

# Exam 1 Review

## Question 1

The piecewise-defined function below represents the shipping charges,  $C(x)$ , in US dollars based on package weight,  $x$  (in pounds).

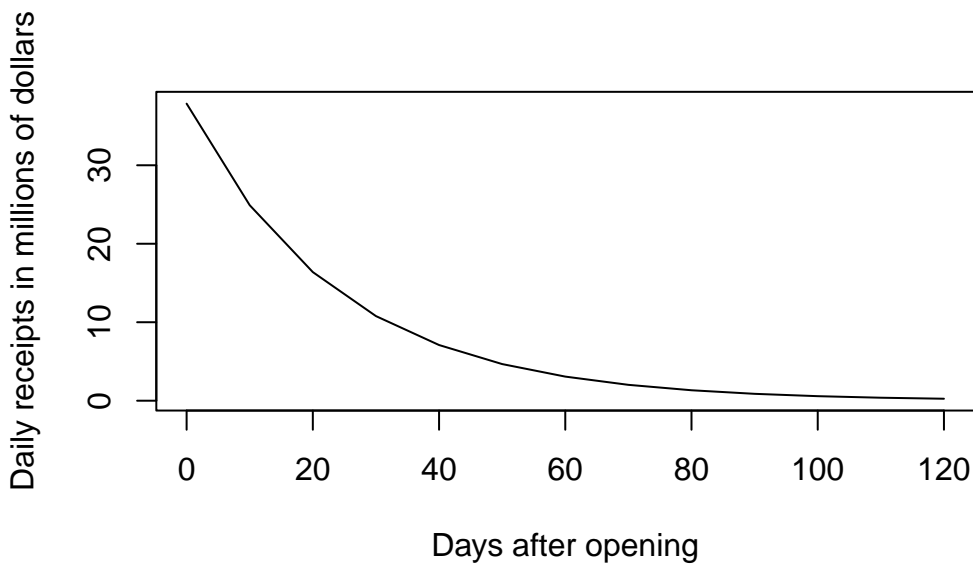
$$C(x) = \begin{cases} 4.50, & 0 < x \leq 3 \\ 0.5(x - 3) + 4.50, & 3 < x \leq 10 \\ 0.5(x - 10)^2 + 7.50, & 10 < x \leq 12. \end{cases}$$

- a. Find  $C(2)$  and interpret its meaning in context.
  
  
  
  
  
  
  
  
  
  
- b. Find  $C(5)$  and interpret its meaning in context.
  
  
  
  
  
  
  
  
  
  
- c. Use a table to find the left limit of the function at  $x = 3$  (i.e.,  $\lim_{x \rightarrow 3^-} C(x)$ ).

- d. Use a table to find the right limit of the function at  $x = 3$  (i.e.,  $\lim_{x \rightarrow 3^+} C(x)$ ).
- e. Based on your work in parts (c) and (d), state whether the limit  $\lim_{x \rightarrow 3} C(x)$  exists and explain why. If the limit exists, find its value.

## Question 2

The daily receipts  $f(t)$  in millions of dollars of the movie “The Hunger Games” after its opening on 23 March 2012 had an exponential model shown below:



- Use the graph to estimate the value of  $f(15)$  and state its meaning in context.
- Without doing any calculations, state whether you expect  $f(25)$  to be greater or less than  $f(35)$ . Explain how you know.

- c. Without doing any calculations, state whether you expect the average rate of change between  $t = 10$  and  $t = 25$  to be greater or less than the average rate of change between  $t = 25$  and  $t = 40$ . Explain how you know.
- d. Without doing any calculations, state whether you expect  $f'(30)$  to be negative or positive. Explain how you know.
- e. Without doing any calculations, state whether you expect  $f''(30)$  to be positive or negative. Explain how you know.
- f. Let  $f(20) = 16.4$  and  $f(30) = 10.8$ . Use these values to estimate  $g'(25)$ . Interpret your answer in context.

### Question 3

A climate model predicts that the average global temperature rise (in degrees Celsius) above pre-industrial levels can be approximated by the function  $T(t) = 0.008t^3 - 0.06t^2 + 0.5t + 0.2$ , where  $t$  represents time in decades since 2000.

- Find  $T(2)$  and interpret its meaning in context.
- Estimate the *average rate of change* in the global temperature rise between  $t = 0$  and  $t = 1$ . Include units in your answer.
- The expression below is for finding the *instantaneous rate of change* in the global temperature rise at  $t = 1$  using the limit definition of the derivative. Write down the next step. No need to evaluate the limit.

$$T'(1) = \lim_{h \rightarrow 0} \frac{T(1+h) - T(1)}{h}$$

d. Use the appropriate derivative rule(s) to find the function  $T'(t)$  then use it to evaluate  $T'(1)$ . Interpret the meaning of  $T'(1)$  in context.

e. Calculate  $T''(3)$  and interpret its meaning in context.

#### Question 4

Use derivative rules to find the derivative function for each of the following functions:

a.  $f(x) = 3x^2 - 4x + 5$

b.  $g(x) = \frac{x^5 + x^3}{x^2}$

c.  $h(x) = \sqrt{x} + x^2$

d.  $k(x) = 5^x + x^5$

e.  $q(x) = e^x + 5x^4$