1 Golden Method

a. Consider the Golden method for $\min g(x)$ in $[a, b]$. Prove that $1/R = (\sqrt{5} - 1)/2$.

Let $[a_k, b_k] \subset [a, b]$ be the reduced interval at the k^{th} step, then its length is $I_k = b_k - a_k$. The $(k+1)^{th}$ step will see the interval reduced to $[a_{k+1}, b_{k+1}]$, with length $I_{k+1} = b_{k+1} - a_{k+1}$.

Then, the ratio is between consequetive intervals is

$$\frac{I_k}{I_{k+1}} = R, \forall k \geq 1$$

b. Use the Golden Method to find the minimum point of function

$$g(x) = 3 \sin x \cos x + 2$$

in $[a_1, b_1] = [1.5, 3.2]$ with $k = 4$ (stop at $k = 4$).

c. Explain two advantages of the Golden Method over a search method with $\rho = 2/3$.

2 Fibonacci Method

a. Consider the Fibonacci method for solving $\min g(x)$ in $[a, b]$. Prove that

$$\frac{I_n}{I_{n-k}} = \frac{1}{F_{k+1}}, k = 1, \ldots, n-1$$

$$\frac{I_k}{I_1} = \frac{F_{n-k+1}}{F_n}, k = 2, 3, \ldots, n,$$

where $F_j, j > 0$ are the Fibonacci numbers.

b. Let stop be at $n = 9$ in the Fibonacci method. Find $I_2/I_1, I_4/I_1, I_8/I_1$.

3 Simplex Method

a. In the Simplex method, give the formulas of locations of $a', a'', a^*, a^{**}, \bar{a}$, and $\bar{b}$.

b. Consider the minimization function

$$g(x, y) = 2x^2 - x - 2y + y^2 + 4.$$

Starting from the initial triangle $\Delta a_0 b_0 c_0$, where $a_0(0.1, 0)$, $b_0(0.0, 0.1)$, $c_0(0.0, 0.0)$, do two steps (i.e. find $\Delta a_2, b_2, c_2$) by the simplex method. What is your approximation to the minimum point? What are the advantages and drawbacks of the simplex method?

4 Steepest Descent Method

Consider the Steepest Descent method for solving the local minimization of $\min g(\vec{x})$ in $\Omega \subset \mathbb{R}^n$.

a. If the previous approximation is $\vec{x}^{(k-1)}$, what is the k'th step search direction $\vec{z}^{(k)}$? Explain briefly why you use this search direction.

b. Write out the Algorithm of the Steepest Descent Method. You need to provide the following details:

5 Application

Let

$$g(x, y) = -\left(x^2 + 4xy + 2y^2\right) e^{-2x^2-y^2}.$$

Use a computer to approximate the local minimization problem of $\min g(x, y)$ in $\mathbb{R}^2$ by *one* of the numerical methods: Newton's method, the Steepest Descent method, or the simplex method.

a. Explain briefly how to solve the problem by the method that you used.

b. Set up a table of numerical results and iteration numbers by using initial guesses and tolerances. Analyze your results.

c. What are the advantages and drawbacks of the method based on your analysis?