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Chaos, oscillation and the evolution of indirect reciprocity in *n*-person games

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Abstract

Evolution of cooperation among genetically unrelated individuals has been of considerable concern in various fields such as biology, economics, and psychology. The evolution of cooperation is often explained by *reciprocity*. Under reciprocity, cooperation can prevail in a society because a donor of cooperation receives reciprocation from the recipient of the cooperation, called *direct reciprocity*, or from someone else in the community, called *indirect reciprocity*. Nowak and Sigmund [1993. Chaos and the evolution of cooperation. Proc. Natl. Acad. Sci. USA 90, 5091–5094] have demonstrated that *directly* reciprocal cooperation in two-person prisoner's dilemma games with mutation of strategies can be maintained dynamically as periodic or chaotic oscillation. Furthermore, Eriksson and Lindgren [2005. Cooperation driven by mutations in multi-person Prisoner's Dilemma. J. Theor. Biol. 232, 399–409] have reported that directly reciprocal cooperation in *n*-person prisoner's dilemma games ($n > 2$) can be maintained as periodic oscillation. Is dynamic cooperation observed only in *direct* reciprocity? Results of this study show that *indirectly* reciprocal cooperation in *n*-person prisoner's dilemma games can be maintained dynamically as periodic or chaotic oscillation. This is, to our knowledge, the first demonstration of chaos in indirect reciprocity. Furthermore, the results show that oscillatory dynamics are observed in common in the evolution of reciprocal cooperation whether for direct or indirect.

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1. Introduction

In social dilemmas (Dawes, 1980), cooperation confers a benefit *b* to another individual while accruing a cost *c* to the donor ($b > c > 0$). What mechanism enables the evolution of the costly behavior, cooperation? *Reciprocity* is a prevailing mechanism for the evolution of cooperation among genetically unrelated individuals. Under reciprocity, cooperative behavior can prevail in a society because a donor of cooperation receives some reciprocal benefits from the recipient of cooperation or from someone else in the community. The former is called *direct* reciprocity and the latter is called *indirect* reciprocity.

The concept of direct reciprocity was first proposed by Trivers (1971), which claims that cooperation can evolve because cooperation is *directly* returned by the recipient of cooperation. Reciprocity has often been studied using two-person iterated prisoner's dilemma games (Axelrod and Hamilton, 1981; Axelrod, 1984; May, 1987; Nowak and Sigmund, 1992). Especially, Axelrod and Hamilton (1981) and Axelrod (1984) have reported, based on results of the tournament of two-person iterated prisoner's dilemma games, that directly reciprocal cooperation can be formed and maintained by the tit-for-tat strategy, which cooperates on the first move and thereafter copies the opponent's previous move, when an interaction is repeated sufficiently many times between the same two individuals. This work is quite influential because that theory, which is based only on the strategic interactions among individuals, does not require kinship to explain the emergence of cooperation. This reciprocity has been considered applicable to cooperation in repeated interactions such as long-term face-to-face interactions.

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On the other hand, the same two individuals do not always encounter again in the real world, especially in human societies (e.g., one-shot anonymous interactions such as those in global markets or on Internet). Evolution of cooperation under non-repeated interactions is often explained by indirect reciprocity (Alexander, 1987), which states that cooperation can evolve because a cooperator *indirectly* receives a return from someone else in the community. Nowak and Sigmund (1998a, b) have formalized the concept using a kind of one-shot two-person prisoner's dilemma (or giving) game in which the *reputation* of each individual affects the decision-making process. Results of those studies have shown that, even without repeated interactions, the discriminating strategy, which cooperates only with individuals having a good reputation, can form indirectly reciprocal cooperation against unconditional defectors. These studies of the evolution of indirect reciprocity via social reputation in two-person games have been extended in various directions (Fishman, 2003; Lotem et al., 1999; Mohtashemi and Mui, 2003; Brandt and Sigmund, 2004, 2005, 2006; Leimar and Hammerstein, 2001; Ohtsuki, 2004; Ohtsuki and Iwasa, 2004, 2005; Panchanathan and Boyd, 2003; Milinski et al., 2001, 2002; Wedekind and Milinski, 2000; Chalub et al., 2006; Pacheco et al., 2006; Suzuki and Toquenaga, 2005).

Here, an interesting point in the evolutionary dynamics of direct reciprocity in two-person games is the fact that the dynamics of the shares of strategies and those of the overall level of cooperation display periodic or chaotic oscillation (Brandt and Sigmund, 2006; Nowak and Sigmund, 1989, 1993; Imhof et al., 2005). In particular, using two-person iterated prisoners dilemma games, Nowak and Sigmund (1993) have investigated the effect of strategic mutation on the evolution of direct reciprocity in a population comprising simple strategies with actions determined by the outcome of the last round. They have demonstrated that, as the mutation rate increases, the evolutionary dynamics change from converging to a fixed point to periodic oscillation to chaos to periodic oscillation and back to converging to a fixed point. Are these dynamics such as periodic oscillation and chaos limited to the evolutionary dynamics of direct reciprocity in two-person games?

Although the studies of the evolution of reciprocity described above have investigated only two-person games, more than two individuals often interact in the real world (e.g., in cooperative management of common-pool resources Hardin, 1968, environmental problems such as global warming or air pollution Milinski et al., 2006, and predator inspection in a school of fish Dugatkin, 1990). Therefore, we believe that not only *two*-person games but also *n*(>2)-person games should be considered as models of interactions in human societies or ecosystems.

Some studies have investigated the evolution of direct and indirect reciprocity in *n*-person games (Joshi, 1987; Boyd and Richerson, 1988, 1992; Molander, 1992; Akimov and Soutchanski, 1992; Yao, 1996; Hauert and Schuster, 1997; Lindgren and Johansson, 2001; Eriksson and Lindgren, 2005; Suzuki and Akiyama, 2005, 2007a, b, 2008). However, few studies discuss periodic or chaotic oscillations like those observed in Nowak and Sigmund (1993). Only Eriksson and Lindgren (2005) has demonstrated that, in *n*-person games, directly reciprocal cooperation can be maintained as periodic oscillation (but not as chaotic oscillation).

In the present study, we investigate the evolutionary dynamics of indirect reciprocity in one-shot *n*-person prisoner's dilemma games including social reputation in a population that includes all the strategies who determine their own action based on the number of opponents whose reputation is good. Especially, as in Nowak and Sigmund (1993) which deals with direct reciprocity in two-person games, we specifically examine bifurcations into periodic or chaotic oscillations depending on the mutation rate.

2. Model

Consider a population comprising an infinite number of individuals. Each individual in the population has a *reputation*, either *G* (*good*) or *B* (*bad*). At the beginning of each generation, the reputation of each individual is presumed as *G*.

Each *generation* comprises a number of *rounds*. After the first round, each subsequent round occurs with probability w ($0 \leqslant w < 1$), i.e., the expected value of the number of rounds in a generation is $1/(1-w)$.

In each round, all individuals are divided randomly into groups, each of which comprises *n*-individuals; all play an *n*-person prisoner's dilemma game in each group. In this game, each individual chooses either to "cooperate (*C*)" or "defect (*D*)". In this study, suppose that the cost and the benefit of cooperation are denoted as c and b, respectively, where $b > c > 0$, and the benefit is shared equally among the $n - 1$ other group members. Then, the payoffs for a cooperator, $V(C|k)$, and that for a defector, $V(D|k)$, where k is the number of opponents cooperating in the group ($0 \leqslant k \leqslant n - 1$), are

$$V(C|k) = \frac{k}{n-1}b - c, \quad V(D|k) = \frac{k}{n-1}b. \quad (1)$$

Moreover, the reputation of opponents affects the decision-making process. For this study, we adopt "*image scoring*" as a *reputation criterion*, which prescribes how to judge the reputation of others based on their past actions. Under image scoring, which was first used in Nowak and Sigmund (1998a, b), the reputation of an individual who has cooperated (defected) in the previous round becomes *G* (*B*).

In this study, each individual is presumed to decide an action based on the number of her opponents having reputation *G* in the group. Such a decision-making rule, called *strategy*, is denoted by an *n*-dimensional vector $\mathbf{P} = (p_0, \ldots, p_{n-1})$ where $p_k \in [0, 1]$, in which p_k indicates the probability that the individual cooperates when the

number of opponents having reputation G is k. For example, in four-person games, strategy $(0, 0, 0, 0)$ called *ALLD* always defect, and strategy $(1, 1, 1, 1)$ called *ALLC* always cooperates. Moreover, strategy $(0, 0, 0, 1)$ called strict discriminator (*stDIS*) cooperates only when all the opponents have reputation G, and $(0, 0, 1, 1)$ or $(0, 1, 1, 1)$ called generous discriminator (*gDIS*) cooperates when at least one or two opponents have reputation G. In all, there are 2^n pure strategies, which we number from 0 to $2^n - 1$. The ith strategy is represented by $\mathbf{P}_i$ (the ith strategy being the binary expression for i). Furthermore, the shares of the respective strategies among the population are denoted, respectively, as $x_0, \ldots, x_{2^n-1}$. In the example of four-person games, $\mathbf{P}_0$ corresponds to *ALLD*, $\mathbf{P}_1$ to *stDIS*, $\mathbf{P}_3$ or $\mathbf{P}_7$ to *gDIS*, and $\mathbf{P}_{15}$ to *ALLC*.

In addition, we assume that, with the probability ε $(0 < \varepsilon \ll 1)$, an individual who intends to cooperate fails to cooperate because of a lack of resources or a mistake, etc.[1] Therefore, we replace 1 in elements of the strategy vectors with $1 - \varepsilon$. For instance, strategy $(1, 0, 1, 0)$ is replaced with $(1 - \varepsilon, 0, 1 - \varepsilon, 0)$.

To investigate the evolution of the shares of the strategies in the population, we consider the natural selection process by which strategies that achieve higher fitness, defined as the expected total payoff, increase their shares among the population. Furthermore, we include mutation, the rate of which is denoted as μ, in the evolutionary dynamics. (The derivation of the fitness for each strategy and the mathematical formulation of the evolutionary dynamics including mutation are described in Appendix A.)

3. Results and discussion

Let us introduce the evolutionary dynamics for the group size of $n = 4$: the benefit of cooperation is $b = 10$; the cost of cooperation is $c = 1$; the probability that each subsequent round occurs is $w = 0.9$; and the probability of implementation error is $\varepsilon = 0.01$. No pure evolutionarily stable strategy exists for this parameter setting (Suzuki and Akiyama, 2008), on the other hand, whether some mixed evolutionarily stable strategies exist or not remains unclear. We also examined the cases of $n = 3, 5$ and confirmed that the result does not change qualitatively.[2] (When n is

extremely large, it is expected that the frequency of cooperation is lower Suzuki and Akiyama, 2005, 2007a.)

Figs. 1(a)–(f) illustrate the numerical calculation of the evolutionary dynamics of the overall level of cooperation under various mutation rates μ. At the initial state of the evolutionary dynamics given in each figure, all strategies are present in equal shares: $x_0 = \cdots = x_{15} = \frac{1}{16}$. We also performed numerical simulations of the evolutionary dynamics starting from 50 random initial states and from the population of *ALLD* (i.e., $x_0 = 1, x_1 = \cdots = x_{15} = 0$). We therefore confirmed that the resulting attractors in the figures do not depend on the initial states as far as the mutation rate, μ, is not extremely small (under the parameter setting in the figure, $\mu > 0.0002$), which suggests that these attractors are global unless μ is extremely small.

As shown in Fig. 1, the evolutionary dynamics change qualitatively as the mutation rate increases. In the case of no mutation ($\mu = 0$, see Fig. 1(a)), the resulting dynamics converges to a fixed point at which the overall level of cooperation is very low. Moreover, as the mutation rate increases, the dynamics displays quasi-periodic oscillation (see Fig. 1(b)). The larger the mutation rate becomes, the more complicated the dynamics is (see Figs. 1(b) and (c)). The dynamics bifurcate into chaotic oscillations, and subsequently revert to periodic oscillations, and to convergence to a fixed point (see Figs. 1(d)–(f)) when the mutation rate increases further. The bifurcation diagram and the chaotic attractor are shown in Fig. 2. These results show that, in n-person games, cooperation based on *indirect* reciprocity can be maintained dynamically as periodic or chaotic oscillation, as seen in Nowak and Sigmund (1993) that deals with *direct* reciprocity in two-person games.

In terms of the effect of mutation rate on the overall level of cooperation, as shown in Fig. 1(g), slight mutation can drastically improve the overall level of cooperation even when cooperation is hardly formed without mutation.[3] However, the additional increase in the mutation rate decreases the level of cooperation, though not so drastically (see Fig. 1(g)). Consequently, there exists a critical mutation rate at which the level of cooperation is highest, and the critical rate is small. Mathematically, mutation has the power to draw the evolutionary dynamics to the center of the state space where the strategy shares are almost equivalent. Therefore, comparing the case of no mutation with that of slight mutation, the dynamics are affected by a defective attractor in the former case, but when the mutation rate exceeds the critical value (i.e., the latter case), a cooperative oscillation attractor is newly formed. In this case, the dynamics escape from the defective attractor and are affected by the cooperative attractor. However, when the mutation rate increases further, the dynamics approach the center of the state space, where the overall level of cooperation is low, from the cooperative state.

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[1] We, as in Panchanathan and Boyd (2003), do not consider errors which cause unintentional cooperation. We confirmed that the resulting dynamics do not change appreciably even if errors causing unintentional cooperation are included. Furthermore, an objective or subjective perception error (Ohtsuki and Iwasa, 2004; Takahashi and Mashima, 2006) is not included.

[2] Concerning the benefit of cooperation, b (the cost c is fixed at 1), similar evolutionary phenomena are observed if b becomes larger (but chaotic oscillation is hardly observed when b is extremely large). On the other hand, if b becomes smaller ($b < 8$), cooperation hardly forms (we show the effect of mutation rate, μ, on the time average of the overall level of cooperation for smaller values of $b = 6, 8$ in Fig. 3). For w and ε, the observed evolutionary dynamics do not essentially change as far as w and $1 - \varepsilon$ are sufficiently close to 1.

[3] Of course, when the benefit of cooperation, b, is extremely large, a high level of cooperation is achieved without mutation.
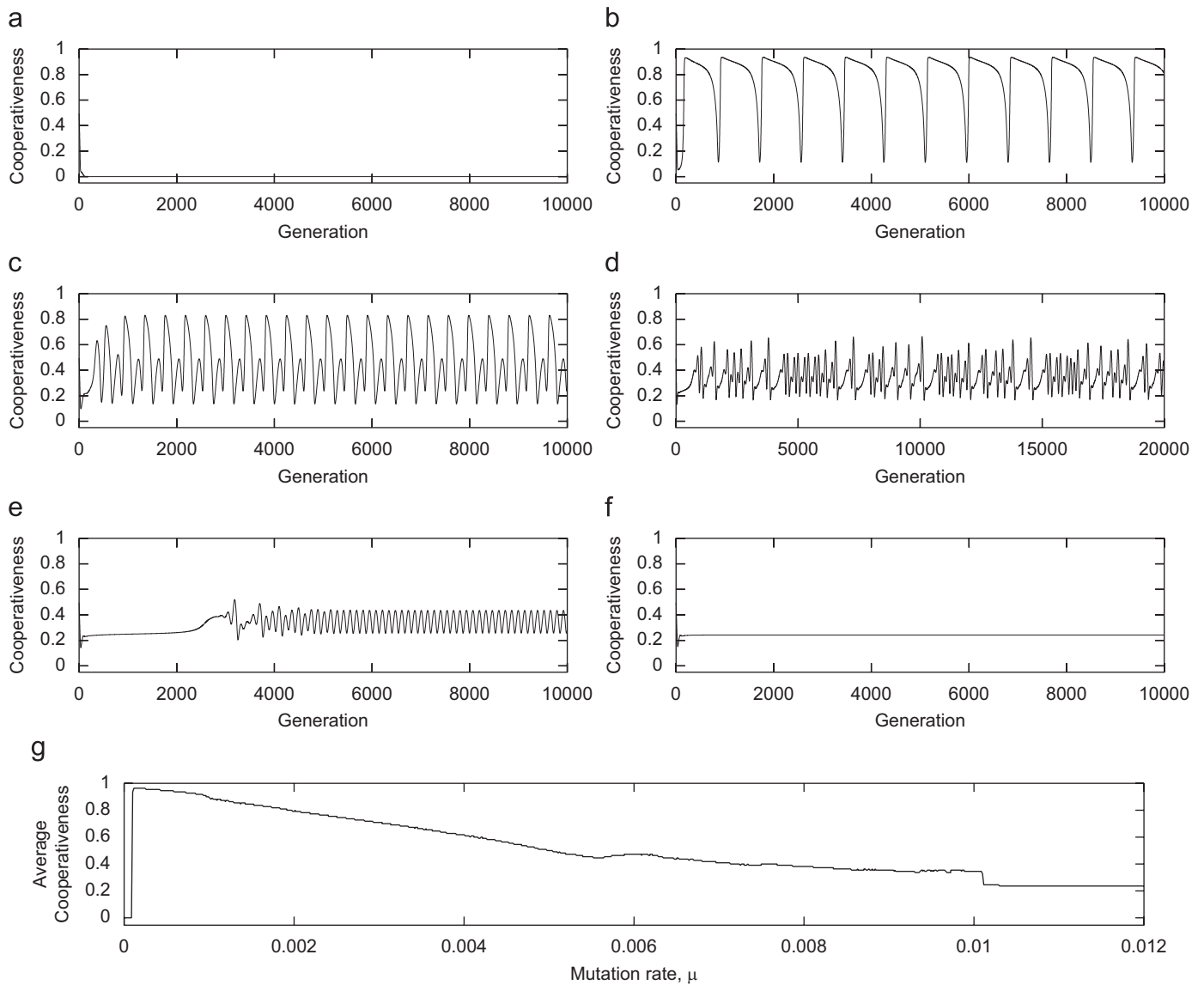
Fig. 1. (a)–(f) Evolutionary dynamics of the overall level of cooperation (the initial state is that $x_0 = \cdots = x_{15} = \frac{1}{16}$). Mutation rate: (a) $\mu = 0.000$, (b) $\mu = 0.002$, (c) $\mu = 0.006$, (d) $\mu = 0.009$, (e) $\mu = 0.010$ and (f) $\mu = 0.011$. (g) Effect of mutation rate, μ, on the time average of the overall level of cooperation.

Here, how are the periodic or chaotic oscillations formed and maintained? We describe the dynamics of the shares of the strategies resulting in the oscillation from the population of unconditional defectors (*ALLD*) to answer this question. First, $(0, 1, 0, 0)$ and $(0, 0, 1, 0)$ strategists invade the *ALLD* population. The $(0, 1, 0, 0)$ and $(0, 0, 1, 0)$ strategists earn almost equal fitness to that for *ALLD* because they do not cooperate both in the first round where all individuals have good reputation and after the first round where all *ALLD* opponents have a bad reputation (note that, in the population almost entirely consisting of *ALLD*, most opponents are *ALLD* strategists whose reputation is good at the first round and bad after the first round). For that reason, they are not exploited by *ALLD*, whereas the other strategists are exploited by

ALLD. Therefore, by the effect of mutation, $(0, 1, 0, 0)$ and $(0, 0, 1, 0)$ strategists increase, whereas *ALLD* strategists decrease. (When slight differences arise in fitness, mutation draws the evolutionary dynamics to the center of the state space where strategy shares are almost equivalent.) Next, once the shares of $(0, 1, 0, 0)$ and $(0, 0, 1, 0)$ strategists increase to some degree, the strictest discriminators, *stDIS*, who cooperate only when all the opponents have good reputation, can increase their share by exploiting the $(0, 1, 0, 0)$ and $(0, 0, 1, 0)$ strategists. When the share of *stDIS* strategists rise to a certain degree, they can overcome the *ALLD* strategists and become predominant because the *stDIS* strategists punish defectors strictly. That is, although *stDIS* cannot invade the *ALLD* population alone (Suzuki and Akiyama, 2007a), they can invade the *ALLD*
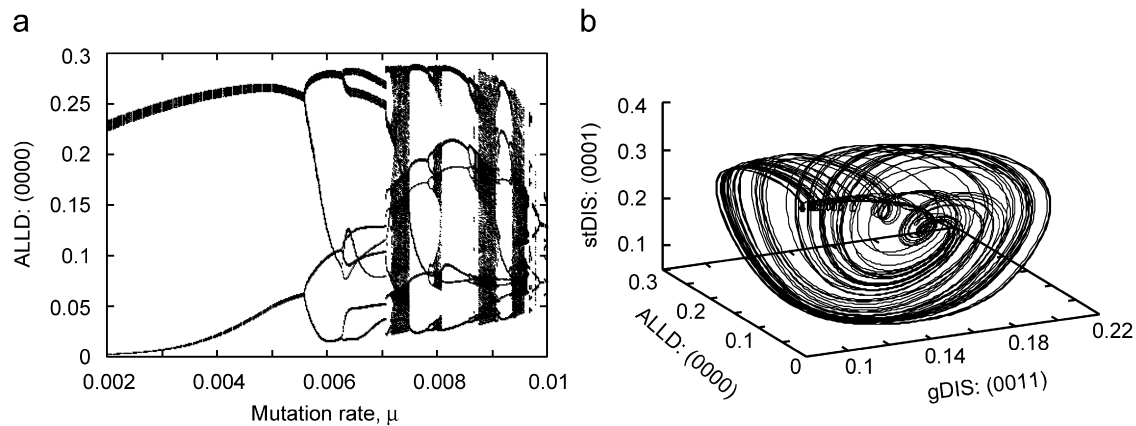
Fig. 2. (a) Bifurcation diagram: the horizontal axis is the mutation rate, μ, and the vertical axis is the share of $ALLD = (0, 0, 0, 0)$ after sufficient time has passed. For each mutation rate, an orbit is traced. Each time $\mathbf{x} = (x_0, \ldots, x_{15})$ crosses the hyper-plane $x_1 = 0.234$, a dot is plotted. (b) Projection of the chaotic attractor when $\mu = 0.009$: $ALLD = (0, 0, 0, 0)$ versus $gDIS = (0, 0, 1, 1)$ versus $stDIS = (0, 0, 0, 1)$.

population by exploiting other strategies which obtain almost the same fitness as that for $ALLD$ in the population mostly consisting of $ALLD$.

However, $stDIS$ strategists, once they become predominant, harm each other as a result of triggering error defections because of their excessive strictness. For that reason, they cannot earn the high level fitness. Therefore, the $stDIS$ strategists are vulnerable to invasion by more generous strategists, generous discriminators ($gDIS$), who cooperate when at least one or two opponents have a good reputation, or by unconditional cooperators ($ALLC$). Finally, once the generous strategists prevail over the population, $ALLD$ strategists invade the population again. This mechanism explains the emergence of the oscillation from the population of $ALLD$. Moreover, once the evolutionary dynamics reach the oscillatory attractor, the share of $ALLD$ strategists is at most 40%, so $stDIS$ strategists can invade the $ALLD$ population without the assistance of $(0, 1, 0, 0)$ and $(0, 0, 1, 0)$ strategists.

The main cycle in the chaotic attractor is similar to that in the periodic oscillation attractor described above: the predominant strategy changes from unconditional defectors to strictest discriminators to generous discriminators or unconditional cooperators, and back to unconditional defectors. In the chaotic attractor, in addition to the generous discriminators, $(0, 1, 0, 1)$ and $(1, 0, 1, 0)$ strategists invade the population predominated by the strictest discriminators, which makes the cycle irregular (chaos).

So far, we have investigated the evolutionary phenomena of indirect reciprocity in n-person games. The analyses have shown that indirectly reciprocal cooperation in n-person games can be formed and maintained dynamically as periodic or chaotic oscillation. This is, to our knowledge, the first demonstration of chaos in indirect reciprocity. Furthermore, the evolutionary dynamics bifurcate as the mutation rate increases.

Similar phenomena have been observed in the evolution of the other type of reciprocity, direct reciprocity, in two-

person games (Nowak and Sigmund, 1993). Moreover, Eriksson and Lindgren (2005) have demonstrated that, in n-person games where the population consists of unconditional cooperators, unconditional defectors and some kinds of reciprocators, directly reciprocal cooperation can be maintained as periodic oscillation. The result of this paper shows that the dynamic cooperation as periodic or chaotic oscillation is observed in common in the evolution of reciprocity, which is a prevailing mechanism for explaining the evolution of cooperation among genetically unrelated individuals, under dilemma games with strategic mutation irrespective of kinds of reciprocity, direct or indirect.[4]

4. Future works

In this study, we have assumed deterministic mutation process by which a strategy is more likely to mutate into a similar strategy in the sense of Hamming distance (see the detailed explanation in Appendix A). However, there is a possibility that the resulting dynamics change essentially depending on types of mutation processes. Therefore, it might be interesting to investigate other types

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[4]Regarding the evolution of indirect reciprocity in two-person games, dynamic cooperation has not been reported (Brandt and Sigmund, 2006) except for the specific case in which the number of rounds in a generation is constant and where implementation error is not included (Nowak and Sigmund, 1998a, b). Note that, for the specific case, Nowak and Sigmund (1998a, b) have demonstrated oscillatory dynamics in indirectly reciprocal cooperation using an individual-based computer simulation model and an evolutionary game model. Few studies have investigated the evolutionary dynamics of indirect reciprocity in two-person games in a population of various strategies (most studies have specifically examined the evolutionary dynamics of only three strategies Nowak and Sigmund, 1998b; Brandt and Sigmund, 2005, 2006; Panchanathan and Boyd, 2003 or dealt only with evolutionary stability Ohtsuki and Iwasa, 2004, 2005). It might be that dynamic cooperation is observed in the evolution of indirect reciprocity in two-person games in the population that includes many strategies (e.g., strategies deciding their own action based on their own and the opponent's reputation).

of random fluctuations and mutations and see how the results are robust.

Furthermore, in this study, as described in Section 2, we have considered all the strategies who determine their own action based on the number of opponents whose reputation is good. On the other hand, as we have shown in Section 3, only the unconditionally defective strategy, the unconditionally cooperative strategy and discriminating strategies play an important role in the oscillatory attractors. So, it might be also worthwhile to analyze the evolutionary dynamics in the population consisting only of the unconditionally defective strategy, the unconditionally cooperative strategy and discriminating strategies. Focusing only on these strategies may allow us to study the case of larger group size.

Appendix A

First, we derive fitness for each strategy defined as the expected value of the total payoff the strategy earns in a generation. To obtain the fitness, we first derive the expected payoff for strategy $\mathbf{P}_i$ in round t, $f_i(t)$.

Given that Eq. (1), $f_i(t)$ is determined by the probability that the focal $\mathbf{P}_i$ strategist cooperates and by the probability that an opponent of the focal $\mathbf{P}_i$ strategist cooperates.

First, we derive the probability that the focal $\mathbf{P}_i$ strategist cooperates. At each round, the possible situations that the individual faces are the following n cases: no opponent exists with G reputation in the group; one exists; $\cdots$; and $n-1$ opponents in the group have reputation G. Denoting the frequency of individuals with reputation G among the whole population as in round t $G(t)$, we can represent the probability by which the focal individual faces each of the above n situations in round t as an n-dimensional vector

$$\mathbf{S}(t) = (s_0(t), \ldots, s_{n-1}(t))$$
$$= \left((1 - G(t))^{n-1}, \ldots, \binom{n-1}{k} G(t)^k (1 - G(t))^{n-1-k}, \right.$$
$$\left. \ldots, G(t)^{n-1} \right). \tag{A.1}$$

The probability that the focal $\mathbf{P}_i$ strategist cooperates is $\mathbf{P}_i \cdot \mathbf{S}(t)$.

Here, we describe the frequency of individuals with reputation G among the whole population in round t, $G(t)$. Presuming that $g_i(t)$ indicates the frequency of individuals

with reputation G among $\mathbf{P}_i$ strategists in round t, then $G(t)$ is given as

$$\sum_{i=0}^{2^n-1} g_i(t) x_i,$$

where x_i represents the share of $\mathbf{P}_i$ strategists in the population. Moreover, $g_i(1) = 1$ and $g_i(t+1) = \mathbf{P}_i \cdot \mathbf{S}(t)$ because all individuals are presumed to have reputation G in the first round and cooperation (defection) is judged to be good (bad) after the first round.

Next, we obtain the probability that an opponent of the focal $\mathbf{P}_i$ strategist cooperates. In each round, an opponent of the focal $\mathbf{P}_i$ strategist faces each of the following n cases: no opponent exists with G reputation in the group; one exists; $\cdots$; and $n-1$ opponents have reputation G in the group, the probabilities of which are represented as an n-dimensional vector

$$\mathbf{S}_i'(t) = \left((1 - g_i(t))(1 - G(t))^{n-2}, \right.$$
$$(1 - g_i(t))(n-2)G(t)(1 - G(t))^{n-3} + g_i(t)(1 - G(t))^{n-2},$$
$$\cdots,$$
$$(1 - g_i(t))\binom{n-2}{k} G(t)^k (1 - G(t))^{n-2-k}$$
$$+ g_i(t)\binom{n-2}{k-1} G(t)^{k-1}(1 - G(t))^{n-1-k},$$
$$\cdots,$$
$$(1 - g_i(t))G(t)^{n-2} + g_i(t)(n-2)G(t)^{n-3}(1 - G(t)),$$
$$\left. g_i(t)G(t)^{n-2} \right). \tag{A.2}$$

Therefore, when an opponent of the focal $\mathbf{P}_i$ strategist uses strategy $\mathbf{P}_j$, the probability that the opponent cooperates is $\mathbf{P}_j \cdot \mathbf{S}_i'(t)$. Because the probability that the opponent uses strategy $\mathbf{P}_j$ is x_j, the probability that an opponent of the focal $\mathbf{P}_i$ strategist cooperates is given as

$$\sum_{j=0}^{2^n-1} \mathbf{P}_j \cdot \mathbf{S}_i'(t) x_j.$$

Considering Eq. (1), the expected payoff for $\mathbf{P}_i$ strategists in round t is

$$f_i(t) = b \sum_{j=0}^{2^n-1} \mathbf{P}_j \cdot \mathbf{S}_i'(t) x_j - c \mathbf{P}_i \cdot \mathbf{S}(t). \tag{A.3}$$

The fitness for strategy $\mathbf{P}_i$, which is defined as the expected total payoff in a generation is

$$f_i = \sum_{t=1}^{\infty} w^{t-1} f_i(t) + c/(1-w),$$

where $w \in (0,1)$ indicates the probability that each subsequent round occurs. (Note that, in order to avoid f_i being negative, we add the constant value, $c/(1-w)$, to the

expected payoff, $\sum_{t=1}^{\infty} w^{t-1} f_i(t)$.) Because it is virtually impossible to get the exact values of f_i, we use approximated values from numerical calculation

$$f_i = \sum_{t=1}^{T} w^{t-1} f_i(t) + c/(1-w)$$

with finite T. The approximation does not essentially change the results as far as T is sufficiently large because $0 \ll w < 1$ and $f_i(t)$ is bounded. For this study, we set $T = 500$. Furthermore we confirmed that the result does not change even in the cases of larger T, $T = 1000$ and 5000.

We adopt the discrete time replicator-mutator dynamics (Hofbauer and Sigmund, 1996) to investigate the evolution of the shares of the strategies. The share of strategy $\mathbf{P}_i$ at the next generation, x_i', is determined as

$$x_i' = \frac{\sum_{j=0}^{2^n-1} x_j f_j q_{ji}}{\sum_{j=0}^{2^n-1} x_j f_j}, \tag{A.4}$$

where q_{ji} is the probability that mutation of the strategy $\mathbf{P}_j$ gives rise to strategy $\mathbf{P}_i$. For this study, we assume that an element of the strategy vector inverts with the probability μ ($0 < \mu \ll 1$), namely $q_{ji} = \mu^{d_H(\mathbf{P}_j, \mathbf{P}_i)}(1-\mu)^{n-d_H(\mathbf{P}_j, \mathbf{P}_i)}$, where $d_H(\mathbf{P}_j, \mathbf{P}_i)$ indicates Hamming distance between $\mathbf{P}_j$ and $\mathbf{P}_i$. For example, in four-person games, the probability that $\mathbf{P}_{15} = (1,1,1,1)$ gives rise to the strategy $\mathbf{P}_3 = (0,0,1,1)$ is $\mu^2(1-\mu)^2$ and the probability that $\mathbf{P}_{15} = (1,1,1,1)$ gives rise to the strategy $\mathbf{P}_7 = (0,1,1,1)$ is $\mu(1-\mu)^3$. Intuitively, this type of mutation implies that a strategy is more likely to mutate into a similar strategy in the sense of Hamming distance.

Appendix B

We show the effect of mutation rate, μ, on the time average of the overall level of cooperation for smaller values of $b = 6, 8$ in Fig. 3.
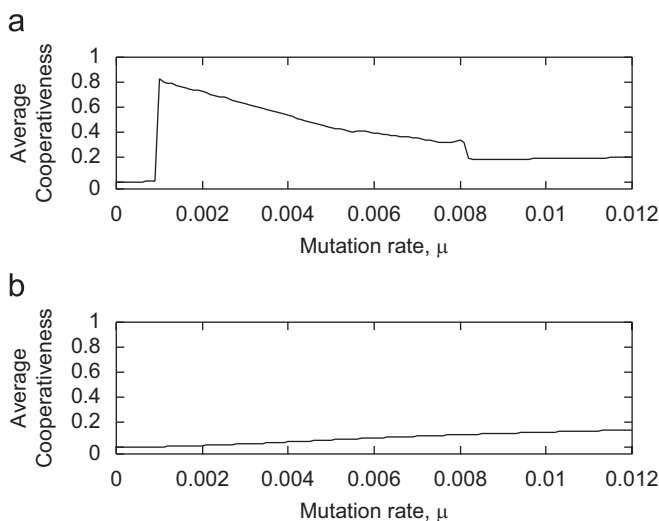

Fig. 3. Effect of mutation rate, μ, on the time average of the overall level of cooperation: (a) $b = 8$ and (b) $b = 6$.

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