1 Changing room temperature

Modify the coffee model to account for a room temperature that starts at 20 °C, and increases at a constant rate to a maximum 26 °C in 2 hours, and then stays at the maximum temperature. Assume that the initial temperature of the cofee is 100 °C and after 10 minutes, the temperature of the coffee is 90 °C. Modify the codes and draw the room temperature and coffee temperature in the same diagram for 6 hours.

1.1 Building model

The independent variable of the model is time t in hours. The observed temperature of the coffee $u_{obs}(t)$ is a function of time, with units °C.

Let u_{sur} be the surruounding temperature, then u_{sur} is given by

$$u_{sur}(t) = \begin{cases} 20 + 3t & 0 \leq t \leq 2 \\ 26 & t > 2 \end{cases}. \tag{1}$$

We use the model presented in class, but modify it to use $u_{sur}(t)$ to get

$$\frac{du}{dt} = c(u_{sur}(t) - u) \tag{2}$$

Now, we discretize the model to get

$$\frac{u_{k+1} - u_k}{\Delta t} = c(u_{sur}(t_k) - u_k \tag{3}$$

where $t_k = k\Delta t$ is the discrete time, at the k^{th} step.

2 When to add milk?

Do you add the milk to coffee straight away? People often ask wheter it is better to add the cold milk to a hot cup of coffee straight away or wait for a while to let it cool down and then add the cold milk.

Estimate the optimal time to add cold milk so that the coffee cools down to the drinkable temperature fastests. Let us assume that the drinable temperature is $50\,°C$. Assume that the initial temperature of the coffee is $100\,°C$ and the insulation of the cup is 0.01282. Room temperature is fixed at 22 °C. Is your result reasonable? Explain it briefly. Based on your result, make suggestion to the people about when to add coffee.

You can use and modify the model and codes we developed in the class for the cooling of the coffee in a constant room temperature. Assume that the temperature of the coffee is independent on special variables. Make further assumptions if needed.

3 Compound interest

You invest \$100 in a savings account paying 6% per year. Let $y(t)$ be the amount in your account after t years. If the interest rate is compounded continuously, then $y(t)$ solves the ODE initial value problem

$$\frac{dy}{dt} = ry$$

where $r = 6\% = 0.06$ and $y(0) = \$100$.

a. What is the analytic solution $y(t)$ to this initial value problem?

   Start with the ensatz $y(t) = Ae^{rt}$, and plug into equation above.

$$\frac{dy}{dt}(t) = rAe^{rt} = ry(t)$$

   Take ensatz at $t \to 0$, to determine constant A,

$$y(0) = Ae^{r\cdot 0} = A \implies A = y(0) = \$100.$$

   The analytic solution is therefore,

$$y(t) = 100e^{rt}. \tag{4}$$

b. What is the balance in your account after 10 years with each of the following methods of compounding interest, yearly, monthly, quarterly, daily, and continuous compounding?

   Compound interest can be solved analytically using 4, as follows:

$$y(10) = 100e^{0.06\cdot 10} = \$182.21.$$