LE/EECS 3172 3.0
Combinatorial Optimization

Workshop 5

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1 A-4 Cane sugar production

Lot	1	2	3	4	5	6	7	8	9	10	11
Loss (kg/h)	43	26	37	28	13	54	62	49	19	28	30
Life span (h)	8	8	2	8	4	8	8	8	6	8	8

1.1 Parameters

Let $W = \{0, \ldots, m-1\}$ enumerate the wagons or lots. Let $S = \{0, \ldots, n-1\}$ enumerate the time slots. Let $k = 3$ denote the number of processing lines in the refinery, with $m = 11$ and $n = \texttt{ceil}(m/k) = 4$.

For $w \in W$ and $s \in S$, let Δ_w denote the hourly loss given in the table above for lot w. Let l_w denote the life span of lot w. Let $d = 2$ denote the time it takes to process one lot.

1.2 Decision variables

For $(w, s) \in W \times S$, let $x_{ws} \in \{0, 1\}$ be a binary decision variable equal to 1 if lot w is processed in slot s, and 0 otherwise.

1.3 Model

We seek to minimize the loss in raw material resulting from fermentation due to delayed processing of a lot. The model is

$$\text{minimize} \sum_{w \in W} \sum_{s \in S} (s+1) d \Delta_w x_{ws},$$

subject to the following constraints:

$$\sum_{s \in S} x_{ws} = 1, \forall w \in W,$$

$$\sum_{w \in W} x_{ws} \leq k, \forall s \in S,$$

$$\sum_{s \in S} (s+1) x_{ws} \leq \frac{l_w}{d}, \forall w \in W.$$

1.4 Results

The optimal solution results in a loss of 1620 kg with the following time slot assignments:

Slot 1	Slot 2	Slot 3	Slot 4
lot 3	lot 1	lot 10	lot 2
lot 6	lot 5	lot 8	lot 4
lot 7	lot 8		

2 A-6 Production of electricity

2.1 Parameters

Let $L = (l_0, \ldots, l_{n-1})$ be the length of the time periods, and let $D = (d_0, \ldots, d_{n-1})$ be the forecasted demand for each period. Let $T = \{0, \ldots, n-1\}$ enumerate the time periods. Let $P = \{0, \ldots, m-1\}$ with $m = 4$ enumerate the types of generators.

For a generator variant $p \in P$, a particular type has minimum base power output when it is running, along with a maximum output capacity. Let these parameters be denoted by ϕ_p and ψ_p respectively. A generator also the following costs associated with it:

$$\lambda_p : \text{startup cost} \qquad \mu_p : \text{running cost} \qquad \nu_p : \text{scalable additional cost}$$

Lastly, the available number of each type of generator p is denoted by a_p.

2.2 Decision variables

For $(p, t) \in P \times T$,

$0 \leq i_{pt} \in \mathbb{N}$: number of generators of type p started in period t

$0 \leq j_{pt} \in \mathbb{N}$: number of generators of type p running in period t

$0 \leq q_{pt} \in \mathbb{R}$: additional scalable power output from generator p in period t.

2.3 Model

The objective is to simply minimize total operating cost.

$$\text{minimize} \sum_{t \in T} \sum_{p \in P} \lambda_p i_{pt} + l_t \left(\mu_p j_{pt} + \nu_p q_{pt} \right),$$

subject to the following constraints:

$$i_{p0} \geq j_{p0} - j_{p(n-1)}, \forall p \in P$$

$$i_{pt} \geq jpt - j_{p(t-1)}, \forall p \in P, 0 < t \in T$$

$$q_{pt} \leq \left(\psi_p - \phi_p \right) j_{pt}, \forall (p, t) \in P \times T$$

$$\sum_{p \in P} \phi_p j_{pt} + q_{pt} \geq d_t, \forall t \in T$$

$$\sum_{p \in P} \psi_p j_{pt} \geq 1.2 d_t, \forall t \in T$$

$$j_{pt} \leq a_p, \forall (p,t) \in P \times T$$

$$i_{pt} \geq 0, j_{pt} \geq 0, q_{pt} \geq 0$$

2.4 Results

The optimal solution was found with a total operating cost of $1,456,810.

Type		**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	No. used	3	4	4	7	3	3	3
	Tot. output	2250	4600	3000	8600	2250	2600	2250
	Add. output	0	1600	0	3350	0	350	0
2	No. used	4	4	4	4	4	4	4
	Tot. output	5750	6000	4200	6000	4950	6000	5950
	Add. output	1750	2000	200	2000	950	2000	1950
3	No. used	2	8	8	8	8	8	4
	Tot. output	4000	16000	16000	16000	16000	16000	8000
	Add. output	1600	6400	6400	6400	6400	6400	3200
4	No. used	0	3	1	3	1	3	1
	Tot. output	0	5400	1800	5400	1800	5400	1800
	Add. output	0	0	0	0	0	0	0

3 C-2 Production of drinking glasses

Table 1: Demands for the planning periods (batches of 1000 glasses)

Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
V1	20	22	18	35	17	19	23	20	29	30	28	32
V2	17	19	23	20	11	10	12	34	21	23	30	12
V3	18	35	17	10	9	21	23	15	10	0	13	17
V4	31	45	24	38	41	20	19	37	28	12	30	37
V5	23	20	23	15	10	22	18	30	28	7	15	10
V6	22	18	20	19	18	35	0	28	12	30	21	23

Table 2: Data for the six glass types

	prod. cost	store cost	init. stock	final stock	W	M	S
V1	100	25	50	10	3	2	4
V2	80	28	20	10	3	1	5
V3	110	25	0	10	3	4	5
V4	90	27	15	10	2	8	6
V5	200	10	0	10	4	11	4
V6	150	20	10	10	4	9	9

3.1 Parameters

Let $n = 12$ and $m = 6$ denote the number of planning periods and number of product variants respectively. Let $W = \{0, \ldots, n-1\}$ enumerate the planning periods. Let $P = \{0, \ldots, m-1\}$ enumerate the different products.

For $p \in P$ and $w \in W$, let

d_{pw} : demand for product p in week w,

λ_p : production cost for each product,

μ_p : storage cost for each product,

I_p : initial stock for each each product at start,

F_p : final stock requirement for each product at end,

δ_p : worker time cost required per product,

π_p : machine time cost required per product,

λ_p : storage area required for production,

$\Delta = 390$: available worker time capacity,

$\Pi = 850$: available machine time capacity,

$\Lambda = 1000$: available production storage area.

3.2 Decision variables

Let $0 \leq x_{pw} \in \mathbb{Z}$ denote the quanitity of production of product p in period w. Let $0 \leq y_{pw} \in \mathbb{Z}$ denote the storage of product p in period w.

3.3 Model

The objective is simply the total production cost, which we seek to minimize.

$$\text{minimize} \sum_{p \in P} \sum_{t \in T} \lambda_p x_{pw} + \mu_p y_{pw}$$

Subject to the following constraints:

$$y_{p0} = I_p + x_{p0} - d_{p0}, \forall p \in P, \quad (1)$$

$$y_{pt} = y_{p(t-1)} + x_{pt} - d_{pt}, \forall p \in T \text{ and } 0 < t \in T \quad (2)$$

$$y_{p(n-1)} \geq F_p, \forall p \in P \quad (3)$$

$$\sum_{p \in P} \delta_p x_{pt} \leq \Delta, \forall t \in T \quad (4)$$

$$\sum_{p \in P} \pi_p x_{pt} \leq \Pi, \forall t \in T \quad (5)$$

$$\sum_{p \in P} \lambda_p x_{pt} \leq \Lambda, \forall t \in T \quad (6)$$

$$x_{pt} \geq 0, y_{pt} \geq 0, \forall (p, t) \in P \times T \quad (7)$$

3.4 Results

A optimal solution is found with a total production cost of $186,076.

Table 3: Production and storage quantities for each product type

	Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	Prod.	0	0	11	34	29	7	23	21	29	29	29	41
	Store	30	8	1	0	12	0	0	1	1	0	1	10
2	Prod.	7	21	14	17	11	10	12	34	21	23	30	22
	Store	10	12	3	0	0	0	0	0	0	0	0	10
3	Prod.	18	35	17	11	8	21	23	15	10	0	13	27
	Store	0	0	0	1	0	0	0	0	0	0	0	10
4	Prod.	16	45	24	38	41	20	20	36	29	11	31	46
	Store	0	0	0	0	0	0	1	0	1	0	1	10
5	Prod.	47	16	34	14	23	24	43	0	26	4	0	0
	Store	24	20	31	30	43	45	70	40	38	35	20	10
6	Prod.	14	17	20	18	18	35	1	27	12	49	28	7
	Store	2	1	1	0	0	0	1	0	0	19	26	10

Note: I choose to solve this as an integer programming model. This is the reason why my results differ slightly from the book.

Table 4: Summary of cutting patterns

Pattern	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
36 x 50	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
24 x 36	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0
20 x 60	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0
18 x 30	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8

4 D-5 Cutting sheet metal

4.1 Parameters

Let $S = \{0, \dots n-1\}$ with $n = 4$ enumerate the sizes $\{\texttt{36x50}, \texttt{24x36}, \texttt{20x60}, \texttt{18x30}\}$. Let $P = \{0, \dots m-1\}$ enumerate the cutting patterns with $m = 16$.

For $s \in S$ and $p \in P$, let

d_s : demand for size s,

c_{sp} : from the table above; with yields of each pattern,

$K = 1$: the cost of each pattern; simply cost of invidual raw sheet material

4.2 Decision variables

Let $0 \leq u_p \in \mathbb{Z}$ be a integer decision variable denoting the number of times pattern p is used.

4.3 Model

The objective is simply the total raw material cost, ie. the number of sheets of metal used.

$$\text{minimize} \sum_{p \in P} u_p K$$

subject to the following constraints:

$$\sum_{p \in P} c_{sp} u_p \geq d_s, \forall s \in S \tag{8}$$

$$u_p \geq 0, \forall p \in P \tag{9}$$

4.4 Results

An optimal solution is found which uses 11 sheets of raw material to satisfy demand, with a objective function value of 11. The following quantities of each pattern are used:

`pattern 1` = 3, `pattern 3` = 5, `pattern 4` = 2, `pattern 7` = 1, and all others are unused.

5 F-1 Flight connections at a hub

5.1 Parameters

Let $P = \{0, \ldots n-1\}$ enumerate all the set of incoming flights with $n = 6$. P enumerates both origins of incoming flights and flights to destinations, since the number of aircraft is fixed.

Let p_{ij} denote the number of passengers arriving from i and having final destination j.

5.2 Decision variables

Let $x_{ij} \in \{0, 1\}$ be a binary variable, indicating plane from city i will depart to city j next when 1.

5.3 Model

We simply wish to maximize the number of passengers that do not have to disembark and transfer to another aircraft. Our objective function is

$$\text{maximize} \sum_{i \in P} \sum_{j \in P} p_{ij} x_{ij}$$

subject to the following constraints:

$$\sum_{j \in P} p_{ij} x_{ij} = 1, \forall i \in P$$

$$\sum_{i \in P} p_{ij} x_{ij} = 1, \forall j \in P$$

5.4 Results

The optimal solution has a value of 112 for its objective function.

The following assignment of aircraft to destination, minimizes passenger inconvenience:

Bordeaux → London 38

Clermon-Ferrand → Bern 8

Marseille → Brussels 11

Nantes → Berlin 38

Nice → Rome 10

Toulouse → Vienna 7