MATH3090: Computational Mathematics

Assignment 1

Due on Monday, October 2 (in-class)

1. Modify the coffee cooling model to account for a room temperature that starts at 20 Celsius and increase at a constant rate to maximum 26 Celsius in 2 hours and then stays at the maximum temperature. Assume that the initial temperature of the coffee is 100 Celsius and after 10 minutes the temperature of the coffee is 90 Celsius. Modify the codes and draw the room temperature and the coffee temperature in the same diagram for 6 hours.

2. Do you add the milk to coffee straight away? People often ask whether it is better to add the cold milk to a hot cup of coffee straight away or wait for a while to let it cool down and then add the cold milk.

   Estimate the optimal time point to add cold milk so that the coffee cools down to the drinkable temperature fastest. Let us assume that the drinkable temperature is 50 Celsius. Assume that the initial temperature of the coffee is 100 Celsius and the insulation of the cup is 0.01282. Room temperature is fixed at 22 Celsius. Is your result reasonable? Explain it briefly. Based on your result, make suggestion to the people about when to add coffee.

   You can use and modify the model and codes we developed in the class for the cooling of the coffee in a constant room temperature. Assume that the temperature of the coffee is independent on special variables. Make further assumptions if needed.

3. You invest $100 in a savings account paying 6% interest per year. Let y(t) be the amount in your account after t years. If the interest is compounded continuously, then y(t) solves the ODE initial value problem

$$\frac{dy}{dt} = r\,y$$

   where r = 6% and y(0) = $100.
   (a) What is the analytic solution y(t) to this initial value problem?
   (b) What is the balance in your account after 10 years with each of the following methods of compounding interest, yearly, monthly, quarterly, daily and continuous compounding?