Exam 1 Review

Question 1

The piecewise function below represents the shipping charges, $C(x)$, for packages based on the package weight, x (in pounds).

$$C(x) = \begin{cases} 4.50, & 0 < x \leq 3 \\ 0.5(x-3) + 4.50, & 3 < x \leq 10 \\ 0.5(x-10)^2 + 7.50, & 10 < x \leq 12. \end{cases}$$

a. Find $C(2)$ and interpret its meaning in context.

b. Find $C(5)$ and interpret its meaning in context.

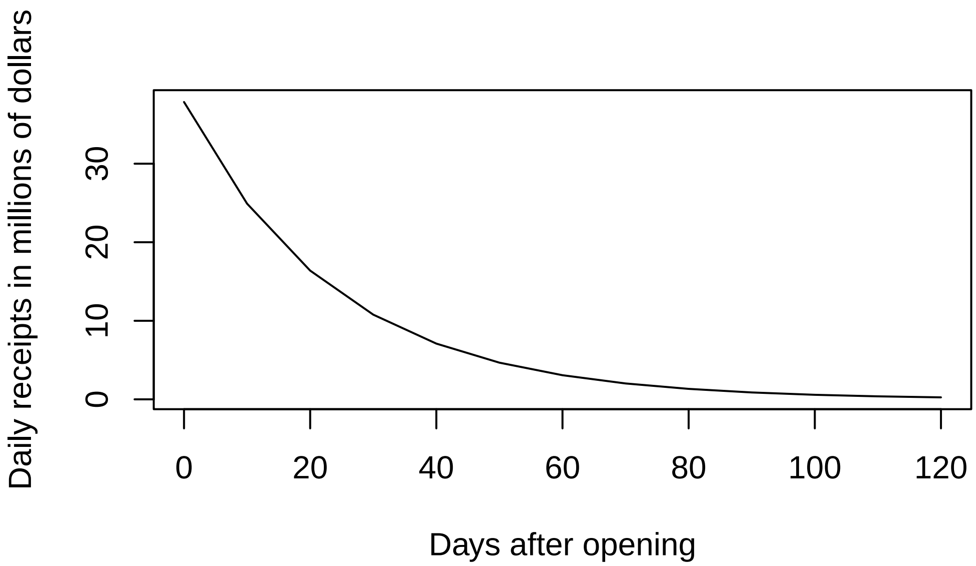
c. Use a table to find the left limit of the function at $x = 3$ (i.e., $\lim_{x \to 3^-} C(x)$).

d. Use a table to find the right limit of the function at $x = 3$ (i.e., $\lim_{x \to 3^+} C(x)$).

e. Based on your work in parts (c) and (d), state whether the limit $\lim_{x \to 3} C(x)$ exists and explain why. If the limit exists, find its value.

Question 2

The daily receipts $f(t)$ in millions of dollars of the movie "The Hunger Games" after its opening on 23 March 2012 had an exponential model shown below:



a. Estimate the value of $f(20)$ and state its meaning in context.

b. Without doing any calculations, state whether you expect $f(25)$ to be greater or less than $f(30)$. Explain how you know.

c. Without doing any calculations, state whether you expect the average rate of change between $t = 15$ and $t = 25$ to be greater or less than the average rate of change between $t = 40$ and $t = 80$. Explain how you know.

d. Without doing any calculations, state whether you expect $f'(30)$ to be negative or positive. Explain how you know.

e. Without doing any calculations, state whether you expect $f''(30)$ to be positive or negative. Explain how you know.

Question 3

A climate model predicts that the average global temperature rise (in degrees Celsius) above pre-industrial levels can be approximated by the function $T(t) = 0.008t^3 - 0.06t^2 + 0.5t + 0.2$, where t represents time in decades since 2000.

a. Find $T(2)$ and interpret its meaning in context.

b. Estimate the *average rate of change* in the global temperature rise between $t = 0$ and $t = 1$. Include units in your answer.

c. The expression below is for finding the *instantaneous rate of change* in the global temperature rise at $t = 1$ using the limit definition of the derivative. Write down the next step. No need to evaluate the limit.

$$T'(1) = \lim_{h \to 0} \frac{T(1+h) - T(1)}{h}$$

d. Use the appropriate derivative rule(s) to find the function $T'(t)$ then use it to evaluate $T'(1)$. Interpret the meaning of $T'(1)$ in context.

e. Calculate $T''(3)$ and interpret its meaning in context.

Question 4

Use derivative rules to find the derivative function for each of the following functions:

a. $f(x) = 3x^2 - 4x + 5$

b. $g(x) = \frac{x^5 + x^3}{x^2}$

c. $h(x) = \sqrt{x} + x^2$

d. $k(x) = 5^x + x^5$

e. $q(x) = e^x + 5x^4$