Fall 2015

Midterm Exam Report

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1 Summary of the two methods

1.1 hedcuter method

1.2 voronoi method

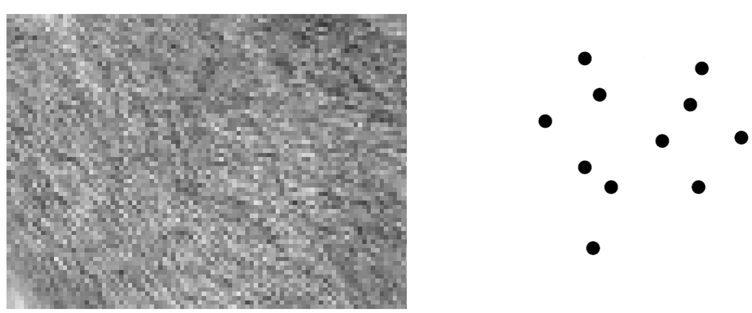
A flow of voronoi method basically follows Lloyd's method, which as follows :
when the centroid of a region is defined as

$$\mathbf{C_i} = \frac{\int_A \mathbf{x}\rho(\mathbf{x})dA}{\int_A \rho(\mathbf{x})dA}, \tag{1}$$

Algorithm 1 Lloyd's method

```
1: while generating point x_i not converged to centroids do
2:     Compute the Voronoi diagram of x_i.
3:     Compute the centroids C_i using equation (1)
4:     Move each generating point x_i to its centroids C_i
5: end while
```

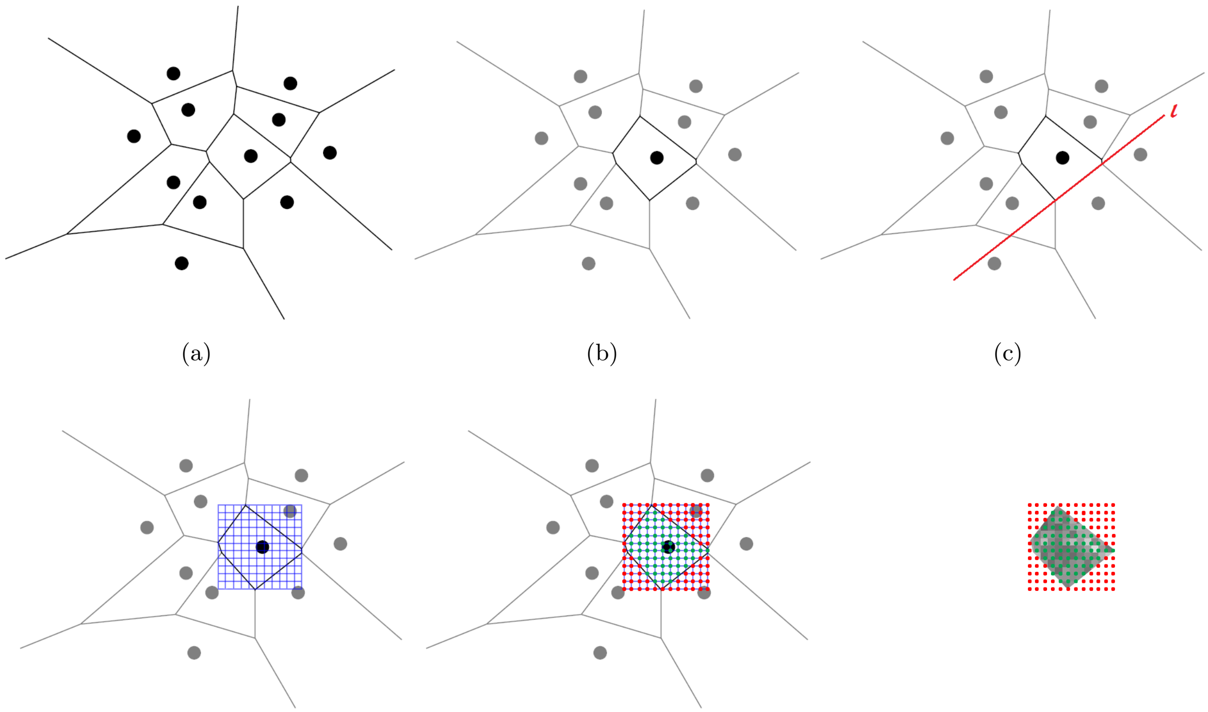
For more simple explanation of details, let's assume that we have a part of image(Fig. 1(a)) from squirrel's back. First, voronoi method samples points on the image randomly(Fig. 1(b)).



(a) (b)

Figure 1: A part of given image and sample points.

Next, voronoi method computes the voronoi region of given sample points(Fig. 2(a)). After creating voronoi diagram of given points, the algorithm computes the centroid of a cell iteratively looping through all these points. For example, with a given cell in Fig. 2(b), voronoi method



(a) (b) (c)

(d) (e) (f)

Figure 2: The progress of computing voronoi diagram and redistributing stipples.

calculates the line which is the extension of voronoi edge and is called clipping line. When two end points of voronoi edge are $\mathbf{x_1} = (x_1, y_1)$ and $\mathbf{x_2} = (x_2, y_2)$, the equation of clipping line is

$$
\begin{aligned}
(y - y_1) &= \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \\
(y - y_1)(x_2 - x_1) &= (y_2 - y_1)(x - x_1) \\
(y_2 - y_1)(x - x_1) - (y - y_1)(x_2 - x_1) &= 0 \\
(y_2 - y_1)x - (x_2 - x_1)y + y_1(x_2 - x_1) - x_1(y_2 - y_1) &= 0 \\
-(y_1 - y_2)x + (x_1 - x_2)y + x_1(y_1 - y_2) - y_1(x_1 - x_2) &= 0
\end{aligned} \tag{2}
$$

$$
\therefore ax + by + c = 0
$$
$$
where\ a = -(y_1 - y_2),\ b = (x_1 - x_2),\ and\ c = x_1(y_1 - y_2) - y_1(x_1 - x_2).
$$

In the same way, we can generate clipping lines for all voronoi edges in a cell.

To obtain the density $\rho(\mathbf{x})$, voronoi method create grid on a cell with the user-specified number of subpixels(Fig. 2(d)). Using the clipping lines(Eq. 2), we can test if a grid point is inside or outside of cell. If a grid point $\mathbf{x_g} = (x_g, y_g)$ satisfies $ax_g + by_g + c < 0$ for all the clipping lines, it locates inside of a cell. The result of test is shown in Fig. 2(e). The red points mean outside and the green points mean inside.
When grid points is inside, the algorithm obtains $\int_A \rho(\mathbf{x})dA$ and $\int_A \mathbf{x}\rho(\mathbf{x})dA$ with intensity $I(\mathbf{x_g})$

which has corresponding location to grid points in grayscale image(Fig. 2(f), Eq. 3).

$$\begin{aligned}\int_A \rho(\mathbf{x})dA &= \int I(\mathbf{x_g}) \\ \int_A \mathbf{x}\rho(\mathbf{x})dA &= \int \mathbf{x}I(\mathbf{x_g}) = \int (x,y)I(\mathbf{x_g}).\end{aligned} \tag{3}$$

Substituting the results of Eq. 3 to Eq. 1, we can generate new centroid $\mathbf{C_i} = (x_c, y_c)$, which satisfies that $x_c = \frac{\int x \cdot I(\mathbf{x_g})}{\int I(\mathbf{x_g})}$ and $y_c = \frac{\int y \cdot I(\mathbf{x_g})}{\int I(\mathbf{x_g})}$. This new centroids will be fed as sites into creating voronoi diagram step. Voronoi method repeats creating voronoi diagram and computing the centroid repeatedly until the average of displacement between points $\mathbf{x_i}$ and centroids $\mathbf{C_i}$ is small enough.

2 Comparison of the two methods

3 Improvement of hedcuter method