Assignment 1 for Advanced Numerical Methods
Fall 2017/2018

Due: Tuesday (class), Oct. 3. Read Chapter 1, Sections 1 - 3.

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1. (i) Consider the Golden method for $\min g(x)$ in [a, b]. Prove that $\frac{1}{R} = \frac{\sqrt{5}-1}{2}$.
   (ii) Use the Golden Method to find the minimum point of function $g(x) = 3\sin x \cos x + 2$ in $[a_1, b_1] = [1.5, 3.2]$ with $k = 4$ (stop at $k = 4$).
   (iii) Explain two advantages of the Golden method over a search method with $\rho = \frac{2}{3}$.

2. (i) Consider the Fibonacci method for solving $\min g(x)$ in [a, b]. Prove that
   1) $\frac{I_n}{I_{n-k}} = \frac{1}{F_{k+1}}, \quad k = 1, 2, \cdots, n-1;$ 2) $\frac{I_k}{I_1} = \frac{F_{n-k+1}}{F_n}, \quad k = 2, 3, \cdots, n,$
   where F_j, $j \geq 0$, are the Fibonacci numbers.

   (ii) Let Stop At $n = 9$ in the Fibonacci method. Find $\frac{I_2}{I_1}$, $\frac{I_4}{I_1}$, and $\frac{I_8}{I_1}$.

3. (a) In the simplex method, give the formulas of locations of a', a'', a^*, a^{**}, $\bar{a}$ and $\bar{b}$ points.

   (b) Consider the minimization of function

   $$g(x, y) = 2x^2 - x - 2y + y^2 + 4.$$

   Starting from the initial triangle $\Delta a_0 b_0 c_0$, where $a_0(0.1, 0), b_0(0, 0.1), c_0(0, 0)$, do two steps (i.e. find $\Delta a_2 b_2 c_2$) by the simplex method. What is your approximation to the minimum point? What are the advantages and drawbacks of the simplex method?

4. Consider the Steepest Descent method for solving the local minimization of $\min g(\vec{x})$ in $\Omega \subset R^n$.

   (a) If the previous approximation is $\vec{x}^{(k-1)}$, what is the k'th step search direction $\vec{z}^{(k)}$? Explain briefly why you use this search direction?

   (b) Write out the Algorithm of the Steepest Descent Method. You need to provide the following details: (i) the proper inputs, (ii) the possible less variables (in order to save memory), (iii) the required informations of formulas including the fomula of approximation α_k, (iv) the effective stop conditions of accuracy $\|\vec{x}^{(k)} - \vec{x}^{(k-1))}\| < TOL$ and maximum iteration $k \leq N$.

5. Let

   $$g(x, y) = -(x^2 + 4xy + 2y^2)e^{-2x^2 - y^2}.$$

Use computer to approximate the local minimization problem of $\min g(x, y)$ in R^2 by ONE of numerical methods: Newton's method, the Steepest Descent method, and the simplex method.

(1) Explain briefly how to solve the problem by the method that you used.

(2) Set up a table of numerical results and iteration numbers by using different initial guesses and tolerances. Analyze your results.

(3) What are the advantages and drawbacks of the method based on your analysis?

(Note: If you choose the Steepest Descent method, you may use/modify the code ALG10.3 of [BF]. If you choose Newton's method, you may use the code ALG 10.1 of [BF].)

Instructions for completing Assignment 1:

(1) Question 5 should be solved by using computer. Please also hand in a copy of your running procedure and the result for one case when you use computer to solve the question.

(2) You can choose to use any computer language. Some existing codes can be found
at the [BF] code web (the detailed information from a link at the bottom of the course web);
OR at the web:
https://sites.google.com/site/numericalanalysis1burden/

OR in a directory on UNIX.aml.yorku.ca computer (detailed information from a link at the bottom of the course web);

(3) Please make sure that you hand in the results with necessary intermediate steps and explanations of your results.