Notes on *EE 219A: Numerical Simulation and Modeling* lectures by Professor Jaijeet Roychowdhury, Spring 2021

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Lecture 1

Introduction

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1.1 Course Coverage

We will be covering

- Models: Mainly circuits, but we will see how this applies to other case
- System equations that arise from connections of modeled circuit components. DAEs will be looked at a lot, differential-algebraic equations
- Biochemical reaction equations (we will see how this looks pretty close to circuits)
- Analysis of noise in systems
- Effects of parameter variability
- Steady state analysis

We will have a more structured format then a standard electronics of ME course. In particular, we have two independent steps:

- Modeling physical systems as nonlinear differential equations
- Solving these differential equations numerically in different ways ("analyses")

The seperation of these two steps gives us a nicer view of the modeling process.

1.2 Why is Simulation Useful?

Motivating example - We might have a super complicated circuit with tons and tons of resistors and we want to determine some waveform with respect to some input. We may desire to do this analytically, but this may not always be feasible or possible. We may proceed by taking analytically simplifications, but this may also be hard and also lead us to oversimplify.

Numerical simulations let us gain insights into a system that may be hard to analytically solve or simplify. We can experiment at will with numerical examples and gain an understanding of the system that may allow us to do well guided analytic simplifications.

The study of modeling also provides some mathematical beauty and lets us identify connetions across many disciplines that may be otherwise hidden.

1.3 First Look at Modeling through Circuits

1.3.1 Why Circuits First?

- It's an EE class
- Historically, modeling/simulation is the most well developed in the world of circuit simulation
  - Starting with SPICE tools introduced in 70's, created at Berkeley

- Later adapted to other domains (biology, mechanics, etc.)
  - Even use circuits to describe things that have nothing to do with circuits,since circuits provide a nice, precise way to specifiy differential equations

1.3.2 EE Devices/Elements

Two Terminal Devices

- Two sides labeled by p and n, stands for positive and negative but that doesn't actually have to be what the potential on the respective ends are
- Some voltage across across the device denoted by V_{pn}
- Current through the device denoted by $\vec{i_{pn}}$
- We have some equation for the relationship between voltage and current (typically voltage is the input) giving us a device model
  - It's typically useful to plot out this relationship

Let's look at some specific devices. We will start with some idealized models.

- Linear resistor, model given by Ohm's Law: $i = \frac{V}{R}$
  - We can see it has a perfect linear $I - V$ relationship
  - The device is memoryless - current output at time t only depends on values of voltage provided at time t
- Linear capacitor, model given by $q = CV$
  - We can take the derivative to get the $I - V$ relationship by taking the derivative of both sides to derive $i(t) = C\dfrac{\mathrm{d}V}{\mathrm{d}t}$
  - Most idealized case is just two electric plates seperated by some insulator
  - This device is not memoryless - the capacitor can be viewed as an integrator, which has some fundemental aspect of memory
- Linear inductor, model given by $\phi = Li$
  - We can take the derivative to get the $I - V$ relationship by taking the derivative of both sides to derive $V(t) = L\dfrac{\mathrm{d}i}{\mathrm{d}t}$
  - We can also view the inductor as the dual of a capacitor
  - Of course, this component will also not be memoryless - we can view it as a differentiator
- Independent sources
  - Can provide voltage and current
  - Either compleletely flat vertical (voltage source) or horizontal (current source) line
    * Thus, the voltage source $I - V$ relationship cannot be expressed as a function of V
- Linear dependent sources
  - This is actually a 4 terminal device, but it is often abstracted to have two terminal
  - The device will act as voltage or current source, with supply equal to some multiple of an input voltage or current
  - Voltage or current through the device depends on load
  - As a guiding principal, p known values are needed to completely specify p inputs and outputs of a system.

Like previously stated, we covered idealized device models

- Linear controlled sources are really unphysical
- R, L, C, etc. parameters are idealized (may be temperature dependent, device limits)

The takeaway is that all models are wrong, but some are useful. It is imporant to be able to know to what extent a model creates some approximation, or else you will be in a pickle.

1.3.3 "System Properties" of Elements

Definition 1.3.1 (Memoryless)**.** The output of a system at time t is dependent only on the input at time t, not values of the input in the past or future.

We will say that memoryless systems have outputs that are functions of the inputs (use $y(t) = f(x)$), or else the outputs are functionals of the inputs (use $y(t) = \chi\{x(t)\}$)

Definition 1.3.2 (Scaling)**.** $x(t) \to y(t) \implies c \cdot x(t) \to c \cdot y(t)$

Definition 1.3.3 (Superpostion)**.** $x_1(t) \to y_1(t), x_2(t) \to y_2(t) \implies x_1(t) + x_2(t) \to y_1(t) + y_2(t)$

Definition 1.3.4. A system is **linear** if it satisfies superpostion and scalaing.

The quickest way to show linearity is to prove

$$x_1(t) \to y_1(t), x_2(t) \to y_2(t) \implies \alpha_1 \cdot x_1(t) + \alpha_2 \cdot x_2(t) \to \alpha_1 \cdot y_1(t) + \alpha_2 \cdot y_2(t)$$

Lecture 2

Systems Continued

2.1 Linearity

We concluded our discussion of linear systems for now and start getting into the interesting stuff.

2.2 Nonlinear Elements

2.2.1 Nonlinear Resistors

A nonlinear resistor might obey the $I - V$ relationship

$$i = f(v)$$

for some nonlinear function f.

It's important for this function f to be well defined, as in it has a defined output for every input over the domain of interest. For good simulation, we should want f to be continuous and infinitely differentiable

Whenever we can express $i = f(v)$, we say we have a **voltage-controlled** element. As you can imagine, the reverse relationship $v = g(i)$ describes as **current-controlled element**.

Not all types of elements will be defined as voltage-controlled or current conrolled. We can sometimes have elements defined in an **implicit form** in which there may not exist a well-defined direct relationship between v and i. For example, consider the (purely hypothetical) relationship

$$i^2 + v^2 = 1$$

The relationship gives us a lot of issues if we try to define this as something voltage-controlled and current-controlled, namely

- For a given v we cannot determined a unique i
- For a given i we cannot determine a unique v
- There are values of v and i for which we do not have a defined output

Although this was just a toy example, we see real world examples in nonlinear elements

Why is this important? The major reason is that this all determines how we write our circuit equations. It can impact solving methods (we can solve voltage controlled sources with nodal analysis) and number of solutions as well.

2.2.2 Ideal Diode Model

The ideal diode model has the $I - V$ relationship

$$i_d = I_s(e^{\frac{V}{V_t}} - 1)$$

and is parameterized by

- I_s, the saturation current (A). Typically takes on values between 10^{-8} (breadboard) to 10^{-12} chip
- V_t, thermal voltage with equation

  $$\frac{kT}{q}$$

  itself with parameters

  - k, Boltzman constant
  - q, unit electric charge

- T, absolute temperature in Kelvin

If we wanted to plot this $I - V$ curve, it would look like a common exponential. We idealize this diode by assuming that any negative current or very low is 0.

We can see that at all points in the curve, $v \cdot i \geq 0$. This means that power is always being dissapted by our device, or alternatively, no power is being generated. It's a nice sanity check to see that this holds. For example. we can see that forgetting the -1 in our equation would make the negative V region of our inputs generate power

2.2.3 Bipolar Junction Transistor (BJT) model: Ebers-Moll

We consider an NPN Transistor (NPN stands for some physical properties of the device). The device has three termianls B, C, E, which stands for base, collector, and emitter.

Before we get into the model, we consider the conservation equations for this device. We have KVL

$$V_{BE} + VBC - V_{CE} = 0$$

and KCL

$$i_B + i_C + i_E = 0$$

We will figure out output current of this device with respect to the input, namly a relationship

$$\begin{bmatrix} i_C \\ i_E \end{bmatrix} = \vec{f}(\begin{bmatrix} V_{BE} \\ V_{BC} \end{bmatrix})$$

We can do KCL at each node to get some starting equations.

Base Node

$$i_B = (1 - \alpha_F)i_F + (1 - \alpha_R)i_R$$

Collector Node

$$i_C = \alpha_F i_F - i_R$$

Note we can get i_F, i_R from our ideal diode equations.

$$i_F = \text{diode}(V_{BE}; I_{SF}) i_R \qquad = \text{diode}(V_{BC}; I_{SR})$$

Now we can explicitly write out a vector system and make it like sort of linear by altering our input vector.

Characteristic Curves

Similarly to how we used plots to analyze our diode model, we can inspect the Characteristic curves of our BJT to analyze its properties. The convention is to plot i_C over V_{CE} for various fixed values of V_{BE}. We call the reigion where i_C increases the **saturation region** and when i_C remains constant the **forward active region**.

2.2.4 Metal Oxide Semiconductor Field-Effect Transistor (MOSFET)

- Just like the BJT transistor, the MOSFET has NPN & PNP types as well
- We will examine the Shichman-Hodges (great people who were at Bell Labs, Dave Hodges used to be at Berkeley) (S-H) model

There are three terminals we look at, Gate (G), Drain (D), Source (S). As before, we can define voltages across these terminals and currents through them. There are two cases we can consider when modeling this device. In the simple case when we have $i_G = 0$, we have

$$i_D = f_{N^+}()$$

Lecture 3

Non-linear Devices Continued

3.1 Shichman-Hodges Model

$$
\begin{aligned}
& i_G = 0 \\
& i_D = f_{Nt}(V_{GS}, V_{DS}) = \begin{cases} 0 & V_{GS} < V_T \text{ (OFF)} \\ \frac{\beta}{2}(V_{GS} - V_T)^2 & V_R \leq V_{GS} < V_{DS} + V_T \text{ (Active/Saturation Region)} \\ \beta[(V_{GS} - V_T) - \frac{V_{DS}}{2}]V_{DS} & V_{GS} \geq V_{DS} + V_T \text{ (Triode/"Linear" Region)} \end{cases}
\end{aligned}
$$

Intuition: As voltage increases, we get more excess electrons which make the device behave more like a resistor. Need in depth physics knowledge to go farther from here.

(β is typically some constant near 10^{-2})

It may be helpful to write things in terms of $V_{GS} - V_T$.

On the characteristic curves, we hold V_{GS} over a varying V_{DS}.

If we reverse the doping (PNP type) the device behaves in an inverse manner: We will be repelling electrons and creating holes.

With $V_{DS} < 0$, S and D will swap. We can label them D', S' now

$$
\begin{aligned}
i_D &= -i'_D \\
&= -f_{N+}(V_{GS'}, V_{D'S'}) \\
&= -f_{N+}(V_{GS} - V_{DS}, -V_{DS}) \\
&= f_{N-}(V_{GS}, V_{DS})
\end{aligned}
$$

And we can create an abstraction now

$$
i_D = \begin{cases} f_{N+}(V_{GS}, V_{DS}) & V_{DS} \geq 0 \\ f_{N-}(V_{GS}, V_{DS}) & V_{DS} < 0 \end{cases}
$$

What changes for the PNP case?

- V_{GS} replaced by $-V_{GS}$, V_{DS} replaced by $-V_{DS}$ for putting in $f_N(\cdot, \cdot)$
- Output from $f(\cdot, \cdot)$ actually gives $i_S = -i_D$

Our current equation is then

$$
i_D = f_P(V_{GS}, V_{DS}) = -f_N(-V_{GS}, -V_{DS})
$$

3.1.1 Recap

- Schichman-Hodes is a basic, 2 parameter model
- This rocked in the old days, but we can do much better now
- Modern models are much more complicated to complement their robustness

3.2 Actual Device Models

Main takeaways:

- A HUGE number of parameters. Hundreds of them! (about 400)
- Many many lines of code
- This isn't even the most modern model!

3.3 Non-linear Capacitors & Inductors

3.3.1 Non-linear Capacitor

Recall the linear capacitor equation

$$q = Cv \wedge i = \frac{\mathrm{d}q}{\mathrm{d}t} \implies i = c\frac{\mathrm{d}v}{\mathrm{d}t}$$

Let's introduce some nonlinear relationships

Voltage-controlled nonlinear capacitor

$$q = f(v)i = \frac{\mathrm{d}q}{\mathrm{d}t} \implies i = f'(v)\frac{\mathrm{d}v}{\mathrm{d}t}$$

Charge-controlled capacitor

$$V = g(q)$$

Implicit capacitor

$$h(v, q) = 0$$

3.3.2 Non-linear Inductor

Example 3.3.1 (Depletion Capacitance)**.**

$$Q_{\text{depl}}(v) = \begin{cases} AC_{JO}\frac{\phi}{1-m}(1-(1-\frac{n}{\phi})^{1-m}) & v \leq \int_c \phi \\ \text{complicated expression} & \text{otherwise} \end{cases}$$

Check out all the parameters!

Example 3.3.2 (Non-linear Inductor)**.** Here we can see a piecewise-linear interpolation which forms a model of a transformer magnetizing curve.

3.4 Continuity, Derivatives, Smoothness

Definition 3.4.1. A function is **smooth** when it and **all** it's derivatives (higher-order & mixed) exist & are well-defined

Why is this importance?

- Numerical algorithms (ie. Newton-Raphson) use functions/derivatives
  - N-R convergence is sensitive to derivatives
    - non-smoothness breaks theoretical (and in practice) assurance of N-R convergence
- Physical systems typically smooth at fine enough resolution
  - Non-smoothness indicative of unphysical model (take this with a grain of ault, just a sanity check)
    - Incorrect modeling of underlying physics/math
      - ex. Modeling a real device by giving it discrete properties
      - Should be a continuous model, no diff eqs
      - Careful with finite-state machines
- Derivatives often have important physical connotations
  - Ex. give us some idea about resistance of the device
- 2^{nd} derivatives (Hessians) help us do optimization
- higher order derivatives can be important in some modeling situations
  - distortion and intermodulation effects

Lecture 4

Continuity and Smoothness

4.1 Ill-defined Models

- Many device models will have discontinuity/domain problems
- Could be because model writers are disocnnected from effect of poor model

Example 4.1.1 (Diode Depletion Cap Expression)**.**

$$q_f(v) = -qN_a x_{po}\sqrt{1 - v/\psi_B}$$

For $v > \psi_B$, we could have problems since the range becomes complex.

Example 4.1.2.

$$\text{diode}(v; I_s, v_t) = I_s(e^{\frac{v}{v_t}} - 1)$$

The ideal diode model is well-defined over all $v \in \mathbb{R}$. By extension, the BJT model is also well-defineded since it is composed of the ideal diode and linear elements. But not all is well for numerical simulation, since

Example 4.1.3 (N-type MOSFET)**.** Need to check

Continuity: Check limits at boundaries

First Derivatives: Check that first derivatives with respect to each argument are the same from each direction at the boundaries

Second Derivatives: In this case, there are going to be three terms

4.2 Problematic Primatives

Example 4.2.1.

$$i = \begin{cases} f_1(v) & v < a \\ f_2(v) & v \geq a \end{cases}$$

One thing we can do is rewrite i using step functions

$$i = u(v - a)f_1(v) + (1 - u(v - a))f_2(v)$$

and then replace the step function with a smooth function that approaches 0 at $-\infty$ and 1 at ∞. A good choice could be activation functions used in neural networks.

$$u(v) = \frac{1 + \tanh kv}{2}$$

where k could represent a "sharpness" parameter. There are tradeoffs with k, and Jaijeet says you have to "finess" it.

The above is an example of discontinuity at the step function primative. We will go through more examples

Example 4.2.2. We can smooth the abs function by providing the a smooth version sabs

$$\text{sabs}(x, \epsilon) = \sqrt[+]{x^2 + \epsilon^2}$$

Example 4.2.3. Similarly, we can smooth the clip function

$$\text{clip}(v) = \begin{cases} 0 & v < 0 \\ v & v \geq 0 \end{cases}$$

using our previous smooth function

$$\text{sclip}(v, \epsilon) = \frac{v + \text{sabs}(v, \epsilon)}{2}$$

We can also use our primatives to fix domain issues

Example 4.2.4.

$$\sqrt{v} \to \sqrt{\text{sclip}(v)}$$

Example 4.2.5.

$$\log(x) \to \log(\text{sclip}(v))$$

Example 4.2.6.

$$\frac{1}{x-a} \to ?$$

4.3 Quanization Issues

First, we must explain double precision

- We have 64 bits to work with
- First bit represents sign of a number
- Next 11 bits represent exponent
- Next 53 bits are mantissa
- Coming together, we have

$$(-1)^{\text{sign bit}} \cdot 2^{\text{exponent bits}} \cdot 1.\text{mantissa bits}$$

We normally work with numbers stored in the double precision format. This format has a maxinum number size, after which we hit the numbers representation of infinity. The limit on infinity seems very large, but we can see that it is pretty easy to reach.

Example 4.3.1.

We also see a loss of precision when performing operations on very large numbers

Example 4.3.2.

4.4 Memristors

- Postulated relationship between charge and flux, called element memristor
  - Someone did their PhD on this
- 2008: Stan Williams claimed to have found one
  - Some really grand claims about applications to computers and devices contribute to hype
  - Apparently a lot of things behave as memristors when approximated, so everyone was saying they found one
- Fundamentally nonlinear