Impact of Population Structure on Cooperation

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# ??

TBD

Allow agents to add their own links. Analyze the resulting network. Is it heterogeneous? Scale-free?

Regular graphs represent unrealistic representations of the real world.

Run simulations, gather statistics about resulting networks, compare to the characteristics for different network types: scale free, etc.

Two different forms up update dynamics (or evolutionary dynamics): synchronous and asynchronous. In synchronous updating, after all games in a generation have been played, the agents then proceed to update their strategies simultaneously. In asynchronous updating, a randomly chosen player plays the game and immediately updates its strategy followed by the next randomly shoes agent and so on. (Tanimoto – Fundamentals of Evolutionary Game Theory and its Applications).

# Social Dilemmas of Cooperation

(insert description of prisoner’s dilemma, snowdrift/chicken/hawk-dove, stag-hunt games here)

prisoner’s dilemma – PD

* T > R > P > S

snowdrift/chicken/hawk-dove game – SG

* T > R > S > P

stag-hunt game – SH

* R > T > P > S

# Network Types

Random graphs

Small-world networks

Single-scale networks

Broad-scale networks

Scale-free networks

Explain heterogeneity of degree distributions…

# Related Work

## Evolution of Cooperation in Well Mixed Populations

Insert (Axelrod/Hamilton 1981) summary here.

## Evolution of Cooperation on a Square Lattice

In [2], the authors consider a scenario in which *n* agents are arranged on a square lattice with each agent connected to its nine immediate neighbors. The edges of the lattice do not wrap around giving agents on its edges fewer neighbors. Unlike other studies reviewed in this section, in each generation, each agent plays the game with itself as well as its nine immediate neighbors. After all games for a generation have been played, the strategy of each agent is replaced with the strategy of the fittest agent among itself and its eight neighbors.

The authors only consider the PD game and use a simplified payout structure for that game with R=1, T=b and S=P=0 where b>1. At the start of each simulation, a specified fraction of cooperators are allocated to nodes in the lattice with the remaining nodes allocated to defectors. Various starting configurations are considered consisting of different fractions of cooperators and different initial placement of those cooperators. The authors run multiple simulations with varying values for *b* and various starting configurations. They find that when , cooperators and defectors coexist in the population.

(insert summary of [3] here)

## Impact of Network Topology on Evolution of Cooperation

In this section, studies are reviewed that demonstrate the sensitivity of cooperation to the topology of the network occupied by the agents. In the cases considered here, the network topology remains fixed during the simulation. Although some of the studies reviewed consider the impact on cooperation in games other than the prisoner’s dilemma, this review focuses on the results reported for PD. All studies, expect[8], use the payout structure used in [2]. In [8], the authors broaden the range of payouts considered for the PD game to include values for S between zero and -1.

In each case, agents are allocated to the nodes of a graph with a fixed number of nodes equal to the number of agents *n*. The graph has a fixed number of edges giving the graph a fixed average connectivity *z*. The agents follow one of two strategies: unconditional cooperation or unconditional defection.

For each generation, each agent plays the social dilemma game being investigated with each of its neighbors achieving a fitness score equal to the sum of the payouts earned from each game. The number of games played by agent *i* is equal to the degree *ki* of the node it occupies and the total number of games played by all agents is equal to the number of edges in the graph. For non-homogeneous network structures, some agents will play more games than other agents.

The evolutionary dynamics are the same as those used in [3] for simulations involving pure strategies except extended to handle graphs with heterogeneous degree. After all games for a generation have been played, the nodes are updated synchronously. For each agent *x* with payout *Px*, one of its neighbors *y* with payout *Py* is selected at random. If then agent *x* maintains its original strategy. Otherwise, agent *x*’s strategy is switched to the strategy of agent *y* with probability *p* defined as follows:

where and *ki* is the degree of node *i*. (The probability calculation can be re-written as a single equation by taking the max of zero and p as defined above).

where For any agent *i*, and … need to get a better understanding of how this equation works.

Initially, an equal number of cooperators and defectors are randomly allocated to the nodes of the graph. After executing 10,000 generations to reach a stationary regime, the final 1000 generations are used to compute the equilibrium frequency of cooperators and defectors in the population (Hauert & Doebeli 2004).

### Fully Connected Networks

The case of agents occupying a fully connected graph corresponds to the well-mixed population case and provides the foundation for the study of the impact of network topology on the evolution of cooperation. A fully connected graph has an average path length *L* of 1, a clustering coefficient *C* of 1, an average degree *z* of and a degree distribution consisting of a single spike located at *z*.

In [8], the authors run experiments on well-mixed populations of agents that occupy a complete graph. The authors reconfirm (need references) that cooperators are driven to extinction when playing the prisoner’s dilemma in a well-mixed population.

### Homogeneous Graphs

A fully connected graph is a special case of a regular graph in which every node is connected to every other node giving it an average degree equal to . Homogeneous regular graphs with lower average degree, have greater average path lengths and lower clustering coefficients than fully connected graphs while preserving a spiked degree distribution. The regular structure of the graph creates correlations between vertices such that the neighbors of a node are more likely to be neighbors of each other. This causes the clustering coefficient to be higher than for an unstructured graph with the same average degree. The increased average path length combined with the moderate clustering coefficient provides the opportunity for some separation between clusters of nodes compared to the fully connected case in which the graph contains a single cluster of nodes.

The authors of [4], [5], [8] and [9] find that, when the agents occupy a regular graph with average degree significantly less than , cooperators can eliminate defectors for small values of *b*. This small window of opportunity exists because the regular spatial structure allows cooperators to form compact clusters that resist invasion by defectors. As expected, increasing *b* decreases the performance of cooperators. In addition, as the average degree of the graph increases the population structure begins to mirror a well-mixed population and cooperation becomes more difficult.

A homogeneous regular graph can be transformed into a homogeneous random graph by swapping the ends of randomly selected edges until all edges have had their ends swapped while ensuring that no duplicate edges or loops have been introduced. This procedure leaves the degree of each node and the degree distribution of the graph unchanged. However, randomly swapping the ends of edges removes vertex correlations and introduces “short cuts” which reduces both the clustering coefficient and the average path length of the resulting graph compared to the original graph. This reduces the possibility of the formation of small semi-isolated clusters of nodes.

In [8], the authors find that when occupying the nodes of a homogeneous random graph, the best cooperators can do is coexist with defectors for small values of *b*. The decrease in cooperation is because the uncorrelated social structure no longer allows cooperators to form tight clusters that resist invasion by defectors. However, the cooperation level is higher than on fully connected graphs demonstrating that the reduction in degree provides some ability for cooperators to resist defectors at least temporarily.

### Heterogeneous Graphs

The results presented in the previous section show that regular graph structure introduces correlations between nodes in the graph that allow agents to form tight clusters that resist invasion by defectors at least for small values of *b*. However, regular graphs are unrealistic representations of most real world networks [4]. This section reviews some graph topologies that provide better representations of real networks.

A small-world graph is a graph that has moderate clustering coefficients similar to regular graphs and small average path lengths similar to random graphs [14]. The degree distribution of these small-world networks is moderately heterogeneous. The Watts-Strogatz algorithm [14] can be used to construct graphs that range from regular graphs to random graphs with the graphs in the middle of this spectrum displaying small-world characteristics. Starting from a regular ring graph, the final graph is produced by randomly rewiring each edge in the graph with probability *p*.

The parameter *p* can be used to control the degree of heterogeneity introduced into the original graph. For *p*=0, the original random graph is preserved. For *p*<0.01, the average path length of the graph decreases quickly as *p* increases while the clustering coefficient remains virtually unchanged. For *p*>0.01, the clustering coefficient decreases rapidly while the average path length remains virtually unchanged. As *p* approaches 1, the graph structure approaches a random graph.

In the studies cited in the previous section, the authors found that cooperators that occupy regular graphs can coexist with defectors for values of *b* up to1.1 when the average degree of the graph is kept low. For small-world networks, in [4] and [6], the authors find that cooperators can coexist with defectors over a broader range of values for *b*. For *p*=0.1, cooperators can coexist for values of *b* up to approximately 1.2. As *p* increases, cooperators can coexist with defectors for larger values of *b* until a maximum of approximately 1.6 is reached for *p*=1 [6]. As with the case of a regular graph, cooperation suffers as the average connectivity *z* of the graph increases and the population approaches a well-mixed population.

These results indicate that cooperators fare the best on small-world networks when the clustering coefficient and average path length are at their smallest. This seems to contradict the results reported in the previous section where decreasing clustering coefficient and decreasing average path length reduced the ability of cooperators to form compact clusters and thus reduced their ability to compete with defectors. It appears that the success of cooperators is due to the heterogeneous degree distributions possessed by these networks.

In [7], the authors introduce a homogeneous small-world graph in order to better understand the impact of heterogeneity on the evolution of cooperation. To construct a homogeneous small-world graph, the authors modify the Watts-Strogatz algorithm so that the degree of each node is preserved during the rewiring process. A parameter *f*, representing the fraction of edges to be rewired, plays a role similar to the role played by *p* in the original algorithm. The authors report that the average path length and clustering coefficient vary relative to *f* in the same way they vary relative to *p* indicating that the resulting graphs provide an reasonable representation of a homogeneous small-world graph.

The authors find that the performance of cooperators on homogeneous small-world graphs is worse than their performance on regular graphs for small values of *b* and only marginally better for larger values of *b*. Performance of cooperators on heterogeneous small-world graphs is significantly better than on the homogeneous variety for all values of *b*. This indicates that most of the performance improvement of cooperators on heterogeneous small-world graphs reported earlier in this section is due to the heterogeneity of these graphs rather than their small-world properties.

To further investigate the impacts of heterogeneity on the evolution of cooperation, the authors in [8] investigate the performance of cooperators on moderately heterogeneous random graphs called single scale-graphs [16]. These graphs are generated using the configuration model [17] to produce a maximally random graph that is compatible with the specified degree distribution. The degree distribution of these graphs is more heterogeneous than the small-world graphs considered in the earlier in this section.

The authors find that cooperators perform significantly better on single scale networks than on regular graphs. Comparison of the results reported [8] with those reported in [4] and [7] show that cooperators fare about the same on single scale networks as on Watts-Strogatz graphs with *p*=1. For single scale graphs, cooperators can eliminate defectors for values of *b* up to approximately 1.16 and coexist up to 1.5. This once again points to heterogeneity providing a significant boost to cooperation.

### Scale Free Networks

The results presented in the previous section indicate that heterogeneity can overcome the problems introduced by low average path length and low clustering coefficients. In a heterogeneous network, some nodes in the network allow agents participate in more interactions than others. These hub nodes give their occupants the ability to accumulate higher payouts and therefore give them a fitness advantage

Both cooperators and defectors can benefit from occupying a hub but, in this case, defectors become victims of their own success. The strategy of the agent that occupies the hub will be propagated to the agents in the neighboring nodes. This means that defectors will surround a defector while cooperators will surround a cooperator. However, both defectors and cooperators perform best when surrounded by cooperators. As more and more defectors surround an incumbent defector, its fitness decreases until a cooperator is able to take over the hub. Once a cooperator holds the hub, its strategy is propagated to neighboring nodes reinforcing its dominant position.

In the previous two sections, vertex correlations and heterogeneity were shown to benefit cooperators. In this section, graph topologies that combine heterogeneity with vertex correlations are investigated.

A scale-free network is a graph in which the degree distribution follows a power-law distribution. The Barabási-Albert algorithm can be used to construct scale free graphs with varying numbers of nodes. This algorithm incorporates two key concepts: growth and preferential attachment. The algorithm begins with a small number of vertices and then grows the graph by adding nodes to the graph one at a time. At each time step, a node is added to the network and attached to *m* different existing nodes. The probability that an existing node is selected as one of the *m* nodes is proportional to node’s degree. Therefore, there is a preference to attach to nodes with a high degree. This preferential attachment produces graphs whose nodes have ”age correlations” in which older vertices have higher degree and are interconnected with other old vertices.

In the studies cited in the previous section, the authors found that cooperators that occupy heterogeneous graphs were able to coexist with cooperators for values of *b* up to 1.5. For scale-free networks, in [4], [5], [6] and [8], the authors find that cooperators are able to eliminate defectors for all values of *b* given that remains small. In addition, they find that, unlike for other graphs types considered, the performance of cooperators improves as the average connectivity *z* of the graph increases up to a critical value at which the population approaches the well-mixed condition.

These results indicate that the introduction of age correlation has a significant positive impact on the ability of cooperators to dominate defectors. The introduction of age correlation increases the ability of cooperators to form compact clusters especially around the nodes with high connectivity. This increased ability to form compact clusters of cooperators increases the performance of cooperators. In addition, the increased correlation between hub nodes makes it easier for a cooperator from one hub to invade a hub controlled by a defector after that defector’s fitness is reduced due to its neighborhood becoming filled with defectors.

In [4], [5] and [9], the authors consider a network grown using uniform attachment to investigate the impacts of growth independent of preferential attachment. To construct such as graph, the authors modify the Barabási-Albert algorithm so that the node selected for attachment is selected from among all nodes with equal probability rather than with probability proportional its degree. This procedure generates a graph with an exponential degree distribution, and fewer large hubs and less age correlation compared to graphs generated using the Barabási-Albert algorithm [5].

The authors find that, in the context of a graph grown using uniform attachment, cooperators are able to maintain control over more than 80% of the population or values of *b* up to approximately 1.4. This is better the single scale graph which can only maintain that level of cooperation for values of b up to approximately 1.16 but significantly worse than the standard scale-free network that is able to sustain a similar fraction of cooperators for all values of *b* considered. This indicates that preferential attachment plays an important role in generating networks that can sustain a high degree of cooperation.

In [5], [6] and [8] the authors consider random scale-free networks in order to analyze the impact of increased heterogeneity provided by the power-law degree distribution independent of the age correlations introduced by Barabási-Albert algorithm. In [5], the authors use the configuration model [17] to construct a random graph with a power-law degree distribution while in [6] and [8] the authors start with a standard scale-free network and then swap edges in order to remove correlations between vertices without changing the degree distribution [18]. In both cases, the resulting lacks any correlations between vertices.

The authors find that cooperators can coexist with defectors on a random scale-free network for all values of *b* considered. Except for values of *b* less than 0.1, cooperators perform significantly better on random scale-free networks than on random Watts-Strogatz and single scale networks. However, cooperators perform better on the uniform attachment network for all values of *b* less than 1.8. This indicates that while the increased heterogeneity of random scale-free networks does benefit cooperation, the presence of age correlation significantly improves cooperator performance especially for large values of *b*.

While the Barabási-Albert algorithm produces age correlations between vertices, some empirical evidence suggests that some real world networks have higher clustering coefficient than those produced by that algorithm [19]. To analyze the performance of cooperators on networks that may more closely match these real world networks, the authors of [5] the minimal model network . This network maintains the power-law distribution of the A-B network but exhibits a significantly larger clustering coefficient (10-3 for A-B, 0.7 for minimal model, 1 for fully connected graph) . The authors note that this model may more accurately reflect real world social and biological networks. This type of network provides an extra boot to cooperation especially for large values of *b*.

In [5], the authors consider networks that are generated using a modification of the A-B algorithm that imposes limits on the maximum degree of any node in the network. This can model the possibility that maintaining connections is expensive and agents may have limits to the number of connections they can maintain. Cooperators actually perform slightly better on these “cut-off” networks than on standard A-B graphs but perform worse than on the minimal model network.

The authors of [4] and [6] also evaluate the ability of a sole defector to invade a population of cooperators. They find that defectors are not able to invade a population of cooperators that occupy a scale-free network even if the defector takes over the most advantageous hub node with the largest connectivity. In order for the defector to take over, the average connectivity z of the network needs to be increased to a high enough level that the network begins to approximate the well-mixed case – a case in which it is well-known that cooperators cannot withstand invasion by a single defector.. In [5] and [6], the authors show that any defectors that remain in the population are driven from nodes with high connectivity and relegated to nodes with moderate to low connectivity.

### Impact of Population Size

[4][5] [9], down to N=128, below this results are unpredictable

The authors of [4] analyze the impact of population size on the success of cooperators. They find that, given a constant average connectivity *z*, the size of the population has little effect on the performance of cooperators. Leading to the insight that the structure of the network is more important than the size of the population. The authors note that for small populations, the graphs constructed using the growth and preferential attachment approach are not scale free. Leading to the insight that growing the network using preferential attachment is more important than the scale-free characteristics.

This study applies the same payout structure as (Nowak/May 1992).

The studies reviewed in this section provide results for several different network topologies:

* Fully connected network: In this case, the network is a fully connected graph where the degree of each node is equal to the graph’s average connectivity . This case corresponds to the so-called *well-mixed* population case.
* Homogeneous structured network: In this case, the network is a regular graph where the degree of each node is equal to the graph’s average connectivity .
* Homogeneous unstructured network: In this case, the network is a random homogeneous graph where the degree of each node is equal to the graph’s average connectivity . Starting from a regular graph with average connectivity *z*, the ends of pairs of randomly chosen edges are swapped without introducing duplicate connections. All pairs of edges are considered [8].
* Watts-Strogatz network: In this case, the network is a small-world network generated using the Watts-Strogatz graph generation algorithm [14]. Starting from a regular ring graph with each node connected to its *k* nearest neighbors, edges in the graph are rewired randomly with probability *p* to produce a Watts-Strogatz graph. When *p*=0, the graph is unchanged while setting *p*=1 produces a graph very similar to a random graph except there are no vertices with degree less than [6]. For intermediate values of p, the graph displays small world characteristics of short path length and cliquishness as measured by the clustering coefficient[[1]](#footnote-1). For *p*<0.1 the path length decreases sharply while the clustering coefficient remains fairly constant while for *p*>0.1 the clustering coefficient decreases quickly.
* Homogeneous small-world network: In this case, the network is generated using a modification of the Watts-Strogatz graph generation algorithm that ensure that the degree of each node remains unchanged. Therefore, the resulting graph is homogeneus. In [7], the authors show that the path length and clustering coefficient of these graphs is very similar to graphs generated using the unmodified Watts-Strogatz algorithm indicating that the main difference between these two types of small world networks is their heterogeneity.
* Single-scale network: In this case, the network is a type of small-world network with moderate heterogeneity where the degree of most nodes does not deviate significantly from the graph’s average connectivity [16]. A single-scale network can be generated using the configuration model (Molloy & Reed) to produce a maximally random graph that is compatible with a specified degree distribution [8].
* Scale-free network: (Barabasi-Albert) In this case, the network is a graph generated using a process that involves preferential attachment. The resulting graph has strong heterogeneity with a degree distribution follows a power-law [15]. The process used to generate these graphs results in a network with large hubs and age correlations between the vertices. The age correlation causes these high connectivity hubs to be interconnected.
* Scale-free network with uniform attachment: In this case, the graph is generated using the same process as scale-free networks except that uniform attachment is instead of preferential attachment.
* Minimal model network: See Dorogotsev Size-dependent degree distribution of a scale-free growing network, Phy Rev E, 63, 2001. Algorithm that generates a graph with the same power-law distribution of scale-free network but with a larger clustering coefficient. (Does it also reduce the total degree of the nodes with the highest connectivity?) This model is referenced in [5].
* Random scale free network: In this case, the network is a scale-free network in which the connections between nodes have been randomized to remove “age correlation” introduced by the process of generating the graph while preserving the power-law degree distribution [6] [8].
* Configuration model network: Used to generate “random-scale free” network? See Molloy & Reed, A critical point for random degree graphs with a given degree sequence, random Struct Alg, 6, 1995. “ensures a maximally random graph compatible with a pre-defined degree distribution”

In the following studies (list references), the authors consider different social dilemma games and evaluate the impact of graph topology on the evolution of cooperation.

The results reported in each study are for *n* = 104 ~~and~~ *~~z~~* ~~= 4~~.

**(Pacheco Santos 2005 Network dependence of the dilemmas of cooperation)**

In [4], the authors consider the cases of agents playing the PD and SG games on different topologies including homogeneous structured networks represented as regular ring-graphs, Watts-Strogatz networks and scale-free networks. The authors use the following payout structure for the PD and SG games. For PD the payout structure is 2>T=b>1, R=1 and P=S=0. For the SG game the payout structure is T=β>1, R=β-1/2, S=β-1 and P=0. For the SG game, the payouts are consolidated into a single parameter representing the cost-to-benefit ratio of mutual cooperation.

PD payout structure: 2>T=b>1, R=1 and P=S=0 [2][4][5][6][7].

**(Santos, Pacheco, Lenaerts 2006 Evolutionary dynamics of social dilemmas in…)**

The authors consider a slightly different payout structure: R = 1, P = 0, and . The authors consider several different network topologies:

* Well-mixed populations: This corresponds to the case when the graph is fully connected resulting in a homogeneous network with average connectivity . For each node *i* in the network, . The authors reconfirm that cooperators are unable to compete with defectors when playing the Prisoner’s Dilemma in a well-mixed population.
* Homogeneous structured populations: In this case, the network is a regular graph with an average connectivity . The authors find that cooperators can outperform defectors in the Prisoner’s Dilemma when T is slightly larger than R and S is only slightly less than P. This small window of opportunity exists because the correlated spatial structure allows cooperators to form small clusters that resist invasion by defectors.
* Homogeneous unstructured populations: In this case, the network is a random graph where the degree *ki* of each node is equal to the average connectivity of the graph. The authors find that the ability of cooperators to outperform defectors is reduced in this case. The reduction occurs because the uncorrelated social structure no longer allows cooperators to form tight clusters that resist invasion by defectors. However, there is still a small window where cooperators can coexist with defectors showing that the reduction in connectedness provides some benefits to cooperation compared to the well-mixed fully-connected case.
* Heterogeneous structured populations: In this case, the network is a non-regular graph where the degree *ki* of each node is not necessarily equal to the average connectivity *z* of the graph. The authors consider two types of heterogeneous networks that fall into the class of small-world networks (Amaral 2000):
  + Single-scale network: a graph with moderate heterogeneity where the degree of most nodes does not deviate significantly from the graph’s average connectivity. The authors find that cooperators can outperform defectors in the Prisoner’s Dilemma when T is slightly larger than R and S is only slightly less than P. This window is slightly larger that the window provided by the homogeneous structured case. The single scale network has characteristics similar to the random graph in the homogeneous unstructured case that prevents compact clusters of cooperators from forming. However, the heterogeneity of the network offsets this effect and leads to an overall improvement in the conditions for cooperation.
  + Scale-free network[[2]](#footnote-2): a graph with strong heterogeneity whose degree distribution follows a power-law. The authors consider two types of scale free networks:
    - Random scale free network: In this scale-free network in which the connections between nodes remain random. The authors find that the introduction of scale-free characteristics into the network significantly improves the chances that cooperators can coexist with defectors in the Prisoner’s Dilemma game. Comparing this to the result obtained for single-scale networks shows that increasing heterogeneity appears to have a positive impact on cooperation. However, the randomness of the connections decreases the ability of cooperators to form tight clusters that resist invasion thus reducing the effectiveness of heterogeneity.
    - Barabási-Albert model: This scale-free network is grown using a process that involves preferential attachment. The process introduces “age correlation” in which older vertices have higher degree and are interconnected with each other. The authors find that the introduction of age correlation has a significant positive impact on the ability of cooperators to dominate defectors. The introduction of age correlation effectively eliminates the randomness that prevents clusters of cooperators from forming.

## Evolution of Network Topology

(Eguiluz Zimmrman 2005 Cooperation and the emergence of role differentiation…)

The authors consider the scenario in which *N* agents occupy the nodes of a graph. The graph contains edges that are initially inserted randomly between pairs of nodes. The constant *K* defines the average connectivity of the nodes in the graph (denoted *z* above). The agents follow one of two fixed strategies: unconditional cooperation or unconditional defection. In each generation, each agent plays the Prisoner’s Dilemma game with each of its neighbors achieving a fitness score equal to the sum of the payouts earned form each game. Following (Nowak/May 1992), the following payouts are used when playing the game: R=1, T=b>1, S=P=0.

After all games for a generation have been played, synchronous updating is used to evolve both agent strategies and the network topology. Following (Nowak/May 1992), after all games for a generation have been played, the strategy of each agent is replaced with the strategy of the fittest agent among itself and its neighbors. In addition, if an agent imitates a neighbor that is a defector then, with probability *p*, the link between the agent an the imitated defector is replaced with a link between the agent and an agent selected randomly from among all agents in the network.

Initially, the graph is populated with 60% cooperators randomly allocated to nodes in the graph. The remaining nodes are populated with defectors. After the simulation reaches a stationary state, the fraction of cooperative agents that exist in the population is computed. The authors report simulation results for various values of *p* and *b*. The values reported are averages over 100 simulation runs.

The authors collected results for the following range of values for *p* and *b*: and . The authors find that for , the fraction of cooperators in the population is kept above 90% for the range of values of *b* considered.

**(Santos, Pacheco, Lenaert PLoS 2006 Cooperation prevails when individuals adjust…)**

The authors consider the scenario in which *N* agents occupy the nodes of a graph with *NE* edges. Initially, each node is randomly connected to other nodes. Each agent follows one of two fixed strategies, unconditional cooperation or unconditional defection, and earns a fitness score that is equal to the sum of the payouts earned when playing social dilemma games against other agents. The payout matrix used for the games fixes R =1 and P = 0 and allows T and S to vary with and . As usual, the fitness score earned by the agents are used to update the strategies those agents follow. In addition, the fitness scores are also used to update the structure of the network. This leads to the co-evolution of the strategy composition of the population and the structure of the network that holds that population.

The evolution of the strategies and network structure occur at different time scales. Let *τe* be the time scale at which strategy updates occur and *τa* be the time scale for network structure updates. In this case, the ratio specifies how frequently structure updates occur relative to strategy updates. For example, if and then indicating that structure updates occur three times as often as strategy updates. The probability *p* that a strategy update occurs during any time step is given by the following formula:

Given this, a structure update occurs with probability . In the example presented earlier, the probability that a strategy update occurs during any time step is while the probability that a structure update occurs is . Therefore, on average one strategy update occurs for every three structure updates.

For both types of updates, an agent *x* is chosen at random and another agent *y* is chosen at random from agent *x*’s neighbors. The two agents play the game with each of its neighbors and earn fitness scores *Px* and *Py* respectively.

In the case of a strategy update, after the games have been played, the strategy of *y* replaces the strategy of *x* with probability *pe* given by the following equation:

This is the *pairwise comparison* process introduced in Traulsen/Nowak/Pacheco Phys Rev E 2006). The parameter *βe* represents the selection strength and determines how strongly the fitness score impacts the decision to replace *x*’s strategy with *y*’s strategy. As , the dynamics approach neutral drift where each strategy has 50% chance of selection. As , the dynamics approach imitation dynamics in which the strategy of the fittest agent is always selected.

In the case of a structure update, agent *x* attempts to rewire the link between it and agent *y* if it is dissatisfied with the current link. An agent is satisfied with a link if the agent on the other end is a cooperator and dissatisfied otherwise. In the case that agent *x* is dissatisfied, the link is switched to a random neighbor of agent *y* with probability *pa* given by the following equation:

This equation is almost identical to the equation for *pe* except that the roles of *Px* and *Py* are switched and selection strength is controlled by the parameter *βa*. To ensure that the network remains connected, the link to agent *y* cannot be removed if this is agent *y*’s only link.

Initially, the structure of the network is a homogeneous random graph in which the degree of each node is equal to the average connectivity *z* of the graph. An equal number of cooperators and defectors are randomly allocated to the nodes of the graph. Each simulation is run until the fraction of cooperators reaches 100% or the number of generations reaches 108. In the case that the fraction of cooperators does not reach 100%, the average fraction of cooperators over the last 1000 generations is used as the result. The authors run simulations with varying values for T, S and W. For each set of parameter values, 100 simulations are run, each with a different starting configuration, and the results are averaged to determine the fraction of cooperators that survive evolution.

The authors find that for *W* = 0 and moderate selection strength, the results reproduce the predictions for finite, well-mixed populations. As *W* increases, it becomes easier for cooperators to survive until *W* reaches a critical value (*Wc*) at which cooperators dominate for any value of T and S. The value of *Wc* increases as *z* increases as expected since there are more links to be rewired in order to reach a state where cooperators can dominate.

The authors find that the value of *W*, as well as the structure of the game payouts, determines the level of heterogeneity that evolves in the network. In general, as *W* increases, the heterogeneity of the evolved network increases. When the payout structure of the game favors defectors but allows cooperators to coexist at low levels, the few cooperators that remain accumulate large numbers of links leading to highly heterogeneous networks. The authors find that large T promotes heterogeneity more than large S.

Finally, the authors find that the value of *Wc* decreases as either of the selection strength parameters (*βe* and *βa*) increase. Smaller selection strength values allow less fit agents to survive. Prior to the network being rewired, cooperators are generally less fit than defectors. Smaller β values allow less fit cooperators to survive long enough to restructure the network into a state where cooperators dominate defectors.

Let *Nk* equal the number of nodes with degree *k* and *kmax* be the maximum degree possessed by any node in the graph. Then, the *degree of heterogeneity* of the graph *h* is given by the following equation:

And the *cumulative degree distribution* of the graph *D(k)* is given by the following equation:

**(Fu Hauert Nowak Wang Phy Rev E 78 2008 Reputation-based partner choice…)**

The authors extend the framework introduced in (Santos/Pacheco/Lenaert PLoS 2006) to allow reputation to influence the link rewiring process. The authors use the *image score* metric introduced in (Nowak/Sigmund 1998) to measure an agent’s reputation.

As in (Santos, Pacheco, Lenaert PLoS 2006), the evolution of strategies and network structure occurs asynchronously. A strategy update proceeds as defined in (Santos, Pacheco, Lenaert PLoS 2006). However, in addition, agent *x*’s reputation is also updated using the following formula:

where is the reputation of agent *x* at time *t*, equals 1 if agent *x* cooperated at time *t* and zero otherwise and for any agent *i*.

The structure update procedure is modified to allow reputation to guide the relinking process. An agent *x* is selected at random to replace the link that exists between itself and its lowest reputation neighbor. With probability *p*, the link that exists between agent *x* and its lowest reputation neighbor is replaced with a link to the agent with the highest reputation among agent *x*’s neighbors’ neighbors while with probability that link is replaced with a link to an agent selected at random from among all agents except agent *x*’s neighbors. To ensure that the network remains connected, the link to agent *x*’s lowest reputation neighbor cannot be removed if this is that agent’s only link.

In their simulations, the authors focus on a payout structure that is consistent with the Prisoner’s Dilemma game. Specifically, they use the payout structure introduced by (May/Nowak 1992): R=1, T=b>1, S=P=0. As in (Santos, Pacheco, Lenaert PLoS 2006), each simulation begins with an equal number of cooperators and defectors randomly allocated to the nodes in a homogeneous random network. For each simulation, a total of 106 generations are executed and the fraction of cooperators that survive evolution is determined by averaging over the last 1000 generation. The data presented in the paper results from averaging over 103 simulation runs.

For moderate parameter settings (N=104, z=10, b=1.2, W=1, p=0.5 and β=0.01), the authors find that cooperation steadily increases with cooperators coming to dominate the population after 25,000 generations. A growth in the frequency of links between cooperators (CC) mirrors the growth in cooperation with a corresponding decrease in the links to defectors. After reaching a steady state dominated by cooperators, as expected based on earlier studies (cite here), the network is highly heterogeneous with a degree distribution that follows a power law indicating that the network has properties similar to a scale free network. As expected, due to introduction of high reputation partner seeking behavior, the authors find a positive correlation between the reputation score of an agent and the degree of the node occupied by that agent.

Consistent with (Santos, Pacheco, Lenaert PLoS 2006), the authors observe a existence of a critical value *Wc* for *W* above which cooperators eliminate defectors from the population and that *Wc* increases with increasing *b* (T). However, the introduction of reputation-based partner switching lowers critical point at which cooperators are able to dominate. While (Santos, Pacheco, Lenaert PLoS 2006) report that for S=0 and T=1.8, *W* must be equal to approximately 2 before cooperators can dominate, the current study finds that, for the same values of S and T, cooperators can dominate for values of *W* as low as approximately 0.3.

The authors also consider a reputation model variation that includes reputation discounting:

Where is the discount rate that diminishes the value or earlier good deeds over time. When the agent’s last action alone determines it reputation while when the original model is restored. The authors find that if the frequency of partner switching is high enough (approximately ), smaller values of δ make it easier for cooperators to dominate. IN this case, when agents react immediately to defection, it makes it harder for defectors to maintain links that allow them to exploit cooperators.

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1. https://en.wikipedia.org/wiki/Clustering\_coefficient [↑](#footnote-ref-1)
2. https://en.wikipedia.org/wiki/Scale-free\_network [↑](#footnote-ref-2)