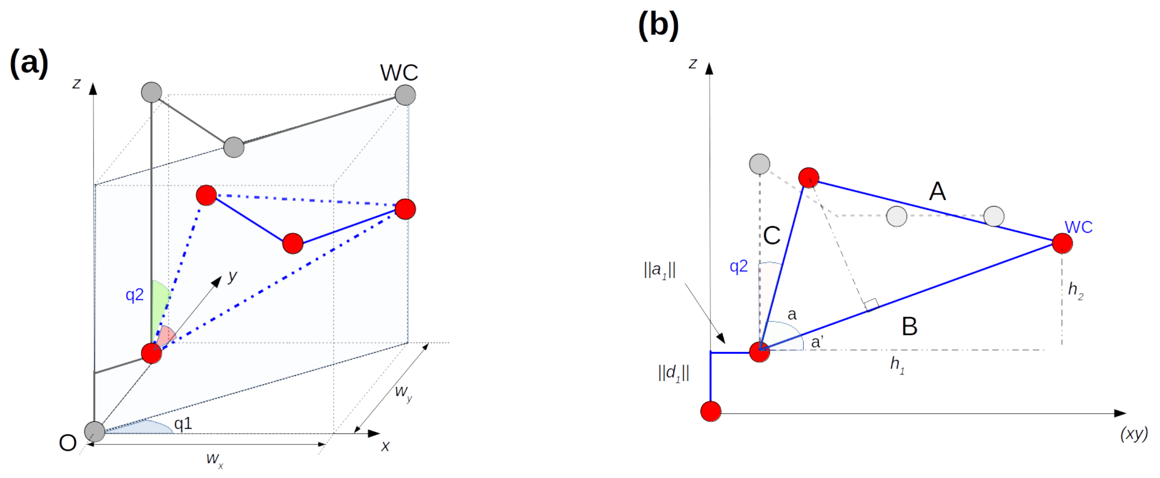


Figure 8: a) Joints configuration when the joint $\{2\}$ is rotated by an angle $q2$. The grey line/dots shows the manipulator geometry when in idle mode. b) Visualization of the joints and links in the plane (Oz, OWC)

Using cosine law, we can determine the angle a:

$$A^2 = B^2 + C^2 - 2BCcos(a) \tag{11}$$

$$\Rightarrow cos(a) = \frac{B^2 + C^2 - A^2}{2B\ C} \quad \text{and} \quad sin(a) = \sqrt{(1 - cos^2 a)} \tag{12}$$

Hence:

$$a = atan2\left(\frac{1 - cos^2 a}{cos(a)}\right) \tag{13}$$

The angle a can take either a positive or a negative value depending on the sign of $sin(a)$. This would result in a elbow up/down configuration.

From Fig. Fig. 8b, h is given by:

$$h^2 = x_c^2 + y_c^2 \quad \Rightarrow \quad h = \sqrt{x_c^2 + y_c^2} \tag{14}$$

$$h_1 + a_2 = \sqrt{x_c^2 + y_c^2} \quad \Rightarrow \quad h_1 = \sqrt{x_c^2 + y_c^2} - a_2 \tag{15}$$

$$\text{and} h2 = z_c - d_1 \tag{16}$$

Hence:

$$a' = atan2\left(\frac{h2}{h1}\right) \tag{17}$$

$$= atan2\left(\frac{z_c - d_1}{\sqrt{x_c^2 + y_c^2} - a_2}\right) \tag{18}$$

Substituting Equation Eq. 13 and Eq. 17 in Eq. 10, we obtain:

$$q2 = \frac{\pi}{2} - atan2\left(\frac{sin(a)}{cos(a)}\right) - atan2\left(\frac{z_c - d_1}{\sqrt{x_c^2 + y_c^2} - a_2}\right) \tag{19}$$

According to the *urdf* file, the joint angle has a upper and a lower safe limit of -45deg and +85deg respectively. We will therefore clip $q2$ to that range of angles.

Joint angle 3: $q3 = \langle \hat{x}_2, \hat{x}_3 \rangle_{z_2}$

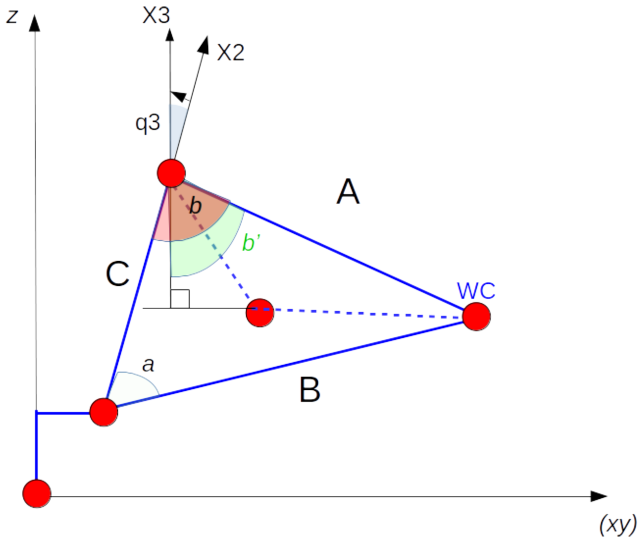


Figure 9: Angle $q3$

$$q3 = -(b - b') \tag{20}$$
$$= -\left(b - atan2\left(\frac{d_4}{a_3}\right)\right) \tag{21}$$

From the cosine law, we get:

$$cos(b) = \frac{A^2 + C^2 - B^2}{2AC} \tag{22}$$
$$sin^2(b) = 1 - cos^2(b) \tag{23}$$

Hence:

$$b = atan2\left(\frac{sin(b)}{cos(b)}\right) \tag{24}$$

Finally, $q3$ is:

$$q3 = -atan2\left(\frac{sin(b)}{cos(b)}\right) + atan2\left(\frac{d_4}{a_3}\right) \tag{25}$$

The angle will be cliped to the range [-210, 65] deg.

Joint angle 4, 5 and 6

The last 3 joint angles are determined using the rotation matrix ${}^{i-1}_{i}R$:

$${}^0_GR = {}^0_1R \ * \ {}^1_2R * \ {}^2_3R * \ {}^3_4R * \ {}^4_5R * \ {}^5_6R * \ {}^6_GR \tag{26}$$

The transformation from frame {6} to frame {G} is a simple translation so ${}^6_GR = I$. Previously, w determined $q1$, $q2$ and $q3$, therefore ${}^0_3R = {}^0_1R(q1) * {}^1_2R(q2) * {}^2_3R(q3)$ can be computed. Hence:

$${}^0_GR = {}^0_3R(q1, q2, q3) \ * \ {}^3_6R(q4, q5, q6) \tag{27}$$

and results in:

$$ {}^{3}_{6}R(q4, q5, q6) = \left({}^{0}_{3}R(q1, q2, q3)^{\intercal}\right) \ * \ \left({}^{0}_{G}R(r, p, y)\right) \tag{28} $$

The general expression of the rotation part of the transformation matrix is:

$$ {}^{i-1}_{i}R = \begin{bmatrix} cos(q_i) & -sin(q_i) & 0 \\ sin(q_i)\ cos(\alpha_{i-1}) & cos(q_i)cos(\alpha_{i-1}) & -sin(\alpha_{i-1}) \\ sin(q_i)\ sin(\alpha_{i-1}) & cos(q_i)\ sin(\alpha_{i-1}) & cos(\alpha_{i-1} \end{bmatrix} \tag{29} $$

From the DH parameter table and Eq. 31, we estimate ${}^{3}_{4}R$, ${}^{4}_{5}R$ and ${}^{5}_{6}R$:

$$ {}^{3}_{4}R = \begin{pmatrix} cos(q4) & -sin(q4) & 0 \\ 0 & 0 & -1 \\ -sin(q4) & -cos(q4) & 0 \end{pmatrix} {}^{4}_{5}R = \begin{pmatrix} cos(q5) & -sin(q5) & 0 \\ 0 & 0 & -1 \\ sin(q5) & cos(q5) & 0 \end{pmatrix} \ {}^{5}_{6}R = \begin{pmatrix} cos(q6) & -sin(q6) & 0 \\ 0 & 0 & -1 \\ -sin(q6) & -cos(q6) & 0 \end{pmatrix} \tag{30} $$

From Eq. 30, the rotation matrix ${}^{3}_{6}R$ is computed:

$$ {}^{3}_{6}R = \begin{bmatrix} c(q_4)c(q5)c(q6) - s(q4)s(q5) & -c(q4)c(q5)s(q6) - c(q5)s(q6) - c(q5)s(q4) & c(q4)s(q5) \\ s(q5)c(q6) & -s(q5)s(q6) & c(q5) \\ -s(q4)c(q5)c(q6) - c(q4)s(q5) & s(q4)c(q5)s(q6) - c(q4)c(q5) & -s(q4)s(q5) \end{bmatrix} \tag{31} $$

${}^{0}_{G}R(r, p, y)$ is the orientation of the Gripper in the reference frame O. It includes the roll, pitch, yaw rotation and a correction rotatiton term to align the *urdf* frame with the DH convention frame. By comparing Eq. 30 and Eq. 28, the joint angles $q4, q5, q6$ are determined:

$$ q4 = atan2\left(-\frac{-s(q4)s(q5)}{c(q4)s(q5)}\right) = atan2\left(-\frac{R[0,2]}{R[2,2]}\right) \tag{32} $$

$$ q5 = atan2\left(\frac{[(s(q5)c(q6))^2 + s(q5)s(q6)]^{0.5}}{c(q5)}\right) = atan2\left(\frac{[R[1,0]^2 + R[1,1]^2]^{0.5}}{R[1,2]}\right) \tag{33} $$

$$ q6 = atan2\left(-\frac{-s(q5)s(q6)}{s(q5)c(q6)}\right) = atan2\left(-\frac{R[1,1]}{R[1,0]}\right) \tag{34} $$

where $R = \left({}^{0}_{3}R(q1, q2, q3)^{\intercal}\right) \ * \ \left({}^{0}_{G}R(r, p, y)\right)$

Each of the joint angles $q4$, $q5$ and $q6$ are clipped to the angle range given in the *urdf* file: q_4-limit = [-350, 350], $q5$-limit=[-125, 125], $q6$-limit=[-350,250]. Note taht $q5$ has multiple solution:positive or negative value.

0.7 Comments

I have implemented all of the parts detailed in this report. However, I have found some bugs that I have not been able to resolved:

- if I clip all the joint angles to their respective values, there is no improvement, in fact no difference, in the behaviour of the manipulator.
- If $q2$ and $q3$ are clipped to their respective safe angle limit, the gripper consistently fails to grap the cylinder. That is to say that the gripper fingers would be placed appropriately around the cylinder, but after clicking [NEXT] to retrieve, the gripper would fail to carry the cylinder off the shelf.

- I also tried to prevent hectic behaviour of the manipulator by comparing current and previous angles. Because some joint angles can have multiple solutions, it is likely that 2 consecutives angle of a joint coudl have solutions that are in 2 different quadrants. To prevent this behaviour, I implemented a function *validate()*. This function takes the current theta value and the old theta value, and select the solution that is in the same quadrant as the previous angle value.