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Midterm Exam Report

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1 Summary of the two methods

1.1 hedcuter method

Hedcuter method consists of two main steps which is slightly different version from Secord's method[2]:

1. Sample initial points $X = \{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \cdots\}$
2. Loop while the displacements don't exceed maximum site displacement and the number of iterations doesn't exceed maximum number of iterations,
   (a) Map the image value at $\mathbf{x_i}$ to stipple level t
   (b) Propagate the stipple level at $\mathbf{x_i}$ to its neighbor
   (c) Compute the centroids
   (d) Move each points $\mathbf{x_i}$ to its new centers of a cell $\mathbf{C_i}$

Sampling

In the loop, the algorithm keeps mapping image using the location of a point(which corresponds to the coordinates of a pixel) as unique identification. This mapping image contains stipple level t at each points with the number of levels l. Unlike Secord's method[2], hedcuter method first computes and stores stipple level t only at given points using Eq. 1. Next, it sort the stipples with stipple level, x values, and y values.

$$t = \frac{l - I(\mathbf{x}) * \rho(\mathbf{x})}{l}, \;\; where \;\; \rho(\mathbf{x}) = 1 \;\; \forall x \tag{1}$$

After propagation, hedcuter method collect pixels covered with a voronoi cell.

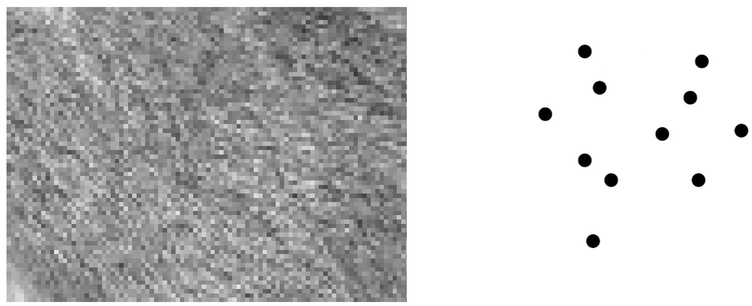
1.2 voronoi method

A flow of voronoi method basically follows Lloyd's method[1], which as follows :
when the centroid of a region is defined as

$$\mathbf{C_i} = \frac{\int_A \mathbf{x}\rho(\mathbf{x})dA}{\int_A \rho(\mathbf{x})dA}, \tag{2}$$

Algorithm 1 Lloyd's method

while generating point $\mathbf{x_i}$ not converged to centroids **do**
  Compute the Voronoi diagram of $\mathbf{x_i}$
  Compute the centroids $\mathbf{C_i}$ using equation (2)
  Move each generating point $\mathbf{x_i}$ to its centroids $\mathbf{C_i}$
end while



(a) (b)

Figure 1: A part of given image and sample points.

For more simple explanation of details, let's assume that we have a part of image(Fig. 1(a)) from squirrel's back. First, voronoi method samples points on the image randomly(Fig. 1(b)).

Next, voronoi method computes the voronoi region of given sample points(Fig. 2(a)). After creating voronoi diagram of given points, the algorithm computes the centroid of a cell iteratively looping through all these points. For example, with a given cell in Fig. 2(b), voronoi method calculates the line which is the extension of voronoi edge and is called clipping line. When two end points of voronoi edge are $\mathbf{x_1} = (x_1, y_1)$ and $\mathbf{x_2} = (x_2, y_2)$, the equation of clipping line is

$$\begin{aligned} (y - y_1) &= \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \\ (y - y_1)(x_2 - x_1) &= (y_2 - y_1)(x - x_1) \\ (y_2 - y_1)(x - x_1) - (y - y_1)(x_2 - x_1) &= 0 \\ (y_2 - y_1)x - (x_2 - x_1)y + y_1(x_2 - x_1) - x_1(y_2 - y_1) &= 0 \\ -(y_1 - y_2)x + (x_1 - x_2)y + x_1(y_1 - y_2) - y_1(x_1 - x_2) &= 0 \end{aligned} \tag{3}$$

$$\therefore ax + by + c = 0$$
$$where\ a = -(y_1 - y_2),\ b = (x_1 - x_2),\ and\ c = x_1(y_1 - y_2) - y_1(x_1 - x_2).$$

In the same way, we can generate clipping lines for all voronoi edges in a cell.

To obtain the integration of density $\rho(\mathbf{x})$, voronoi method creates grid on a cell with the user-specified number of subpixels(Fig. 2(d)). Using the clipping lines(Eq. 3), we can test if a grid point is inside or outside of cell. If a grid point $\mathbf{x_g} = (x_g, y_g)$ satisfies $ax_g + by_g + c < 0$ for all the clipping lines, it locates inside of a cell. The result of test is shown in Fig. 2(e). The red points mean outside and the green points mean inside.
When grid points are inside, the algorithm obtains $\int_A \rho(\mathbf{x})dA$ and $\int_A \mathbf{x}\rho(\mathbf{x})dA$ with intensity $I(\mathbf{x_g})$

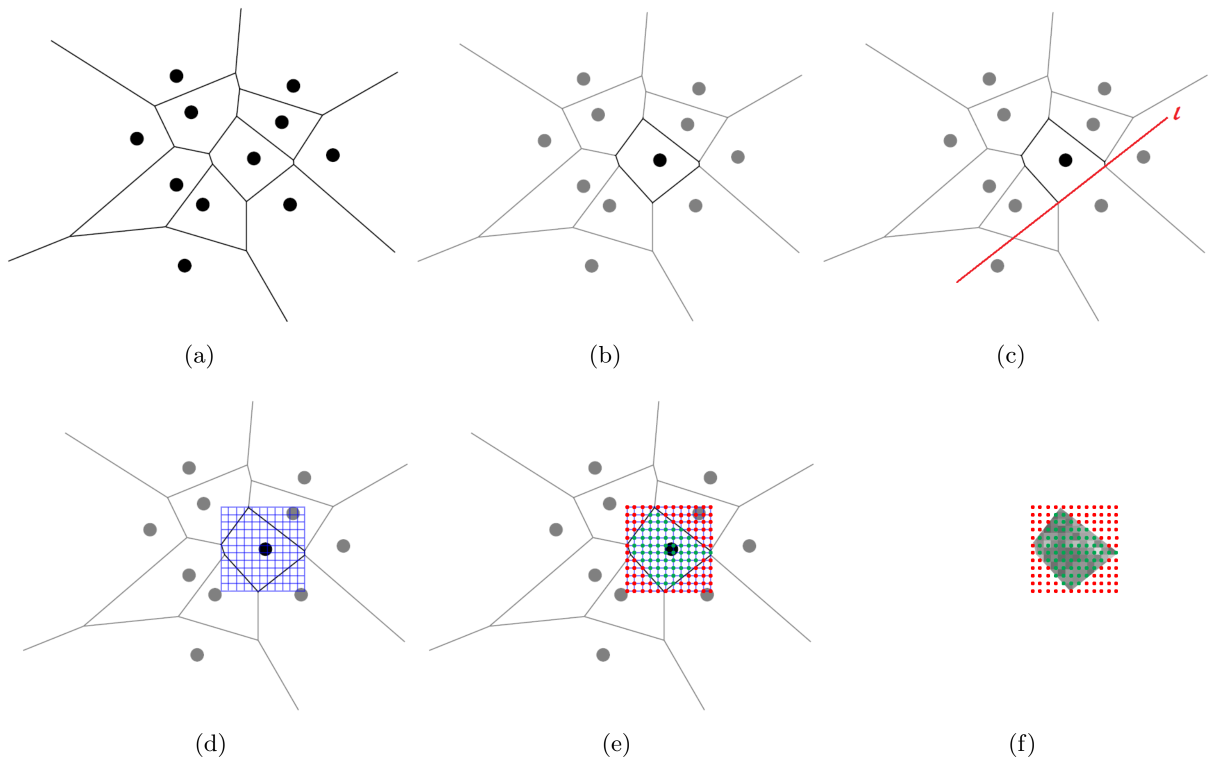


Figure 2: The progress of computing voronoi diagram and redistributing stipples.

which has corresponding location to grid points in grayscale image(Fig. 2(f), Eq. 4).

$$\begin{aligned} \int_A \rho(\mathbf{x})dA &= \int I(\mathbf{x_g}) \\ \int_A \mathbf{x}\rho(\mathbf{x})dA &= \int \mathbf{x}I(\mathbf{x_g}) = \int (x,y)I(\mathbf{x_g}). \end{aligned} \tag{4}$$

Substituting the results of Eq. 4 to Eq. 2, we can generate new centroid $\mathbf{C_i} = (x_c, y_c)$, which satisfies that $x_c = \frac{\int x \cdot I(\mathbf{x_g})}{\int I(\mathbf{x_g})}$ and $y_c = \frac{\int y \cdot I(\mathbf{x_g})}{\int I(\mathbf{x_g})}$.This new centroids will be fed as sites into creating voronoi diagram step. Voronoi method repeats creating voronoi diagram and computing the centroid repeatedly until the average of displacement between points $\mathbf{x_i}$ and centroids $\mathbf{C_i}$ is small enough.

2 Comparison of the two methods

3 Improvement of hedcuter method

References

[1] Atsuyuki Okabe, Barry Boots, and Kokichi Sugihara. *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams.* John Wiley & Sons, Inc., New York, NY, USA, 1992. 1.2

[2] Andrian Secord. Weighted voronoi stippling. In *2nd International Symposium on Non-Photorealistic Animation and Rendering (NPAR'02)*, pages 37–43, Annecy, France, June 3-5 2002. 1.1, 1.1