Exam 2 Review

Question 1

For each of the following functions, find $f'(x)$. You do not need to simplify the expressions for $f'(x)$.

a. $f(x) = 3x^7 + 5x - 7$

b. $f(x) = \frac{x^2+4}{x^2-4}$

c. $f(x) = 3e^{x^3}$

d. $f(x) = x^2 e^{2x}$

Question 2

At what point is the tangent line to the curve $f(x) = x^3 - 3x^2 + 4$ horizontal?

Question 3

Consider the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x$. Find the critical values of $f(x)$. Use the first derivative test to find the local minimum and maximum of the function.

Question 4

Suppose a company models its profit P in thousands of dollars as a function of the number of units (in hundreds) sold x using the function $P(x) = x^3 - 6x^2 + 9x$.

a) Find $P(2)$ and interpret its meaning in context.

b) Find $P'(2)$ and interpret its meaning in context.

c) Using the first or second derivative test, find the production level that maximizes the profit if the company produces between 0 and 350 units. What is the maximum profit made?

d) If the company must produce a minimum of 50 units and a maximum of 500 units per day, find the absolute minimum and maximum profit made in that range.

Question 5

A herring swimming along a straight line has traveled $s(t) = \frac{t^2}{t^2+2}$ feet in t seconds.

a) How fast is the herring travelling after 2 seconds?.

b) How far has the herring traveled after 3 seconds?

c) At three seconds, is the herring speeding up or slowing down? Justify your answer.

Question 6

Owners of a car rental company have determined that if they charge customers p dollars per day to rent a car, where $0 \leq p \leq 200$ the number of cars n they rent per day can be modeled by the linear function $n(p) = 1000 - 5p$. If they charge \$50 per day or less, they will rent all their cars. If they charge \$200 per day or more, they will not rent any cars. Assuming the owners plan to charge customers between \$50 per day and \$200 per day to rent a car, use the calculus tools learned in the course to find how much they should charge to maximize their revenue. What is the maximum revenue?