

LE/EECS 3172 3.0
Combinatorial Optimization

Workshop 5

Jacques Nel

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1 A-4 Cane sugar production

Lot	1	2	3	4	5	6	7	8	9	10	11
Loss (kg/h)	43	26	37	28	13	54	62	49	19	28	30
Life span (h)	8	8	2	8	4	8	8	8	6	8	8

1.1 Parameters

Let $W = \{0, \dots, m-1\}$ enumerate the wagons or lots. Let $S = \{0, \dots, n-1\}$ enumerate the time slots. Let $k = 3$ denote the number of processing lines in the refinery, with $m = 11$ and $n = \text{ceil}(m/k) = 4$.

For $w \in W$ and $s \in S$, let Δ_w denote the hourly loss given in the table above for lot w . Let l_w denote the life span of lot w . Let $d = 2$ denote the time it takes to process one lot.

1.2 Decision variables

For $(w, s) \in W \times S$, let $x_{ws} \in \{0, 1\}$ be a binary decision variable equal to 1 if lot w is processed in slot s , and 0 otherwise.

1.3 Model

We seek to minimize the loss in raw material resulting from fermentation due to delayed processing of a lot. The model is

$$\text{minimize } \sum_{w \in W} \sum_{s \in S} (s+1) d \Delta_w x_{ws},$$

subject to the following constraints:

$$\begin{aligned} \sum_{s \in S} x_{ws} &= 1, \forall w \in W, \\ \sum_{w \in W} x_{ws} &\leq k, \forall s \in S, \\ \sum_{s \in S} (s+1) x_{ws} &\leq \frac{l_w}{d}, \forall w \in W. \end{aligned}$$

1.4 Results

The optimal solution results in a loss of 1620 kg with the following time slot assignments:

Slot 1	Slot 2	Slot 3	Slot 4
lot 3	lot 1	lot 10	lot 2
lot 6	lot 5	lot 8	lot 4
lot 7	lot 8		

2 A-6 Production of electricity

2.1 Parameters

Let $L = (l_0, \dots, l_{n-1})$ be the length of the time periods, and let $D = (d_0, \dots, d_{n-1})$ be the forecasted demand for each period. Let $T = \{0, \dots, n-1\}$ enumerate the time periods. Let $P = \{0, \dots, m-1\}$ with $m = 4$ enumerate the types of generators.

For a generator variant $p \in P$, a particular type has minimum base power output when it is running, along with a maximum output capacity. Let these parameters be denoted by ϕ_p and ψ_p respectively. A generator also the following costs associated with it:

$$\lambda_p : \text{startup cost} \quad \mu_p : \text{running cost} \quad \nu_p : \text{scalable additional cost}$$

Lastly, the available number of each type of generator p is denoted by a_p .

2.2 Decision variables

For $(p, t) \in P \times T$,

$0 \leq i_{pt} \in \mathbb{N}$: number of generators of type p started in period t

$0 \leq j_{pt} \in \mathbb{N}$: number of generators of type p running in period t

$0 \leq q_{pt} \in \mathbb{R}$: additional scalable power output from generator p in period t .

2.3 Model

The objective is to simply minimize total operating cost.

$$\text{minimize } \sum_{t \in T} \sum_{p \in P} \lambda_p i_{pt} + l_t (\mu_p j_{pt} + \nu_p q_{pt}),$$

subject to the following constraints:

$$\begin{aligned} i_{p0} &\geq j_{p0} - j_{p(n-1)}, \forall p \in P \\ i_{pt} &\geq j_{pt} - j_{p(t-1)}, \forall p \in P, 0 < t \in T \\ q_{pt} &\leq (\psi_p - \phi_p) j_{pt}, \forall (p, t) \in P \times T \\ \sum_{p \in P} \phi_p j_{pt} + q_{pt} &\geq d_t, \forall t \in T \end{aligned}$$

$$\sum_{p \in P} \psi_p j_{pt} \geq 1.2 d_t, \forall t \in T$$

$$j_{pt} \leq a_p, \forall (p, t) \in P \times T$$

$$i_{pt} \geq 0, j_{pt} \geq 0, q_{pt} \geq 0$$

2.4 Results

The optimal solution was found with a total operating cost of \$1,456,810.

Type		1	2	3	4	5	6	7
1	No. used	3	4	4	7	3	3	3
	Tot. output	2250	4600	3000	8600	2250	2600	2250
	Add. output	0	1600	0	3350	0	350	0
2	No. used	4	4	4	4	4	4	4
	Tot. output	5750	6000	4200	6000	4950	6000	5950
	Add. output	1750	2000	200	2000	950	2000	1950
3	No. used	2	8	8	8	8	8	4
	Tot. output	4000	16000	16000	16000	16000	16000	8000
	Add. output	1600	6400	6400	6400	6400	6400	3200
4	No. used	0	3	1	3	1	3	1
	Tot. output	0	5400	1800	5400	1800	5400	1800
	Add. output	0	0	0	0	0	0	0

3 C-2 Production of drinking glasses

Table 1: Demands for the planning periods (batches of 1000 glasses)

Week	1	2	3	4	5	6	7	8	9	10	11	12
V1	20	22	18	35	17	19	23	20	29	30	28	32
V2	17	19	23	20	11	10	12	34	21	23	30	12
V3	18	35	17	10	9	21	23	15	10	0	13	17
V4	31	45	24	38	41	20	19	37	28	12	30	37
V5	23	20	23	15	10	22	18	30	28	7	15	10
V6	22	18	20	19	18	35	0	28	12	30	21	23

Table 2: Data for the six glass types

	prod. cost	store cost	init. stock	final stock	W	M	S
V1	100	25	50	10	3	2	4
V2	80	28	20	10	3	1	5
V3	110	25	0	10	3	4	5
V4	90	27	15	10	2	8	6
V5	200	10	0	10	4	11	4
V6	150	20	10	10	4	9	9

3.1 Parameters

Let $n = 12$ and $m = 6$ denote the number of planning periods and number of product variants respectively. Let $W = \{0, \dots, n - 1\}$ enumerate the planning periods. Let $P = \{0, \dots, m - 1\}$ enumerate the different products.

For $p \in P$ and $w \in W$, let

d_{pw} : demand for product p in week w ,

λ_p : production cost for each product,

μ_p : storage cost for each product,

I_p : initial stock for each each product at start,

F_p : final stock requirement for each product at end,

δ_p : worker time cost required per product,

π_p : machine time cost required per product,

λ_p : storage area required for production,

$\Delta = 390$: available worker time capacity,

$\Pi = 850$: available machine time capacity,

$\Lambda = 1000$: available production storage area.

3.2 Decision variables

Let $0 \leq x_{pw} \in \mathbb{Z}$ denote the quantity of production of product p in period w . Let $0 \leq y_{pw} \in \mathbb{Z}$ denote the storage of product p in period w .

3.3 Model

The objective is simply the total production cost, which we seek to minimize.

$$\text{minimize } \sum_{p \in P} \sum_{t \in T} \lambda_p x_{pw} + \mu_p y_{pw}$$

Subject to the following constraints:

$$y_{p0} = I_p + x_{p0} - d_{p0}, \forall p \in P, \quad (1)$$

$$y_{pt} = y_{p(t-1)} + x_{pt} - d_{pt}, \forall p \in T \text{ and } 0 < t \in T \quad (2)$$

$$y_{p(n-1)} \geq F_p, \forall p \in P \quad (3)$$

$$\sum_{p \in P} \delta_p x_{pt} \leq \Delta, \forall t \in T \quad (4)$$

$$\sum_{p \in P} \pi_p x_{pt} \leq \Pi, \forall t \in T \quad (5)$$

$$\sum_{p \in P} \lambda_p x_{pt} \leq \Lambda, \forall t \in T \quad (6)$$

$$x_{pt} \geq 0, y_{pt} \geq 0, \forall (p, t) \in P \times T \quad (7)$$

3.4 Results

A optimal solution is found with a total production cost of \$186,076.

Table 3: Production and storage quantities for each product type

	Week	1	2	3	4	5	6	7	8	9	10	11	12
1	Prod.	0	0	11	34	29	7	23	21	29	29	29	41
	Store	30	8	1	0	12	0	0	1	1	0	1	10
2	Prod.	7	21	14	17	11	10	12	34	21	23	30	22
	Store	10	12	3	0	0	0	0	0	0	0	0	10
3	Prod.	18	35	17	11	8	21	23	15	10	0	13	27
	Store	0	0	0	1	0	0	0	0	0	0	0	10
4	Prod.	16	45	24	38	41	20	20	36	29	11	31	46
	Store	0	0	0	0	0	0	1	0	1	0	1	10
5	Prod.	47	16	34	14	23	24	43	0	26	4	0	0
	Store	24	20	31	30	43	45	70	40	38	35	20	10
6	Prod.	14	17	20	18	18	35	1	27	12	49	28	7
	Store	2	1	1	0	0	0	1	0	0	19	26	10

Note: I choose to solve this as an integer programming model. This is the reason why my results differ slightly from the book.

4 D-5 Cutting sheet metal

5 F-1 Flight connections at a hub

5.1 Parameters

Let $P = \{0, \dots, n-1\}$ enumerate all the set of incoming flights with $n = 6$. P enumerates both origins of incoming flights and flights to destinations, since the number of aircraft is fixed.

Let p_{ij} denote the number of passengers arriving from i and having final destination j .

5.2 Decision variables

Let $x_{ij} \in \{0, 1\}$ be a binary variable, indicating plane from city i will depart to city j next when 1.

5.3 Model

We simply wish to maximize the number of passengers that do not have to disembark and transfer to another aircraft. Our objective function is

$$\text{maximize } \sum_{i \in P} \sum_{j \in P} p_{ij} x_{ij}$$

subject to the following constraints:

$$\sum_{j \in P} p_{ij} x_{ij} = 1, \forall i \in P$$

$$\sum_{i \in P} p_{ij} x_{ij} = 1, \forall j \in P$$

5.4 Results

The optimal solution has a value of 112 for its objective function.

The following assignment of aircraft to destination, minimizes passenger inconvenience:

Bordeaux \rightarrow London 38

Clermon-Ferrand \rightarrow Bern 8

Marseille \rightarrow Brussels 11

Nantes \rightarrow Berlin 38

Nice \rightarrow Rome 10

Toulouse \rightarrow Vienna 7