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# A Framework for Indirect Reciprocity

With *direct reciprocity*, an agent responds in-kind to its direct experiences with another agent. The game theoretic framework of the prisoner’s dilemma is often used to investigate the evolution of cooperation through direct reciprocity. In the seminal work on the evolution of cooperation through direct reciprocity, the authors of [1] showed that, under certain conditions, cooperation based on direct reciprocity could become established in a population of unrelated individuals. This work prompted many follow on studies where the results have been debated and refined.

With *indirect reciprocity* an agent responds based on observations of interactions involving agents other than itself. The driving force behind indirect reciprocity is reputation [2]. An agent’s reputation contributes to its perception as trustworthy by other agents. Agents learn the reputations of other agents and thereby learn which agents can be trusted to cooperate and which are likely to defect.

In one of the seminal works on the subject, the authors of [3] introduced a theoretical framework for analyzing the evolution of cooperation through indirect reciprocity. Through contributions made in publications following its original introduction, a refined and well-defined framework has emerged. Here that framework is defined as consisting of five elements: a simple game known as the *indirect reciprocity game*, a *reputation score* for measuring an agent’s reputation and assigning a moral status to that agent, an *assessment rule* for evaluating the morality of an agent action, and an *action rule* that defines the action a donor should take in each possible situation.

## Indirect Reciprocity Game

The indirect reciprocity game forms the foundation for the framework. In this game, two agents are paired together with one agent taking the role of the donor and the other agent taking the role of the recipient. The donor can choose to donate at a cost of *c* to itself or abstain from donating. If the donor donates, the recipient receives a benefit *b > c*. Otherwise, both agents receive zero payout. The payouts for the game are described in the following table.

|  |  |  |
| --- | --- | --- |
| **Donor Action** | **Donor Payout** | **Recipient Payout** |
| Contribute | -*c* | *b* |
| Do not contribute | 0 | 0 |

Since *b* > *c*, when the game is played repeatedly, all agents stand to benefit if all agree to cooperate. However, the temptation to defect and exploit the largess of cooperators threatens the stability of community-wide cooperation.

Because the population is large, the probability that two agents meet twice during a round is very small and thus the game can be used to investigate indirect reciprocity. Since the driving force behind reciprocity is reputation, the rest of the framework deals with the management of agent reputations.

## Population Structure

The population can be structure as a single, well-mixed group or divided into “islands” [10]. Although not review here, the population can also be organized into a single population with structure. For example, the population could be organized into a lattice structure or a graph structure.

## Population Evolution

This indirect reciprocity game is used in the context of evolutionary game theory to evaluate the fitness of strategies followed by agents in a large population. During each generation, the indirect reciprocity game is played multiple times. For each repetition of the game, one agent is selected at random to play the donor role and a second agent is randomly selected to play the recipient role. The payout earned by each agent over the course of the round is tracked and used to determine the fitness of each strategy. For the next generation, the proportion of agents in the population that follow a particular strategy is proportional to the fitness of that strategy in the previous generation.

Fitness is either tracked at the individual level [3] or at the strategy level [5].

Mutations [3][5][7][10]

Sometimes a fixed number or rounds are played per generation. In other cases, a probability of another round is used to determine whether another round will be played in a generation.

Different approaches are used to evolve the population resulting is slight differences in how strategies are passed on from generation to generation.

When the population is divided into groups, the assessment rules used by each group may evolve as well [10].

## Reputation Score

The reputation score is an integer value used to measure the reputation of an agent. In most cases, lower and upper bounds are defined for the score. Common choices for bounds are [-5, 5] and [0, 1]. An agent’s action is deemed either good or bad by an action assessment rule (defined below). An agent’s reputation score is increased by one if its action is deemed to be good and decreased by one otherwise. If changing the agent’s score would cause it to move outside the established bounds then its score remains unchanged.

A threshold is used to determine whether an agent’s moral status is bad or good. If the agent’s reputation score is greater than or equal to the threshold then the agent’s moral status is good otherwise it is bad. Combining a reputation score whose bounds are [0, 1] with a threshold equal to 1 results in a binary reputation score where the agent’s moral status only depends on that last action it has taken.

The reputation of an agent can publically known by all agents [5][8][9][10] or privately known [7]. The publically know case is covered by the “indirect observation model” described in [8].

An agent can witness all actions taken by all other agents or only s subset of those actions. In some cases a probability of a perception error is introduced. If a perception error occurs then the agent assigns the wrong reputation to an agent (???).

Probability that an agent observes an interaction [3][7]

Probability that the interaction is perceived correctly [3]

Assignment error - Probably that reputation is assigned correctly[8][10]

Probability that an agent knows the reputation of another agent [3][6]

Probability that an agent assumes another agent is good prior to having any knowledge of that agent’s actions[6]

Assume an agent is good prior to having any knowledge of that agent’s actions [6]

In some cases, different thresholds are used to determine whether the donor and the recipient are “bad”.

Examples:

* [+5, -5] with ? threshold (look up thresholds used in [3][5])
* [0,1] with threshold equal to 1 (binary good/bad reputation) [3][5][6][7][8][9]

## Assessment Rule

An assessment rule determines whether an action taken by an agent is “good” or “bad”. An assessment rule assigns a moral value to all possible situations that an agent may encounter. The order of an assessment rule depends on the level of granularity that is used to distinguish situations to be assessed. A first-order assessment rule only considers the donor action and therefore only distinguishes two situations. A third-order action assessment rule considers the donor action, donor reputation and recipient action and therefore distinguishes eight situations. In general, an *n*-order action assessment rule can distinguish situations. An *n*-order assessment rule assigns one of the two moral values (good or bad) to each of the situations it can distinguish. Therefore, there are different *n*-order assessment rules.

All agents can share the assessment rule or each agent can have its own assessment rule. In some cases, a subset of the population shares an assessment rule (i.e., in the island model).[10]

Perception errors – with some probability, the agent fails to correctly perceive the action taken by another agent and assigns the wrong reputation to an agent [5][7]

Different perception error – donor believe it has donated while the recipient perceives that there was no donation [6]

Probability that a bad agent does not regain good reputation after performing an action that should provide a good moral status (modeling the observation that sometimes it is hard to overcome a bad reputation) [9]

Examples:

* Scoring [3][7][8][9][10] – donors are bad if they refuse to donate with any agent
* Weak Standing/Simple Standing [5][6][7][8][9][10] – donors are bad if they refuse to donate to good agents
* Strict Standing [8][10] – same as standing except bad agents cannot be come good by cooperating with bad agents – however, good agents remain good if they cooperate with a bad agent
* Strong Standing/Judging/Stern-Judging [4][7][10] – donors are bad if they refuse to donate to good agents or if they donate to bad agents, same as standing except that cooperating with a bad agent always makes an agent bad

## Action Rule

An action rule determines the action an agent will take in all possible situations that the agent may encounter. As with the assessment rule, the order of an action rule depends on the level of granularity that is used to distinguish situations. A first-order action rule only considers the reputation of one of the agents while a second-order rule considers the reputations of both agents. There are two different kinds of first-order action selection rules, those that consider the donor’s reputation and those that consider the recipient’s reputation. Similar to assessment rules, there are possible *n*-order action selection rules.

Execution error – with some probability, the agent fails to take the actions specified by the action rule and instead takes the opposite action. [5][7][10]

Implementation error – similar to execution error except that it only impacts an attempted donation (resulting in a failure to donate) and both donor and recipient are aware that the error occurred [6][8]

Examples:

* CO/Disc/RDisc - Cooperate is recipient is good [3][5][6][7][8]
* And – cooperate if recipient is good AND donor is bad [3][5][7]
* Or/CTFT – cooperate if recipient is good OR donor is bad [3][4][6][7]
* Self – cooperate if donor is bad [3][5][7][8]
* AllC
* AllD

## Parameters to be Varied

Benefit b

Cost c

Number of interaction per generation or alternatively the probability that another interaction will occur

Mutation rate

Execution errors

Perception errors

Non-random assortment (kin selection) is required for cooperation to overtake a population of defectors. If individuals get to seek out their partners can they acquire the knowledge necessary to sort themselves into groups with cooperators.

Researchers have taken different approaches to investigate the evolutionary stability of various strategies often introducing subtle differences into the overall basic framework. These small variations often make it difficult to directly compare results published by different researchers.

There are variations on exactly how the population is evolved after each round.

Using these scales for measuring reputation, the authors proposed a simple rule for assessing an action taken by a donor. The rule, now known as *scoring* in the literature, stipulates that cooperating is always good and defecting is always bad regardless of the reputations of the agents involved in the interaction.

Using the image score reputation scale and the scoring assessment rule, the authors investigated the ability of several donor strategies to promote the emergence of cooperation:

* Donate if, and only if, the image score of the potential recipient is greater than a threshold *k*
* Donate if the image score of the potential donor is greater than a threshold *k* AND the image score of the donor is less than a threshold *h*
* Donate if the image score of the potential recipient is greater than a threshold *k* OR the image score of the donor is less than a threshold *h*
* Donate if, and only if, the image score of the potential donor is less than a threshold *h*

Using computer simulations, the authors found that, under certain conditions, the first three strategies lead to the emergence of cooperation while the last strategy did not.

Using the binary image scale and the scoring assessment rule, the authors investigated the evolutionary stability of several donor strategies:

* Donate if, and only if, the image of the potential recipient is good

Using this simplified reputation model, the authors derived equations that describe the population dynamics. From these equations, the authors determined conditions under which cooperation will emerge.

Since the publication of this paper much debate has ensued regarding whether the proposed scoring assessment rule and proposed donor strategies provide a good model of how cooperation through indirect reciprocity emerged in human societies. The following objections are often raised in the literature that cites this paper:

* The factor that determines whether an agent will receive donations in the future is its own reputation. However, an agent following the first donor strategy listed above does not consider its own reputation when deciding whether to donate [5].
* The scoring assessment rule dictates that refusing to donate is bad regardless of the reputation of the potential donor. However, the first donor strategy dictates that an agent should refuses to donate to a bad agent. Following this strategy is irrational since it causes the agent itself to become bad and thus puts the agent in a situation where it will not receive donations in the future [7].
* Given the previous critique, the rational strategy is for the agent to donate to all agents regardless of their reputations. However, this rewards defectors and thus jeopardizes the chances that cooperation will emerge as a dominant strategy [?].

An alternative strategy, attributed to Sugden [4], is the *standing* strategy. The assessment rule used by the standing strategy discriminates between justified and unjustified defections. Defecting against a bad agent is labeled as justified while defecting against a good agent is labeled as unjustified. An agent is given a bad reputation if they engage in unjustified defection but otherwise labeled as good. An agent following the standing strategy focuses on maintaining its good standing. Therefore, the action rule used by agents following this strategy dictates that the agent should cooperate if they have a bad reputation or if the recipient has a good reputation.

This standing strategy seems to overcome the deficiencies of the scoring strategy and several authors have analyzed the relative performance of the two strategies using computer simulations.

Out of the debate that has transpired since the publication [3], a more refined and well-defined framework for investigating the evolution of cooperation through indirect reciprocity has emerged. The framework consists of the following three elements:

of concerning which assessment rule and action strategy most likely accounts for the evolution of cooperation through indirect reciprocity in human societies, a more refined and precisely defined framework for evaluating indirect reciprocity has emergedhas led to a refinement and more precise definition of the framework originally described in [3].

tracking scales described above the authors investigated a stra

To use the reputation tracking scales described above, a reputation *assessment rule* must be used to determine whether the action taken by an agent is deemed bad or good. The authors propose a simple assessment rule that is now referred to as *scoring*. The scoring assessment rule considers defection to be bad and cooperation to be good regardless of the reputations of the agents participating in the interaction.

To use the agent reputation measures derived by an assessment rule, an action rule must be used to determine how to act given the reputations of the agents involved in the interaction.

When an agent takes an action that is deemed to be “bad”, its image is set to bad.

When the agent takes an action that is deemed to be “bad”, its score is reduced by one unless the agent’s score is already equal to the minimum score. When the agent takes an action that is deemed to be “good”, its score is increased by one unless its score is already equal to the maximum score.

The authors introduce a simple strategy now referred to as *scoring* in the literature. The strategy consists of a simple assessment rule for tracking agent reputations and a simple action rule that an agent can use to decide whether to donate to the potential recipient.

The simple assessment rule takes two forms. The first form involves tracking agent reputations using an *image score* that ranges from -5 to +5. Each agent starts with an image score equal to zero. The agent’s image score is increased by one (up to +5) each time the agent cooperates and decreased by one (down to -5) each time the agent defects. The second form simplifies the image score down to two *images*: *bad* and *good*. Each agent starts with a good image and then its image is determined based on the last action it has taken. If the agent cooperates, its image becomes good. If the image defects, its image becomes bad.

The authors proposed a simple action rule that only depends on the reputation of the recipient. When using the image score form of reputation assessment, the rule specifies to cooperate when the recipient’s reputation is greater than a specified threshold *k*. When using the simplified good/bad image form of reputation assessment, the rule specifies to cooperate if the agent has a good reputation and defect otherwise.

(the authors do consider strategies that take into account the recipient’s reputation)

(briefly describe results)

Obviously, different assessment rules and action rules are possible and, since the publishing of this paper, much debate has ensued regarding which assessment rule and action rule combination is most likely to lead to a cooperative outcome[2][3][4].

The image scoring strategy only takes into account the reputation of the potential recipient: an agent should donate to a potential recipient if that recipient has a good reputation and refuse to donate otherwise. There are at least two criticisms that can be leveled at this strategy. First, the factor that determines whether the agent will receive donations in the future is its own reputation. However, an agent following the scoring strategy does not consider its own reputation when deciding whether to donate or not [3]. Second, the strategy dictates that an agent refuses to donate to a bad agent. However, following this strategy causes the agent itself to become bad and thus puts the agent in a situation where it will not receive donations in the future [5]. Third, the assessment rule gives agents a good score for cooperating regardless of the reputation of the recipient. (…) Because of these two issues, it is questionable whether a rational agent would actually follow the scoring strategy.

Much of that debate has focused on whether scoring or standing constitutes an evolutionarily stable strategy that promotes cooperation.

An alternative strategy, attributed to Sugden [2], is the *standing* strategy. The assessment rule used by the standing strategy discriminates between justified and unjustified defections. Defecting against a bad agent is labeled as justified while defecting against a good agent is labeled as unjustified. An agent is given a bad reputation if they engage in unjustified defection but otherwise labeled as good. An agent following the standing strategy focuses on maintaining its good standing. Therefore, the action rule used by agents following this strategy dictates that the agent should cooperate if they have a bad reputation or if the recipient has a good reputation.

This standing strategy seems to overcome the deficiencies of the scoring strategy and several authors have analyzed the relative performance of the two strategies using computer simulations.

Some authors have analytically evaluated the possible strategies… (MORE HERE…)

Analytic strategies make simplifying assumptions in order to make the problem tractable…

Here I am concerned about the approaches taken by the authors to evaluate the strategies and not about the results of the experiments.

In [8] the authors analyze the evolutionary stability of all 4096 assessment rule and action rule pairs. The authors analyzed the evolutionary stability of each action rule given a fixed, shared assessment rule and publically known and shared reputation scores for agents. Therefore, strategies do not need to compete against agents that have a alternative view of reputation.

In [3], the authors consider a model in which the population is divided among *g* groups each consisting of *n* members. During each generation, *m* rounds of donor-recipient interactions are played. For each round, two individuals are chosen randomly with one playing the role of the potential donor and the other playing the role of the potential recipient. The donor chooses to donate or not based on its action rule. Its decision may depend on its own reputation, the reputation of the potential recipient or both. The reputations of agents are public and determined based on an assessment rule that is shared by all agents. If the potential donor provides help to the potential recipient, then the usual pay-offs are provided to the players.

After the completion of *m* interactions, the local within-group and global cross-group reproductive probabilities for each action rule are determined as follows. For each group, the pay-offs earned by each action rule in the previous generation are summed and normalized to produce the local within-group reproductive probability for each action rule. To produce the global cross-group reproductive probabilities, the pay-offs earned by each action rule are summed and normalized across all groups.

The action rule followed by each individual in the next generation is determined as follows. With probability p, the individual’s action rule is derived from the local group using the within-group reproductive probability. With probability 1-*p*, the individual’s action rule is derived using the global cross-group reproductive probability. When determining the action rule for an individual in the next generation, a mutation occurs with probability *μ*. When a mutation occurs, the individual’s action rule is selected from among all available action rules with equal probability.

In order to improve the fidelity of the model as it relates to real world scenarios, the authors introduce a probability ε that an agent will take an action that is different from the action prescribed by its strategy. (Implementation error)

(perception error)

The authors conduct two different sets of experiments. In the first set of experiments, they take the scoring assessment rule as a given and evaluate the evolutionary stability of several different action rules.

The authors investigate two assessment rules: *scoring* and *standing*. For the scoring assessment rule, the form of the rule in which an agent’s reputation is an integer between -5 and +5 is used. For the standing assessment rule, the binary good/bad assessment is used.

In the context of the scoring assessment rule, the authors consider three different actions rules:

* The agent provides help if its own image score is less than a threshold *h*
* The agent provides help if the recipient’s image score is greater than or equal to a threshold *k*. This is the action rule proposed in [1].
* The agent provides help if its own image score is less than a threshold *h* and the recipient’s image score is greater than or equal to a threshold *k*
* (Other strategies not studied by [1])

The authors evaluate the success of these strategies using computer simulations.

In the context of the standing assessment

In [5], the authors use a variation of the approach used in [3]. The assessment rule is limited to the simplified binary version in which the agent’s reputation can be one of two values: good or bad. In addition, each agent tracks a private assessment of other agents’ reputations and this assessment is not necessarily based on observing all interactions.

The authors clearly define a strategy as consisting of two parts: assessment rule and action rule. The assessment rule determines how an agent assigns a new reputation to a donor based on the agent’s last action and the reputation of the two participants when the action was taken. Given two possible actions that a potential donor can take, two possible reputation scores for the potential donor, two possible reputation scores for the potential recipient and two possible reputation scores that can be assigned to the potential donor there are 256 possible assessment rules to consider. The authors consider three assessment rules: scoring, standing and judging. The composition of each of these rules is described in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Observed Situation** | | | **Assigned Reputation** | | |
| **Donor Reputation** | **Recipient Reputation** | **Donor Action** | **Scoring** | **Standing** | **Judging** |
| Good | Good | Donate | Good | Good | Good |
| Good | Bad | Donate | Good | Good | Bad |
| Bad | Good | Donate | Good | Good | Good |
| Bad | Bad | Donate | Good | Good | Bad |
| Good | Good | Refuse | Bad | Bad | Bad |
| Good | Bad | Refuse | Bad | Good | Good |
| Bad | Good | Refuse | Bad | Bad | Bad |
| Bad | Bad | Refuse | Bad | Bad | Bad |

The action rule determines the action that an agent should take based on the reputation of the two participants. Given two possible reputation scores for the potential donor, two possible reputation scores for the potential recipient and two possible actions that an agent can take there are 16 possible action rules to consider. The authors consider six of these assessment rules: CO, SELF, AND, OR, ALLD and ALLC. The composition of each of these rules is described in the following table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Situation** | | **Action to Take** | | | | | |
| **Donor Reputation** | **Recipient Reputation** | **SELF** | **CO** | **AND** | **OR** | **ALLC** | **ALLD** |
| Good | Good | Refuse | Donate | Refuse | Donate | Donate | Refuse |
| Good | Bad | Refuse | Refuse | Refuse | Refuse | Donate | Refuse |
| Bad | Good | Good | Donate | Donate | Donate | Donate | Refuse |
| Bad | Bad | Good | Refuse | Refuse | Donate | Donate | Refuse |

Considering all possible combinations to the three 3 assessment rules and 6 actions rules listed above, 14 strategies can be constructed. The authors use computer simulations to evaluate the evolutionary stability of these 14 strategies in three different ways. First the authors conduct experiments in which all 14 strategies are present. In this case, they evaluate the performance of the strategies for different cost benefit ratios. In the second set of experiments, the authors keep the assessment rule fixed and evaluate the performance of the different actions rules in that environment. In the final set of experiments, the authors keep the action rule fixed and evaluate the performance of the different assessment rules in that environment.

Given this model, the assessment rules are fixed and not subject to evolutionary pressures.

In [6], the authors investigate the co-evolution of assessment rules and action rules in the context of the door-recipient game. The model employed by the authors considers evolution at two levels. At the base level, the authors consider the evolution of action strategies in the context of a fixed assessment rule. On top of this base level, the authors consider the evolution of social norms in the context of competition between groups of agents called tribes.

Let be a tribe of agents and be the *jth* member of that tribe. Let be the social norm used by tribe *Ti*, be the strategy followed by agent *aij* and be the reputation of agent *aij*. The reputation of each agent is considered public shared information.

The simulation proceeds in rounds and each round consists of two stages. During the first stage, each agent participates in one donor-recipient game with every other member of its tribe. The payouts received by each agent are tracked in order to calculate the fitness of each individual in the tribe.

Public goods games can also benefit from such a theoretical framework… ???

This framework has served as the basis for

The form of the basic donor-recipient game investigated in [1] is the following.

In order to avoid negative pay-offs, the amount c is added to the pay-offs for both the donor and recipient. At the beginning of each generation, the pay-offs of all group members have a pay-off *u0*, which can be zero or positive.

Since the experiments do not incorporate any strategies that use the round number as the basis for decisions, we will not be concerned with the “end effect” which removes the incentive to help in the last round and leads to cooperation unraveling in all previous rounds.

The donor-recipient game provides a framework for investigating indirect reciprocity.

## Critique, Observations, Improvements and Extensions

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