Skype chat with Taeho and Kater

Logistics

- Time
  - Wed., 3/20/19
  - 4 PM

- Locations
  - Skype
  - Harvard Square

To-do list

- **(1) Run experiments.**
  - **Goal: Observe the wiggling of the ridge in the plot of the RHS against theta_f and theta_a.**

- **(2) Binning**
  - **(A)**
    - **Set the bin width to l = 1, then 1.1, then 1.2, …**
    - **Identify the greatest bin width at which the bound is sensible (at which LHS - RHS > 0).**
  - **(B) Ensure that r = 0 always sits in the center of a bin.**
    - **Start with a bin width l.**
    - **Center a bin on r = 0.**
    - **Place bins leftward of that first bin, and place bins rightward of it.**
    - **Keep placing bins until hitting approximately r = ±50. (Ok to stop at, e.g., r = ±49.6 or ±52.)**

- **(3) Redo some of the slides that show what the weak measurement adds to entropic uncertainty relations.**
  - **When the LHS or LHS-RHS, at each point on the plot,**
    - **solve for the rho that makes the bound tightest.**
    - **evaluate the function (LHS or LHS-RHS) on that state.**
  - **When you send these plots, please include the plots of the RHS, even though those plots need no changing.**

- **(4) The anomaly of the crosses**

- **(A) Plot the RHS against theta_a.**
  Check whether this RHS plot looks identical to the LHS-RHS plot.
- **(B) Use the redone slides that show**
  what the weak measurement adds to entropic uncertainty relations
  [see (3)].
  Look at the LHS plot.
  Focus on the region near theta_f = pi/2.
  Look for wiggles.

Contents

(I) Experiments

- *Need to improve calibration*

- *Will run these experiments over the next month*
    - *The person who usually uses the necessary equipment will be out of town.*

(II) Binning: (1) (How) does the binning strategy affect the bound? (2) Discrepancy between Mathematica and IGOR

- *(A) Where the binwidth appears*
  - *(i) Explicitly in the LHS of the Mathematica calculation*
  - *(ii) Implicitly in the RHS*
    - *(a) p_r*
      - *Is larger when l is larger*
      - *Doesn't depend on l explicitly*
    - *(b) g_r*
      - *= analytical expression that depends on sqrt{l} explicitly*
  - *(iii) Implicitly in the LHS of the IGOR calculation*
    - *Why not explicitly*
      - In IGOR and in the lab, you construct a discrete probability: prob(r lies between r_1 and r_2) = (# of measured r-values that lie between r_1 and r_2) / (total # of measured r-values).
      - The latter probability is dimensionless.
      - To turn the probability into a probability density, would divide by the bin width.
      - But we multiply by the bin width, so might as well not bother dividing.

- *(B)* ***Possible explanations for the negativity of LHS-RHS in the Mathematica and IGOR calculations at large bin widths***
  - ***(i) The codes contain bugs.***
  - ***(ii) Using a large bin width, we approximate the probability distribution too poorly (we coarse-grain too much) for the approximation to obey the bound.***

- *(C) To do*
  - *(i)*
    - *Set the bin width to l = 1, then 1.1, then 1.2, …*
    - *Identify the greatest bin width at which the bound is sensible (at which LHS - RHS > 0).*
  - *(ii) Ensure that r = 0 always sits in the center of a bin.*
    - *Start with a bin width l.*
    - *Center a bin on r = 0.*
    - *Place bins leftward of that first bin, and place bins rightward of it.*
    - *Keep placing bins until hitting approximately r = ±50.*
      (Ok to stop at, e.g., r = ±49.6 or ±52.)

(III) What weak measurements add to the story of EUR

(III.1) Premeeting thinking

- (A) The weak measurement nudges the theta_f value at which the RHS's minimum obtains.
  - Creates ridges in the RHS plot's and the LHS-RHS plot's trough
  - Reference: results reported by Taeho in 2/13/19 message in email chain "Skype follow-up" — https://mail.google.com/mail/u/0/?fs=1&source=ig&tf=1&shva=1#search/ltaeho7%40gmail.com/QgrcJHrjCFpTvFndZZFXJXSgHjHbTvMPbwV

- (B) The weak measurement raises both sides of the bound.

  - (i) Why raises the LHS: The weak measurement changes one H_{vN} from an entropy over a single-variable probability distribution $\{p(f)\}$ into an entropy over a joint distribution $\{p(r, f)\}$. (r, f) can assume more values than f can. The entropy associated with (r, f) tends to be greater than the entropy associated with f.

  - (ii) Why the weak measurement raises the RHS

    - (a) Math
      - The RHS's weak-value term must be positive.
      - Have Taeho double-check.
      - Raising of bound by weak value is great…even though the weak measurement weakens LHS-RHS

    - (b) Physics, part (1)
      - Why *should* the weak measurement raise the RHS?
      - The physical explanation that came to mind
        implied that the weak measurement
        would lower the RHS:
        We set I = Z, while F = X. Z and X are known to disagree maximally. So an uncertainty relation for Z and X has a high (tight) RHS (bound). We change the X measurement into a weak measurement of an A that lies between Z and the xy-plane, followed by a strong measurement of X. Since A lies between Z and the xy-plane, our weak-and-strong measurement may be expected to disagree with Z less than X does. So we should expect the bound to weaken.

- (c) Physics, part (2)
  - Temporarily forget about the weak measurement. I = Z and F = X disagree maximally (have a mutually unbiased basis).
  - The weakly measured observable, A, lies between I and F on the Bloch sphere.
  - **If we regard the uncertainty bound's magnitude**
    as a measure of two quantum operators' disagreement,
    a weak measurement of A
    followed by a strong measurement of F = X
    disagrees with a measurement of I = Z
    more than X does,
    even though X and Z disagree maximally by having mutually unbiased bases.

- (C) The weak measurement extends entropic uncertainty relations beyond strong measurements of observables to more-general POVMs.
  - This might be the first experimental test of such a generalized EUR. Must check thoroughly.

(III.2) Understanding agreed upon during meeting:
What weak measurements add to the story of entropic uncertainty relations

- *References*
  - *(A) "Write-ups from others" folder —> "TL - 3/13/19 - what wk meas adds"*
  - *(B) 3/20/19: Taeho will redo the calculation, setting rho to whatever state tightens the bound that most at each point on each LHS-dependent plot.*

- ***(A) The weak measurement increases the RHS.***
  - *Reasons*
    - *Physics: Introducing a weak measurement introduces another physical degree of freedom—another measurement pointer—that behaves randomly. The new randomness raises the unpredictability of the measurement process that involves F. The associated entropy rises.*
    - *Math: The weak measurement changes one H_{vN} from an entropy over a single-variable probability distribution {p(f)} into an entropy over a joint distribution {p(r, f)}. (r, f) can assume more values than f can. The entropy associated with (r, f) tends to be greater than the entropy associated with f.*
  - *The increase's magnitude depends on the bin width.*

- ***(B) The weak measurement raises the RHS [except in the anomalous-weak-value regime**; see below].*
  - *Mathematical reason: In the absence of a weak measurement, the RHS equals $\min_{i,f} \left\{ -\log \left(| \langle f | i \rangle |^2 \right) \right\}$. In the presence, that dominant term acquires a p_r: The negative log becomes $-\log p_r - \log \left(| \langle f | i \rangle |^2 \right)$. The p_r < 0, so the $-\log p_r$ > 0, raising the bound.*
    - ***Is the weak-value term positive?***
  - *The rise's magnitude depends on the bin width.*

- ***(C) When the weak value goes anomalous (when $\theta_f \approx 0$, such that F ≈ I),***
  - ***(i) the LHS rises above the weak-measurement-free LHS slightly more than the LHS does at other settings.***
  - ***(ii) the RHS approaches $-\infty$.***
    - *The bound becomes trivial.*

- *(D)* ***The weak measurement raises LHS-RHS, weakening the bound.***
    - *Why?*

- *(E) T**he weak measurement makes two maximally disagreeing quantum operations—a measurement of I=Z and a measurement of F=X, measurements of observables whose eigenbases are mutually unbiased—agree more.***
    - *(i) <u>How this phenomenon manifests</u>*
        - *In the wriggling of the ridge at $\theta_f = \pi/2$ on the RHS plot*
        - *In the absence of any weak measurement, the RHS maximizes at $\theta_f = \pi/2$. The RHS maximizes near $\theta_f = \pi/2$ in the presence of a weak measurement.*
    - *(ii) <u>Reason</u>*
        - *When $\theta_f = \pi/2$, F = X. X disagrees maximally with I = Z: The operators' eigenbases are mutually unbiased. So an uncertainty relation for Z and X has a high RHS. Consider changing the X measurement into a weak measurement of an A that lies between Z and the xy-plane, followed by a strong measurement of X. Since A lies between Z and the xy-plane, our weak-and-strong measurement should disagree with Z less than X does. So the uncertainty bound should decline under the weak measurement.*
        - *Think of A as a middleman between debaters I = Z and F = X.*
    - *(iii)* ***On the other hand, the weak measurement takes two maximally disagreeing quantum operations—a measurement of I = Z and a measurement of F = X—and raises the operations' disagreement, by increasing the uncertainty bound. See (B).***

- *(F)* ***The weak measurement shifts the F at which the bound is tightest.***
    - *(i) <u>How this shift manifests</u>: in the wriggling of the ridge of the LHS-RHS plot*
    - *(ii) <u>Reason</u>: See (E).*

(IV) The anomaly of the crosses

- *(1) Reference: slides "Anomaly of the crosses - 3/20/19"*

- (2) Old to-do-list item: Figure out whether the anomalous crosses reflect interesting weak-measurement physics.
  - Study the anomalous regions near $\theta_a = 1.1$ and $\theta_a = 1.2$.
  - Near $\theta_a = 1.2$, Mathematica reports that $\theta_\rho = 3.13725$ achieves the min. Set θ_ρ to π, while keeping Mathematica's preferred value of φ_ρ. How much does the LHS change, e.g., by $\sim 10^{-3}$?
  - The LHS-RHS plot displays no anomalies, does it? Just the LHS plot does?

- *(3) Why does LHS-RHS show no anomaly, while the LHS does?*
  - *The LHS wave oscillates across a tiny range of y-values, a range of width* 5×10^{-5}.
  - *The LHS-RHS wave oscillates across a much greater range, a range of width* 10^{-2}.
  - *The LHS oscillations shouldn't be visible in the LHS-RHS.*

- *(4) Implications*
  - *The LHS-RHS should look identical to the RHS.*
  - *We should plot the RHS and check.*

- *(5) How we might tell whether the LHS's anomaly comes from physics or from a bug*
  - *(A) Redo some of the slides that show what the weak measurement adds to entropic uncertainty relations.*
    - *When the LHS or LHS-RHS, at each point on the plot,*
      - *solve for the rho that makes the bound tightest.*
      - *evaluate the function (LHS or LHS-RHS) on that state.*
  - *(B) Look at the LHS plot.*
    Focus on the region near theta_f = pi/2.
    Look for wiggles.