CHAPTER 13

Erosion of the Size-Density Relation

In Chapter 12 it appeared that slopes **more** negative than -2/3 were the result of including cities in the analysis. In this chapter we examine the reverse condition: slopes **less** negative than -2/3, slopes tending toward zero (which is to say tending toward no slope or relation at all).

At the conclusion of the article in which we presented the results of the last chapter, we added the findings shown in the next table, comparing our latest slopes with the earliest ones for which we could obtain data.[1] We were attempting to see if other countries might have experienced the kind of "erosion" of a -2/3 size-density slope which we saw earlier in England (Chapter 7).

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Nation/Yr n       b       p

Belgium
 1816      9   -0.27    .035
 1961      9   -0.13   <.001
France
 1872     90   -0.14   <.001
 1962     90   -0.29   <.001
Italy
 1861     13   -0.14    .087
 1961     19    0.32   <.001
Netherlands
 1849     11   -0.12    .067
 1960     11   -0.09    .009
Portugal
 1878     19   -0.64    .832
 1960     18   -0.44    .002
Spain
 1833     48   -0.68    .931
 1960     50   -0.42    .002
```

It is evident that in every case the earlier slope is closer to -2/3. The earlier slopes for Italy,

Netherlands, Portugal and Spain, in fact, conform to the hypothesis by the p > .05 criterion. Even in the case of France, the p-value improved from .00000005 to .00008, suggesting that, if we had still earlier data, *départements* which were intended to be uniformly small still might have shown some conformity to the size-density law.

We developed a tentative explanation for the tendency of slopes to erode toward zero. If **boundaries remain fixed** while the population experiences redistribution, then any relation which once existed will, obviously erode; you can't change the independent variable while holding the dependent variable constant and expect the original relation to persist.

More particularly, suppose we have a set of territorial divisions which do fit the size-density law. Now fix the boundaries of the territorial divisions once and for all. This could happen through historical inertia (e.g., England) or technological change (e.g., the automobile and U.S. counties). Under normal processes of population growth ***high-density places will tend to become even more dense***. This is the law of proportionate effect: nothing succeeds like success, the rich get richer, big population centers draw more people than small ones (gravity model). Ordinarily such growth would lead to further subdivision (down to some technologically/economically minimal size), but here no further subdivision occurs. The result is erosion of the slope.

This is easily simulated An initial group of 20 data points, scattered around a regression line (exact-fit points were computed, then a random number was added or subtracted from the vertical coordinate).
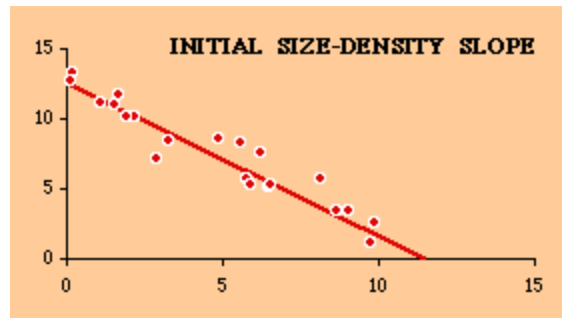
Next we move data points to the right. The arrow points have the same vertical coordinates as the original data points (no change in area), but the horizontal coordinates have been increased. The amount of increase is **proportional to the original value**, plus a positive or negative random number. The new data points resulting from the rightward moves are shown in green.
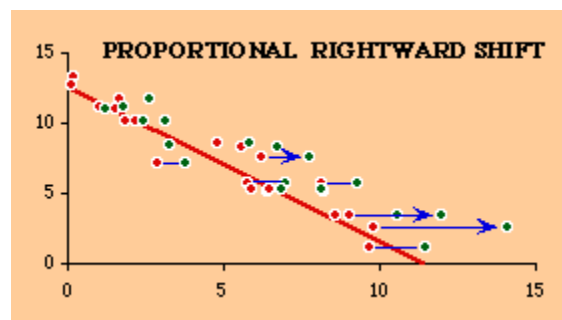
Finally, we compute a new regression line for the generated points. The slope has eroded has clearly eroded toward zero.
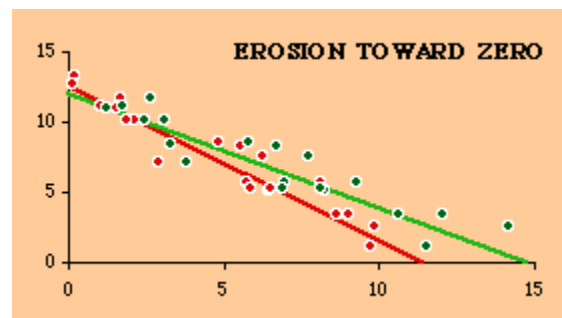
As originally put forward, the erosion hypothesis was only suggested, along with the data in Table 13-1 confirming that earlier slopes tended to be more negative. In 1982, with the help of my wife Karen,[2] I conducted a direct test of the erosion hypothesis.

The question was: Does the slope become less negative as population becomes more concentrated?

We needed area and population data for stable sets of territorial divisions over significant periods of time. Ideally, stability would mean no change whatever in the boundaries of divisions throughout the period of study. We found such data[3] for

Austria (9 *Bundesländer:* 1910-1961)
Belgium (9 Provinces: 1816-1961)
England and Wales (54 Counties: 1801-1961)
France (90 *Départements:* 1872- 1962)
Netherlands (11 Provinces: 1830-1960)
Switzerland (24 Cantons: 1850-1960)
United States (48 States: 1900- 1970)

We also included studies[4] of three nations where the number of changes in territorial divisions was small relative to the number of divisions

Italy (14 *Regioni:* 1861-1911; 15: 1921-1951)
Portugal (19 *Distritos:* 1878-1920; 20: 1930-1960)
Spain (48 *Provincías:* 1838-1897; 50: 1900-1960)

We computed size-density slopes for each of the years indicated in the accompanying table:
.

NATION/yr	S.D.	Slope	NATION/yr	S.D.	Slope
AUSTRIA			NETHERLANDS		
1910	1.593	-0.633	1830	0.521	-0.123
1923	1.562	-0.630	1840	0.510	-0.125
1934	1.537	-0.648	1849	0.491	-0.122
1951	1.422	-0.709	1859	0.487	-0.129
1961	1.411	-0.724	1869	0.490	-0.134
			1879	0.511	-0.128
BELGIUM			1889	0.551	-0.123
1816	0.393	-0.272	1899	0.577	-0.123
1831	0.469	-0.219	1909	0.597	-0.124
1846	0.611	-0.099	1920	0.620	-0.131
1856	0.599	-0.094	1930	0.660	-0.115

Year		
1866	0.611	-0.084
1876	0.653	-0.083
1880	0.658	-0.083
1890	0.681	-0.077
1900	0.710	-0.078
1910	0.731	-0.084
1920	0.739	-0.094
1930	0.763	-0.104
1947	0.787	-0.121
1961	0.803	-0.133

ENGLAND & WALES

Year		
1801	0.144	-0.604
1811	0.149	-0.560
1821	0.151	-0.534
1831	0.157	-0.475
1841	0.164	-0.436
1851	0.173	-0.392
1861	0.187	-0.337
1871	0.198	-0.299
1881	0.216	-0.261
1891	0.230	-0.245
1901	0.231	-0.249
1911	0.243	-0.244
1921	0.249	-0.234
1931	0.253	-0.229
1951	0.258	-0.243
1961	0.251	-0.288

FRANCE

Year		
1872	0.609	-0.426
1881	0.634	-0.423
1891	0.649	-0.418
1901	0.692	-0.386
1911	0.725	-0.367
1921	0.755	-0.348
1931	0.794	-0.326
1946	0.792	-0.320
1954	0.838	-0.308
1962	0.848	-0.292

ITALY

Year		
1861	0.540	-0.142
1871	0.520	-0.177
1881	0.518	-0.159
1901	0.526	-0.125
1911	0.543	-0.101
1921	0.593	-0.042
1931	0.579	-0.043
1936	0.577	-0.039
1951	0.573	0.005

Year		
1940	0.679	-0.114
1950	0.687	-0.109
1960	0.815	-0.098

PORTUGAL

Year		
1878	0.795	-0.644
1890	0.780	-0.638
1900	0.784	-0.634
1911	0.776	-0.631
1920	0.783	-0.620
1930	0.830	-0.604
1940	0.837	-0.600
1950	0.855	-0.585
1960	0.898	-0.548

SPAIN

Year		
1833	0.533	-0.676
1850	0.559	-0.671
1857	0.564	-0.659
1877	0.583	-0.624
1887	0.589	-0.614
1897	0.610	-0.594
1900	0.608	-0.606
1910	0.618	-0.596
1920	0.647	-0.548
1930	0.682	-0.507
1940	0.711	-0.478
1960	0.815	-0.419

SWITZERLAND

Year		
1850	0.763	-0.615
1870	0.792	-0.610
1880	0.778	-0.639
1888	0.818	-0.586
1900	0.843	-0.525
1910	0.855	-0.554
1920	0.857	-0.514
1930	0.860	-0.492
1941	0.847	-0.487
1950	0.864	-0.470
1960	0.917	-0.413

UNITED STATES

Year		
1900	1.607	-0.477
1910	1.474	-0.526
1920	1.454	-0.531
1930	1.470	-0.518
1940	1.438	-0.526
1950	1.422	-0.533
1960	1.393	-0.541
1970	1.403	-0.539

There are a variety of concentration measures which might have been computed[5] We chose the standard

deviation of log D (referred to in the table as S.D.). It seems to have escaped the attention of most authors[6] that any measure of concentration can in the end be shown to be based upon this mundane statistic. A standard deviation of zero indicates no concentration (even densities), with increasing values indicating increasing concentration. The standard deviation suffers from the fact that we cannot appropriately compare it across sets with different numbers of divisions. But it is perfectly suitable for our purposes.

Figs. 13-1 through 13-10 show historical values for slopes and standard deviations. To the left of each historical graphs is a scatter diagram showing the relation between the two. The size-density slopes are shown in green; the standard deviations (concentration measures) are shown in red. It will be obvious that the two are strongly related (increasing concentration leading to erosion of the slope). A following statistical table summarizes the results of our analysis.
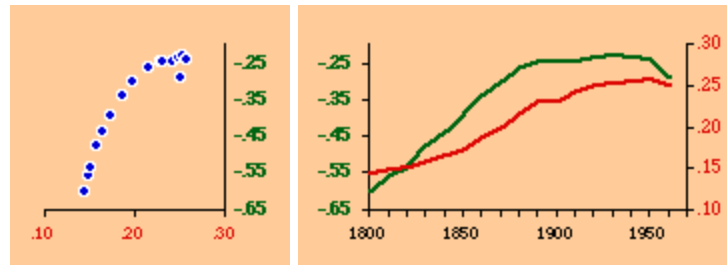
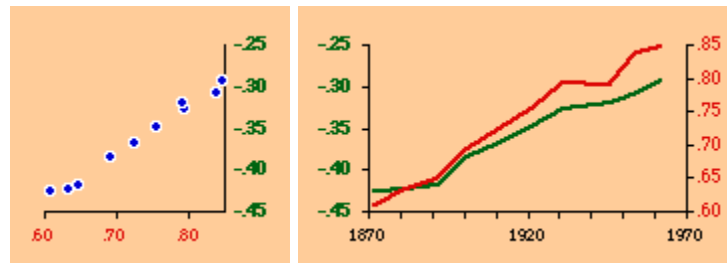**FIG. 13-1. AUSTRIA**   (see commentary below)
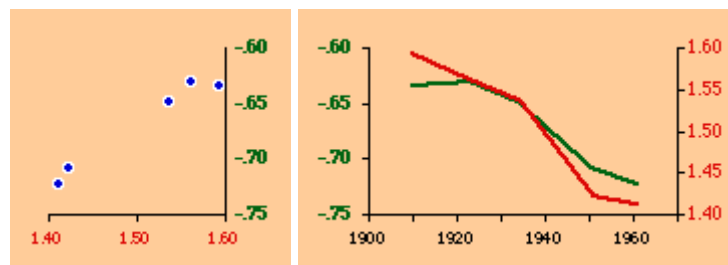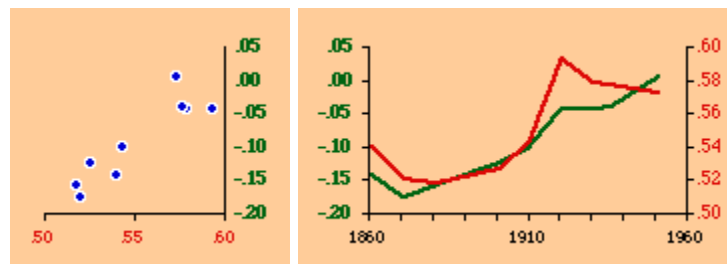
FIG. 13-2. BELGIUM
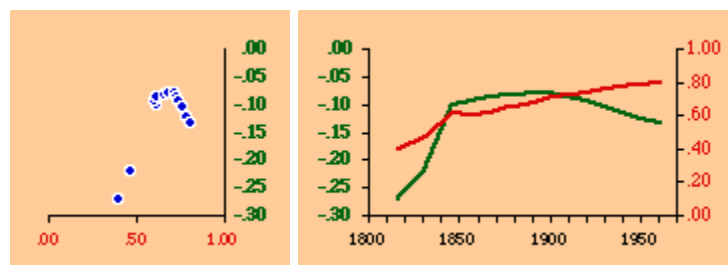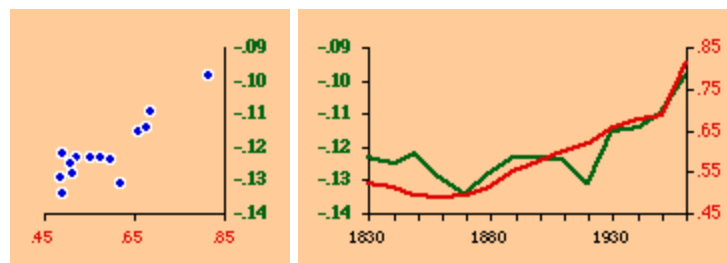
FIG. 13-3. ENGLAND & WALES
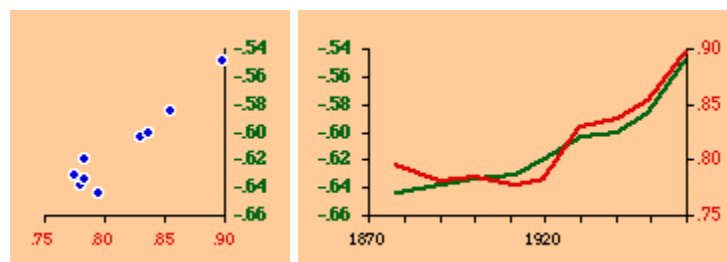
FIG. 13-4. FRANCE
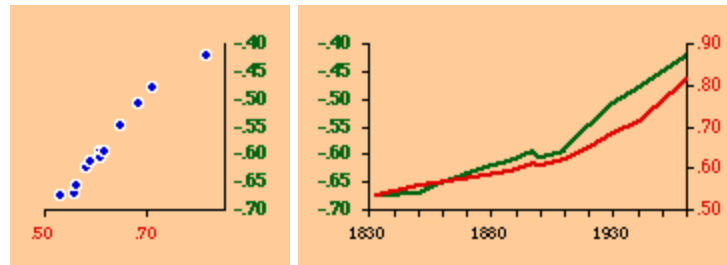

FIG. 13-5. ITALY

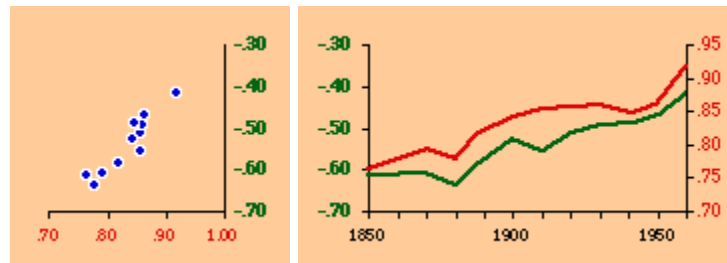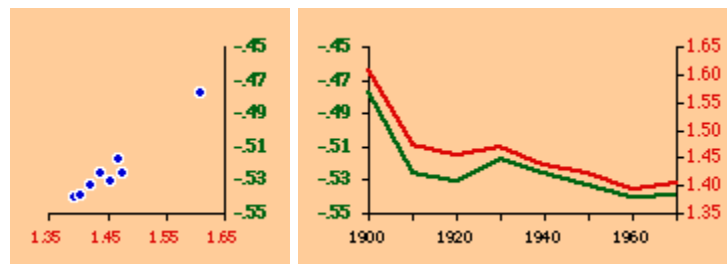**FIG. 13-6. NETHERLANDS**

FIG. 13-7. PORTUGAL

FIG. 13-8. SPAIN

FIG. 13-9. SWITZERLAND

FIG. 13-10. UNITED STATES

The following table summarizes several statistics
which confirm what the visual presentation has
already shown.

Nation	r^2	$p\{r^2=0\}$	rho	$p\{\rho=0\}$	tau	$p\{t=0\}$
Austria	.97	.0024	.90	.0719	.80	.0500
Belgium	.50	.0040	.09	.7423	.03	.8695
England & Wales	.86	.0001	.94	.0003	.87	.0001
France	.99	.0001	.99	.0030	.96	.0001
Italy	.83	.0007	.82	.0209	.61	.0218
Netherlands	.74	.0001	.71	.0110	.55	.0062
Portugal	.96	.0001	.75	.0339	.61	.0218
Spain	.97	.0001	.99	.0010	.97	.0001
Switzerland	.88	.0001	.92	.0037	.82	.0005
United States	.98	.0001	.92	.0146	.82	.0044

The first is a test of significance, against the null
hypothesis, for the linear correlation coefficient[7]. In

some cases, e.g. Belgium, the relation is clearly not linear. Nothing in our erosion hypothesis suggests that it should be; we only expect a monotonic increase of the one with the other, i.e., an ordinal relation.

For this reason two additional test statistics are provided, Spearman's rho, and Kendall's tau. Hays and Winkler[8] review the advantages and disadvantages of each as an alternative to r-square, concluding

> ...each index supplies information of importance about the general monotone relation that may exist between variables, quite irrespective of the true value of the correlation coefficient.... For moderately large samples, ... tau seems to provide a better test of the hypothesis of no association....

Austria, Fig. 13-1, interestingly, shows support for the hypothesis but in the reverse historical order expected. Over time the population has become less concentrated, and the size-density slope has become steadily more negative, even passing the theoretically expected - 2/3 value. A similar pattern shows up in the United States, Fig. 13-10: here the decreasing concentration reflects increased settlement in large states such as California. This may be more a problem of the level of measurement of area and density, i.e., the state vs. the county.

The historical charts for the other nations show remarkable conformity to the notion that size-density tends to erode toward zero over time. The chart for France, incidentally, projected back another century, would produce a slope slightly more negative than -1/2, supporting our earlier conjecture that — even when a deliberate effort is made to produce uniformly size divisions — the size-density relation tends to surface.

Next Chapter

NOTES:

[1] B. R. Mitchell, *European Historical Statistics*, New York: Columbia University, 1975.

[2] The study was published as G. Edward Stephan and Karen H. Stephan, "Population Redistribution and changes in the Size- Density Slope", *Demography*, 21:35:40, 1984

[3] The data for Austria, Belgium, France, Netherlands and Switzerland came from Mitchell, *Op. Cit.*; England and Wales data were from B. R. Mitchell and H. C. Jones, *Second Abstract of British Historical Statistics*, Cambridge: Cambridge University Press, 1971; United States data was from the census.

[4] Mitchell, *Op. Cit.*

[5] I have reviewed a number of these measures and shown that each can be given a probability interpretation: G. Edward Stephan, "Measures of Population Distribution: A Probability Interpretation", *Western Sociological Review*, 7:3-10, 1976.

[6] except notably Maurice G. Kendall and Alan Stuart, *The Advanced Theory of Statistics* (3 vols.; 2nd ed.), New York: Hafner, 1963.

[7] (this will be in the methodological appendix)

[8] William L. Hays and Robert L. Winkler, *Statistics: Probability, Inference and Decision*, 849, New York: Holt, Rinehart and Winston, Inc., 1971.