Market Concentration and the Productivity Slowdown

Jane Olmstead-Rumsey[*]

August 10, 2020

Abstract

Since around 2000, U.S. aggregate productivity growth has slowed and product market concentration has risen. I present new evidence that average patent quality has fallen over the same period, particularly among laggard firms. I develop an endogenous growth model with innovations of heterogeneous quality. Depending on distance to the technology frontier, firms face draws from different innovation quality distributions. Consistent with the data, lowering the expected quality of laggards' innovations generates wider average productivity gaps between leaders and followers in steady state, increased concentration, and slower aggregate productivity growth. In the estimated model, between a quarter and a half of the slowdown is due to firms' endogenous responses to changes in the patent quality distribution. The model's industry structure allows me to explore alternative hypotheses about rising market power or the emergence of superstar firms, but I conclude that declining patent quality provides a better fit for the data.

1   Introduction

Beginning around 2000, U.S. productivity growth slowed and industry leaders began capturing an increasingly large share of sales. In this paper I illustrate a new mechanism that can explain both of these trends, along with patterns of markups, profitability, and research and development expenditures across firms. This mechanism is the declining ability of laggard firms to catch their more productive competitors through innovation.

To support the existence of this mechanism in the data I provide new evidence that patent quality, one measure of innovativeness, has fallen sharply among industry laggards since 2000. Looking over a long time horizon, average patent quality varies considerably, with periods of high patent quality coinciding with the arrival of general purpose technologies.

I propose a general equilibrium, quality ladder model of directed innovation along the lines of Aghion et al. (2001) to model the effects of changes in the ability of laggards to catch up with or leapfrog over leaders through innovation over time. The model, estimated on Compustat data for U.S. firms, can explain the entirety of the productivity slowdown and the bulk of the rise in concentration by matching the decline in patent quality I observe in the data. Laggards respond to a lower chance of attaining market leadership by investing less in research and development. Facing a diminished probability of being overtaken, market leaders invest slightly more. Together, these decisions lead to wider productivity gaps between leaders and laggards in steady state. The model has a direct mapping from productivity gaps to leaders' market shares such that sales concentration is higher in the new steady state, as are markups and the profit share of total output. The real interest rate declines.

The decline in productivity growth can be decomposed into endogenous and exogenous forces. When average quality improvements are smaller, the economy grows more slowly. However, firms' endogenous responses amplify the decline in economic growth. Sectors with wider productivity gaps innovate less in total than sectors with close rivals. Moreover, in the model, also consistent with evidence on heterogeneity in patent quality by firm size, leaders make more incremental innovations than followers on average. Thus there is also a composition effect that more R&D is carried out by industry leaders contributing to the decline in aggregate productivity growth. Decomposing the endogenous versus endogenous effects reveals that between a quarter and a half of the productivity slowdown in the model is due to the endogenous

responses of the firms.

The model is a tractable general equilibrium model of strategic interactions in both innovation and pricing decisions. Most neo-Schumpeterian growth models assume goods within sectors are perfect substitutes so that each sector has just one producer in each period (see Klette and Kortum (2004), Lentz and Mortensen (2008), Acemoglu and Cao (2015), and Akcigit and Kerr (2018), for leading examples). Because of this, these models take matching firm-level moments seriously, but are not well-suited to address industry-level moments.[1] Introducing a duopoly allows me to make unified predictions both about market concentration at the industry level and firm-level innovation rates, and makes market concentration an endogenous outcome of the innovation process.

The duopoly formulation also brings together previously distinct strands of literature in macroeconomics concerned with (i) slowing growth (ii) changes in market structure and potentially market power and (iii) superstar firms. Many papers studying the recent rise of large firms have made passing references to the potentially harmful dynamic effects of these large firms on productivity growth but do not articulate this link theoretically (OECD 2018). This model provides a theoretical foundation for the link between the two.

Strands (ii) and (iii) typically rely on opposing assumptions. According to the literature on rising market concentration, incumbent firms exercise greater market power now than in the past and this is reflected in rising markups and profitability (de Loecker and Eeckhout 2017). On the other hand, the literature on superstar firms typically contends that greater import competition and greater consumer price sensitivity due to better search technology like online retail have *increased* competitive pressures and reduced the market power of incumbent firms, resulting in reallocation to the most productive (superstar) firms (Autor et al. 2017). The model resolves this seeming contrast by demonstrating how markups can rise at the same time as there is reallocation to the most productive firms without any changes at all to consumer preferences.

Empirical evidence suggests that these large firms are often some of the most productive in their industry (hence the "superstar firm" label), so growth in sales of these firms should increase measured aggregate productivity, all else equal. However, there

---

[1]Aghion et al. (2001) were the first to introduce a duopoly in a neo-Schumpeterian model with directed innovation. However, the paper does not quantify the model or attempt to match industry moments and there is no heterogeneity in innovation step size.

are also dynamic considerations: when the technology gap between the largest firm and its rivals widens, the large firm might "rest on its laurels" rather than invest in further productivity-enhancing technologies that simply replace its own technology (this is Arrow (1962)'s replacement effect). Thus a dynamic model of productivity growth at the firm level with multiple firms operating heterogeneous technologies in the same sector is needed to untangle the balance of these two forces.

This paper contributes a novel mechanism to the large and growing literature linking trends in concentration, productivity growth, and business dynamism using models of endogenous growth. Many of these papers emphasize the increasing importance of intangible assets and information and communications technology (ICT) as a possible explanation (Aghion et al. 2019, de Ridder 2020, Corhay, Kung, and Schmid 2020). Non-technological explanations include demographic changes (Hopenhayn, Neira, and Singhania 2018, Peters and Walsh 2019, Karahan, Pugsley, and Sahin (2019), Engbom 2017, Eggertsson and Mehrotra 2014, Bornstein 2018) or declining real interest rates (Liu, Mian, and Sufi 2019, Chatterjee and Eyigungor 2019). The most closely related explanation is the one in Akcigit and Ates (2019c) and Akcigit and Ates (2019a) that diffusion of knowledge from leaders to laggards is slowing down, either because of ICT and the increasing importance of data in firms' production processes or because of anti-competitive use of patents.

Rather than emphasizing particular features of ICT, the theory presented here instead suggests that general purpose technologies can affect firm dynamics and market structure. Past fluctuations in patent quality and productivity growth have been attributed to waves of innovation due to the arrival of general purpose technologies (GPTs). Bresnahan and Trajtenberg (1995) identify characteristics of GPTs. First, GPTs are *pervasive*, meaning they are applicable in a wide range of sectors. Second, GPTs involve *innovational complementarities*: the productivity of downstream research and development increases as a result of innovation in the GPT. Due to these complementarities, the gains of which are diffuse from the perspective of the sector creating the GPT, their model rationalizes the lags involved in the commercialization of GPTs. For example, the ICT revolution arguably began in the 1970s with the invention of the microprocessor but the biggest gains for productivity growth did not occur until the 1990s. As the GPT wave subsides, inventions again become more incremental.

This paper's finding that laggards' patent quality has declined since 2000 is consistent with Bloom et al. (2019), who show that despite increasing inputs (expenditures, workers) to R&D production, outputs in terms of productivity improvements have

4

declined. Cavenaile, Celik, and Tian (2020) estimate an endogenous growth model and find that declining R&D productivity of small firms can explain a large share of the rise in concentration and the productivity slowdown. Anzoategui et al. (2017) also infer a decline in R&D productivity using indirect inference in a DSGE model with endogenous productivity growth. Several empirical papers have also documented that laggard firms are less likely to overtake market leaders in recent years (Pugsley, Sedlacek, and Sterk 2020, Andrews, Criscuolo, and Gal 2016).

Many papers have studied rising concentration and the productivity slowdown (Hall (2015), Syverson (2016)) in isolation from one another. Rising concentration is mainly a within-sector phenomenon, according to Hsieh and Rossi-Hansberg (2019). A variety of explanations for rising sales concentration have been proposed, from the introduction of ICT that creates winner-take-all markets and enables the growth of superstar firms (see for example Bessen (2017), Crouzet and Eberly (2018), van Reenen (2018))[2], to excessive regulations that erect barriers to entry and create unnatural monopolies (Gutierrez and Philippon (2017)), to increased mergers and acquisitions activity, possibly due to weak antitrust enforcement (Grullon, Larkin, and Michaely (2017)). This rise is not just due to the increasing market share of foreign firms and resultant mismeasurement of industry sales in Compustat: Gutierrez and Philippon (2017) construct an import-adjusted Herfindahl index for the U.S. and a similar rise can be seen in this metric. Using a more sophisticated definition of firms' competitors using text analysis of product descriptions in firms' 10-K forms from Hoberg and Phillips (2010), Pellegrino (2020) finds that Compustat firms also face greater market concentration among their direct competitors. However, while many papers find that concentration is growing in *product* markets, Rossi-Hansberg, Sarte, and Trachter (2020) and Berger, Herkenhoff, and Mongey (2019) both find that concentration in local geographic markets has fallen.

The rest of the paper is organized as follows. In section 2 I present additional empirical motivation about changes in patent quality and evidence of the correlation between productivity growth, productivity gaps, and concentration at the sector level. Section 3 presents the model and section 4 lays out results from a numerical exercise comparing the growth rate and other features of the economy in two different steady states with higher and lower patent quality of laggard firms. In section 5 I discuss

---

[2]Kehrig and Vincent (2020) dispute the idea of durable market leadership for superstar firms, instead arguing that industries are characterized by "shooting stars" with temporarily low labor share and high markups. However, they find evidence that low labor shares have become more persistent in recent years.

the role of the elasticity of substitution within sectors, comparing explanations for increased markups and profits in recent years to the superstar firm hypothesis that greater price sensitivity has driven the growth of large, productive firms. Within the model, neither story matches the data as well as a decline in laggards' patent quality.

2 Empirical Evidence

The main contribution of the paper is the model, which is presented in section 3. This section presents additional empirical motivation for the model. I first find that patent quality has declined relative to the 1990s along various metrics, particularly for laggard firms. I then show that laggard firms are less likely to catch up to the leading firm in their industry now than in the past. I discuss evidence that productivity gaps, concentration, and the productivity slowdown are correlated at the sector level. Finally, I describe productivity and R&D patterns in Compustat.

To establish the main empirical motivation for the paper, figure 1 shows that among U.S. public companies, the largest firm's average market share of sales has risen significantly since the late 1990s.[3] Both labor productivity and total factor productivity (TFP) differences between so-called "frontier" or "superstar" firms and their competitors have been growing since 2000, particularly in information and communications technology (ICT) intensive industries (figure 2). Static models of the rise of superstar firms suggest that sales growth of these firms improves allocative efficiency (Autor, Katz, and Dorn (2017)), yet aggregate productivity growth has fallen over the same period (figure 1).[4]

2.1 Trends in patent quality

Various measures of patent quality show substantial heterogeneity at any given time (Akcigit and Kerr 2018). Recent evidence from patent data also points to changes in quality over time, particularly in the right tail of the quality distribution. Kelly

---

[3]See Grullon, Larkin, and Michaely (2017) and Council of Economic Advisers (2016) for overviews of trends in market concentration. More than 75% of U.S. industries have experienced an increase in the Herfindahl-Hirschman index.

[4]Some argue that productivity growth has not actually slowed, that it has just been persistently mismeasured recently. Syverson (2016) challenges these hypotheses' ability to explain the majority of the measured slowdown using four separate analyses.
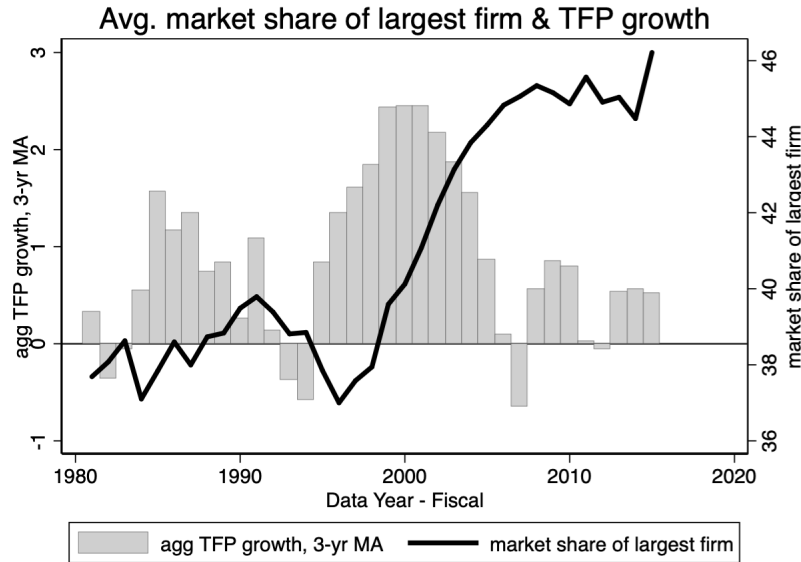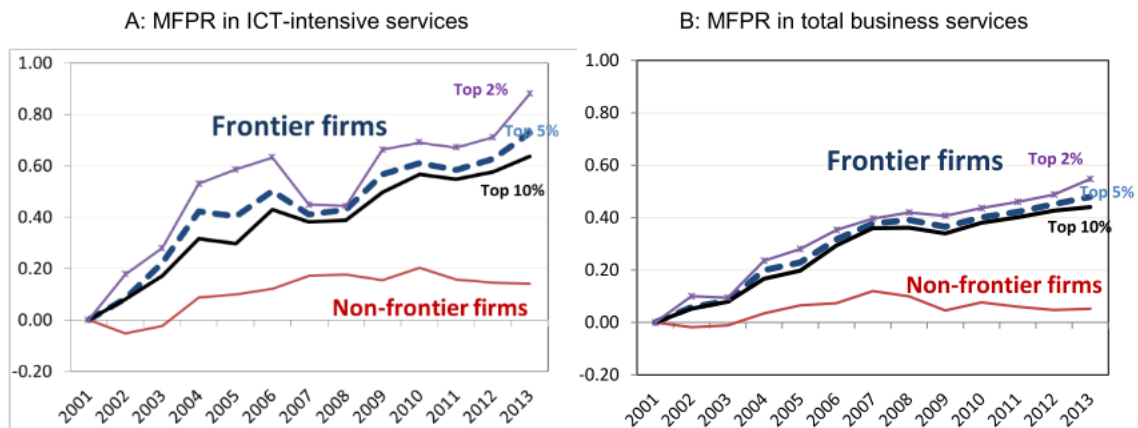
Figure 1: Source: Market share of largest firm (by sales) in 4-digit SIC industries from Compustat (weighted by industry sales); utilization-adjusted total factor productivity (TFP) growth from Fernald (2014), three year moving average.

Notes: the global frontier group of firms is defined by the top 5% of companies with the highest MFPQ levels within each 2-digit industry. Laggards capture the all the other firms. Unweighted averages across 2-digit industries are shown for MFPQ, separately for manufacturing and services, normalized to 0 in the starting year. Time period is 2001-2013. Services refer to non-financial business services. ICT-intensive services refer to the information and communication sector (industry code J in NACE Rev. 2) and postal and courier activities (53). See details in Sections 3.3 and 4.4.

Source: Authors' calculations based on the recent update of the OECD-Orbis productivity database (Gal, 2013)

Figure 2: Source: Andrews, Criscuolo, and Gal (2016). MFPR is revenue-based multi-factor productivity. MFPQ is markup-corrected multi-factor productivity.

et al. (2018) create a text-based measure of patent quality, identifying "breakthrough" patents as those where the patent's text differs from the text of past patents but is similar to the text of future patents. Plotting the median, 75th, 90th, and 95th percentile of the patent quality distribution using this metric shows periods of high patent quality coincide with timelines of general purpose technologies, including the ICT revolution in the 1990s, consistent with Bresnahan and Trajtenberg (1995)'s "innovational complementarities" from GPTs (figure 3).
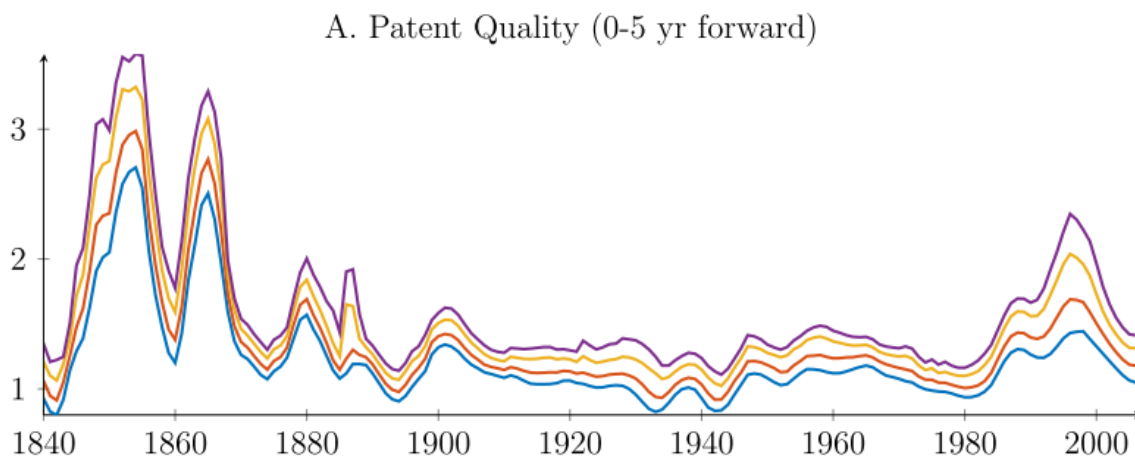

A. Patent Quality (0-5 yr forward)

Figure 3: Percentiles of text-based patent quality distribution over time. Blue = P50, Red = P75, Yellow = P90, Purple = P95. Source: from Kelly et al. (2018) figure 3a.

To explore heterogeneity in the decline in patent quality across firms, I use the measure of patent quality from Kogan et al. (2017) that estimates the market value of all patents issued in the U.S. and assigned to public firms from 1926-2010 using excess returns around patent approval dates. Kelly et al. (2018) document the strong correlation between the market- and text-based measures at the patent level as well as the correlation of these measures with forward citation-weighted measures. All three measures show a sharp uptick in average patent quality and in the right tail during the 1990s and a subsequent decline beginning in the late 1990s.

In the model presented in section 3, firms make innovations that grow the quality of their product variety by a random amount. I use the dollar value estimates of Kogan et al. (2017) to construct a measure of each public firm's "knowledge stock" as the cumulative value of all past patents.[5] With this measure in hand, I show how much

---

[5]Some depreciation can be applied to the patent stock measure. For example Peters and Taylor (2017) use the Bureau of Economic Analysis' R&D expenditure depreciation rates by sector, ranging from 5-20% per

a marginal innovation increases the knowledge stock of the firm over time, splitting the sample into market leaders (largest firms by sales in 4-digit SIC industries) and followers (all other firms).
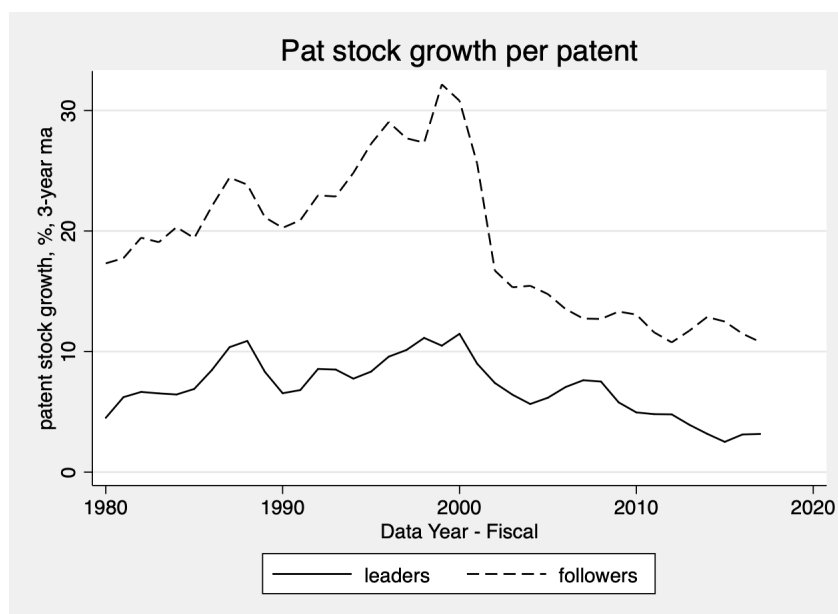

Figure 4: Contribution of average new patent to firm's existing stock of patents, using estimated patent values from Kogan et al. (2017). Leader indicates sales leaders in 4-digit SIC industries and followers are all other firms.

The striking fact of figure 4 is that laggard firms' patents make a much smaller contribution to the firm's knowledge stock now than in the 1990s, consistent with the patent quality wave pattern of figure 3. The innovations of relatively smaller firms seem to be more incremental now than in the past, or the market internalizes the fact that it is harder for small firms to make a dent in the advantage of their leading competitor than before.

One concern with figure 4 is that the raw number of patents issued has grown over this period, so it may be that a single idea is embodied in multiple patents or firms patent more often but the overall growth of their knowledge stock is unchanged. However, looking at the total annual growth of leaders' and followers' patents shows the same pattern as figure 4. This finding is also robust to restricting the sample to

year to construct a measure of firms' intangible capital stock. Applying depreciation rates in this range increases the level of the estimated quality improvements but does not affect the magnitude of the slowdown or the differential decline across leaders vs. laggards.

firms that have been public for at least ten years, so it is not just driven by the decline in IPOs/entry of young, innovative firms over the same period.

Figure 5 shows the empirical distribution of patent qualities using this measure for all public firms from 1994-2003 and 2004-2017 separately.[6] In both periods, most innovations are incremental and grow the patent stock of the firm by around 5%. But this is especially true from 2004-2017, when the mass of the distribution shifts leftward toward more incremental innovations and features fewer radical innovations.[7]
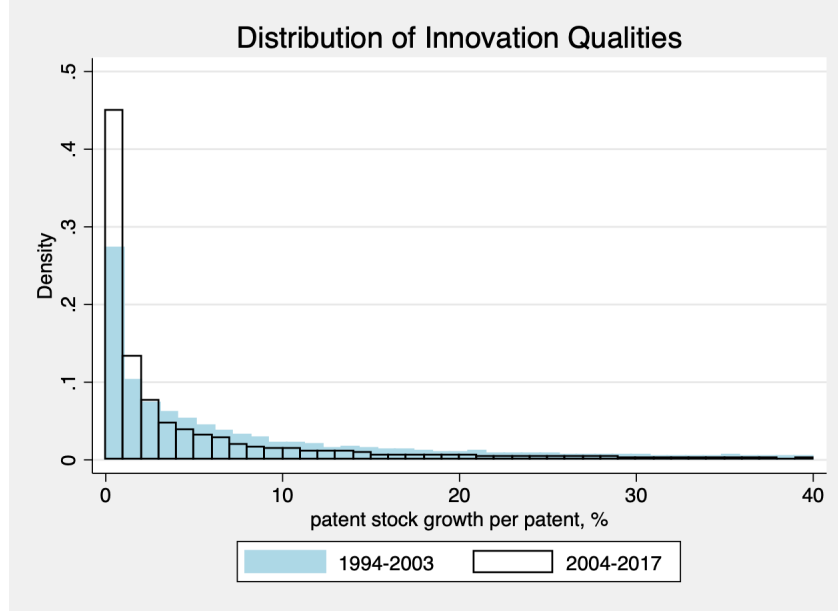

Figure 5: Empirical distributions of patent stock growth per patent based on Kogan et al. (2017) market-based patent quality estimates.

2.2 Declining dynamism and catchup speeds

Firm-level data also shows that the "advantage of backwardness" has fallen relative to the 1990s, consistent with the idea that it is now harder to catch up through innovation than it was in the 1990s. Andrews, Criscuolo, and Gal (2016) show that in a regression of firm-level productivity growth on a variety of explanatory variables, the coefficient

---

[6]The figure is truncated at 40% in both cases for readability.

[7]The use of "radical innovation" in this paper to describe a relatively large quality improvement differs from some other papers in the literature such as Acemoglu and Cao (2015) who use "radical innovation" to refer to an entrant replacing an incumbent.

on the lagged productivity gap to the technology frontier has been declining over the 2000s (figure 6), suggesting that distance to the productivity frontier is becoming a less important predictor of future growth.[8]

Estimated convergence parameter from neo-Schumpeterian model and 95% confidence intervals

Notes: The marker line shows the evolution over time of the estimated $GAP_{i,t-1}$ coefficient from the firm level MFP growth regression in Table B1 Appendix B (Column 1, Panel A), while the dashed line provides the 95% confidence interval around this coefficient estimate.
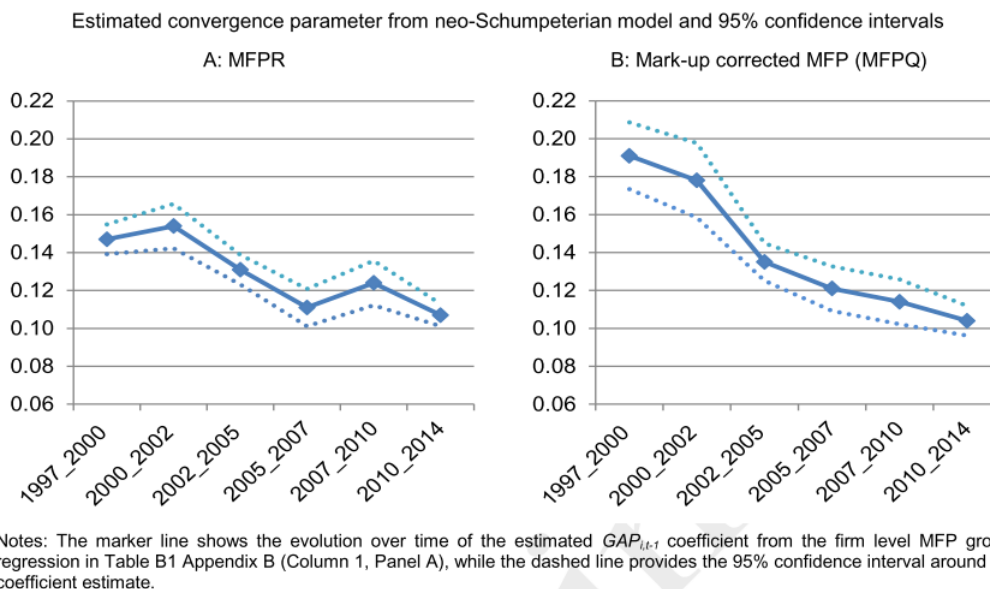
Figure 6: Speed of technological convergence is slowing down. Plotted coefficient is the coefficient on a firm's lagged multi-factor productivity (MFP) gap in a regression of current MFP on lagged MFP gap to the frontier. Source: Andrews, Criscuolo, and Gal (2016).

Turnover at the technology frontier also appears to be slowing. Figure 7 shows that the share of 4-digit SIC industries with a new sales leader has fallen from around 15% per year in the late 1990s to around 9% in recent years. This is also true globally: according to Andrews, Criscuolo, and Gal (2016), from 2011-2013, more frontier firms survived an additional year at the frontier, and those moving into the productivity frontier were more likely to already come from the top 10% or 20% of the productivity distribution in the previous year compared to 2001-2003.

---

[8]However, this empirical observation is endogenous according to the model, because it may be a result of both structural change to catchup speeds and to the endogenously lower innovation effort by laggard firms since their regression does not control for innovation effort (R&D investment).
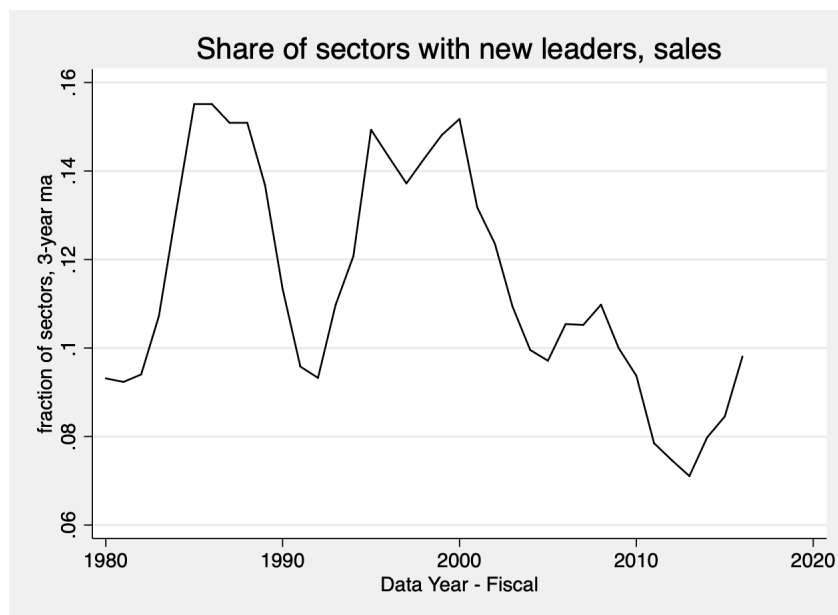
Figure 7: Share of 4-digit SIC sectors with a new sales leader each year. Source: author's calculations from Compustat.

2.3 Sector-level correlation between productivity gaps, concentration, and the productivity slowdown

An exploration of the causal relationships among productivity growth, productivity gaps, and concentration is beyond the scope of this paper, but some recent sector-level evidence suggests at least a correlation, consistent with the predictions of the model. Gutierrez and Philippon (2017) find that R&D expenditure has slowed down more in more concentrated sectors. Autor, Katz, and Dorn (2017) generally find that sectors with large superstar firms have higher productivity growth over long horizons, but don't investigate changes in the relationship between concentration and productivity growth over time. When taking these changes into account, Gutierrez and Philippon (2017) find positive correlations between concentration and TFP growth "only before 2002, but an insignificant and sometimes negative correlation after 2002." In another paper they find that the contribution of superstar firms to aggregate productivity growth has fallen by more than a third since 2000 (Gutiérrez and Philippon (2019)). Autor, Katz, and Dorn (2017) also find that technology diffusion is slower in more concentrated sectors. More concentrated industries have also seen the greatest slowdown in investment, which has accompanied the aggregate labor productivity

12

Figure 8: Source: Author's calculations from Compustat and BEA Integrated Industry-Level Production Accounts. 3-digit NAICS sectors, comparing 1994-2004 average to 2005-2017 average.

slowdown that began in the early 2000s (Gutierrez and Philippon (2016); Hall (2015); Crouzet and Eberly (2018)).

To directly study the productivity *slowdown* rather than the correlation between productivity growth and concentration, I use data from the Bureau of Economic Analysis estimates of multifactor productivity[9] at the 3-digit NAICS level and data from Compustat to check the association between the change in the leader's market share in Compustat and the change in the sector's average productivity growth rate from 1995-2004 to 2005-2017 at the sector level. Figure 8 shows that sectors experiencing greater slowdowns in average productivity growth rates between 1995-2004 and 2005-2017 also saw greater increases in concentration on average.

2.4 Firm-level productivity patterns

In two companion papers Andrews, Criscuolo, and Gal (2015) and Andrews, Criscuolo, and Gal (2016) summarize characteristics of firms at the global productivity frontier (defined as either the top 100 or the top 5% of firms by estimated productivity in each

---

[9]https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems

industry-year). The first paper highlights the growing productivity gap between this frontier and other firms. Globally, frontier firms' productivity has grown at a rate of 3.6% per year while non-frontier firms' productivity grew at just 0.4% over the 2000s.[10] The authors identify two distinct periods: from 2001-2007, frontier firms' productivity grew 4-5% per year and other firms grew 1%, but since the global financial crisis frontier firms have seen productivity growth of just 1% per year while the productivity growth of non-frontier firms was flat.

To demonstrate that this global fact is also true within the U.S., figure 9 shows average productivity gaps for U.S. public firms in Compustat, either from the industry's market leader in terms of sales or to the most productive firm within its industry.[11] Gaps are defined as the difference between log-TFP of the two firms. I then compute the (unweighted) average of these gaps for all firms within an industry and compute the economy-wide average weighted by industry size. Both measures of the gap are growing over time. The average gap to the largest firm nearly doubled in size between 1980 and 2000, from around 22 log points to 42.

According to the standard Olley and Pakes (1996) decomposition, aggregate total factor productivity growth could be slowing down for two reasons. First, average TFP growth across all firms could be slowing down. Second, reallocation to the most productive firms (i.e. the growth rate of productive firms) could be slowing down. A careful decomposition by Baqaee and Farhi (2017) shows that within-firm growth has contributed essentially nothing to aggregate TFP growth since the late 1990s. Broad-based below-trend productivity growth, not increasing misallocation among U.S. firms, seems to be driving the aggregate slowdown, lending support to explanations focusing on the incentives of existing firms to improve productivity, like the hypothesis I propose here.

2.5 R&D investment patterns

The consensus is that innovation and technology adoption drive productivity at the firm, in addition to random shocks (see Griliches (2001) for a survey of the relation-

---

[10]Productivity is measured in their papers using both labor productivity and multi-factor productivity to account for the fact that frontier firms tend to be more capital intensive. These numbers refer to labor productivity.

[11]Appendix A provides details of the firm-level TFP estimation procedure for the TFP estimates I refer to throughout the paper. I follow the estimation strategy of de Loecker and Warzynski (2012) to estimate TFP in Compustat, using the variable construction of de Loecker and Eeckhout (2017) in Compustat.
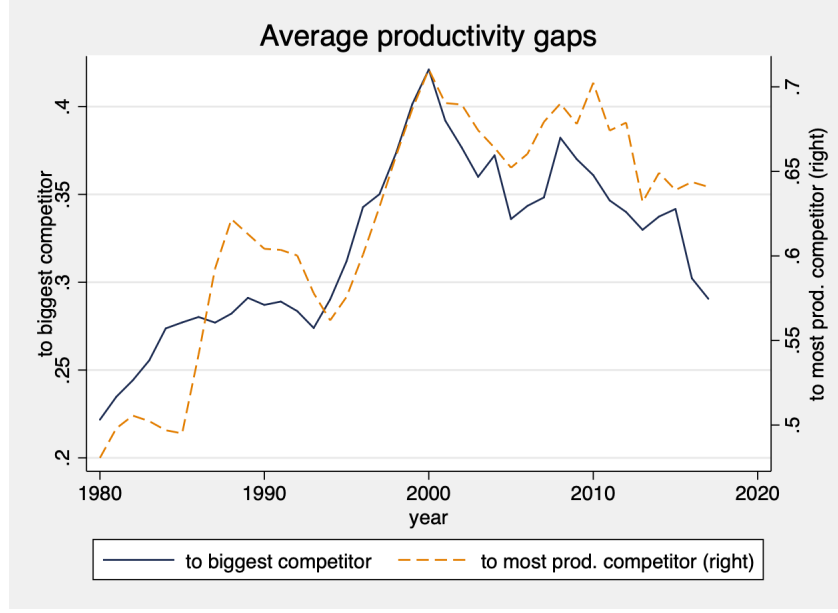
Figure 9: (Unweighted) average productivity gap to either the largest (left axis) or most productive (right axis) competitor in 4-digit SIC, sale-weighted across industries. Compustat.

ship between R&D and productivity at the firm level and Zachariadis (2003) for a leading empirical test). Has the productivity slowdown been accompanied by a slowdown in research inputs to improve productivity? Aggregate R&D's share of sales in Compustat has been roughly flat since 1999, before which it had been rising steadily since 1980 (figure 10).

Similar to a firm's decision to invest in physical capital, many factors may influence the decision to invest in innovation. For investment, q-theory suggests that firms should invest when the market value of their assets exceeds the book (replacement) value.[12] I replicate the exercise of Alexander and Eberly (2018) for intangible capital using R&D expenditure as the outcome variable to check whether R&D is slowing down *relative to what theory would predict*. The regression is:

$$\log\left(\frac{R\&D}{assets}_{i,t}\right) = \alpha + \eta_t + X_i + \beta_1 \log\left(\frac{cashflow}{assets}_{i,t}\right) + \beta_2 \log(q_{i,t}) + \varepsilon_{i,t}$$

I find that R&D has also declined relative to what would be predicted by the theory (figure 11).

---

[12]Formally, I construct Tobin's q in Compustat as $\frac{\text{assets (AT)+shares outstanding (CSHO)∗share price (PRCCF)-common equity (CEQ)}}{\text{assets (AT)}}$
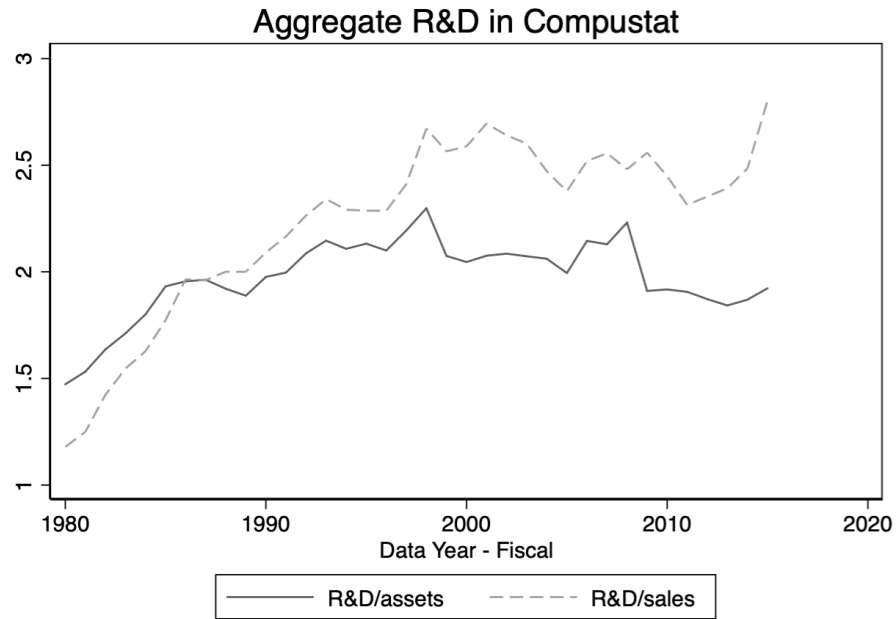
Figure 10: Source: Author's calculations from Compustat summing R&D (XRD) for all firms and dividing by the sum of sales (SALE) or assets (AT) for all firms.
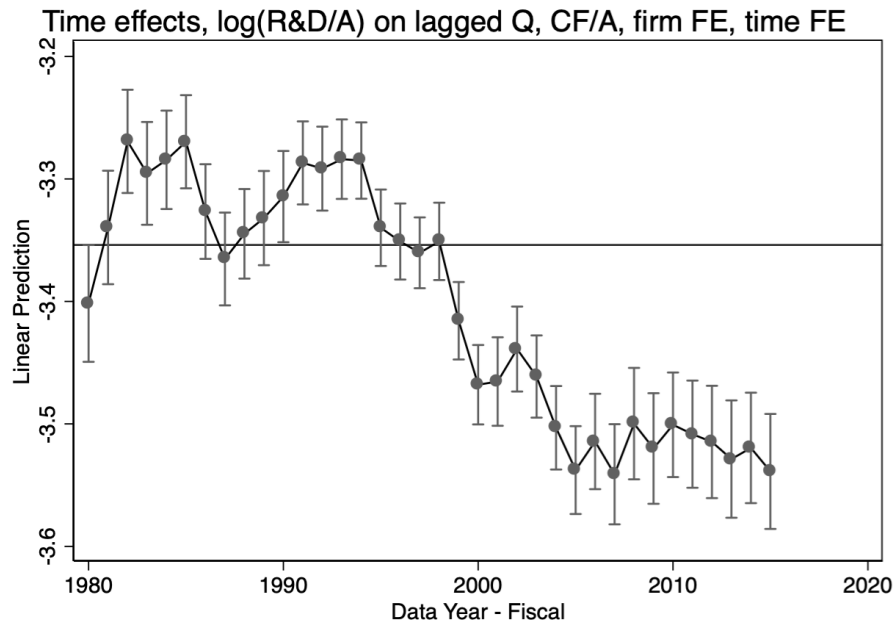

Figure 11: Source: Author's calculations from Compustat.

Similarly, in studying the investment slowdown that has partially driven the growth slowdown recently, Gutierrez and Philippon (2016) point out that net investment is low among U.S. public firms despite high value of Tobin's q and explore potential explanations. Increasing concentration appears to be one of the strongest factors correlated with the investment slowdown. Substituting intangible investment or R&D for physical capital investment does not change their results.
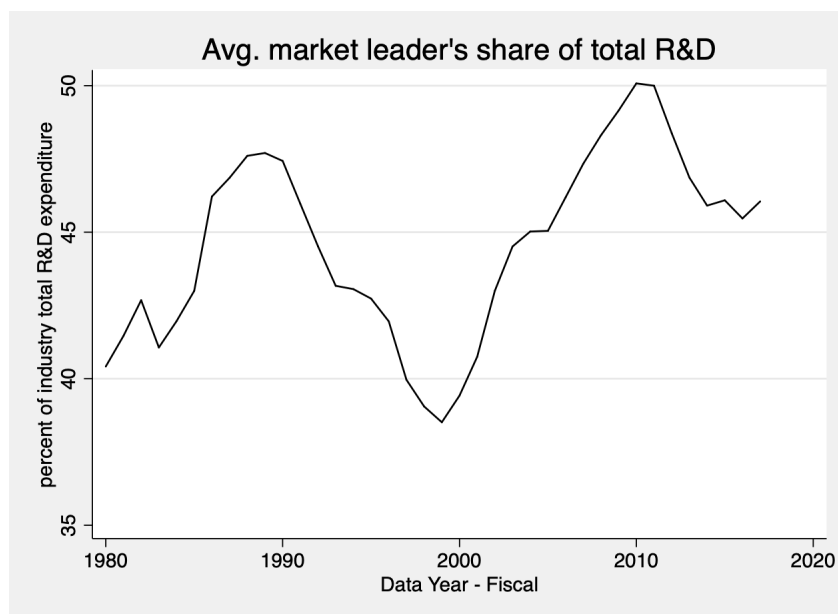
Figure 12: Source: Compustat. Leader indicates sales leaders in 4-digit SIC industries. Average across industries, sale-weighted by industry size.

Finally, I find that market leaders now perform a larger share of total R&D expenditures in their industries than during the 1990s (figure 12). I next present a model that can capture the above findings parsimoniously.

3   Model

The model is of a closed economy in continuous time. There are three types agents: a representative household, a representative competitive final good firm, and intermediate goods firms producing capital goods. This section presents the model by going step-by-step through each type of agent in the economy, then analyzes the equilibrium of the model.

3.1 Households

A representative household consumes, saves, and supplies labor inelastically to maximize:

$$U_t = \int_t^\infty \exp(-\rho(s-t)) \frac{C_s^{1-\psi}}{1-\psi} ds$$

subject to:

$$r_t A_t + W_t L = P_t C_t + \dot{A}_t$$

Households own all the firms, and the total assets in the economy are:

$$A_t = \int_0^1 \sum_{i=1}^2 V_{ijt} dj$$

Where V_{ijt} is the value of intermediate good firm i in sector j at time t. These value functions are explained in greater detail in section 3.3. The number of firms per sector (two) and the measure of sectors (one) are imposed exogenously. On a balanced growth path with constant growth rate of output g this yields the standard Euler equation $r = g\psi + \rho$.

3.2 Final Good Producers

The competitive final goods sector combines intermediate goods and labor to create the final output good which is used in consumption, research, and intermediate good production. The final good firm's technology is as follows:

$$Y = \frac{1}{1-\beta} \left(\int_0^1 K_j^{1-\beta} dj \right) L^\beta$$

where K_j is a composite of two intermediate good firms' products within sector j described below. β determines both the elasticity of substitution across sectors ($\frac{1}{\beta}$) and the labor share. For now consider the final good firm's problem of hiring sector composite goods K_j and labor:

$$\max_{K_j, L} P \frac{1}{1-\beta} \left(\int_0^1 K_j^{1-\beta} dj \right) L^\beta - P_j K_j - WL$$

where P is the price of the final good and P_j is the price of the sector j composite good and W is the nominal wage. The first order condition for sector j's composite good

given sector j's composite price index P_j will give the demand for sector j's good:

$$K_j = \left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L$$

and the real wage is equal to the marginal product of labor:

$$\beta \frac{Y}{L} = \frac{W}{P}$$

Now to derive the demand curve for each firm i within sector j we need to define the sector composite K_j explicitly:

$$K_j = \left((q_{1j}k_{1j})^{\frac{\epsilon-1}{\epsilon}} + (q_{2j}k_{2j})^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

where q_{ij} is the quality of firm i's product (equivalently as firm i's productivity) and k_{ij} is the output of firm i purchased by the final good producer.[13] $\epsilon > \frac{1}{\beta}$ is the elasticity of substitution between products in the same sector.

Within each sector j the final good producer will seek to minimize (dropping the j subscript to focus on a single sector):

$$\min_{\{k_i\}_{i=1}^2} \sum_{i=1}^{2} p_i k_i$$

subject to:

$$K = \left((q_1 k_1)^{\frac{\epsilon-1}{\epsilon}} + (q_2 k_2)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \geq \underline{K}$$

Taking the first order condition for either k_i yields:

$$p_i = P_j K_j^{\frac{1}{\epsilon}} q_i^{\frac{\epsilon-1}{\epsilon}} k_i^{-\frac{1}{\epsilon}}$$

So, plugging in for K_j we get inverse demand:

$$p_i = P_j \left(\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L\right)^{\frac{1}{\epsilon}} q_i^{\frac{\epsilon-1}{\epsilon}} k_i^{-\frac{1}{\epsilon}}$$

$$= k_i^{-\frac{1}{\epsilon}} q_i^{\frac{\epsilon-1}{\epsilon}} P_j \left(\frac{P_j}{P}\right)^{-\frac{1}{\beta\epsilon}} L^{\frac{1}{\epsilon}}$$

Rearranging, the demand function is:

$$k_i = q_i^{\epsilon-1} \left(\frac{p_i}{P_j}\right)^{-\epsilon} \left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L \tag{1}$$

---

[13]This is the sense in which productivity and quality are equivalent: doubling the quality q_{ij} of both firms has the same effect on final output as doubling the output k_{ij} of both firms.

That is, demand is increasing in the firm's quality, decreasing in its price relative to the sector j price index, and decreasing in the sector's price index relative to the price index in the economy as a whole.

3.3 Intermediate Goods Producers

In this section I discuss production and competition in the intermediate goods market and then the innovation technology. In both cases I present the firms' problem first and then the solution. Each intermediate good sector is a duopoly and there is no entry margin. This assumption precludes the possibility of analyzing the contribution of entry to output and productivity growth. Empirical evidence summarized in Bartelsman and Doms (2000) suggests that incumbents are responsible for around 75% of industry-level TFP growth in the U.S. so the model still captures a large share of productivity growth dynamics.

Similarly, the decision to model R&D as a process of own-product quality improvement by incumbents is consistent with the evidence in Garcia-Macia, Hsieh, and Klenow (2019) that: (i) incumbents are responsible for most employment growth in the U.S., and this share has increased in recent years; (ii) growth mainly occurs through quality improvements rather than new varieties; (iii) creative destruction by entrants and incumbents over other firms' varieties accounted for less than 25% of employment growth from 2003-2013.

3.3.1 Production and Price Setting

The intermediate goods producers purchase final goods to transform them into differentiated intermediate goods. Each unit of intermediate output requires $\eta < 1$ units of the final good to produce. There are no other inputs to intermediate good production.

Facing the demand for their product from the final good producer given in equation 1, I assume the technology *follower*, the firm with lower quality q_i, must set price equal to marginal cost. Understanding this, the leader chooses its optimal price in response to the price of the follower.[14] If the firms are neck-and-neck I assume both set price equal to marginal cost. Since this competition assumption plays an important role in the mechanisms of the model I take a quick detour to discuss it here. It helps me match the level of leader market share in the data.

---

[14]Solving the model where both firms set prices a la Bertrand is also possible. See Appendix B for a discussion and results. The main results in section 4 are mostly unchanged.

First, this pricing assumption is actually similar to the assumptions made in other quality ladder models (Klette and Kortum (2004), Acemoglu and Cao (2015), Akcigit and Kerr (2018)), with the caveat that my model features the presence of a lower-quality substitute to the market leader's product. In those other models, quality improvement over a product line confers a fully enforceable patent on the product until the next innovation occurs. I similarly assume the firm with the higher quality has a fully enforceable patent on its product, but I introduce the notion of sectors and include a second lower quality product in each sector. Another way to micro-found this assumption is by introducing a cost to filing a patent that is sufficiently high that only the leader, who exercises some additional market power by possessing the higher quality and thus earns higher profits, would be willing to pay. Most Schumpeterian growth models provide no micro-foundation for the monopolist pricing assumption.

If one further assumes that there is free entry to the production of the follower's product, the pricing decisions are the fully optimal outcomes of Bertrand pricing. Under that assumption, the number of followers is indeterminate since the intermediate good production technology is constant returns to scale. The simplest way to resolve this indeterminacy is to assume the lower quality product is produced by a single firm with zero profits.

This assumption plays an important role in determining the shape of the innovation policy as a function of technology differences, specifically the hump shape. This shape has been suggested theoretically in the work of Aghion et al. (2005), Akcigit, Ates, and Impullitti (2018), and Schmidt (1997) and found in a variety of studies including Aghion et al. (2005) and Carlin, Schaffer, and Seabright (2004). Intuitively the hump shape appears in this model because the pricing assumption means that the greatest incremental gain in flow profits comes from obtaining quality leadership, so innovation effort will be highest when firms have equal quality.

Finally, the assumption generates empirically plausible predictions about profit shares: the largest U.S. public firms (by sales) capture by far the largest share of industry profits (see figure 13).[15]

I now proceed to the solution of the leader's pricing problem. Some definitions

---

[15]TFP and size are correlated, and the figure looks similar if one uses a productivity ranking instead of sales-based ranks.
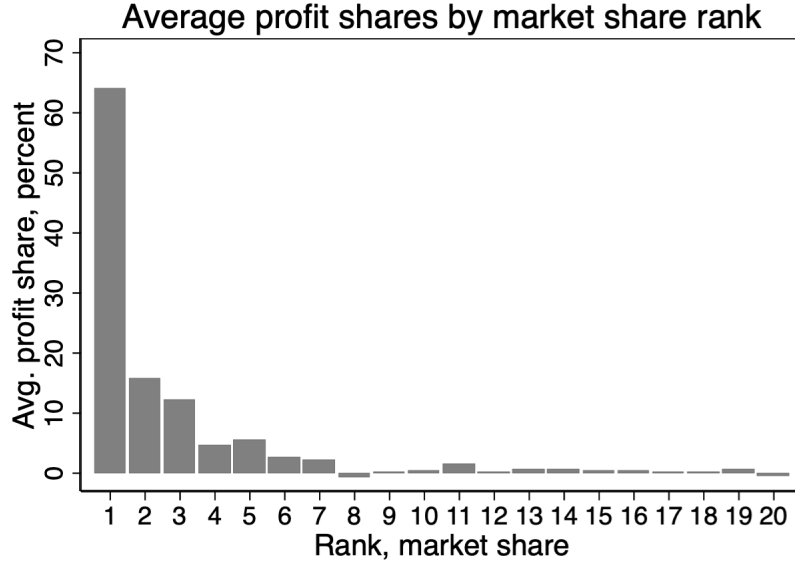
Figure 13: Source: Compustat, 1975-2015. Firms are ranked by market share (sales) within 4-digit SIC industries, and these ranks are compared to profit shares (firm's own operating income as a share of industry-total operating income). The figure averages across 4-digit sectors.

are needed. First, sector j's price index:

$$P_j = \left(\sum_{i=1}^{2} q_{ij}^{\epsilon-1} p_{ij}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

Let s_i be firm i's market share of sales in sector j, plugging in the final good firm's demand for k_{ij} (again dropping js to focus on a single sector):

$$s_i = \frac{p_i k_i}{\sum_{i=1}^{2} p_i k_i} = \frac{q_i^{\epsilon-1} p_i^{1-\epsilon} P_j^{\epsilon-\frac{1}{\beta}} P^{-\frac{1}{\beta}} L}{\sum_{i=1}^{2} q_i^{\epsilon-1} p_i^{1-\epsilon} P_j^{\epsilon-\frac{1}{\beta}} P^{-\frac{1}{\beta}} L} = q_i^{\epsilon-1} \left(\frac{p_i}{P_j} \right)^{1-\epsilon} = \frac{p_i}{P_j} \frac{\partial P_j}{\partial p_i}$$

The final equality holds because:

$$\frac{\partial P_j}{\partial p_i} = \frac{1}{1-\epsilon} P_j^{\epsilon}(1-\epsilon) q_i^{\epsilon-1} p_i^{-\epsilon} = q_i^{\epsilon-1} \left(\frac{p_i}{P_j} \right)^{-\epsilon}$$

Now we are ready to consider the pricing problem of the technology leader i in sector j:

$$\max_{p_i} p_i k_i - \eta k_i$$

22

subject to the inverse demand:

$$k_i = q_i^{\epsilon-1} \left(\frac{p_i}{P_j}\right)^{-\epsilon} \left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L$$

Taking the first order condition for the price and using the definition of market share above yields the optimal pricing policy:

$$p_i = \frac{\epsilon - (\epsilon - \frac{1}{\beta})s_i}{\epsilon - (\epsilon - \frac{1}{\beta})s_i - 1}\eta$$

The optimal price is the standard one for two-layered constant elasticity of demand structures (nested CES): a variable markup that rises in market share. This is easiest to see for the two extreme cases where market share is 0 or 1. When market share is 0, the firm is atomistic with respect to the sector and charges a markup $\frac{\epsilon}{\epsilon-1}$, the CES solution for an elasticity of substitution equal to ϵ. On the other hand, if the market share is 1, the firm only weighs the elasticity of substitution across sectors and sets a markup $\frac{1}{1-\beta} > \frac{\epsilon}{\epsilon-1}$ since products are less substitutable across sectors than within sectors.

It will be important for the (tractable) solution of the model that firms' prices not depend on their quality level, only on their quality *relative* to their competitor, referred to as the *technology gap* between the two firms. The technology gap is defined by the ratio $\frac{q_1}{q_2}$ for firm 1 and $\frac{q_2}{q_1}$ for firm 2. Below I show that this is the case. First, this is clearly satisfied for the technology follower who always sets price equal to marginal cost η regardless of absolute quality.

Second, for the leader, use the definition of the market share and the price index to solve for the market share of the leader i in sector j ($-i$ denotes the follower):

$$s_i = q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{1-\epsilon}$$

$$= \frac{q_i^{\epsilon-1}p_i^{1-\epsilon}}{q_i^{\epsilon-1}p_i^{1-\epsilon} + q_{-i}^{\epsilon-1}\eta^{1-\epsilon}}$$

$$= \frac{1}{1 + \left(\frac{q_{-i}}{q_i}\right)^{\epsilon-1}\left(\frac{p_i}{\eta}\right)^{\epsilon-1}}$$

Now using the pricing decision of the leader $p_i = \frac{\epsilon-(\epsilon-\frac{1}{\beta})s_i}{\epsilon-(\epsilon-\frac{1}{\beta})s_i-1}\eta$:

$$s_i = \frac{1}{1 + \left(\frac{q_{-i}}{q_i}\right)^{\epsilon-1}\left(\frac{\epsilon-(\epsilon-\frac{1}{\beta})s_i}{\epsilon-(\epsilon-\frac{1}{\beta})s_i-1}\right)^{\epsilon-1}}$$

23

Thus there is a mapping from technology gaps to market shares and prices that is independent of quality levels. The market shares and prices for firms with a given technology gap are shown in figure 14. The x-axis values m are a way of capturing the number of quality steps ahead/behind a firm is from its competitor described in detail in the next section. The particular parameterization used to generate the figure is given in table 1 in section 4. The slope of these figures is sensitive to ϵ, the elasticity of substitution between firms in the same sector. The leader's optimal price p_i rises as the technology gap widens (that is, as the leader's relatively quality improves). Most of the effect of increased quality appears in the leader's output k_i, so the market share of the leader ($\frac{p_i k_i}{\sum_{i=1}^{2} p_i k_i}$) rises more dramatically in quality than does the price. In this particular parameterization, market share rises from around 30% of sales with one quality step ahead to 80% of the market at 16 steps ahead. The follower, who must sell at $p = \eta$, has a large market share (due to their relatively low price) that also increases as the follower's relative quality improves.
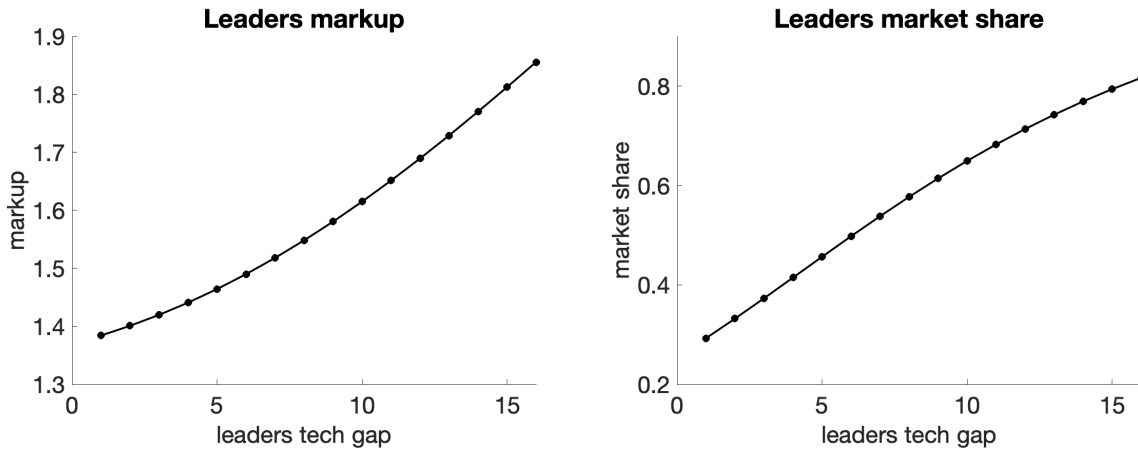
Figure 14: Markups and resulting market shares as a function of the technology gap (ratio of firm qualities).

Obtaining quality leadership in the model causes a drop in market share but, crucially, a rise in profits which is the payoff-relevant object of the firm. Growing market share itself is not an objective of the firm. I focus on the market share of the quality leader in the numerical results because the model's two-firm setup has no direct analogy to industry-level data with more firms per sector. From the discussion of the pricing assumption, an alternative interpretation is that a competitive mass of small firms produce the lower quality product, but a single firm, the quality leader, has the ability to produce the higher quality product.

24

3.3.2 Innovation

The innovation process for improving the quality of intermediate goods follows Akcigit, Ates, and Impullitti (2018). Intermediate good producers choose the amount of research spending R of the final good to maximize the discounted sum of expected future profits. Innovations arrive randomly at Poisson rate x which depends on research spending according to the function:

$$x = \left(\frac{\gamma R}{\alpha}\right)^{\frac{1}{\gamma}} q_i^{\frac{1-\frac{1}{\beta}}{\gamma}}$$

that is, since $\beta < 1$, at higher quality levels more research spending is needed to achieve the same arrival rate of innovations x. γ and α are R&D technology parameters.

Conditional on innovating the size of the quality improvement is random. Formally, conditional on innovating,

$$q_{i,(t+\Delta t)} = \lambda^{n_{i,t}} q_{i,t}$$

where $\lambda > 1$ is the minimum quality improvement and $n_{i,t} \in \mathbb{N}$ is a random variable. Note that each competitor improves over their own quality when they innovate.[16] Initial qualities of all firms are normalized to 1. Let $N_{i,t} = \int_0^t n_{i,s} ds$ denote the total number of step size improvements over a product line i since the beginning of time. The technology gap from firm 1's in sector j's perspective at moment t is:

$$\frac{q_{1,j,t}}{q_{2,t}} = \frac{\lambda^{N_{1,t}}}{\lambda^{N_{2,t}}} \equiv \lambda^{m_{1,t}}$$

Given λ, $m_{i,j,t}$ parameterizes the technology gap between the two firms, representing the number of λ steps ahead or behind its competitor firm i is. This formulation allows me to restrict attention to a finite set of technology gaps described below. m_{ijt} turns out to be the only state variable for the firm, so having a finite set of gaps is needed for a tractable solution of the model. For numerical tractability I impose a maximal technology advantage $\bar{m}$, but in calibrating the model I will set the parameters so that this maximal gap rarely occurs in steady state. I assume that the only spillover in the model between firms occurs when a firm at the maximal gap innovates. In that case, both the innovating firm and its competitor's quality increase by

---

[16]Luttmer (2007) provides an additional rationale for this type of assumption: entrants are typically small and enter far from the productivity frontier, implying that imitation of other firms' technologies is difficult.
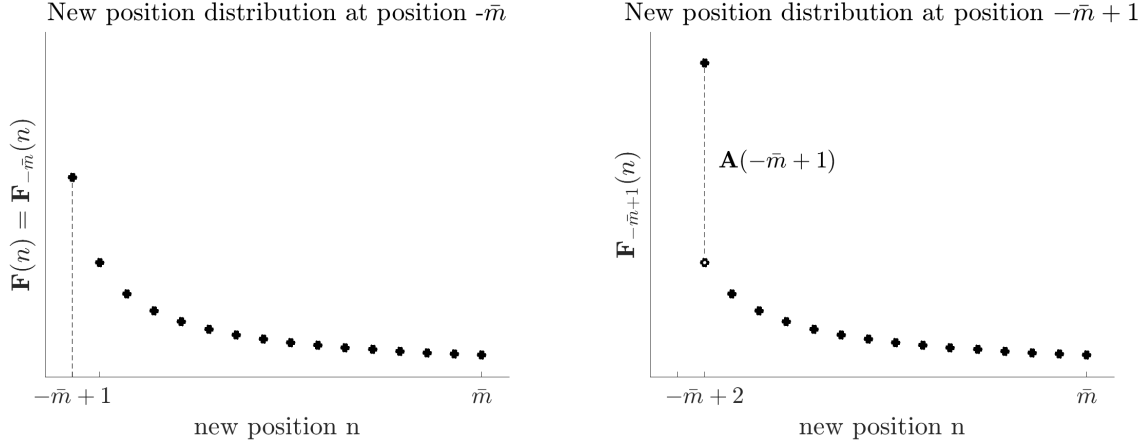
Figure 15: Examples of new position distributions for positions $-\bar{m}$ and $-\bar{m}+1$

the factor λ, keeping the technology gap unchanged but raising the absolute quality of the sector composite good.

The probability distribution of possible quality improvements depends on the firm's current relative quality. As in Akcigit, Ates, and Impullitti (2018), I assume there exists a fixed distribution $\mathbb{F}(n) \equiv c_0(n + \bar{m})^{-\phi}$ for all $n \in \{-\bar{m}+1, \ldots, \bar{m}\}$, shown in the left panel of figure 15, that applies to firms that are the furthest possible distance behind their competitor and describes the probability that they move to each position in technology gap space. The curvature parameter ϕ is critical in the model and determines the speed of catchup by increasing or decreasing the probability of larger innovations. A higher ϕ means a lower probability of these "radical" improvements.[17] c_0 is simply a shifter to ensure $\sum_n \mathbb{F}(n) = 1$.

Given this fixed distribution for the most laggard firm, the new position distribution specific to each technology gap $m > -\bar{m}$ is given by:

$$\mathbb{F}_m(n) = \begin{cases} \mathbb{F}(m+1) + \mathbb{A}(m) & \text{for } n = m+1 \\ \mathbb{F}(s) & \text{for } n \in \{m+2, \ldots, \bar{m}\} \end{cases}$$

where $\mathbb{A}(m) \equiv \sum_{-\bar{m}+1}^{m} \mathbb{F}(n)$. This distribution is shown in the right panel of figure 15. Simply put, all the mass of the fixed distribution on steps down from current quality is instead put on one-step ahead improvements. This formulation captures the feature that laggard firms make larger improvements than leaders on average.

---

[17] As noted by Akcigit, Ates, and Impullitti (2018), this formulation converges to the less general step-by-step model as $\phi \to \infty$.

3.3.3 Value Functions

An intermediate good firm's value function with quality q_t and gap to its rival m_t at moment t is defined as[18]:

$$r_t V_{mt}(q_t) - \dot{V}_{mt}(q_t) = \max_{x_{mt}}\{\pi(m, q_t) - \alpha \frac{(x_{mt})^\gamma}{\gamma} q_t^{\frac{1}{\beta}-1}$$

$$+ x_{mt} \sum_{n_t=m+1}^{\bar{m}} \mathbb{F}_m(n_t)[V_{nt}(\lambda^{n_t-m} q_t) - V_{mt}(q_t)]$$

$$+ x_{(-m)t} \sum_{n_t=-m+1}^{\bar{m}} \mathbb{F}_{-m}(n_t)[V_{(-n)t}(q_t) - V_{mt}(q_t)]\} \tag{2}$$

The firm chooses the arrival rate of innovations x_{mt}. The first line denotes the flow profits and the research cost R_{mt} given the choice of x_{mt}. The second line denotes the probability the firm innovates and sums over the possible states the firm could move to using the distribution of steps sizes and the firm's new value function with higher quality and a larger quality advantage over its rival. The final line denotes the chance the firm's rival innovates and the change in the firm's value because its relative quality falls when the rival innovates.

Now consider the flow profits of the firm, denoting the optimal price of the leader at technology gap m as $p(m)$. We want to eliminate q_t from the value function for tractability, so that each technology gap is associated with a value and the specific firm value function scales in q_t or some function of q_t.

$$\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ (p(m) - \eta)k_i & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}$$

Use the fact that $k_i = q_i^{\epsilon-1} p(m)^{-\epsilon} P_j^{\epsilon-\frac{1}{\beta}} P^{-\frac{1}{\beta}} L$. Normalize P and L to 1. Expanding the definition of the sectoral price index P_j: $k_i = q_i^{\epsilon-1} p(m)^{-\epsilon} (q_i^{\epsilon-1} p(m)^{1-\epsilon} + q_{-i}^{\epsilon-1} \eta^{1-\epsilon})^{\frac{\epsilon-\frac{1}{\beta}}{1-\epsilon}}$. This further simplifies (by factoring out $q_i^{\epsilon-1}$ from the price index) to:

$$\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ q_i^{\frac{1}{\beta}-1}(p(m) - \eta)p(m)^{-\epsilon}(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1}\eta^{1-\epsilon})^{\frac{\epsilon-\frac{1}{\beta}}{1-\epsilon}} & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}$$

So $V_{mt}(q_t) = v_{mt} q_t^{\frac{1}{\beta}-1}$. It can be shown by a guess-and-check approach that this is the case.

---

[18] I give the slightly altered equations for firms at the minimum and maximum gaps in Appendix C.

The firm's optimal arrival rate x_{mt} (which I refer to as effort in subsequent discussions) is the solution to the first order condition of equation (2), which gives:

$$x_{mt} = \begin{cases} \left(\dfrac{\sum_{n=m+1}^{\bar{m}} \mathbb{F}_m(n_t)[(\lambda^{n_t-m})^{\frac{1}{\beta}-1}v_{nt}-v_{mt}]}{\alpha} \right)^{\frac{1}{\gamma-1}} & \text{for } m < \bar{m} \\[3ex] \left[\frac{1}{\alpha}(\lambda^{\frac{1}{\beta}-1}-1)v_{\bar{m}t} \right]^{\frac{1}{\gamma-1}} & \text{for } m = \bar{m} \end{cases}$$

The model delivers the predictions that R&D intensity is independent of size (sales) [conditional on productivity gaps] and heterogeneous across firms in the same sector, consistent with the empirical evidence discussed in Klette and Kortum (2004).

3.4   Equilibrium Output

Below I solve for output Y plugging in the intermediate goods firms' output decisions to illustrate the components of output growth:

$$\begin{aligned} Y &= \frac{1}{1-\beta} \left(\int_0^1 K_j^{1-\beta} dj \right) L^\beta \\ &= \frac{1}{1-\beta} \left(\int_0^1 \left(\sum_{i=1}^2 q_i^{\frac{\epsilon-1}{\epsilon}} (q_i^{\epsilon-1} \left(\frac{p_i}{P_j} \right)^{-\epsilon} \left(\frac{P_j}{P} \right)^{-\frac{1}{\beta}} L)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}1-\beta} dj \right) L^\beta \\ &= \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \left(\int_0^1 P_j^{\epsilon(1-\beta)-\frac{1-\beta}{\beta}} \left(\sum_{i=1}^2 q_i^{\epsilon-1} p_i^{1-\epsilon} \right)^{\frac{\epsilon(1-\beta)}{\epsilon-1}} dj \right) \\ &= \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \left(\int_0^1 P_j^{-\frac{1-\beta}{\beta}} dj \right) \end{aligned}$$

The demand shifter $P^{\frac{1}{\beta}}L$ index is common to all firms and can be taken out entirely (and normalized to one since I assume zero population growth). The price index P_j of each sector falls as the qualities of the two firms in the sector grow, and the exponent is negative for all $\beta \in (0,1)$ so Y is growing in qualities.

Common to all firms with a particular technology gap m are the prices $p(m)$ of the firm at gap m and its competitor at $-m$, $p(-m)$. At time t, Y can be expressed as:

$$Y_t = \frac{1}{2}\frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}} \left(\int_0^1 \left(q_{it}^{\epsilon-1} p_i(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p_{-i}(-m)^{1-\epsilon} \right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i \in \mu_{mt}\}} di \right)$$

$$Y_t \equiv \frac{1}{2}\frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}} Q_{mt} \tag{3}$$

Where Q_{mt} is defined as:

$$Q_{m,t} = \int_0^1 \left(q_{it}^{\epsilon-1} p(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p(-m)^{1-\epsilon} \right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i \in \mu_{mt}\}} di$$

$$= \left(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon} \right)^{\frac{1-\beta}{\beta(\epsilon-1)}} \int_0^1 q_{i,t}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_{mt}\}} di \qquad (4)$$

Here, μ_{mt} is the measure of firms at each technology gap m at time t (normalizing measure of firms to one) and Q_{mt} is a particular index of the qualities of all firms at gap m. The change in output therefore depends on the changes $\dot{Q}_{mt}$ for each technology gap m which in turn depend on the innovation arrival rates x_{mt} chosen by firms and the exogenous distribution of quality improvement sizes $F(n)$. The final component determining output will be the measure of firms at each technology gap μ_{mt} that is itself an endogenous object. The next section describes how to solve for the measures μ_{mt}.

3.5 Stationary Distribution Over Technology Gaps

Firms move to technology gap m through innovation from a lower technology gap, or because their competitor innovates to gap $-m$. The distributions $F_n(m)$ and $F_{-n}(-m)$ determine these probabilities, combined with the innovation efforts of firms at n and $-n$. The outflows from gap m are due to the firm at m or its competitor at $-m$ innovating. Putting this together into the Kolmogorov forward equations for the evolution of the mass of firms at each gap:

$$\dot{\mu}_{mt} = \sum_{n=-\bar{m}}^{m-1} x_n F_n(m) \mu_{nt} + \sum_{n=m+1}^{\bar{m}} x_{-n} F_{-n}(-m) \mu_{nt} - (x_m + x_{-m}) \mu_{mt}$$

The highest and lowest technology gaps are special cases because of spillovers: if the firm at the highest gap innovates both firms remain at the same gap in the next instant.

$$\dot{\mu}_{-\bar{m}t} = \sum_{n=-\bar{m}+1}^{\bar{m}} x_{-n} F_{-n}(\bar{m}) \mu_{nt} - x_{-\bar{m}} \mu_{-\bar{m}t}$$

$$\dot{\mu}_{\bar{m}t} = \sum_{n=-\bar{m}}^{\bar{m}-1} x_n F_n(\bar{m}) \mu_{nt} - x_{-\bar{m}} \mu_{\bar{m}t}$$

On a balanced growth path, $\mu_{mt} = \mu_m$ for all m, t. Replacing the left hand side of the above equations with zero change in equilibrium and the measures on the right

hand side with the constants μ_n, μ_m defines a system of $2\bar{m}+1$ equations in $2\bar{m}+1$ unknowns that determine the steady state distribution of firms over possible technology gaps. There are several additional restrictions on the solution to this system. First, for each firm at m there is a firm at $-m$ (that is, the stationary distribution is symmetric). Second, I impose the restriction that the measure of all the firms sums to one.

3.6 Output growth

I next describe how to solve for the growth rate on a balanced growth path. From equation 3 I obtain:

$$\frac{\dot{Y}_t}{Y_t} = g_{Yt} = \frac{1}{2}\frac{1}{1-\beta} \sum_{m=-\bar{m}}^{\bar{m}} \frac{\dot{Q}_{mt}}{Y_t}$$

It's useful to define:

$$\tilde{Q}_{mt} = \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_{mt}\}} di$$

So that:

$$g_{Yt} = \frac{1}{2}\frac{1}{1-\beta} \sum_{m=-\bar{m}}^{\bar{m}} (p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1}p(-m)^{1-\epsilon})^{\frac{1-\beta}{\beta(\epsilon-1)}} \frac{\dot{\tilde{Q}}_{mt}}{Y_t}$$

I will focus on a balanced growth path where $\frac{\dot{\tilde{Q}}_{mt}}{Y_t}$ is constant for all m. The next step is to study how $\tilde{Q}_m$ evolves between t and $t + dt$ for all m. These expressions are similar to those for the stationary distribution but account for the quality improvements that occur because of innovation.

Assuming fixed distribution $\mu_{mt} = \mu_m$ for all m, t:

$$\dot{\tilde{Q}}_{mt} = \int_0^1 q_{m,t+dt,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_m\}} di - \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_m\}} di$$

Consider an arbitrary $m \in (-\bar{m}, \bar{m})$ ($-\bar{m}$ and $\bar{m}$ are special cases because of spillovers). A portion of firms at m at t innovate to a different gap, and another portion leave gap m because their competitor innovates. Because all firms at gap m choose the same arrival rate x_m, these are a random sample of the firms at gap m at time t. The outflows from $\dot{\tilde{Q}}_m$ are:

$$-(x_m + x_{-m}) \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_m\}} di = -(x_m + x_{-m})\tilde{Q}_m$$

The inflows to m's quality index come from two sources. First, some firms innovate into position m from a lower position n, improving their quality by λ^{m-n}. The probability they innovate and reach gap m is given by $x_n F_n(m)$. Some firms fall back to m from a higher gap n because their competitor innovates to $-m$. The probability their competitor reaches $-m$ is given by $x_{-n} F_{-n}(-m)$. So cumulative inflows are:

$$\sum_{n=-\bar{m}}^{m-1} x_n F_n(m)(\lambda^{(m-n)})^{\frac{1-\beta}{\beta}} \tilde{Q}_n + \sum_{n=m+1}^{\bar{m}} x_{-n} F_{-n}(-m) \tilde{Q}_n$$

So, putting it together:

$$\dot{\tilde{Q}}_{mt} = \sum_{n=-\bar{m}}^{m-1} x_n F_n(m)(\lambda^{(m-n)})^{\frac{1-\beta}{\beta}} \tilde{Q}_n + \sum_{n=m+1}^{\bar{m}} x_{-n} F_{-n}(-m) \tilde{Q}_n - (x_m + x_{-m}) \tilde{Q}_m \quad (5)$$

For lowest gap there are spillovers when competitor innovates:

$$\dot{\tilde{Q}}_{-\bar{m}t} = \sum_{n=-\bar{m}+1}^{\bar{m}} x_{-n} F_{-n}(\bar{m}) \tilde{Q}_n + x_{\bar{m}}(\lambda^{\frac{1-\beta}{\beta}} - 1) \tilde{Q}_{-\bar{m}} - x_{-\bar{m}} \tilde{Q}_{-\bar{m}} \quad (6)$$

For highest gap you do not exit that gap if you innovate:

$$\dot{\tilde{Q}}_{\bar{m}t} = \sum_{n=-\bar{m}}^{\bar{m}-1} x_n F_n(\bar{m})(\lambda^{(m-n)})^{\frac{1-\beta}{\beta}} \tilde{Q}_n + x_{\bar{m}}(\lambda^{\frac{1-\beta}{\beta}} - 1) \tilde{Q}_{\bar{m}} - x_{-\bar{m}} \tilde{Q}_{\bar{m}} \quad (7)$$

Given equations 5, 6, and 7, on a balanced growth path where $\frac{\dot{\tilde{Q}}_{mt}}{Y_t}$ is constant, it's sufficient to assume $\frac{\tilde{Q}_{mt}}{Y_t}$ is constant over time for all $m \in [-\bar{m}, \bar{m}]$. Differentiating $\frac{\tilde{Q}_{mt}}{Y_t}$ with respect to time yields:

$$\left(\frac{\dot{\tilde{Q}}_m}{Y}\right) = \frac{\dot{\tilde{Q}}_m}{Y} - \frac{\tilde{Q}_m}{Y}\frac{\dot{Y}}{Y}$$
$$= \frac{\dot{\tilde{Q}}_m}{Y} - g\frac{\tilde{Q}_m}{Y}$$

Imposing that the left hand side is zero implies:

$$\frac{\dot{\tilde{Q}}_m}{Y} = g\frac{\tilde{Q}_m}{Y}$$

The vector on the left hand side is defined above by the flow equations (5), (6), and (7) divided by GDP. Use those equations to form a matrix A that captures the flow equations:

$$\frac{\dot{\tilde{Q}}_m}{Y} = A\frac{\tilde{Q}_m}{Y} = g\frac{\tilde{Q}_m}{Y}$$

31

The values in A depend on λ, ϕ, and x_m. The above equation means that the growth rate g is an eigenvalue of the matrix A and $\frac{\tilde{Q}_m}{Y}$ is the corresponding eigenvector of A. If there is only one positive, real eigenvalue there is only one such balanced growth path where the contribution of the growth of each technology gap to the total growth rate is constant and the growth rate of the economy is constant. I next define the full set of conditions for a balanced growth equilibrium in this economy.

3.7 Equilibrium

Let $R_t = \int_0^1 \sum_{i=1}^2 R_{ijt} dj$ denote total research and development spending, C_t total consumption, and $K_t = \int_0^1 \sum_{i=1}^2 \eta k_{ijt} dj$ total purchases of final goods for production of intermediate goods.

An equilibrium is an allocation $\{k_{ijt}, K_t, x_{ijt}, R_t, Y_t, C_t, L, \mu_{mt}, Q_{mt}, A_t\}_{i\in\{1,2\},j\in[0,1],m\in[-\bar{m},\bar{m}]}^{t\in(0,\infty)}$ and prices $\{r_t, W_t, p_{ijt}\}_{i\in\{1,2\},j\in[0,1]}^{t\in(0,\infty)}$ such that for all t:

1. Intermediate goods firms solve their innovation and price-setting problems (price-setting optimally for the leader only)

2. Final goods firms solve their problem to hire labor and intermediate goods

3. Households solve their consumption-savings problem

4. Goods market clears: $Y_t = C_t + R_t + K_t$

5. Asset market clears, pinning down r_t via the household's Euler equation

6. Labor market clears, pinning down the wage rate from the final good producers' problem

7. μ_{mt}, Q_{mt} are consistent with firms' optimal innovation decisions

4 Results

In this section I present preliminary results from a numerical exercise with the model. I briefly describe the solution algorithm for finding the steady state and the estimation strategy before presenting the results. The exercise compares properties of the economy in two different steady states of the model with different values of the probability of radical innovations parameter ϕ. This is meant to capture the fact that radical innovation probabilities change over time depending on general purpose technologies like the internet and other ICT. Changing ϕ is an appropriate representation of the

changing impact of a GPT since it affects research productivity in all sectors of the economy. Recall also that figure 2 shows that productivity divergence is particularly pronounced in ICT intensive sectors, pointing to ICT as a potential cause. Bessen (2017) also shows that ICT intensity is correlated with rising market concentration.

The two values for ϕ are chosen to match the average patent stock growth per patent in 1994-2003 and from 2004-2017 for all firms. I show that the leader's market share, leader's share of total R&D, productivity gaps, markups, and the profit share are all higher when radical innovations are less likely. The growth rate and R&D as a share of GDP are lower.

I then decompose the difference in growth rates in the two steady states into the contributions from the exogenous effect of reducing the average size of quality improvements and the endogenous effect of reduced effort by firms and the resulting change in the distribution of sectors over technology gaps. I find that the endogenous forces account for between a quarter to a half of the total reduction in the growth rate between the two steady states depending on the order of the decomposition, with the remainder due to the fact that quality improvements due to innovating are simply smaller on average, regardless of the firms' policies.

4.1 Solution Algorithm

The solution algorithm involves first guessing a steady state interest rate. Given this interest rate, solve the value functions for each technology gap by policy function iteration using the fact that $\dot{v}_{mt} = 0$ on a balanced growth path. This process yields the optimal innovation policies of firms at each technology gap. Given the policy functions the stationary distribution of firms over technology gaps can be obtained by solving the system of equations described in section 3.5. To obtain the growth rate of GDP, solve the system described in section 3.6. Check whether this growth rate is consistent with the interest rate guess using the household's Euler equation: $r = g\psi + \rho$. Update the guess of the interest rate and repeat until the interest rate guess and the interest rate implied by the resulting growth rate and the Euler equation are consistent.

To obtain the micro-level moments, I then simulate a discrete version of the model with ten subperiods per year for a panel of firms (3000 for the results presented here) for a long time series (400 years) and take averages.

33

4.2 Estimation

Table 1 shows the parameters I use for the numerical exercises. Four parameters are calibrated outside the model. The intertemporal elasticity of substitution is set to 1. The labor share, β, is set to 0.6. This also implies an elasticity of substitution across sectors of $\frac{1}{\beta} = \frac{5}{3}$, within the range of upper-level elasticities of substitution estimated in Hobjin and Nechio (2019). The curvature of the R&D cost function, γ, is calibrated outside to match the empirical evidence on the elasticity of patenting to R&D expenditures, discussed in Acemoglu et al. (2018). The maximum technology gap, $\bar{m}$, is set to 16 (arbitrarily).

The rest of the parameters are estimated using a simulated method of moments approach to match six targets for the 1990s equilibrium (1994-2003). These targets are given in table 2. The data sources and computation methods for the data moments are given in appendix F.

Parameter	Value	Meaning/source
ψ	1	Intertemporal elasticity of substitution
ρ	**0.133**	Rate of time preference (annual)
β	0.6	Labor share/Nechio & Hobijn (2017)
ϵ	**4.4033**	Elasticity of substitution within sectors
η	**0.6554**	Marginal cost of intermediate producers
λ	**1.0683**	Min. qual. improvement
γ	2	Curvature of R&D function
α	**4.1383**	R&D technology
$\bar{m}$	16	Maximum number of steps ahead
ϕ	**1.0047**	**estimated**

Table 1: Model parameters for numerical exercises (estimated parameters in bold).

The estimation performs well in matching the growth rate, aggregate R&D share of GDP, R&D intensity (R&D/sales of the average firm), and the average patent stock growth per patent but falls somewhat short in matching the level of average leader market share (concentration) and the rate of sales leadership turnover.

To calibrate the change in ϕ between the two equilibria I keep all the parameters fixed and estimate ϕ to match the slowdown in average patent stock growth per patent

Targeted moments	Data	Model
Avg. TFP growth, %, 1994-2003	1.7	1.71
Avg. leader market share, %, 1994-2003	42.99	47.82
Avg. R&D/sales, %, 1994-2003	5.42	5.18
Avg. R&D share of GDP, %, 1994-2003	1.8	1.8
Avg. patent stock growth per patent, %, 1994-2003	23.44	23.46
Avg. leadership turnover, %, 1994-2003	13.73	3.64

Table 2: Targeted moments from estimation of 6 parameters

from section 2.1. This estimation procedure results in $\phi = 1.6652$.[19]

4.3 Numerical Results

Table 3 shows non-targeted moments in the data in the 1990s (1994-2003) and the 2000s (2004-2017) and how the two steady states of the model with different catchup probabilities compare to the data.

Targeting just the slowdown in patent stock growth per patent documented in section 2.1 in the model matches the two main objects of interest, rising concentration and slowing productivity growth, very closely. This experiment explains around 96% of the rise in concentration and 113% of the productivity slowdown.

The model also performs well in matching other non-targeted moments: the rise in markups and the profit share, the rise in the industry leader's share of total industry R&D, and the slowdown in leadership turnover. One puzzle is why R&D expenditures have risen in the data without yielding the same productivity gains as in the past. One explanation suggested by Bloom et al. (2019) is that ideas are getting harder to find, that is, R&D productivity is declining. Note that I hold α, the efficiency of R&D expenditures, constant between the two equilibria.

The rise in concentration, markups, and the profit share in the 2000s model equilibrium is driven purely by a composition effect because productivity gaps get wider on average (figure 16) and leaders with higher relative quality capture a larger share of industry sales and choose to set higher markups. Productivity gaps get larger on average because innovation effort shifts significantly from followers in the 1990s equi-

---

[19]The new position distributions $F(n)$ for quality improvements under the two values of ϕ are plotted in Appendix D.

Moment	Data			Model		
	1990s	2000s	Chg. (pp)	$\phi = 1.01$	$\phi = 1.67$	Chg. (pp)
TFP growth, %	1.7	0.5	-1.2	1.71	0.45	-1.26
Leader market share, avg, %	42.99	48.04	5.05	47.82	52.69	4.87
Pat stock growth/patent, avg, %	23.44	11.71	-11.73	23.46	11.71	-11.75
Markup, avg, %	26.16	29.88	3.72	24.09	26.36	2.27
R&D share of GDP, %	1.8	1.89	0.09	1.8	0.62	-1.2
R&D intensity, avg, %	5.42	6.98	1.56	5.18	1.62	-3.56
Profit share of GDP, %	5.24	6.61	1.37	6.54	8	1.46
Leader's share of R&D, avg, %	41.32	47.04	5.72	24.88	68.34	43.46
Leadership turnover, %	13.73	9.3	-4.43	3.64	0.49	-3.15

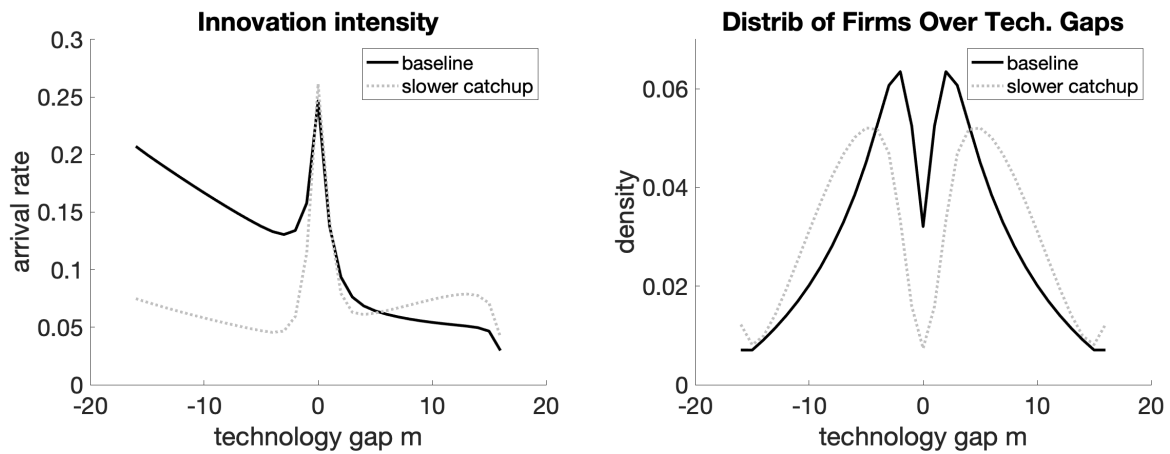Table 3: Model and data comparison, main results

Figure 16: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), two catchup regimes.

librium to leaders in the 2000s equilibrium (figure 16). Note that fixing innovation effort at its 1990s level in the model but increasing ϕ (decreasing the average innovation size) results in a *tighter* distribution around the neck-and-neck state, because firms pull far away from each other less frequently under a higher ϕ regime (counterfactual plotted in figure 22 in Appendix E).

The growth rate falls in the 2000s equilibrium for several reasons. First, the location of effort changes endogenously, slowing the growth of the quality indexes de-

scribed in equation 4 even if ϕ is held fixed. A larger share of innovations are done by leaders in the 2000s equilibrium, which represent smaller quality improvements than the innovations of the followers on average. This result is in line with recent evidence from: (i) Akcigit and Ates (2019b) that patents are more concentrated among a small number of firms than in the past; (ii) Autor, Katz, and Dorn (2017)'s finding that technology diffusion is slowed in more concentrated sectors; (iii) the evidence in section 2.5 that leaders do more of total R&D now than in the past. The total investment in R&D as a share of GDP in model also declines. To understand the role of firms' effort in the slowdown, I hold ϕ fixed at its 1990s value and calculate the growth rate with the innovation polices x_m from the 2000s equilibrium. This can explain about 43% of the total slowdown in the data (table 4).

However, an alternative decomposition considers the first order effect of raising ϕ, which means firms make smaller quality improvements conditional on innovating, keeping the innovation policies of the firms fixed at their 1990s equilibrium values. This alone can account for around 73% of the decline in the growth rate (table 4).

Decomposition	% of slowdown explained
Role of effort (ϕ fixed, x changes)	42.87
First order effect (x fixed, ϕ changes)	72.5

Table 4: Growth decompositions

5 Role of Elasticity of Substitution

The elasticity of substitution within sectors ϵ turns out to play an important role in the determination of the growth rate and to a lesser extent productivity gaps. In this section I explore both directions of change in ϵ compared to the baseline exercise in section 4, as each can capture (in a very reduced form) different structural changes in the U.S. economy that have been suggested in the literature recently. This illustrates how the model can be used to unify the neo-Schumpeterian endogenous growth literature with the literature on superstar firms and rising market power. Neither change matches the direction of all the moments of interest that the baseline exercise regarding patent quality does, though the superstar firm experiment gets closer to the data than increased market power. This turns out to be because the model has the stan-

dard Schumpeterian feature than increased market power gives a greater incentive for innovation.

5.1 Increasing Market Power?

Recent research has focused on the potential costs of rising market power and markups (see de Loecker and Eeckhout (2017), Eggertsson, Robbins, and Wold (2018) and Edmond, Midrigan, and Xu (2018) for example) for growth and welfare. Can an increase in market power generate the same predictions for the macroeconomic changes experienced in the U.S. in recent years as a change in the probability of radical innovations in the model? I model an increase in market power as a decrease in the substitutability of products in the same sector, ϵ.

I keep the calibration the same as in table 1 and set $\phi = 1.0047$ (1990s case). I decrease ϵ from 4.4033 in the baseline to 3. Average markups rise by about 10 percentage points, about a third of the total rise estimated by de Loecker and Eeckhout (2017). With the exception of markups and the profit share, the results from this exercise are the opposite of what has happened in the data. Because of greater market power, markups and profits are higher when the firm has market leadership and this induces more innovation effort by laggard firms as they try harder to overtake the market leader (R&D/GDP rises from 1.8% to 2.6%, see table 5 and figure 17). This results in a higher growth rate. There is also greater turnover in market leadership and average productivity differences go down (figure 17).
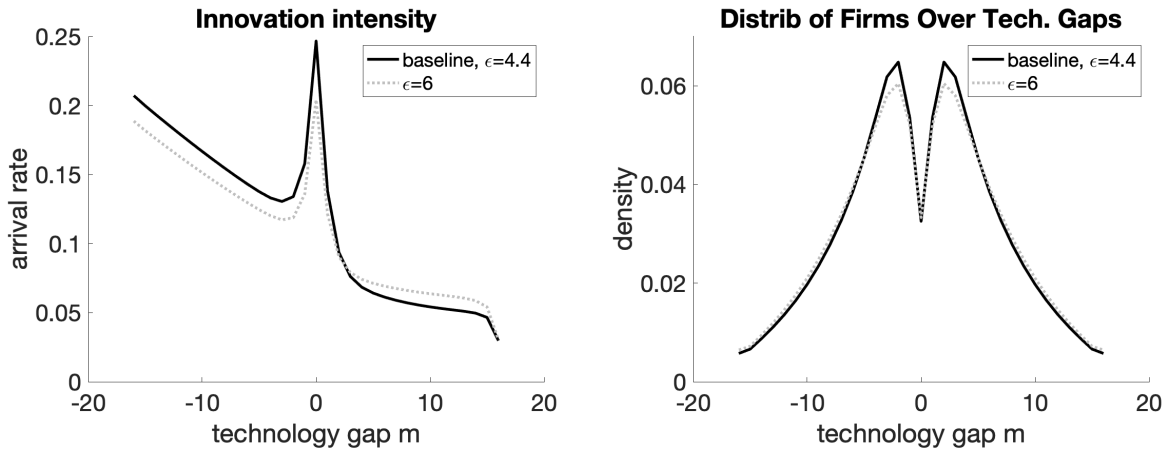

Figure 17: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), two market power regimes.

Moment	Model	
	$\epsilon = 4.4$	$\epsilon = 3$
TFP growth, %	1.7	2.06
Leader market share, avg, %	47.82	42.25
Pat stock growth/patent, avg, %	23.46	23.15
Markup, avg, %	24.09	33.93
R&D share of GDP, %	1.8	2.61
R&D intensity, avg, %	5.32	6.62
Profit share of GDP, %	6.54	6.89
Leader's share of R&D, avg, %	24.88	19.86
Leadership turnover, %	3.64	4.58

Table 5: Model comparison, market power

5.2 Superstar Firms?

Seminal work on the macroeconomic effects of superstar firms is Autor, Katz, and Dorn (2017). The main focus of their work is to point out that reallocation to more productive, already large firms can cause a decline in the labor share because these firms produce more with fewer workers than their competitors. In their static industry model, firms produce products that are imperfect substitutes within an industry. Firms draw labor productivities from a distribution and then make an entry decision. Upon entering, firms set prices a la Bertrand.

The force for reallocation to more productive firms in their model is an increase in product substitutability between firms in the same industry. The authors argue that this increase could represent more fierce import competition from abroad, particularly China, in recent years or increased price sensitivity due to better search technology such as online retail. Keeping the exogenous productivity distribution fixed, an ancillary result is that a sector's measured TFP will rise unambiguously when substitutability increases because of two forces: first, the minimum productivity threshold for entrants rises, and second, more productive firms grow their sales share.

I first show that this static result is usually true in my two-firm model without entry. Because there is no entry, sector TFP depends on the covariance between relative quality and leader's market share. Figure 18 plots market shares as a function of the technology gap for different values of ϵ for the baseline parameterization of the model

given in table 1. For most values of the technology gap, increasing the substitutability of products statically increases the leader's market share which increases sector TFP using market shares.[20]

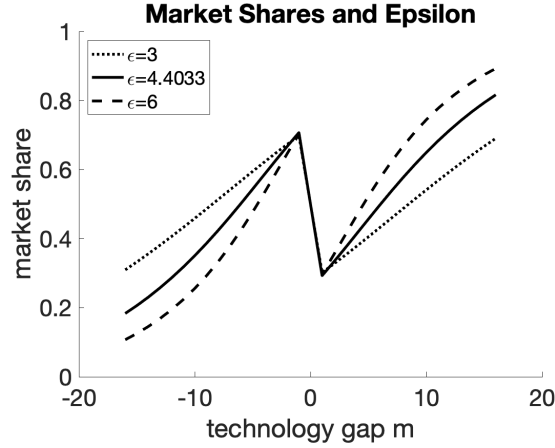

Market Shares and Epsilon

Figure 18: Market shares in leader-follower model with different values of elasticity of substitution within sectors.

On impact, therefore, increasing the substitutability of the firms' products will raise measured TFP as in Autor, Katz, and Dorn (2017). However, dynamically, this change reduces the markup that the leader charges and thus reduces both firms' expected future profits. The effect of this change on TFP *growth* is unclear ex-ante.

To answer this question I compare the steady state of the model with higher ϵ to the baseline. Under this parameterization, raising ϵ lowers the growth rate while increasing concentration dramatically. The rise in concentration comes from two forces. First, the static reallocation force operates: even if technology gaps were unchanged from one steady state to another, these same gaps would generate a higher average leader market share according to figure 18. Second, changes in effort shown in figure 19 cause the average technology gap to grow. With higher ϵ the value of overtaking the leader falls since leader markups are lower so followers and firms in the neck-and-neck position innovate less (figure 19). On the other hand, markups and profits become more elastic in the technology gap when ϵ is higher, while the likelihood of being overtaken falls along with the interest rate so leading firms discount the future

---

[20]With Bertrand pricing this is always true. But it is not necessarily true under the assumption that the follower sets price equal to marginal cost: if relative quality differences are small, increasing ϵ can cause a drop in the leader's market share.

less and leaders innovate more than before. In combination these changes in effort cause technology gaps to get larger on average.

Moment	Model	
	$\epsilon = 4.4$	$\epsilon = 6$
TFP growth, %	1.7	1.59
Leader market share, avg, %	47.82	54.19
Pat stock growth/patent, avg, %	23.46	23.78
Markup, avg, %	24.09	20.25
R&D share of GDP, %	1.8	1.56
R&D intensity, avg, %	5.32	5.18
Profit share of GDP, %	6.54	6.72
Leader's share of R&D, avg, %	24.88	30.37
Leadership turnover, %	3.64	3.26

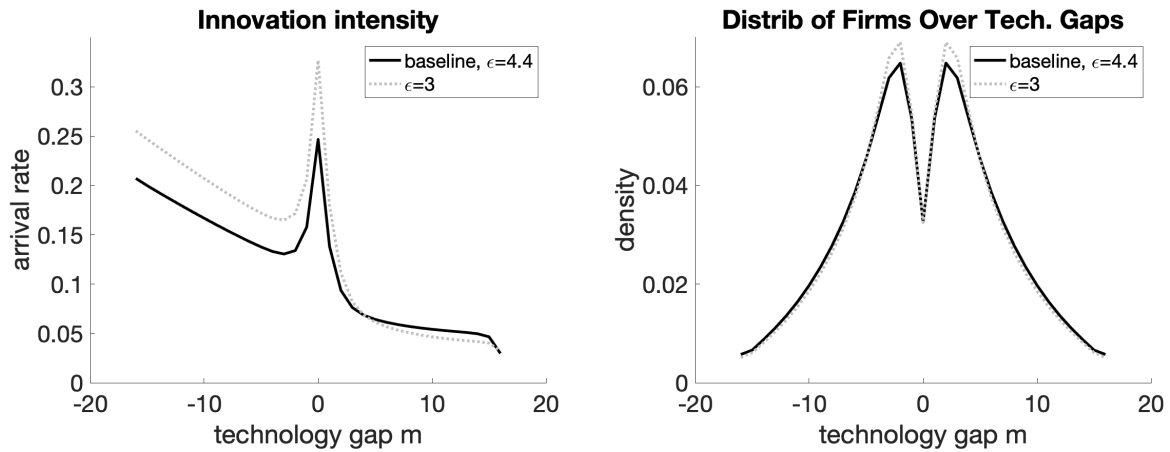Table 6: Model comparison, superstar firms

Figure 19: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), superstar firms.

This experiment demonstrates how the rise of superstar firms can coincide with slowing productivity growth through an increase in product substitutability. The dynamic dimension with endogenous productivity overturns the Autor, Katz, and Dorn (2017) result that increased market share of superstar firms necessarily raises measured TFP. Transition dynamics following a permanent positive shock to ϵ will likely

show first an increase in TFP growth due to greater allocative efficiency then a decline as the within-firm contribution to growth slows. However, non-targeted moments go in the wrong direction: average markups fall, unlike in the data.

6   Conclusion

I have presented a general equilibrium model of innovation and growth where multiple firms are active in a sector in each period and goods within sectors are imperfect substitutes. Through the lens of the model I unify the Schumpeterian endogenous growth literature with the growing literature on rising concentration, markups and market power in the U.S. Crucially, incorporating a nested constant elasticity of substitution demand structure also allows me to study the dynamic, endogenous growth analog to Autor et al. (2017)'s recent work on superstar firms. From this experiment the model demonstrates that the rise of superstar firms can result in slower productivity growth.

A quantitative version of the model, incorporating new evidence on declining patent quality since the late 1990s, can fully explain the decline in productivity growth and explain almost the entire rise in concentration over the same period. The main driver of these changes is how firms, particularly laggard firms, respond to a lower probability of catching their leading rival through innovation. Laggard firms choose to invest less in R&D as a share of sales than before. Leaders are less likely to be displaced and so innovate more frequently than before, but their improvements tend to be more incremental. Together these forces result in wider productivity gaps on average, meaning sales leaders capture a larger share of total industry sales, and the economy as a whole grows more slowly. Markups and the profit share also rise. A change in the elasticity of substitution in the quantitative model, capturing either increased market power or the rise of superstar firms, cannot generate the same fit for data.

References

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr. 2018. "Innovation, Reallocation, and Growth." *American Economic Review* 108 (11): 3450–3491.

Acemoglu, Daron, and Dan Cao. 2015. "Innovation by entrants and incumbents." *Journal of Economic Theory* 157:255–294.

Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J. Klenow, and Huiyu Li. 2019. "A Theory of Falling Growth and Rising Rents." *Federal Reserve Bank of San Francisco, Working Paper Series*, no. October:01–43.

Aghion, Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. 2005. "Competition and Innovation: An Inverted-U Relationship*." *Quarterly Journal of Economics* 120 (2): 701–728.

Aghion, Philippe, Christopher Harris, Peter Howitt, and John Vickers. 2001. "Competition, imitation and growth with step-by-step innovation." *Review of Economic Studies* 68 (3): 467–492.

Akcigit, Ufuk, and Sina Ates. 2019a. "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory." *National Bureau of Economic Research*.

———. 2019b. "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory."

———. 2019c. "What Happened to U.S. Business Dynamism?"

Akcigit, Ufuk, Sina Ates, and Giammario Impullitti. 2018. "Innovation and Trade Policy in a Globalized World." *NBER Working Paper*.

Akcigit, Ufuk, and William R. Kerr. 2018. "Growth Through Heterogeneous Innovations." *Journal of Political Economy* 126, no. 4.

Alexander, Lewis, and Janice Eberly. 2018. "Investment Hollowing Out." *IMF Economic Review* 66 (1): 5–30.

Andrews, Dan, Chiara Criscuolo, and Peter Gal. 2016. "The Global Productivity Slowdown, Technology Divergence and Public Policy: A Firm Level Perspective." *The Future of Productivity: Main Background Papers*, pp. 1–50.

Andrews, Dan, Chiara Criscuolo, and Peter N Gal. 2015. "Frontier Firms, Technology Diffusion and Public Policy: Micro Evidence From OECD Countries." *The Future of Productivity: Main Background Papers*, p. 38.

Anzoategui, Diego, Diego Comin, Mark Gertler, and Joseba Martinez. 2017. "Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence." *NBER Working Paper*, no. 22005:1–52.

Arrow, Kenneth J. 1962. "The Economic Implications of Learning by Doing." *The Review of Economic Studies* 29 (3): 155–173.

Autor, David, David Dorn, Lawrence F Katz, and Christina Patterson. 2017. "Concentrating on the Fall of the Labor Share." 107 (1476): 180–185.

Autor, David, Lawrence F. Katz, and David Dorn. 2017. "The Fall of the Labor Share and the Rise of Superstar Firms." *NBER Working Paper Series*, no. 23396.

Baqaee, David Rezza, and Emmanuel Farhi. 2017. "Productivity and Misallocation in General Equilibrium." *Working Paper*, pp. 1–76.

Bartelsman, Eric J., and Mark E. Doms. 2000. "Understanding Productivity: Lessons from Longitudinal Microdata." *Journal of Economic Literature* XXXVIII (September): 569–594.

Berger, David W., Kyle F. Herkenhoff, and Simon Mongey. 2019. "Labor Market Power." *NBER Working Paper Series* 25719:1–80.

Bessen, James. 2017. "Information Technology and Industry Concentration." *Boston University School of Law Law & Economics Paper* 17, no. 41.

Bloom, Nicholas, Charles Jones, John Van Reenen, and Michael Webb. 2019. "Are Ideas Getting Harder to Find? (slides)."

Bornstein, Gideon. 2018. "Entry and Profits in an Aging Economy: The Role of Consumer Inertia." pp. 909–927.

Bresnahan, Timothy F., and M. Trajtenberg. 1995. "General purpose technologies 'Engines of growth'?" *Journal of Econometrics* 65 (1): 83–108.

Carlin, Wendy, Mark Schaffer, and Paul Seabright. 2004. "A minimum of rivalry: Evidence from transition economies on the importance of competition for innovation and growth." *Contributions to Economic Analysis and Policy* 3 (1): 241–285.

Cavenaile, Laurent, Murat Alp Celik, and Xu Tian. 2020. "Are Markups Too High? Competition, Strategic Innovation, and Industry Dynamics."

Chatterjee, Satyajit, and Burcu Eyigungor. 2019. "The Firm Size and Leverage Relationship and Its Implications for Entry and Concentration in a Low Interest Rate World." 18, no. 19.

Corhay, Alexandre, Howard Kung, and Lukas Schmid. 2020. "Q: Risk, Rents, or Growth ?"

Council of Economic Advisers. 2016. "Benefits of Competition and Indicators." *Issue Brief*, no. May.

Crouzet, By Nicolas, and Janice Eberly. 2018. "Intangibles, Investment, and Efficiency." *AEA Papers and Proceedings* 108:426–431.

de Loecker, Jan, and Jan Eeckhout. 2017. "The Rise of Market Power and the Macroeconomic Implications." *Working Paper*, pp. 1–44.

de Loecker, Jan, and Frederic Warzynski. 2012. "Markups and Firm-Level Export Status: Appendix." *American Economic Review* 102 (6): 2437–2471.

de Ridder, Maarten. 2020. "Market Power and Innovation in the Intangible Economy."

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu. 2018. "How Costly Are Markups?" no. September.

Eggertsson, Gauti, and Neil Mehrotra. 2014. "A Model of Secular Stagnation." *NBER Working Paper*, no. 20754.

Eggertsson, Gauti, Jacob Robbins, and Ella Getz Wold. 2018. "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States." pp. 1–69.

Engbom, Niklas. 2017. "Firm and Worker Dynamics in an Aging Labor Market." *Working Paper*, pp. 1–115.

Fernald, John. 2014. "A quarterly, utilization-adjusted series on total factor productivity." *Federal Reserve Bank of San Francisco*.

Garcia-Macia, Daniel, Chang-Tai Hsieh, and Peter J. Klenow. 2019. "How Destructive Is Innovation?" *Econometrica* 87 (5): 1507–1541.

Griliches, Zvi. 2001. *R&D, Education, and Productivity: A Retrospective*. Cambridge, Mass.: Harvard University Press.

Grullon, Gustavo, Yelena Larkin, and Roni Michaely. 2017. "Are US Industries Becoming More Concentrated?" *Working Paper*, pp. 1–68.

Gutierrez, German, and Thomas Philippon. 2016. "Investment-Less Growth: an Empirical Investigation." *NBER Working Paper*.

———. 2017. "Declining Competition and Investment in the U.S." *NBER Working Paper*.

Gutiérrez, Germán, and Thomas Philippon. 2019. "Fading Stars." no. January.

Hall, Robert E. 2015. "Quantifying the Lasting Harm to the US Economy from the Financial Crisis." *NBER Macroeconomics Annual* 29 (1): 71–128.

Hoberg, Gerard, and Gordon Phillips. 2010. "Product market synergies and competition in mergers and acquisitions: A text-based analysis." *Review of Financial Studies* 23 (10): 3773–3811.

Hobjin, Bart, and Fernanda Nechio. 2019. "Sticker shocks: using VAT changes to estimate upper-level elasticities of substitution." *Journal of the European Economic Association* 17 (3): 799–833.

Hopenhayn, Hugo, Julian Neira, and Rish Singhania. 2018. "From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share." *National Bureau of Economic Research Working Paper Series*, pp. 1–55.

Hsieh, Chang-Tai, and Esteban Rossi-Hansberg. 2019. "The Industrial Revolution in Services." \, pp. 139–158.

Karahan, Fatih, Benjamin W. Pugsley, and Aysegul Sahin. 2019. "Demographic Origins of the Startup Deficit." *SSRN Electronic Journal*, no. 888.

Kehrig, Matthias, and Nicolas Vincent. 2020. "The Micro-Level Anatomy of the Decline of the Labor Share." *NBER Working Paper*, no. 25275:1689–1699.

Kelly, Bryan T., Dimitris Papanikolaou, Amit Seru, and Matt Taddy. 2018. "Measuring Technological Innovation over the Long Run." *SSRN Electronic Journal*.

Klette, Tor Jakob, and Samuel Kortum. 2004. "Innovating Firms and Aggregate Innovation." *Journal of Political Economy* 112 (5): 986–1018.

Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman. 2017. "Technological innovation, resource allocation, and growth." *Quarterly Journal of Economics*, pp. 665–712.

Lentz, Rasmus, and Dale Mortensen. 2008. "An Empircal Model of Growth Through Product Innovation." *Econometrica* 76 (6): 1317–1373.

Liu, Ernest, Atif R. Mian, and Amir Sufi. 2019. "Low Interest Rates, Market Power, and Productivity Growth."

Luttmer, Erzo G.J. 2007. "Selection, Growth, and the Size Distribution of Firms." *Quarterly Journal of Economics*, no. August:1103–1145.

OECD. 2018. "Market Concentration." *OECD Issue Paper* 46:1527–1527.

Olley, G. Steven, and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica* 64 (6): 1263–1297.

Pellegrino, Bruno. 2020. "Product Differentiation , Oligopoly , and Resource Allocation."

Peters, Michael, and Conor Walsh. 2019. "Declining Dynamism, Increasing Markups and Missing Growth: The Role of the Labor Force." *SSRN Electronic Journal*, pp. 1–49.

Peters, Ryan H., and Lucian A. Taylor. 2017. "Intangible capital and the investment-q relation." *Journal of Financial Economics* 123 (2): 251–272.

Pugsley, Benjamin W., Petr Sedlacek, and Vincent Sterk. 2020. "The Nature of Firm Growth."

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Nicholas Trachter. 2020. "Diverging Trends in National and Local Concentration." *NBER Macroeconomics Annual* 35:1–39.

Schmidt, Klaus M. 1997. "Managerial Incentives and Product Market Competition." *The Review of Economic Studies* 64 (2): 191–213.

Syverson, Chad. 2016. "Challenges to Mismeasurement Explanations for the U.S. Productivity Slowdown." *Working Paper*.

van Reenen, John. 2018. "Increasing Differences Between Firms: Market Power and the Macro-Economy." *CEP Discussion Paper*, no. 1576.

Zachariadis, Marios. 2003. "R&D, innovation, and technological progress: a test of the Schumpeterian framework without scale effects." *Canadian Journal of Economics/Revue Canadienne d'Economique* 36 (3): 566–586.

A TFP Estimation

I use Compustat data on U.S. public firms from 1962-2017 to estimate revenue-based total factor productivity (TFPR) and markups at the firm level. I focus on the non-farm, non-financial sector and exclude utilities and firms without an industry classification. I keep only those companies that are incorporated in the U.S. The sample includes around 3,000 firms per year, though this number varies over time.

I construct each firm's capital stock $K_{i,t}$ by initializing the capital stock as PPEGT (total gross property, plant, and equipment) for the first year the firm appears. I then construct $K_{i,t+1}$ recursively:

$$K_{i,t+1} = K_{i,t} + I_{i,t+1} - \delta K_{i,t}$$

where PPENT (total net property, plant, and equipment) is used to capture the last two terms (net investment). I deflate the nominal capital stock using the Bureau of Economic Analysis (BEA) deflator for non-residential fixed investment.

In de Loecker and Warzynski (2012) the authors show that under a variety of pricing models firm i's markup at time t, μ_{it}, can be computed as a function of the output elasticity θ_{it}^V of any variable input and the variable input's cost share of revenue[21] :

$$\mu_{it} = \theta_{it}^V \frac{P_{it} Q_{it}}{P_t^V V_{it}} \tag{8}$$

where P_{it} is the output price of firm i's good at time t, Q_{it} its output, P_t^V the price of the variable input and V_{it} the amount of the input used.

Following de Loecker and Eeckhout (2017) I use COGS (cost of goods sold) deflated by the BEA's GDP deflator series as the real variable input cost $M_{i,t}$ of the firm. While the number of employees is well measured in Compustat and would be sufficient to estimate productivity, the wage bill is usually not available and would be needed to compute the labor cost share needed to compute the markup simultaneously with productivity.

For the results presented in this paper, I assume a Cobb-Douglas production function[22] for firm i in 2-digit SIC sector s in year t so that factor shares may vary across sectors but not over time:

---

[21]This approach requires several assumptions. First, the production technology must be continuous and twice differentiable in its arguments. Second, firms must minimize costs. Third, prices are set period by period. Fourth, the variable input has no adjustment costs. No particular form of competition among firms must be assumed.

[22]Alternative estimation of a translog production function yielded similar estimates.

$$Y_{i,s,t} = A_{i,s,t} M_{i,s,t}^{\beta_{M,s}} K_{i,s,t}^{\beta_{K,s}}$$

I use the variable SALE to measure firm output $Y_{i,s,t}$. I deflate SALE using the GDP deflator series to obtain real revenue at the firm level. I include firm and time fixed effects and obtain revenue-based TFP in logs (lower case variables denote variables in logs) by computing the residual (including fixed effects) of the following regressions for each 2-digit sector:

$$y_{i,t} = \alpha + \eta_t + \delta_i + \beta_{M,s} m_{i,t} + \beta_{K,s} k_{i,t-1} + \varepsilon_{i,t}$$

In the above equation, $\beta_{M,s}$ captures the sector specific variable output elasticity, so I use equation 8 to obtain the markup from the estimated $\hat{\beta}_{M,s}$ and the inverse cost share $\frac{SALE}{COGS}$.

B   Alternate Model with Bertrand Pricing

It is also possible to solve the full dynamic model under the assumption that both firms exercise market power over their variety i of sector j good and set prices a la Bertrand rather than requiring that the follower set price equal to marginal cost. The analogy from the model to the data becomes less obvious under this assumption, since the laggard firm can no longer be thought of representing a competitive fringe composed of many firms producing generic products that are perfectly substitutable with other generic products but imperfectly substitutable with the brand produced by the leader. In the Bertrand setup the leader always has at least 50% market share, unlike in the data. This assumption also gives empirically counterfactual predictions that the profit shares of total industry profits of the market leader and the other firm in the industry are relatively similar, contradicting the pattern shown in figure 13.

Nonetheless, many of the main results carry through under this alternate assumption. Before describing these alternate results, I return to the pricing problem of the firms assuming the follower can now choose its optimal markup. Using the same derivation as in section 3.3.1 it can be shown that both firms follow the pricing policy the leader follows in the baseline model:

$$p_i = \frac{\epsilon - (\epsilon - \frac{1}{\beta})s_i}{\epsilon - (\epsilon - \frac{1}{\beta})s_i - 1}\eta$$

where

$$s_i = q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{1-\epsilon}$$

I look for a Markov perfect equilibrium with balanced growth where each firm's price is the best response to its competitor's price at time t. The algorithm for finding the steady state remains the same, plugging in the pricing functions of the firms, plotted in figure 20.

Table 7 gives the results of the same experiment as in section 4.3 under the alternate pricing strategies with the same parameters as in table 1 and figure 21 shows the policy functions and stationary distributions. Note that innovation intensity is greatest for the most laggard firm under this pricing assumption. As before, changing ϕ has a level effect on total innovation effort but also changes the location of R&D from laggard firms to leading firms.

The level of the growth rates and the change in the growth rate from one steady state to the other under Bertrand pricing are very similar to the baseline model with
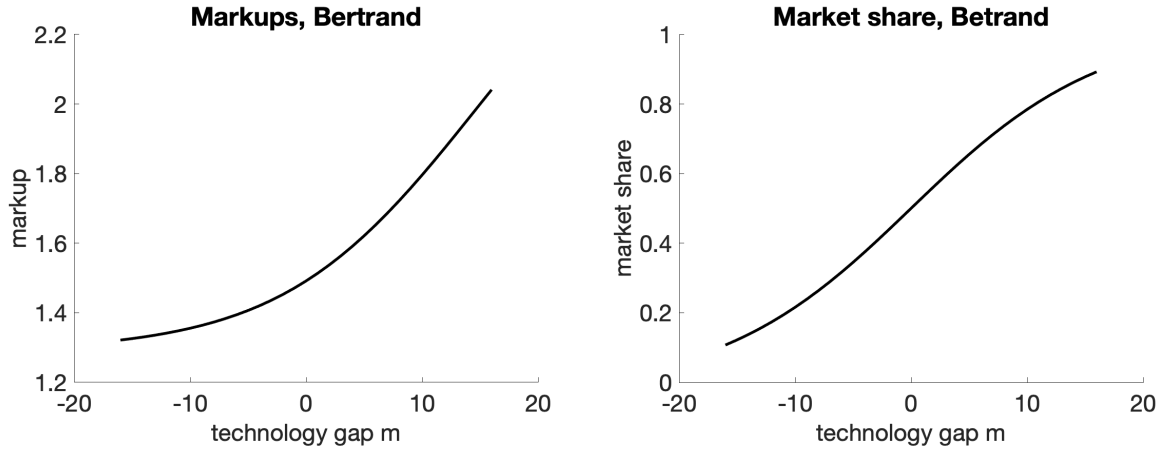
Figure 20: Markups and resulting market shares as a function of the technology gap (ratio of firm qualities), Bertrand pricing.

marginal cost pricing of the follower. The change in concentration is much smaller since the change in technology gaps is not as dramatic as in the main case (figure 21), though technology gaps do increase modestly. Markups are basically unchanged. This is partly due to the fact that as the leader's markup rises when the gap gets larger, the follower's markup now falls whereas before it would have been unchanged (at zero). The profit share rises, though more modestly than before. As for the growth decomposition, the effects of the firms' innovation responses is larger, and the first order effect of lowering the probability of radical innovations is roughly the same as in the baseline.
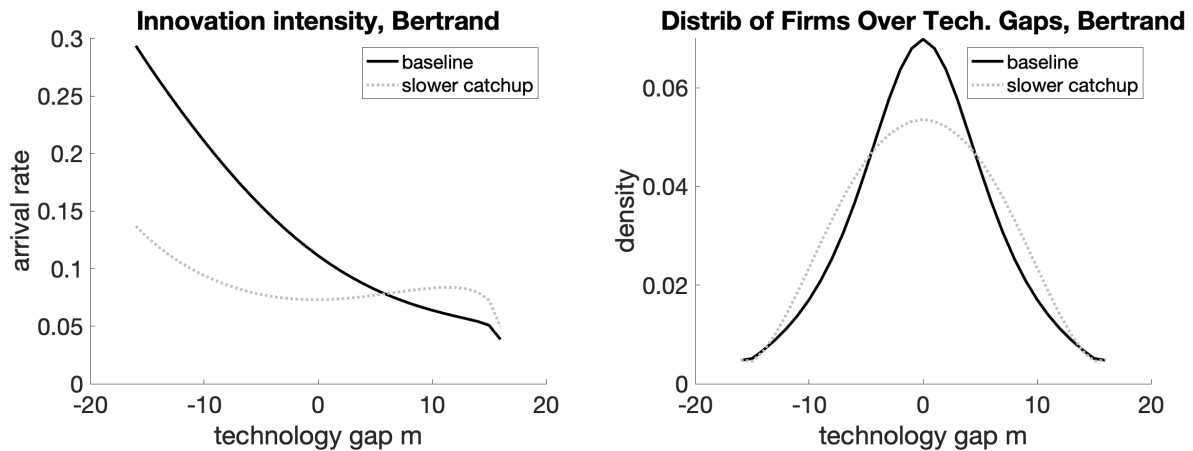

Figure 21: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), Bertrand pricing.

51

Moment	Data			Model		
	1990s	2000s	Chg. (pp)	$\phi = 1.01$	$\phi = 1.67$	Chg. (pp)
TFP growth, %	1.7	0.5	-1.2	1.81	0.54	-1.27
Leader market share, avg, %	42.99	48.04	5.05	64.33	66.08	1.75
Pat stock growth/patent, avg, %	23.44	11.71	-11.73	22.51	11.02	-11.49
Markup, avg, %	26.16	29.88	3.72	52.3	52.9	0.6
R&D share of GDP, %	1.8	1.89	0.09	2.24	0.92	-1.32
R&D intensity, avg, %	5.42	6.98	1.56	8.74	2.84	-5.9
Profit share of GDP, %	5.24	6.61	1.37	14.47	14.62	0.15
Leader's share of R&D, avg, %	41.32	47.04	5.72	29.46	53.21	23.75
Leadership turnover, %	13.73	9.3	-4.43	3.8	1.08	-2.72

Table 7: Model and data comparison, **Bertrand**

Decomposition	% of slowdown explained
Role of effort (ϕ fixed, x changes)	62.63
First order effect (x fixed, ϕ changes)	73.5

Table 8: Growth decompositions, Bertrand

C Value Function Boundary Equations

For the firm that's furthest behind (at gap $-\bar{m}$ with quality q_t):

$$r_t V_{-\bar{m},t}(q_t) - \dot{V}_{-\bar{m},t}(q_t) = \max_{x_{-\bar{m},t}} \{0 - \alpha \frac{(x_{-\bar{m},t})^\gamma}{\gamma} q_t^{\frac{1}{\beta}-1}$$

$$+ x_{-\bar{m},t} \sum_{n_t=-\bar{m}+1}^{\bar{m}} \mathbb{F}_m(n_t)[V_{nt}(\lambda^{n_t+\bar{m}} q_t) - V_{-\bar{m},t}(q_t)]$$

$$+ x_{\bar{m},t}(V_{-\bar{m},t}(\lambda q_t) - V_{-\bar{m},t}(q_t))\}$$

The difference between this and equation 2 is in the last line, where if the firm's competitor innovates, there is a spillover that causes the firm at gap $-\bar{m}$ to improve its quality by λ.

For a firm at gap $\bar{m}$ the value function is:

$$r_t V_{\bar{m},t}(q_t) - \dot{V}_{\bar{m},t}(q_t) = \max_{x_{\bar{m},t}} \{\pi(\bar{m}, q_t) - \alpha \frac{(x_{\bar{m},t})^\gamma}{\gamma} q_t^{\frac{1}{\beta}-1}$$

$$+ x_{\bar{m},t}(V_{\bar{m},t}(\lambda q_t) - V_{\bar{m},t}(q_t))$$

$$+ x_{-\bar{m},t} \sum_{n_t=-\bar{m}+1}^{\bar{m}} \mathbb{F}_{-\bar{m}}(n_t)[V_{nt}(q_t) - V_{\bar{m},t}(q_t)]\}$$

Where:

$$\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ q_i^{\frac{1}{\beta}-1}(p(m) - \eta)p(m)^{-\epsilon}(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1}\eta^{1-\epsilon})^{\frac{\epsilon-\frac{1}{\beta}}{1-\epsilon}} & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}$$

D   Step Size Distributions for Results

Here I include a plot of the fixed distributions $F(n)$ discussed in section 3.3.2 under the two values of the radical innovation parameter ϕ used for the steady state comparisons in section 4.3. Recall that for a particular step size (size of quality improvement) $F(n) = c_0(n + \bar{m})^{-\phi}$. For the most backward firm at position $-\bar{m}$, the probability of getting a larger than one step improvement is about 25 percentage points higher in the baseline case than in the higher ϕ case.

E   Role of Effort
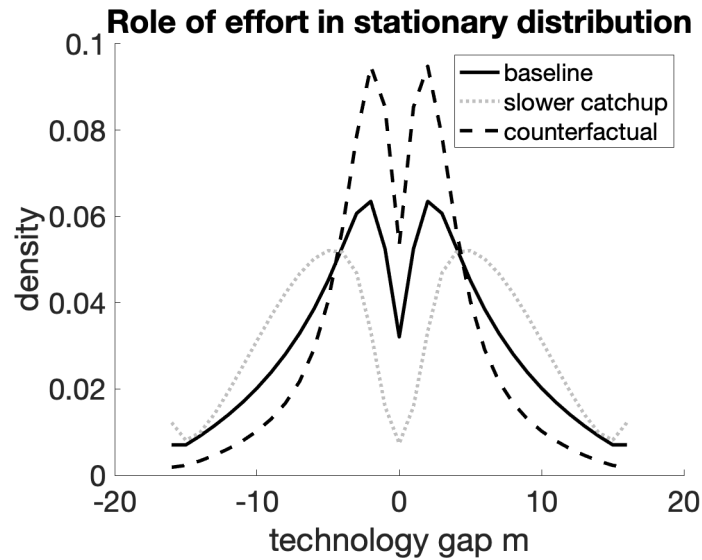
Role of effort in stationary distribution
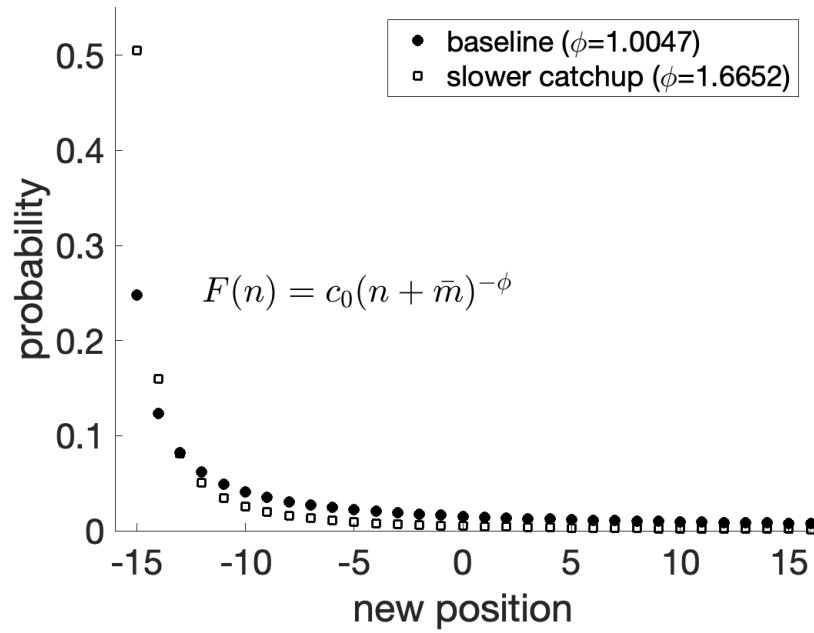
Figure 22: Stationary distribution of firms over technology gaps in two equilibria (baseline, slower catchup), plus distribution assuming firm innovation effort is fixed at the baseline equilibrium while ϕ changes to its value in the slow catchup equilibrium (counterfactual).

F Data Sources and Moment Computations

The table below lists the source and, where necessary, computation method for each target moment from the data.

Moment	Source	Computation/Series Name
TFP growth	Fernald (2014)	Utilization-adjusted annual total factor productivity growth
Leader market share	Compustat	Average of sales share of largest firm in each 4-digit SIC industry (weighted by industry size)
Patent stock growth per patent (psgpp)	Kogan et al. (2017)	$rTsm_{it} = \frac{Tsm_{it}}{GDPdefl_t}$ is the real value of firm i's patents issued in year t. $psgpp_{it} = \frac{\frac{rTsm_{it}}{fNpats_{it}}}{\sum_{s=1}^{t-1} rTsm_{is}}$; s =first year in Compustat
Markups	Compustat	Median est. markup among all firms; estimation details in Appendix A.
R&D share of GDP	OECD Main Science and Technology Indicators	Business Expense R&D (private)/GDP
R&D intensity	Compustat	XRD/SALE, mean across all firms with real sales over 1 million in 2012 USD.
Profit share of GDP	Bureau of Economic Analysis/FRED	Profits after tax with inventory valuation and capital consumption adjustments/Gross domestic income
Leader's share of R&D	Compustat	Average sales leader share of total R&D in 4-digit sector (weighted by industry size)
Leadership turnover	Compustat	Share of 4-digit SIC industries with new sales leader per year

Table 9: Data sources and computation method for each moment used in the text.