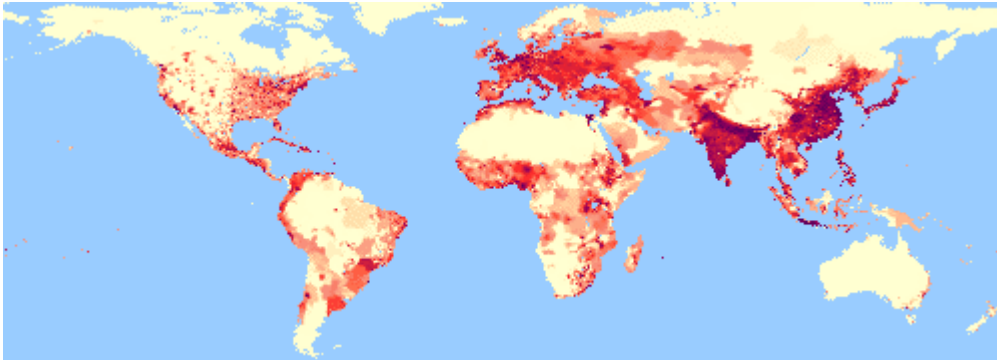CHAPTER 5

The Size-Density Hypothesis

> We shall not cease from exploration
> And the end of all our exploring
> Will be to arrive where we started
> And know the place for the first time.
> — T. S. Eliot, *Four Quartets - Little Gidding*

The next phase of the research was less pre-planned than the work I had done so far. After arriving at Western Washington University, I was browsing through a world atlas[1] when I began to wonder about the administrative divisions of some of the nations shown there. Would the model developed to account for size variation in U.S. counties have anything to say, for example, about variation in county size in the British Isles? What about the *départements* of France? The *provincias* of Spain?

I couldn't approach such a problem directly because I didn't have the kinds of historical maps used in the case of the United States, Figs. 4-1 and 4-2. Still, some aspects of a map like Fig. 5-1 were suggestive. Ireland was originally divided into counties in 1167 when Henry II established English courts at Dublin. There is a rough size- distance relation, county size increasing with distance from Dublin. Northern Ireland, which only separated from the remainder in 1922, became more heavily populated with modern industrialization, but the presence of a size-distance relation suggests that Ireland may once have fit the historical U.S. pattern.

FIG. 5-1. BRITISH ISLES
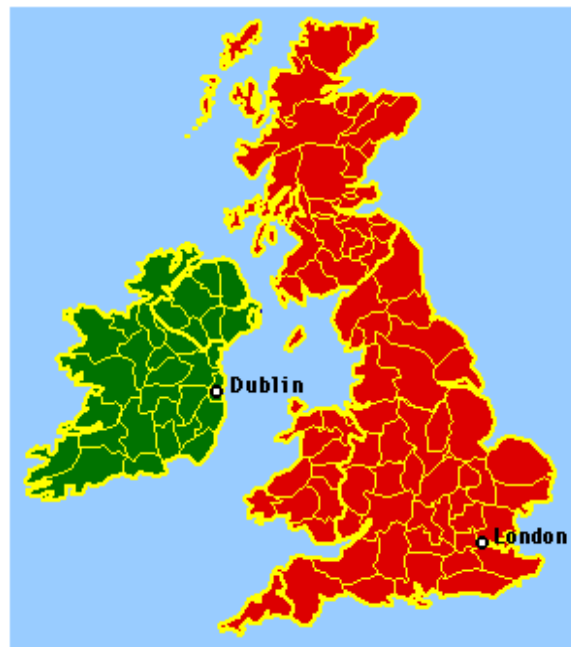
A similar observation can be made for England and Wales. The English counties emerged gradually from ancient kingdoms. Those of Wales were created shortly after 1284 when Edward I placed Wales under the English crown. Again, it is apparent that there is a size-distance relation for the region as a whole, with London as its focus, larger counties to the north and southwest. In Scotland, where "sheriffdoms" were established in the 12th century, the determining factor is clearly geography; the lightly settled highlands contain much larger counties than the more intensely settled lowland region.

The difficulty is already apparent. Testing the model on other nations requires information which may be lost in pre-history. Even in the cases just mentioned, there is not sufficient demographic and cartographic information to answer whether county division in the British Isles was in any sense similar to county division in the United States. I had to develop an alternative approach — something other than the historical-geographic — if I wanted to study administrative divisions of other nations.

I asked myself what data available *today* would enable me to test the model developed in Chapter 3? If a long-term process of subdivision had produced administrative boundaries, what relationships would I expect to observe at present? The size-distance relation would be one such finding, but it need

not show up elsewhere. It had, after all, disappeared in areas of the United States fully settled prior to the introduction of the automobile.

If one or more seats of power had once been the focus of size-distance relations, and these had all been brought together in a larger territorial unit, what would one expect to see at present? In general, it seemed to me, regions which had already been intensely settled should continue to add to their settlement intensity. Regions more lightly settled in the past should still be relatively lightly settled. This was what I later came to call the "size-**density** hypothesis" — *smaller units should show up in regions of higher population density, with larger units in regions of lower density.*

The relation between the new hypothesis and the old is shown in Fig. 5- 2. Fig. 5-2-A is the same as Fig. 3-8-I, representing expansion of the settlement area from the original settlement area (lower left hand corner). There appears to be a size-distance relation since the partially settled units are not yet fully subdivided. Assuming that older areas continue to grow, their densities would increase over time, whether or not subdivision had gone to completion. If this were the case, the result would be as shown in Fig. 5-2-B, with yellow-orange shades indicating higher density. Fig 5-2-C is what we would expect if we computed densities directly from data obtained from modern territories (density = population/area).

**FIG. 5-2. SIZE-DIVISION
AND SIZE-DENSITY**
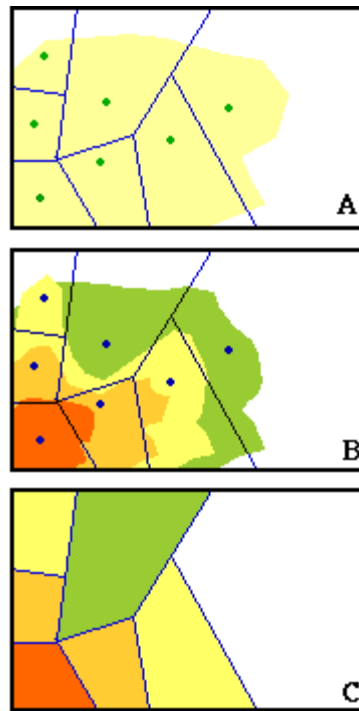
The situation shown in 5-2-A does not require the situation
shown in 5- 2-C . But the 5-2-C would be consistent with the
5-2-A, assuming continued growth of the original settlement
area (the lower left of 5-2-A). A size-density relation (assuming
there is any variation in size to begin with) is consistent with
the notion of an earlier process of subdivision in response to
changes in population distribution.

The data needed to test the size-density hypothesis were
available in the *Britannica World Atlas,* the maps of which had
originally led me to wonder about international application of
the division model. There is a section of the *Atlas,*
"Geographical Summaries", which gives a variety of statistical
information for nations from Aden to Zambia. Part of a typical
entry is illustrated in Fig. 5-3, the territorial divisions of Algeria

FIG. 5-3. ATLAS ENTRY FOR ALGERIA [2]

Algeria Democratic and Popular Republic of Algeria / Africa

Form and States of Government: Republic. *Head of Government and Chief of State:* President.
International Associations: UN, Arab League, Organization of African Unity.
Predominant Languages: Arabic, French. *National Anthem:* "Kassamen."

**Political subdivision, area, population, capital city**

POLITICAL SUBDIVISION	AREA square kilometers	POPULATION 1954 census	1960 census	Density sq.km.	Urban	% Urban	CAPITAL CITY	Population 1960
Total	2,381,743	9,529,726	10,784,309	4.5	2,726,964	25.7	Algiers	722,066
Départements								
Alger	3,393	1,114,362	1,560,000	459.8	868,017	55.6	Algiers	722,066
Annaba	25,367	735,966	761,233	30.0	221,935	29.2	Annaba	135,150
Batna	38,494	478,165	567,455	14.7	98,196	17.3	Batna	18,114
Constantin	19,899	1,218,410	1,410,522	70.9	305,935	21.7	Constantine	159,071
El Asnan	12,257	631,109	693,656	56.6	867,110	12.5	El Asnan	38,607
Médéa	50,331	630,573	757,181	15.0	35,007	4.6	Médéa	13,348
Mostaganem	11,432	613,748	696,015	60.9	142,514	20.4	Mostaganem	64,786
Oran	16,438	872,314	1,020,339	62.1	537,532	52.7	Outahran	350,087
Saïda	60,114	191,963	193,908	3.2	21,396	11.0	Saïda	21,396
Sétif	17,405	1,005,789	1,081,747	62.2	186,692	17.2	Sétif	82,340
Tagdempt	25,997	267,409	320,884	12.3	50,391	15.7	Tagdempt	36,322
Tizi-Ouzou	5,806	803,693	751,178	29.4	54,263	7.2	Tizi-Oouzou	25,367
Tlemçen	8,100	375,531	382,622	47.2	119,376	31.2	Tlemçen	70,930
Saharan Départements								
LaSaoura	789,660	156,664	166,124	0.2	—	—	Colomb-Béchar	16,408
Oasis	1,297,050	333,830	421,445	0.3	—	—	Ouargla	11,279

This shows the size and density for each territorial unit. By inspection, the first *département,* Alger, has the highest density and the smallest size of any *département.* Among the regular *départements,* Saïda has the lowest density and the largest size. Including the Saharan *départements,* we find still larger sizes together with still lower densities.

> The charts that follow were generated with StatView, though there are many equivalent applications now. For a look at how I did much of the work originally, you might want to look at Data Processing in the '70s and Later

If we attempt to graph the relation between size and density overall, however, as in Fig. 5-4, we encounter a difficulty. The largest of the units, Oasis, is so large that nearly all the others are pushed down along the horizontal axis. It is difficult to observe variation in size, relative to that of the largest unit. By the same token the highest density, that for Alger, is so high that it pushes most of the other dots together near the vertical axis.

FIG. 5-4. SIZE AND DENSITY IN ALGERIA
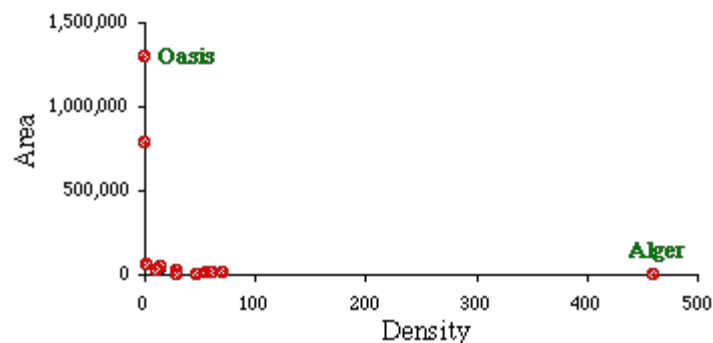
What produces such extremes? The theory says that increasing density should be associated with further subdivision. Yet there is no doubt a **lower limit** to size beyond which subdivision ceases. If a large region of a country remains unsettled while all the rest is more-or-less settled, we obtain extreme values for size. A related argument can be developed for density. Most regions would show whatever density is "normal" for their country. If one area becomes a focus of migration, attracting more and more population to itself, it would "out-distance" the normal units, resulting in extreme density. This appears to have been the case with the Algerian *département* containing Algiers, the national capital.

One way of reducing the effects of these extreme values is to toss them out of the analysis. While this might make some sense in the case of Algeria's Saharan *départements,*[3] it is hard to justify throwing out Alger solely because it is highly dense. That, after all, is what the theory is about: ***variation in size*** being related to ***variation in density***. We can retain extreme cases without compacting the rest of the distribution by logarithmic transformation [?], as in Fig. 5-5.

FIG. 5-5. LOGARITHMIC TRANSFORMATION OF FIG. 5-4
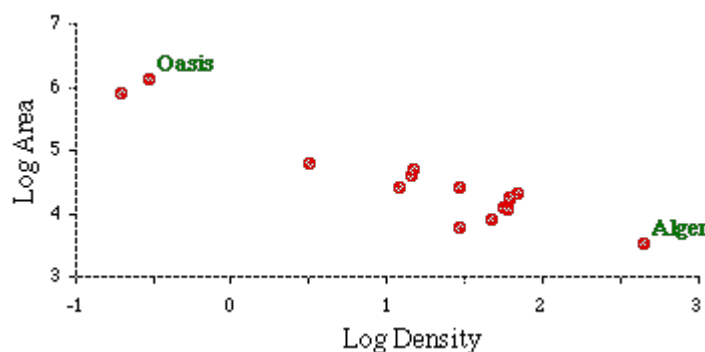
Logarithmic transformation produced an observation which I

had not originally anticipated[4]: the scatter of data points now formed an approximately straight line. My argument thus far had been only that higher densities ought to be associated with smaller territorial size. The form of the relation had not been specified. It now appeared that the "negative-ness" of the relation could be specified quantitatively, as the **slope** of a line fitting the scatter of data points.

FIG. 5-6. SIZE-DENSITY REGRESSION LINE FOR ALGERIA
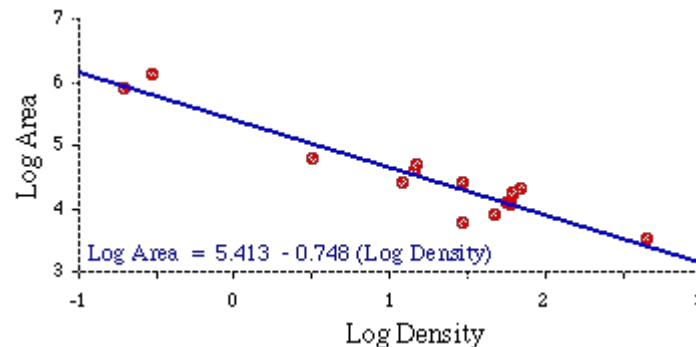
Such a line is shown in Fig. 5-6; its formula was obtained through ordinary least-squares regression analysis[?]. Incidentally, dropping out the two desert *départements* results in only slightly different values for the intercept and slope (a = 5.217, b = -0.62).

View all the Atlas data, c. 1960

There were 98 nations in the *Atlas* for which suitable data were given. Several were too small to show territorial divisions; in several others data were seriously incomplete. At the time I was conducting this analysis I wanted to see if nations showed the expected **negative slope** relating the logarithms of size and density. I needed a decision criterion to decide whether the observed slopes *departed significantly from zero in the negative direction*. This implied a one-way test of significance[?] against the null hypothesis: does the slope depart in a negative direction?

The results of these tests are shown in Table 5-1. This shows the number of divisions in each country (N), the computed slope relating log-size to log-density (b), and the probablity of obtaining a slope that negative, assuming the null hypothesis

($\beta = 0$). The tests of significance are summarized in Fig. 5-7. In all but 4 of the 98 nations the slopes were negative, as expected. Among the 94 negative slopes, 78 show departures from the null hypothesis which are statistically significant ($p<.05$). 50 slopes, over half the total number, were significant at the .0005 level. None of the four positive slopes (Haiti, Italy, Malawi, and Yugoslavia) was significant.

Table 5-1. Size-Density Slopes for 98 Nations

Nation	N	b	p{β=0}	Nation	N	b	p{β=0}
Albania	27	-0.55	<.0005	Japan	46	-0.50	<.0005
Algeria	15	-0.75	<.0005	Jordan	8	-0.28	n.s.
Angola	13	-0.71	<.025	Kenya	13	-1.09	<.0005
Argentina	23	-0.58	<.0005	Korea N	11	-0.81	<.005
Australia	8	-0.74	<.025	Korea S	11	-0.96	<.005
Austria	9	-0.73	<.005	Lebanon	5	-1.05	<.0005
Belgium	9	-0.13	n.s.	Luxembourg	4	-0.84	<.05
Bolivia	9	-0.27	n.s.	Malagasy Republic	5	-0.76	<.005
Brazil	28	-0.64	<.0005	Malawi	3	0.16	n.s.
British Honduras	6	-0.36	n.s.	Malaysia	13	-0.99	<.0005
Bulgaria	28	-0.52	<.0005	Mexico	32	-0.59	<.0005
Burma	7	-0.47	<.05	Mongolia	18	-0.89	<.0005
Cambodia	21	-0.89	<.0005	Morocco	18	-0.71	<.0005
Canada	15	-0.30	<.025	Netherlands	11	-0.09	n.s.
Central African Rep	13	-0.36	<.005	New Zealand	13	-0.19	n.s.
Ceylon	9	-0.33	<.0005	Nicaragua	16	-0.77	<.0005
Chad	11	-0.51	<.0005	Nigeria	5	-1.55	<.005
Chile	26	-0.45	<.0005	Norway	20	-0.89	<.0005
China	27	-0.64	<.0005	Pakistan	18	-0.32	<.025
Colombia	29	-0.44	<.0005	Panama	9	-0.32	n.s.
Congo	13	-1.19	<.0005	Paraguay	17	-0.59	<.0005
Costa Rica	7	-0.48	<.05	Peru	24	-0.69	<.0005
Cuba	6	-0.47	n.s.	Philippines	57	-0.64	<.0005
Czechoslovakia	11	-1.00	<.0005	Poland	22	-1.18	<.0005
Denmark	24	-0.79	<.0005	Portugal	29	-0.86	<.0005
Dominican Republic	25	-0.21	n.s.	Rhodesia	6	-0.48	<.05
Ecuador	16	-0.10	n.s.	Romania	18	-1.06	<.0005
El Salvador	14	-0.39	<.025	Saudi Arabia	4	-1.13	<.005
Finland	12	-0.58	<.05	Sierra Leone	4	-1.58	<.005
Formosa	24	-0.95	<.0005	South Africa	4	-0.93	n.s.
France	90	-0.29	<.0005	Spain	50	-0.42	<.0005
Gabon	9	-0.55	<.05	Sudan	9	-1.05	<.0005
Germany E	15	-0.78	<.0005	Sweden	25	-0.78	<.0005
Germany W	10	-1.53	<.0005	Switzerland	25	-0.61	<.005
Ghana	9	-0.95	<.0005	Syria	12	-0.76	<.0005
Greece	9	-0.17	n.s.	Tanzania	17	-0.80	<.005
Guatemala	22	-0.60	<.0005	Thailand	4	-0.39	n.s.
Haiti	5	0.76	n.s.	Togo	4	-0.72	n.s.
Honduras	18	-0.58	<.005	Tunisia	13	-0.81	<.0005
Hungary	24	-1.07	<.0005	Turkey	67	-0.38	<.0005
Iceland	5	-0.33	n.s.	Uganda	4	-0.68	<.005
India	20	-0.13	n.s.	USSR	43	-0.71	<.0005
Indonesia	27	-0.60	<.0005	United Arab Rep	38	-0.69	<.0005
Iran	19	-0.47	<.025	United Kingdom	101	-0.09	n.s.

Iraq	14	-0.82	<.0005	United States	48	-0.53	<.0005
Ireland	26	-0.36	<.025	Uruguay	19	-0.48	<.0005
Israel	6	-0.78	<.0005	Venezuela	24	-0.56	<.0005
Italy	19	0.32	n.s.	Yugoslavia	9	0.74	n.s.
Ivory Coast	4	-0.81	<.01	Zambia	8	-0.69	<.005

FIG. 5-7. TESTS OF THE NULL HYPOTHESIS
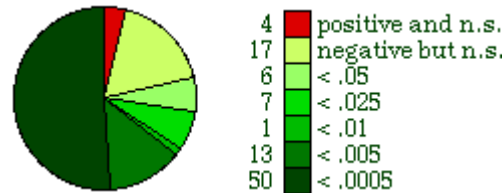
Having established that there was a negative relation between size and density, my interest shifted to "variation in negativity" among the nations. Fig. 5-8 shows that the negative slopes (ignoring the four positive ones) range from Sierra Leone's -1.58 to the practically non- negative -0.09 shared by Netherlands and the United Kingdom. Was this variation of any interest? What did it mean to be very negative, or moderately negative, or hardly negative at all?

FIG. 5-8. DISTRIBUTION OF NEGATIVE SIZE-DENSITY SLOPES

I wanted to describe some central value in this range, to distinguish nations which were more negative from those which were less negative. The data for the earlier analysis had been punched on IBM cards, one for each territorial division, the set grouped behind a lead card identifying the nation. I removed the national identifier cards, in effect creating one world with 1,764 territorial divisions. (see, One Worlders? It's just that easy!) The analysis of this set yielded the "world regression line":

log Area = 5.0656 - 0.65 log Density

which, with t = -36.5 (completely off the chart for 1762 degrees of freedom), clearly rejected the null hypothesis for the world as a whole.

I next intended testing the degree to which individual nations fit this world pattern, but before doing that I modified the

world slope somewhat. Though I had computed a value of b = -0.6514, I found myself referring to it as the "-2/3 slope". This was part convenience, part simplicity, part esthetics. I think I also sensed (maybe I'm a closet Pythagorean) that simple ratios (1/2, 2/3, 3/4) tend to be theoretically interesting, though that certainly wasn't my conscious focus at the time. At any rate, I checked the degree to which the world regression line conformed to the simpler -2/3 value: t = 0.855 which, with N = 1764, is practically zero. So everything from here on is with reference to -2/3.[5]

Table 5-2 shows the result of two-tailed t-tests of the hypothesis β = -2/3. Two-tailed because nations could be more or less negative than -2/3. Nations are arranged in order, from most positive to most negative slope. As can be seen, many more nations are in agreement with the -2/3 size-density hypothesis than were in agreement with the null hypothesis in table 5-1.

Table 5-2. Size-Density Hypothesis: β = -2/3

Nation	N	b	p{β=-2/3}	Nation	N	b	p{β=-2/3}
Haiti	5	0.76	n.s.	Chile	25	-0.59	n.s.
Yugoslavia	8	0.45	n.s.	Korea N	11	-0.59	n.s.
Italy	19	0.32	<.001	Paraguay	17	-0.59	n.s.
Malawi	3	0.16	n.s.	Guatemala	22	-0.60	n.s.
Netherlands	11	-0.09	<.01	Indonesia	27	-0.60	n.s.
United Kingdom	101	-0.09	<.001	Switzerland	25	-0.61	n.s.
Ecuador	16	-0.10	<.01	Venezuela	22	-0.62	n.s.
India	20	-0.13	n.s.	China	27	-0.64	n.s.
Belgium	9	-0.13	<.001	Brazil	28	-0.64	n.s.
Greece	9	-0.17	n.s.	Uganda	4	-0.68	n.s.
New Zealand	13	-0.19	<.05	Zambia	8	-0.69	n.s.
Dominican Republic	25	-0.21	<.02	USSR	43	-0.71	n.s.
France	89	-0.26	<.001	Angola	13	-0.71	n.s.
Bolivia	9	-0.27	n.s.	Morocco	18	-0.71	n.s.
Jordan	8	-0.28	<.05	Togo	4	-0.72	n.s.
Cambodia	17	-0.28	<.001	Austria	9	-0.73	n.s.
Panama	9	-0.32	n.s.	Australia	8	-0.74	n.s.
Ceylon	9	-0.33	<.001	Mexico	30	-0.75	n.s.
Iceland	5	-0.33	n.s.	Algeria	15	-0.75	n.s.
Peru	23	-0.34	<.05	Syria	12	-0.76	n.s.
Pakistan	16	-0.34	<.001	Malagasy Republic	5	-0.76	n.s.
Ireland	26	-0.36	n.s.	Nicaragua	16	-0.77	n.s.
British Honduras	6	-0.36	n.s.	Israel	6	-0.78	n.s.
Turkey	67	-0.38	<.01	Sweden	25	-0.78	n.s.
Thailand	4	-0.39	n.s.	Germany E	15	-0.78	n.s.
El Salvador	14	-0.39	n.s.	Denmark	24	-0.79	n.s.
Central African Rep	13	-0.39	<.01	Tanzania	17	-0.80	n.s.
Spain	50	-0.42	<.01	United Arab Rep	16	-0.80	n.s.

Colombia	29	-0.44	<.02	Tunisia	13	-0.81	n.s.
Portugal	18	-0.44	<.01	Luxembourg	4	-0.84	n.s.
Canada	12	-0.45	n.s.	Norway	20	-0.89	<.001
Philippines	57	-0.46	<.01	South Africa	4	-0.93	n.s.
Cuba	6	-0.47	n.s.	Ghana	9	-0.95	<.05
Iran	19	-0.47	n.s.	Korea S	11	-0.96	n.s.
Burma	7	-0.47	n.s.	Kenya	8	-0.97	n.s.
Uruguay	19	-0.48	<.01	Malaysia	13	-0.99	n.s.
Rhodesia	6	-0.48	n.s.	Formosa	18	-1.00	n.s.
Costa Rica	7	-0.48	n.s.	Czechoslovakia	11	-1.00	<.01
Mongolia	17	-0.49	n.s.	Lebanon	5	-1.01	<.02
Japan	46	-0.50	<.05	Sudan	9	-1.05	n.s.
Chad	11	-0.51	n.s.	Romania	17	-1.06	n.s.
Ivory Coast	4	-0.51	n.s.	Hungary	24	-1.07	<.01
Bulgaria	28	-0.52	n.s.	Iraq	14	-1.10	<.02
United States	48	-0.53	n.s.	Saudi Arabia	4	-1.13	<.05
Gabon	9	-0.55	n.s.	Congo	13	-1.17	<.001
Albania	27	-0.55	n.s.	Poland	21	-1.19	<.001
Finland	12	-0.58	n.s.	Germany W	10	-1.53	<.02
Honduras	18	-0.58	n.s.	Nigeria	5	-1.55	<.05
Argentina	23	-0.58	n.s.	Sierra Leone	4	-1.58	<.02

Variation around the -2/3 value is shown in Fig. 5-9. There were 44 nations with slopes more negative than -2/3 (i.e., toward the -1.53 value for Sierra Leone), of which 31 were not significantly different. There were 54 nations with slopes less negative (more toward zero), and of these 34 were not significantly different. Thus, **65 nations show agreement with the -2/3 slope** at the .05 level of significance.

FIG. 5-9. TESTS OF THE -2/3 SIZE-DENSITY HYPOTHESIS
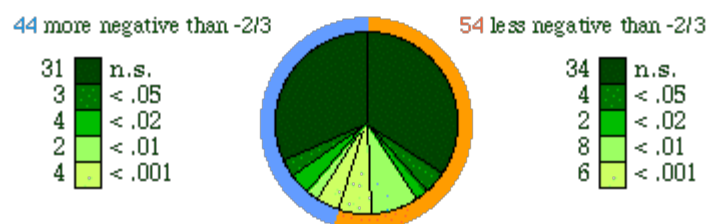
Historical Tests of Size-Density

The earlier study of county size in the United States had been conducted entirely with maps; no statistics were computed. The international study was conducted entirely from a statistical approach. In an effort to relate the two, to get a sense of the connection, I conducted a statistical analysis of the growth of Oregon counties.

FIG. 5-10. OREGON COUNTIES
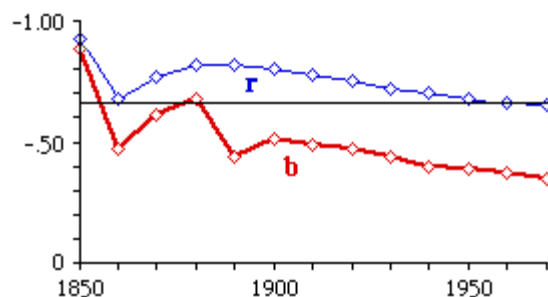
I wanted to do this for the entire United States, but there were
not area figures for U.S. counties prior to 1930. Since I had
prepared maps for Oregon earlier, by tracing these on fine
graph paper I was able to estimate historical county areas by
counting millimeter graph squares in each county. (It would
only be some time later that I discovered the possibility of
measuring areas with a planimeter, rather than counting these
tiny squares ... my eyes still ache.) The results of the analysis
are shown in Fig. 5- 10. The slope is initially near the -2/3 value
shown for the world, but it drifts toward zero generally
throughout the period, and steadily from 1900.

A parallel study of the states of the United States (shown in
Fig. 5-11) showed a tendency to decline below the -2/3 value,
but the slope has, in general, been steady since 1800, with a
slight tendency to move toward -2/3 since about 1870.

FIG. 5-11. STATES OF THE U.S.
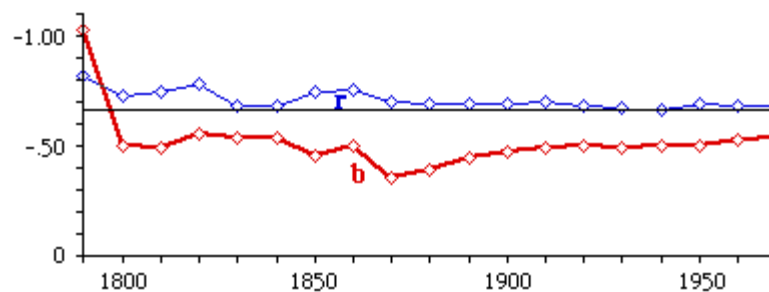
In addition to these, I was examining a map of the British Isles
like the shown in Fig. 5-1, speculating about the possibility of
size-density relations based on possible size-distance ones.

I combined the material in this chapter into a paper titled
"Historical and International Tests of the Size-Density
Hypothesis", and sent it to the *American Sociological Review* in
July of 1971. The reviewers had two criticisms. First, they said
there was too much material in it (odd criticism); they wanted

me to throw out everything but the international study.

The second criticism (which would pop up often later on) was that the size-density hypothesis was only a *tautology,* an artifact, since area occurs as both the dependent variable and in the denominator of the independent variable (density = population / area). Large areas have low density by definition .

I agreed to limit the scope of the paper, and I submitted a quote which seemed to relieve the editors of the concern over tautology. The quote was from Snedecor's seminal statistics text[6], referring to correlations between variables of the form Y and X/Y:

> Having observed some unwarranted interpretations of such correlations, Karl Pearson dubbed them `spurious', and this rather derogatory title has led people to distrust them. Of course, it is the interpretation that may be spurious. The correlations are on the same footing as any others.

This hardly constituted an argument. It was an appeal to authority. But it satisfied the reviewers and the article did get published.[7] I suggested at the end of the article, partly in response to this concern, that future work might use a different independent variable, "population potential". I take up this suggestion myself in Chapter 7.

Next Chapter

NOTES:

[1] *Britannica World Atlas*. London: Encyclopedia Britannica, Inc. 1967.

[2] *Britannica World Atlas*. 199.

[3] They are distinctly different from the rest of the nation. All the other units cluster along the coastline; these two are largely empty desert regions, stretching off to the south. Their size is the result of being empty space out to the national boundary.

[4] My original purpose in the logarithmic transformation was simply to make graphing possible, to get the dots away from the axes. The mathematical-theoretical significance of the resulting logarithmic equation came later.

[5] I ignored the intercept in this and most later studies. The intercept's value is a function of the units in which area is measured (square kilometers, square miles, acres, hectares, etc.), so the actual number is more conventional than theoretical.

[6] George Snedecor, *Statistical Methods* (4th edition) 162, Ames, Iowa: Iowa State College Press, 1946

[7] G. Edward Stephan, "International Tests of the Size-Density Hypothesis," *American Sociological Review,* 37:365-8. 1972