Determination of Higgs branching ratios in $H \to W^+W^- \to \ell\nu jj$ and $H \to ZZ \to \ell^+\ell^- jj$ channels

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The Higgs boson is an elusive boson that is being searched for at the Large Hadron Collider located near Geneva, Switzerland. This particle has a very short lifetime and will not be directly observed in the scope of this experiment, its decay products however, will be observed using very large and intricate detectors placed around the ring of the LHC. The two that are most actively involved in the Higgs search are called ATLAS and CMS. This paper draws from early results and theoretical papers from both detectors to show how to generate branching ratios of the Higgs boson in the scope of Quantum Field Theory. The branching ratios for the Higgs to ZZ and Higgs to WW decays are specifically analyzed here.

INTRODUCTION

To understand the decay patters of the Higgs boson, one must first understand the piece of the standard model that requires the existence of a Higgs field, and therefore a Higgs boson, namely the electroweak sector. This sector is described as an SU(2)XU(1) gauge theory[2] the framework of which will be described later. There are many notable decay channels for the Higgs boson to follow, after a discovery of a standard model Higgs particle it will be important to verify a decay excess in several of the possible channels to be sure the production rates are as predicted. The branching ratios of the Higgs increase as the possible Higgs mass increases, making the identification of a heavy Higgs ($> 400GeV/c^2$) much more difficult than a lighter Higgs. The Large Hadron Collider experiment is uniquely suited to produce Higgs bosons in the proton proton collisions that occur while the experiment is running due to the very high energy of the beams. Currently the LHC is running at a center of mass energy of $\sqrt{s_{pp}} = 8$TeV. At this energy, there will be a strong chance of gluon fusion, a process that produces Higgs particles at high rate. Gluon fusion dominates the Higgs production spectrum at the maximum energy of the LHC ($\sqrt{s_{pp}} = 14$ TeV) for all possible masses of the Higgs. This paper deals with two decay channels that are most prevalent to a heavy Higgs boson namely, the WW and ZZ decays.[2]

The W and Z bosons always decay themselves after being created by the Higgs. There are several decay channels they can follow. For the Z boson, the easiest to distinguish from background is the 4-lepton channel.

$$H \to ZZ \to \ell^+\ell^-\ell^+\ell^- \quad (1)$$

A 4-lepton (4ℓ) final state has almost no background in the scope of the LHC experiment and is very easy to reconstruct with high precision. It is, however a very rare decay channel (less than 0.5% of the total Higgs→ZZ decays will continue on to a 4ℓ decay. The most common final state is the 4-jet final state. A jet is a large spray of particles that originate from a single point, like a cone of debris. Particle jets however are part of a large background signal produced by the numerous QCD processes that occur much more frequently in every collision than Higgs production.[3] A slight middle ground is offered by the semi-leptonic final state

$$H \to ZZ \to \ell^+\ell^- q\bar{q} \quad (2)$$

This channel has a branching ratio more than 20 times larger than the 4ℓ final state and the presence of the leptons allow for removal of some background signal that is associated with the 4 jet final state.[3] The decay channel,

$$H \to W^+W^- \to \ell\nu jj \quad (3)$$

where j is a jet from a quark in the W decay. This has a branching ratio just below 30% which is about 50 times higher than the 4ℓ decay from Z pairs. However there is a large background due to direct W production and $t\bar{(t)}$ production. All of these channels require the Higgs mass to be at least twice the mass of the Z boson, which is considered a heavy Higgs, (greater than 190 GeV). In this paper we will explore the process of calculating the Higgs boson decay rate into the WW and ZZ intermediate states.

THE STANDARD MODEL AND THE ELECTROWEAK SECTOR

The standard model can be divided into two pieces: the Quantum Chromodynamics (QCD) piece which governs the strong interactions of quarks and gluons through color charge. The other sector governs electroweak theory which includes weak and electromagnetic interactions. When developed with a Higgs mechanism, this sector provides mass terms for all the gauge bosons except the photon, which remains massless.

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW} \quad (4)$$

The electroweak Lagrangian is symmetric under SU(2)XU(1) gauge transformations. The U(1) group represents global phase transformations, such as ones that have been touched on in class. This symmetry forces all bosons governed under it to have zero mass. This is experimentally not the case. This feature is not present when spontaneous symmetry breaking is introduced into the theory, as long as the final state is still invariant under local phase transformations (the SU(2) symmetry group). In this way we can obtain a Lagrangian density that is gauge-invariant, re-normalizable and contains mass terms for the $W^{\pm}$ and Z^0 bosons while the photon remains massless. [1] This electroweak Lagrangian density can be further broken up into

$$\mathcal{L}_{EW} = \mathcal{L}_{leptons} + \mathcal{L}_{bosons} \tag{5}$$

where the separated densities describe the leptonic and bosonic parts of the field respectively. As it stands, the mass terms for all bosons in this field will vanish, so we must spontaneously break the symmetry of this state by introducing a Higgs field, a scalar field with a non vanishing vacuum expectation value. This field will not be invariant under the gauge transformations that the Lagrangian is invariant under.[1]

THE HIGGS MECHANISM

Since we now want to break the symmetries of the Lagrangian density spontaneously, we pick a scalar Higgs field in an arbitrary guage

$$\Phi(x) = \begin{pmatrix} \phi_a(x) \\ \phi_b(x) \end{pmatrix} \tag{6}$$

a general SU(2) transformation creates

$$\begin{aligned} \Phi(x) \rightarrow \Phi'(x) =& e^{\frac{ig\tau_j\omega_j(x)}{2}}\Phi(x) \\ \Phi^\dagger(x) \rightarrow \Phi^{\dagger'}(x) =& \Phi^\dagger(x) e^{\frac{-ig\tau_j\omega_j(x)}{2}} \end{aligned} \tag{7}$$

and a U(1) transformation,

$$\begin{aligned} \Phi(x) \rightarrow \Phi'(x) =& e^{ig'Yf(x)}\Phi(x) \\ \Phi^\dagger(x) \rightarrow \Phi^{\dagger'}(x) =& \Phi^\dagger(x) e^{-ig'Yf(x)} \end{aligned} \tag{8}$$

where Y is the weak hypercharge of the field $\Phi(x)$[1]. Now we can add this Higgs field and its interactions with the fields representing gauge bosons to the Lagrangian density. The simplest way to do this is

$$\mathcal{L}_{EW} = \mathcal{L}_{leptons} + \mathcal{L}_{bosons} + \mathcal{L}_{Higgs} \tag{9}$$

where

$$\mathcal{L}_H = [\delta^\mu\Phi(x)]^\dagger[\delta_\mu\Phi(x)] - \mu^2\Phi^\dagger(x)\Phi(x) - \lambda[\Phi^\dagger(x)\Phi(x)]^2. \tag{10}$$

This Higgs field $\Phi(x)$ must have a nonzero vacuum expectation value and must also spontaneously break symmetry, we can do so by choosing an initial value for $\Phi(x)$

$$\Phi_0 = \begin{pmatrix} \phi_a^0 \\ \phi_b^0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} + h(x) \end{pmatrix} \tag{11}$$

where

$$v = (-\frac{\mu^2}{\lambda})^{1/2} \tag{12}$$

which is always greater than zero. This Φ_0 is allowed because any other choice of Φ_0 is related to (11) a global phase transformation. This ground state of the Higgs field (11) is not necessarily invariant under SU(2)XU(1) transformations, but it must be invariant under U(1) gauge transformations. This set of U(1) gauge transformations includes all electromagnetic types of transform, this allows a zero mass term for the electromagnetic boson i.e. the photon and allows for charge conservation globally. Using a Φ_0 such as (11) requires the bosonic $W^{\pm}$ and Z fields to acquire a mass

$$m_W = \frac{v}{2}g \quad \& \quad m_Z = \frac{v}{2}\sqrt{g^2 + g'^2} \tag{13}$$

also, a new physical particle has been introduced described by the $h(x)$ in (11) with mass

$$m_H = \sqrt{2}\mu = \sqrt{2\lambda}v \tag{14}$$

This is the Higgs boson. The Lagrangian density given in (9) includes only terms for the free fields. We can also add interaction terms between all pairs of the following 3 field types, fermionic, bosonic and Higgs fields, also the interaction terms of each field with itself can be added as well. For the scope of this paper, we will take only the interaction term between fermions and the Higgs field.

$$\mathcal{L}_{FH} = -\frac{1}{v}m_f\bar{\psi}_f\psi_f\sigma \tag{15}$$

A sum over all fermions is implicit in (15). The m_f term here represents the mass of whichever fermion is being used in the calculation. As can be seen from (15) the Lagrangian density of the interactions between fermions and the Higgs field are proportional to their mass. This is how the conclusion is drawn that he Higgs boson will couple to fermions based on their mass and perhaps dictates mass, this term disappears for neutrinos if they are massless. [2]

DECAY RATES AND BRANCHING FRACTIONS

Starting with the information given in [1] where the Feynman rules for Higgs decay verticies are shown, we take the $H \rightarrow ZZ$ vertex and the $H \rightarrow W^+W^-$ vertex.

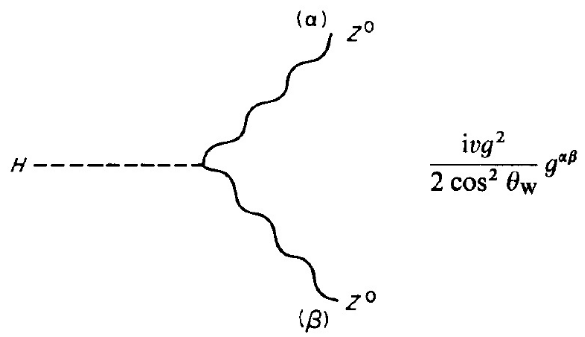

FIG. 1: Branching diagram for Higgs to ZZ decay with Feynman amplitude rule. [1]

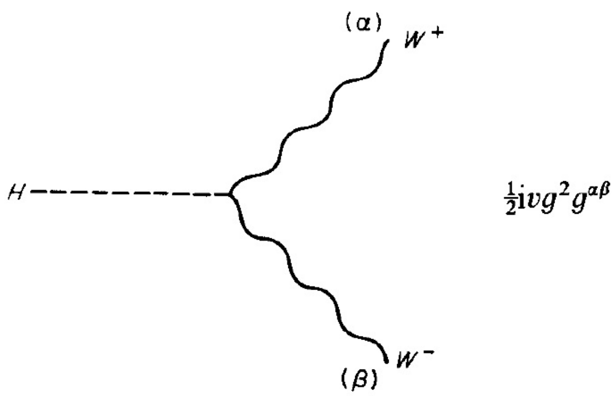

FIG. 2: Branching diagram for Higgs to WW decay with Feynman amplitude rule [1]

using these Feynman rules, and the masses for the W and Z bosons from (13) we can see that the Feynman amplitude matrix element is

$$i\mathcal{M}_{H\to ZZ} = \sum_{\lambda,\rho} ig\frac{m_z}{\cos\Theta_W} g_{\mu\nu}\epsilon^{*\mu}_{1\lambda}\epsilon^{*\nu}_{2\rho} \tag{16}$$

where the ϵ's are based on the polarization and λ, ρ are the polarization indices. For this, a two body, equal mass final state, the decay rate is given as

$$\frac{d\Gamma}{d\Omega} = \frac{\sqrt{\lambda\left(m_H^2, m_Z^2, m_Z^2\right)}}{64\pi^2 m_H^3}|\mathcal{M}|^2\mathcal{S} \tag{17}$$

with λ in a quadratic form

$$\sqrt{\lambda\left(m_H^2, m_Z^2, m_Z^2\right)} = m_H^2\sqrt{1-\frac{2m_Z}{m_H}^2} \tag{18}$$

and

$$\mathcal{S} = \prod_k n_k^{-1} = 2^{-1} \tag{19}$$

where n is the number of identical final state particles of type k i.e. 2 in the Z case, 1 in the W case. The squared matrix element from (16) can now be written as

$$|\mathcal{M}|^2 = \left(\frac{gm_Z}{\cos\Theta_W}^2 \sum_{\lambda,\rho} g_{\mu\nu}\epsilon^{*\mu}_{1\lambda}\epsilon^{*\nu}_{2\rho}g_{\alpha\beta}\epsilon^{*\alpha}_{1\lambda}\epsilon^{*\beta}_{2\rho}\right) \tag{20}$$

The summation of the three polarization states of the Z boson is

$$\sum_\lambda \epsilon^\mu_\lambda\epsilon^{*\nu}_\lambda = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_Z^2} \tag{21}$$

if one were to transform to the rest frame of the Z boson, this would look like 3 orthonormal vectors so that the sum would become

$$\sum_\lambda e^i_\lambda e^j_\lambda = \delta^{ij} \tag{22}$$

now simplify the matrix element $\mathcal{M}$

$$\begin{aligned}
|\mathcal{M}|^2 &= \left(\frac{gm_Z}{\cos\Theta_W}\right)^2 g_{\mu\nu}\left(-g^{\mu\alpha}+\frac{p^\mu p^\alpha}{m_Z^2}\right)g_{\alpha\beta}\left(-g^{\nu\beta}+\frac{q^\nu q^\beta}{m_Z^2}\right)\\
&= \left(\frac{gm_Z}{\cos\Theta_W}\right)^2\left(-g^\alpha_\nu+\frac{p_\nu p^\alpha}{m_Z^2}\right)\left(-g^\nu_\alpha+\frac{q^\nu q_\alpha}{m_Z^2}\right)\\
&= \left(\frac{gm_Z}{\cos\Theta_W}\right)^2\left(4-\frac{p_\alpha p^\alpha}{m_Z^2}-\frac{q^\nu q_\nu}{m_Z^2}+\frac{p_\nu q^\nu p^\alpha q_\alpha}{m_Z^4}\right)\\
&= \left(\frac{gm_Z}{\cos\Theta_W}\right)^2\left(2+\frac{(p\cdot q)^2}{m_Z^4}\right)
\end{aligned} \tag{23}$$

where p and q are the 4 momentums of the two Z bosons. Because they both were produced by the same Higgs, we require

$$\begin{aligned}
m_H^2 &= (p+q)^2\\
&= p^2+q^2+2p\cdot q\\
&= 2m_Z^2+2p\cdot q\\
&\quad so\\
p\cdot q &= \frac{m_H^2-2m_Z^2}{2}
\end{aligned} \tag{24}$$

This property further reduces $\mathcal{M}$

$$\begin{aligned}
|\mathcal{M}|^2 &= \left(\frac{gm_Z}{\cos\Theta_W}\right)^2\left(2+\frac{\left(m_H^2-2m_Z^2\right)^2}{4m_Z^4}\right)\\
&= \left(\frac{gm_H^2}{2m_Z\cos\Theta_W}\right)^2\left(1-\frac{4m_Z^2}{m_H^2}+\frac{12m_Z^4}{m_H^4}\right)\\
&= \left(\frac{gm_H^2}{2m_W}\right)^2\left(1-\frac{4m_Z^2}{m_H^2}+\frac{12m_Z^4}{m_H^4}\right)
\end{aligned} \tag{25}$$

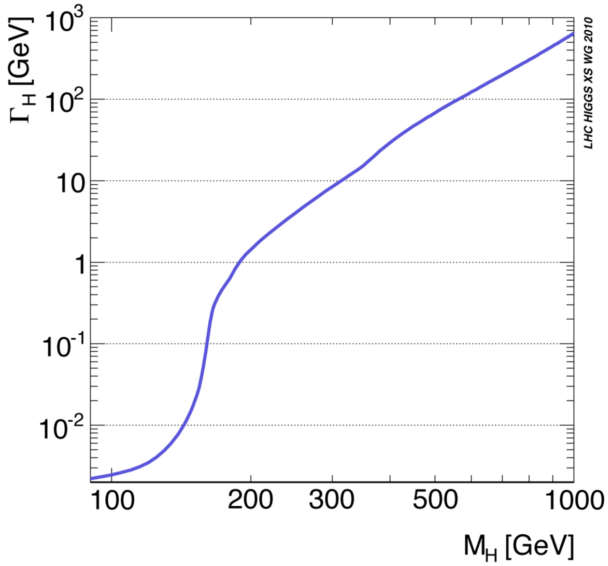

FIG. 3: Total Higgs decay rate as a function of m_H. [4]

Then the partial decay rate to ZZ is

$$\begin{aligned}\Gamma_{H\to ZZ} &= \int d\Omega \frac{d\Gamma}{d\Omega} \\ &= \frac{g}{512\pi^2}\frac{m_H^3}{m_W^2}\sqrt{1-\left(\frac{2m_Z}{m_H}\right)}\left(1-\frac{4m_Z^2}{m_H^2}+\frac{12m_Z^4}{m_H^4}\right)\int d\Omega \\ &= \frac{g}{128\pi}\frac{m_H^3}{m_W^2}\sqrt{1-\left(\frac{2m_Z}{m_H}\right)}\left(1-\frac{4m_Z^2}{m_H^2}+\frac{12m_Z^4}{m_H^4}\right)\end{aligned} \tag{26}$$

The partial decay rate for $H \to W^{\pm}$ is exactly the same, except for a factor of one half taken away in (19) because W^+ and W^- are not identical particles.[2]

In order to get a Branching ratio into ZZ and WW we need to take take the ratio between the partial decay rates of the individual final states and the total Higgs decay rate into all final states. This total decay rate (ideally) includes all possible decay channels for the Higgs boson at the correct center of mass energy (in this case the maximum energy of the LHC). This is very difficult to derive, and will simply be shown here as a function of Higgs mass. and shown here is a graph of all results of branching ratios, the two that we calculated, $H \to ZZ$ and $H \to W^{\pm}$ are labeled.

SUMMARY

This is not a completely final representation of the products of Higgs production in the LHC. As was mentioned in the introduction, the W and Z bosons themselves are not stable, especially when produced by a Higgs particle with mass below $2m_W \& 2m_Z$ respectively. The

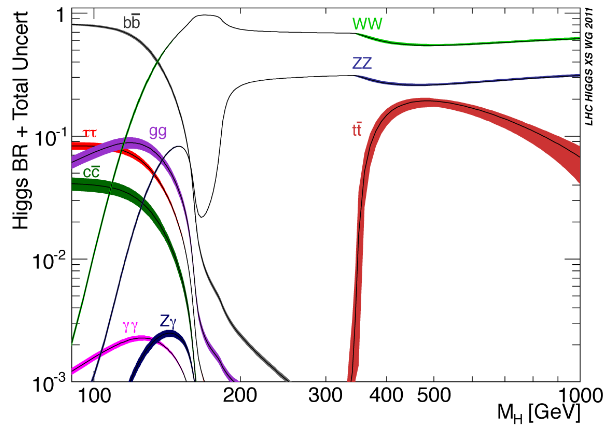

FIG. 4: Higgs branching ratios for a range of m_H. [4]

W and Z particles then decay into different channels like the quad jet, 4ℓ and di-jet discussed earlier. However these two decays form the majority of expected Higgs products, especially for a heavy Higgs boson. The search for a Higgs boson extends very far beyond the standard model of particle physics and the discovery of a heavy higgs would imply the presence of new physical processes that are unpredicted by the theory in it's current form. With an increase in luminosity delivered to the Atlas and CMS detectors in coming years, the sensitivity of the 4ℓ channel is expected to rise and it is likely to become the dominant search channel in the high mass range.

[1] F. Mandl and G. Shaw *Quantum Field Theory*, **Ch 13-14**, John Wiley and Sons Ltd., 1984.

[2] Ulrik Egede *The search for a standard model Higgs at the LHC and electron identification using transition radiation in the ATLAS tracker*, **Ch 8**, (1997), Lund University Sweden.

[3] Francesco Pandolfi *Search for the Standard Model Higgs Boson in the $H \to ZZ \to \ell^+\ell^- q\bar{q}$decay channel at CMS* **Ch 1**, (2011), University of Rome

[4] CERN Higgs Cross Section Working Group *https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CrossSections* (2012), CERN and Contributing Authors