Evolution of Strategies for Public Goods Games

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In the context of a standard framework for indirect reciprocity [?], the authors of [9] mathematically analyzed all possible combinations of assessment modules and action models in order to identify those combinations that are evolutionarily stable. 25 strategies were identified as evolutionarily stable. Of these 25 stable strategies, eight strategies were able to maintain a high level of cooperation - earning more that 90% of the maximum possible payout. The authors labeled these eight strategies the “leading eight”.

The study conducted in [9] contains the following limitations:

* Only the stability of a strategy is considered. The stability of a strategy is its ability to resist invasion by other strategies once it has become established in a population. The study assumes that a population composed of a majority of agents following one action rule is invaded by a small number of agents following one other action rule.
* All the agents in the population are assumed to share the same assessment module

In previous experiments, the “leading eight” evolutionarily stable strategies were applied to the public goods game. In computer simulations, it was found that, in the context of a public goods game, none of the “leading eight” strategies met the criteria of robustness, stability and initial viability. However, the analysis performed in this study did not impose the same limitations that were imposed in original analysis that identified the “leading eight”. Each strategy was evaluated in an environment that included unconditional cooperators and unconditional defectors in addition to agents following the subject strategy.

In [12] and [13], the authors consider a population of agents divided into groups. Each agent is bestowed with a private action rule. However, the agents in each group share a common assessment module. After the generation of a new generation of agents, the assessment rules followed by each group were evolved based on the overall fitness achieved by each group. The authors found that the assessment module followed by groups evolved into the stern-judging assessment module shown in the following table.

|  |  |  |  |
| --- | --- | --- | --- |
| **Observed Situation** | | | **Stern-Judging** |
| **Donor Reputation** | **Recipient Reputation** | **Donor Action** |
| Good | Good | Donate | Good |
| Good | Good | Refuse | Bad |
| Good | Bad | Donate | Bad |
| Good | Bad | Refuse | Good |
| Bad | Good | Donate | Good |
| Bad | Good | Refuse | Bad |
| Bad | Bad | Donate | Bad |
| Bad | Bad | Refuse | Good |

The CO and OR action rules, shown in the following table, were the most common action rules to co-evolve along with the stern-judging assessment rule. These two action modules are cooperative.

|  |  |  |  |
| --- | --- | --- | --- |
| **Situation** | | **Action to Take** | |
| **Donor Rep.** | **Recipient Rep.** | **CO[[1]](#footnote-1)** | **OR[[2]](#footnote-2)** |
| Good | Good | Donate | Donate |
| Good | Bad | Refuse | Refuse |
| Bad | Good | Donate | Donate |
| Bad | Bad | Refuse | Donate |

As an extension to the study performed in [?], the group evolution procedure employed in [12] and [13] will be used to evaluate the effectiveness of different strategies in the context of the public goods game.

Within each group, the indirect observation model is used to assign and distribute a shared opinion of each agent’s reputation among all members of the group.

# Related Work

In [6], the authors consider indirect reciprocity in a population that is divided among *g* groups each consisting of *n* members. During each generation, *m* rounds of donor-recipient interactions are played. For each round, two individuals are chosen randomly with one playing the role of the donor and the other playing the role of the recipient. The reputations of agents are public and determined based on an assessment module that is shared by all agents. The donor chooses to donate or not based on its action module. Its decision may depend on its own reputation, the reputation of the recipient or both. There is a small probability ε that an agent will take an action that is different from the action prescribed by its action module. If the donor provides help to the recipient, then the usual pay-offs are provided to the agents.

After the completion of *m* interactions, the local within-group and global cross-group reproductive probabilities for each strategy are calculated. Summing and normalizing the pay-offs earned by agents in a group that followed a strategy in the previous generation produces that strategy’s local within-group reproductive probability. Summing and normalizing the pay-offs earned by all agents that followed a strategy in the previous generation produces that strategy’s global cross-group reproductive probability.

With probability p, the strategy followed by an agent in the next generation is derived locally while with probability 1-*p* the agent’s strategy is derived globally. When determining the strategy for an agent in the next generation, a mutation occurs with probability *μ*. When a mutation occurs, the agent’s strategy is selected from among all available strategies with equal probability (independent of the strategy fitness).

The authors conduct two different sets of experiments. In the first set of experiments, they take the scoring assessment module as a given and measure agent reputations on a scale from -5 to +5. In this context they evaluate the evolutionary stability of the following types of action modules:

* The agent provides help if its own image score is less than a threshold *h*. This is the SELF action module.
* The agent provides help if the recipient’s image score is greater than or equal to a threshold *k*. This is the CO action module.
* The agent provides help if its own image score is less than a threshold *h* and the recipient’s image score is greater than or equal to a threshold *k*. This is the AND action module.

In the second set of experiments, they take the standing assessment module as given and measure agent reputations using the binary image scale. In addition, they factor in a small probability δ that an agent misperceives an action taken by another agent and therefore potentially assigns the wrong reputation to that agent. This can lead to a divergence of the opinion on the reputations of agents in the population. In this context, they evaluate the evolutionary stability of the following types of action modules.

In the first experiment in this set, the authors show that the CO action module with *k=0* can be invaded by the SELF-action module with *h=1*. This shows that the strategy combining the scoring assessment rule with the CO action module is not evolutionarily stable.

In a second experiment in this set, the authors show that the AND action module with *k=0* and *h=1* can also be invaded by the SELF-action module with *h=1*. This shows that the strategy combining the scoring assessment rule with the AND action module is not evolutionarily stable.

The authors investigate two assessment rules: *scoring* and *standing*. For the scoring assessment rule, the form of the rule in which an agent’s reputation is an integer between -5 and +5 is used. For the standing assessment rule, the binary good/bad assessment is used.

In the context of the scoring assessment rule, the authors consider three different actions rules:

* (Other strategies not studied by [1])

The authors evaluate the success of these strategies using computer simulations.

In the context of the standing assessment

In [8], the authors use a variation of the approach used in [6]. The assessment rule is limited to the simplified binary version in which the agent’s reputation can be one of two values: good or bad. In addition, each agent tracks a private assessment of other agents’ reputations and this assessment is not necessarily based on observing all interactions.

The authors clearly define a strategy as consisting of two parts: assessment rule and action rule. The assessment rule determines how an agent assigns a new reputation to a donor based on the agent’s last action and the reputation of the two participants when the action was taken. Given two possible actions that a potential donor can take, two possible reputation scores for the potential donor, two possible reputation scores for the potential recipient and two possible reputation scores that can be assigned to the potential donor there are 256 possible assessment rules to consider. The authors consider three assessment rules: scoring, standing and judging. The composition of each of these rules is described in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Observed Situation** | | | **Assigned Reputation** | | |
| **Donor Reputation** | **Recipient Reputation** | **Donor Action** | **Scoring** | **Standing** | **Judging** |
| Good | Good | Donate | Good | Good | Good |
| Good | Bad | Donate | Good | Good | Bad |
| Bad | Good | Donate | Good | Good | Good |
| Bad | Bad | Donate | Good | Good | Bad |
| Good | Good | Refuse | Bad | Bad | Bad |
| Good | Bad | Refuse | Bad | Good | Good |
| Bad | Good | Refuse | Bad | Bad | Bad |
| Bad | Bad | Refuse | Bad | Bad | Bad |

The action rule determines the action that an agent should take based on the reputation of the two participants. Given two possible reputation scores for the potential donor, two possible reputation scores for the potential recipient and two possible actions that an agent can take there are 16 possible action rules to consider. The authors consider six of these assessment rules: CO, SELF, AND, OR, ALLD and ALLC. The composition of each of these rules is described in the following table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Situation** | | **Action to Take** | | | | | |
| **Donor Reputation** | **Recipient Reputation** | **SELF** | **CO** | **AND** | **OR** | **ALLC** | **ALLD** |
| Good | Good | Refuse | Donate | Refuse | Donate | Donate | Refuse |
| Good | Bad | Refuse | Refuse | Refuse | Refuse | Donate | Refuse |
| Bad | Good | Good | Donate | Donate | Donate | Donate | Refuse |
| Bad | Bad | Good | Refuse | Refuse | Donate | Donate | Refuse |

Considering all possible combinations to the three 3 assessment rules and 6 actions rules listed above, 14 strategies can be constructed. The authors use computer simulations to evaluate the evolutionary stability of these 14 strategies in three different ways. First the authors conduct experiments in which all 14 strategies are present. In this case, they evaluate the performance of the strategies for different cost benefit ratios. In the second set of experiments, the authors keep the assessment rule fixed and evaluate the performance of the different actions rules in that environment. In the final set of experiments, the authors keep the action rule fixed and evaluate the performance of the different assessment rules in that environment.

Given this model, the assessment rules are fixed and not subject to evolutionary pressures.

In [13], the authors investigate the co-evolution of assessment rules and action rules in the context of the door-recipient game. The model employed by the authors considers evolution at two levels. At the base level, the authors consider the evolution of action strategies in the context of a fixed assessment rule. On top of this base level, the authors consider the evolution of social norms in the context of competition between groups of agents called tribes.

Let be a tribe of agents and be the *jth* member of that tribe. Let be the social norm used by tribe *Ti*, be the strategy followed by agent *aij* and be the reputation of agent *aij*. The reputation of each agent is considered public shared information.

The simulation proceeds in rounds and each round consists of two stages. During the first stage, each agent participates in one donor-recipient game with every other member of its tribe. The payouts received by each agent are tracked in order to calculate the fitness of each individual in the tribe.

## Critique, Observations, Improvements and Extensions

# References

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1. The CO strategy is also referred to as the “discriminator” strategy. [↑](#footnote-ref-1)
2. The OR strategy is also referred to as the “contrite tit-for-tat” strategy. [↑](#footnote-ref-2)