

Overview of Linacs

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What?
When ?
How?

What is a linac

- LINear ACcelerator : single pass device that increases the energy of a charged particle by means of a (radio frequency) electric field and it is equipped with magnetic elements (quadrupoles, solenoids, bending magnets) to keep the charged particle on a given trajectory.
- Motion equation of a charged particle in an electromagnetic field

$$\frac{d\vec{p}}{dt} = q \cdot (\vec{E} + \vec{v} \times \vec{B})$$

\vec{p} = momentum = $\gamma m_0 \vec{v}$
 q, m_0 = charge, mass
 \vec{E}, \vec{B} = electric, magnetic field
 t = time
 \vec{x} = position vector
 $\vec{v} = \frac{d\vec{x}}{dt}$ = velocity

What is a linac-cont'ed

$$\frac{d}{dt} \left(\gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left(\vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right)$$

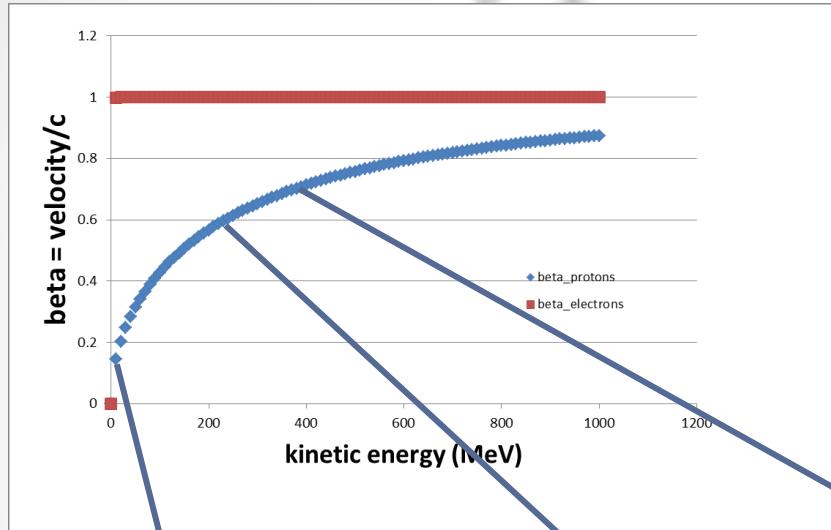
type of particle :
charge couples with the field, mass slows the acceleration

Relativistic or not

type of focusing

type of RF structure

Type of particles



- electron, mass 0.511 MeV, quickly relativistic,easi(er) to accelerate
- proton, mass 938.28 MeV, $q/m=1$
- carbon ion , mass 11.3 GeV , $q/m=1/3$ and then $\frac{1}{2}$.

At 7 MeV
beta =0.12,
gamma=1.01

At 250 MeV
beta =0.6 ,
gamma=1.26

At 450 MeV
beta =0.74 ,
gamma=1.48

Electron linacs

To order these specialized brazed products contact us today.

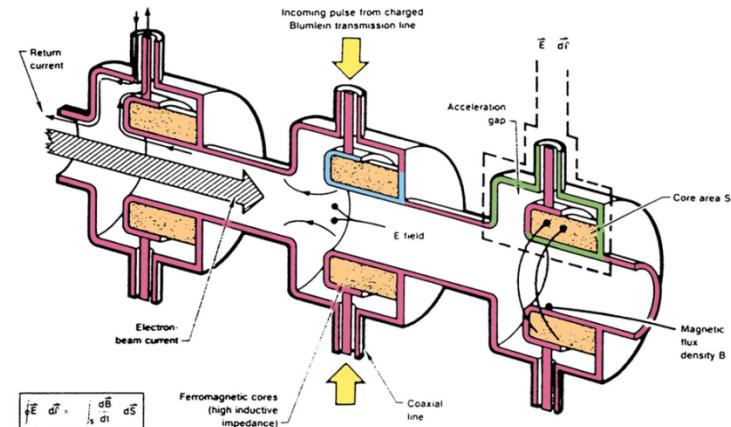


- Energy range of linacs: 4-25 MeV
- Electrons are accelerated by microwaves (10^3 - 10^4 MHz)
- Philips SL-75/5: S-band 2856 MHz, MW cavities dimensions - lenght 3 cm, radius 5 cm, electrons 5 MeV, tungsten target



For sake of completeness

- Static- Efield Linacs :
- device which provides a constant potential difference (and consequently electric field) ..
- acceleration is limited to few MeV. Limitation comes from electric field breakdown
- still used in the very first stage of acceleration when ions are extracted from a source.
- Time varying- Efield Linacs :
- Induction linacs : based on the magnetic induction principle.



The magnetic induction accelerator principle

Electrostatic field



750 keV Radio Frequency
accelerator (2m long, 0.5 m across)

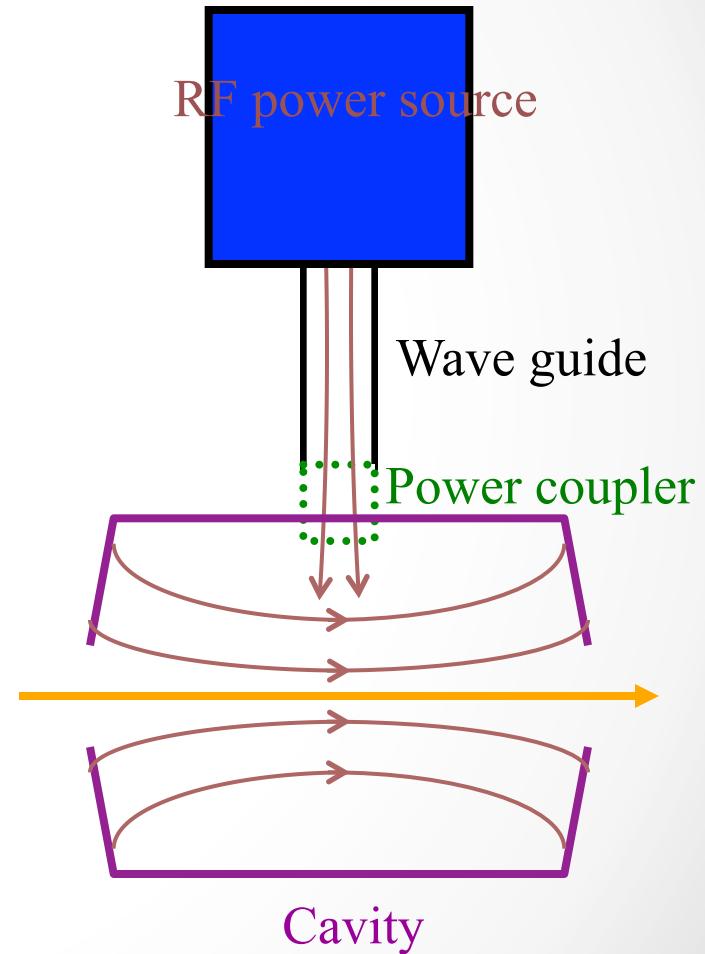
750 kV Cockcroft-Walton

When ? A short history

- Acceleration by **time varying** electromagnetic field overcomes the limitation of static acceleration
- First experiment towards an RF linac : Wideroe linac 1928 on a proposal by Ising dated 1925. A bunch of potassium ions were accelerated to 50 keV in a system of drift tubes in an evacuated glass cylinder. The available generator provided 25 keV at 1 MHz.
- First realization of a linac : 1931 by Sloan and Lawrence at Berkeley laboratory
- From experiment to a practical accelerator : Wideroe to Alvarez
 - to proceed to higher energies it was necessary to increase by order of magnitude the frequency and to enclose the drift tubes in a RF cavity (resonator)
 - this concept was proposed and realized by Luis Alvarez at University of California in 1955 : A 200 MHz 12 m long Drift Tube Linac accelerated protons from 4 to 32 MeV.
 - the realization of the first linac was made possible by the availability of high-frequency power generators developed for radar application during World War

How? principle of an RF linac

- 1) **RF power source**: generator of electromagnetic wave of a specified frequency. It feeds a
- 2) **Cavity** : space enclosed in a metallic boundary which resonates with the frequency of the wave and tailors the field pattern to the
- 3) **Beam** : flux of particles that we push through the cavity when the field is maximized as to increase its
- 4) **Energy.**



Brief description of Radio Frequency Linear Accelerators for protons and light ions for medical applications with the purpose of understanding

- 1) Why medical linacs are designed the way they are
- 2) what are the important issues should you ever attempt to design one.

What is a linac-cont'ed

$$\frac{d}{dt} \left(\gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left(\vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right)$$

$q/m=1/1$ to $q/m=1/3$

From 1 to 1.26

type of focusing

type of RF
structure

Types of RF structures

Type of structure	Used
Radio Frequency Quadrupole	HIT,CNAO, MEDAUSTRON, ADAMS,TULIP2.0
Interdigital-H structure	HIT CNAO MEDAUSTRON
Drift Tube Linac	IMPLART (ENEA FRASCATI) / TULIP2.0
Cell Coupled Linac also called Side Coupled Linac	ADAMS, TULIP

wave equation -recap

- Maxwell equation for E and B field:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = 0$$

- In **free space** the electromagnetic fields are of the *transverse electro magnetic*,TEM, type: the electric and magnetic field vectors are \perp to each other and to the direction of propagation.
- In a **bounded medium** (cavity) the solution of the equation must satisfy the boundary conditions :

$$\begin{aligned}\vec{E}_{//} &= \vec{0} \\ \vec{B}_{\perp} &= \vec{0}\end{aligned}$$

TE or TM modes

- TE (=transverse electric) : the electric field is perpendicular to the direction of propagation. in a cylindrical cavity

$$TE_{nml}$$

n : azimuthal,

m : radial

1 longitudinal component

- TM (=transverse magnetic) : the magnetic field is perpendicular to the direction of propagation

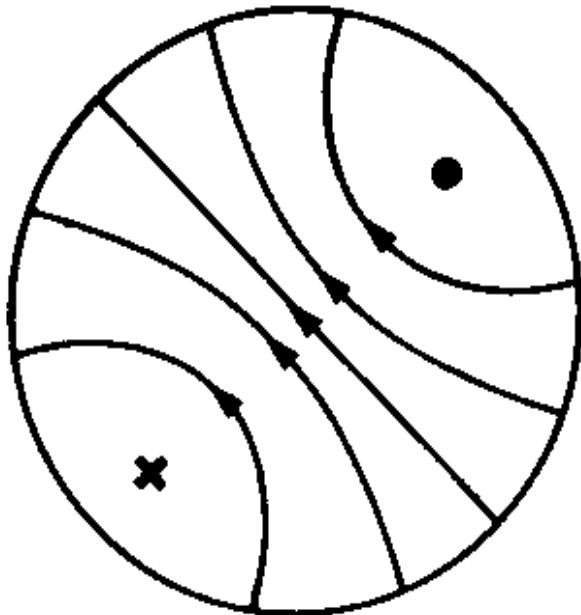
$$TM_{nml}$$

n : azimuthal,

m : radial

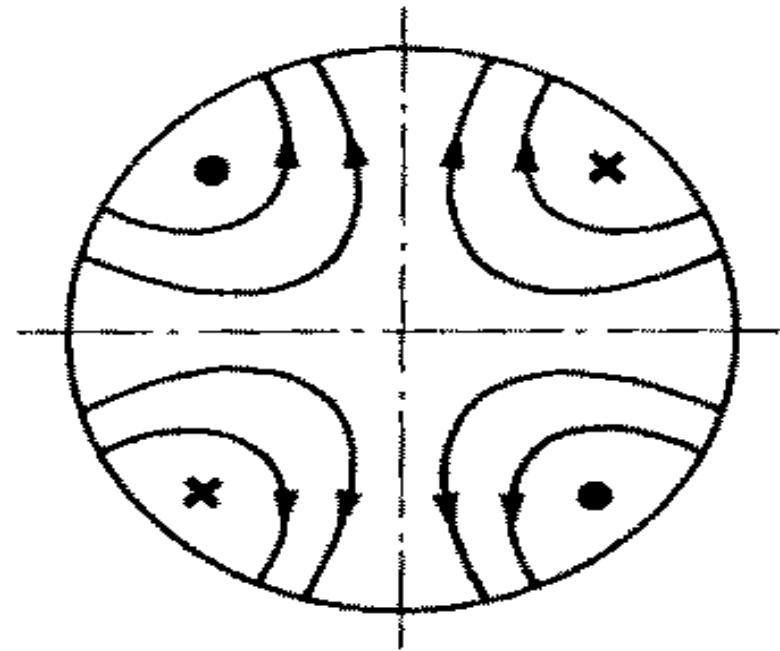
1 longitudinal component

TE modes



Empty cavity; mode TE_{11}

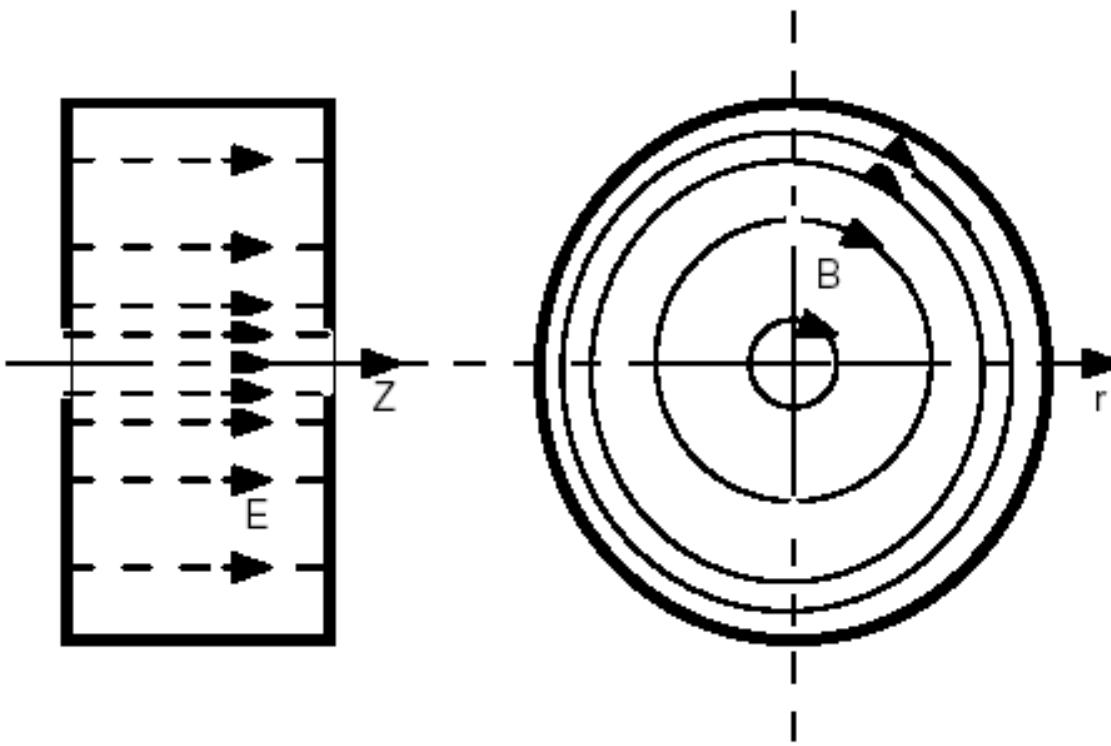
dipole mode used in the IH structures



Empty cavity; mode TE_{21}

quadrupole mode used in Radio Frequency Quadrupole

TM modes



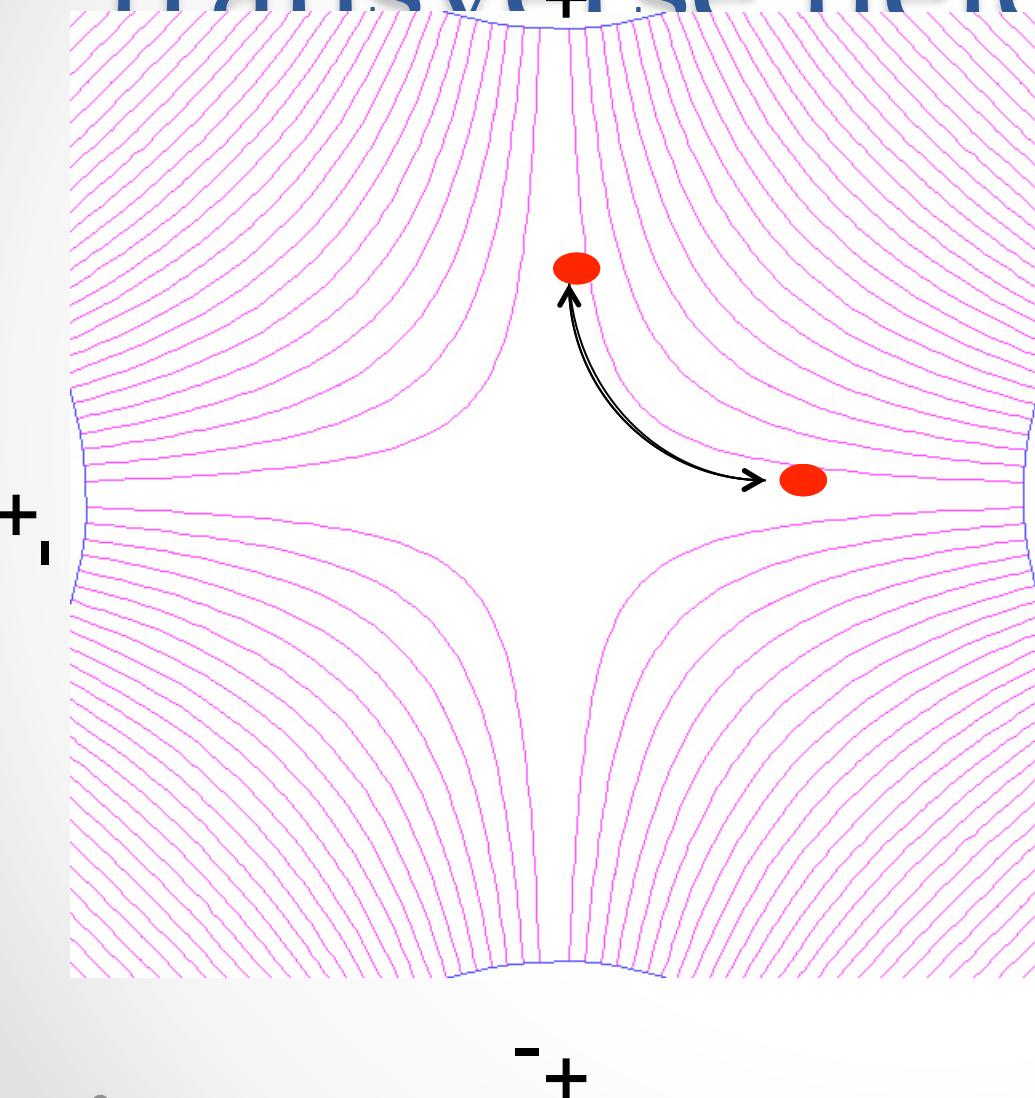
TM010 mode , most commonly used accelerating mode

Radio Frequency Quadrupoles

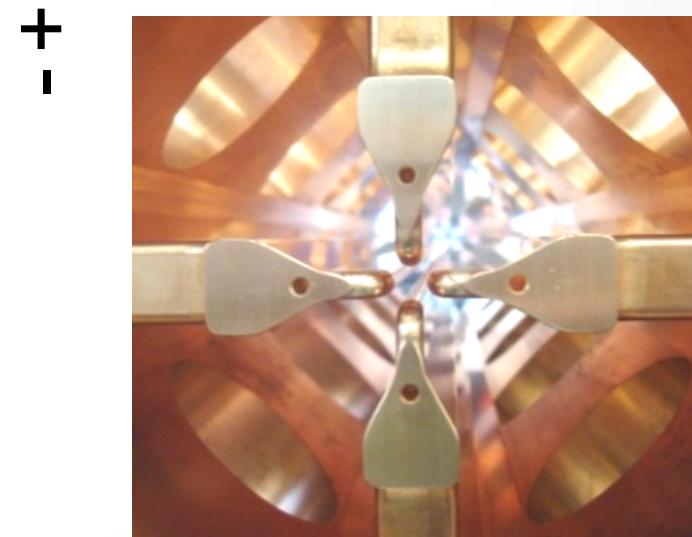


TE or TM?

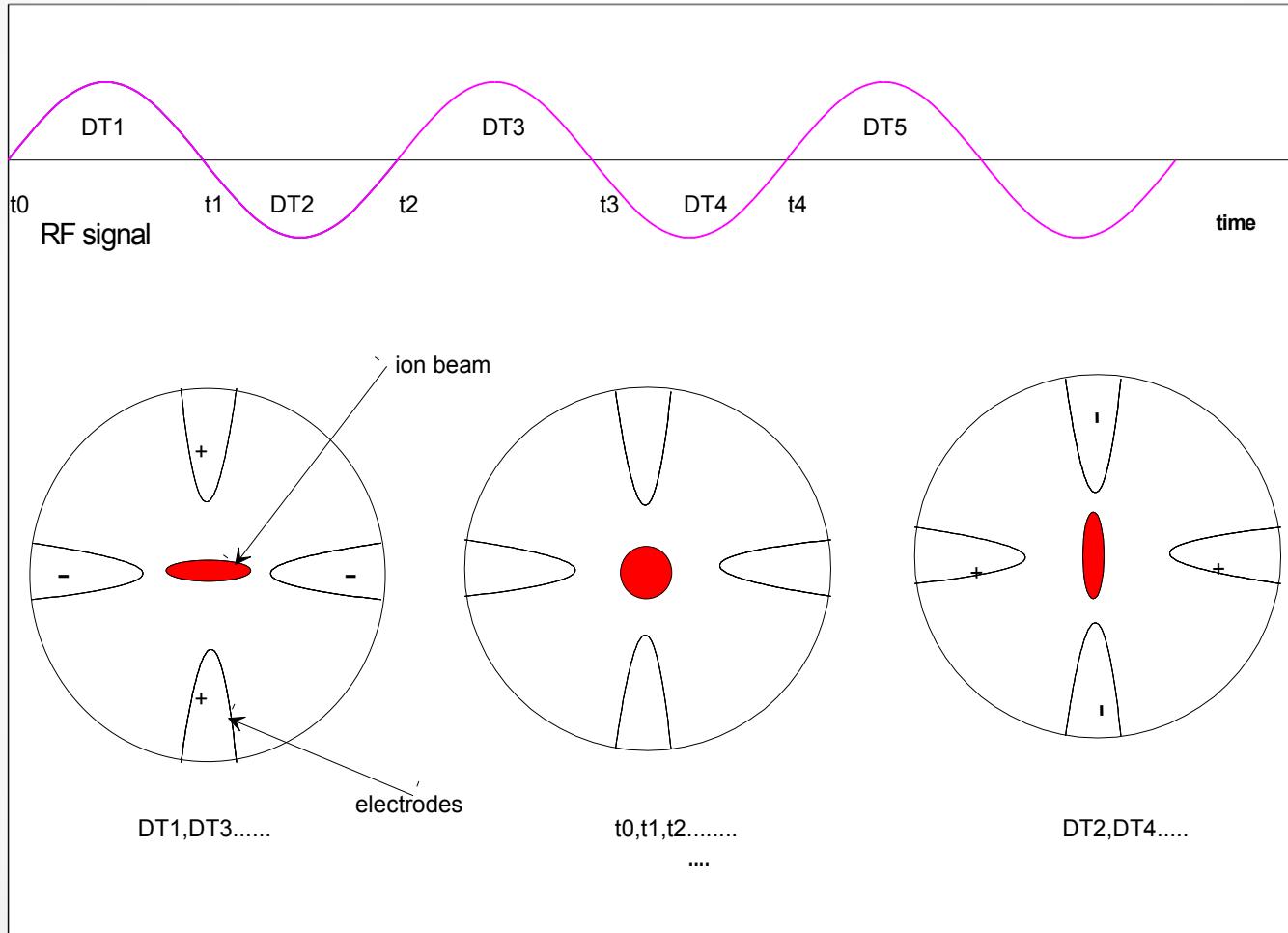
transverse field in an RFQ



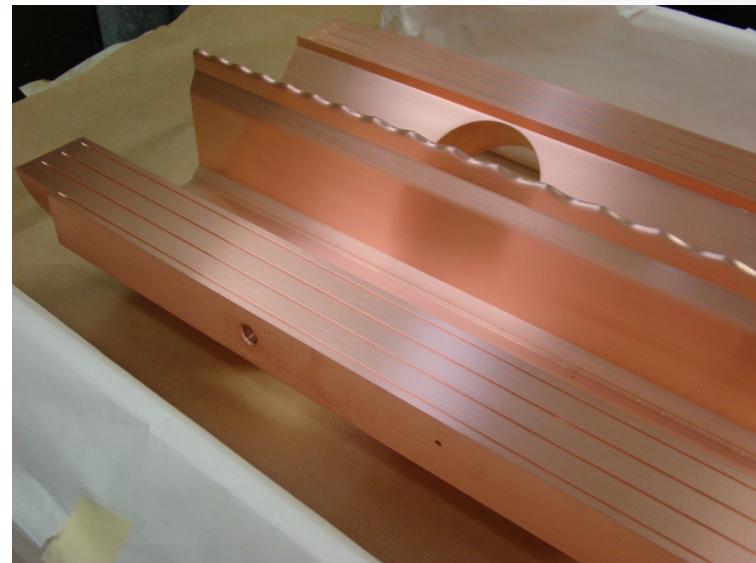
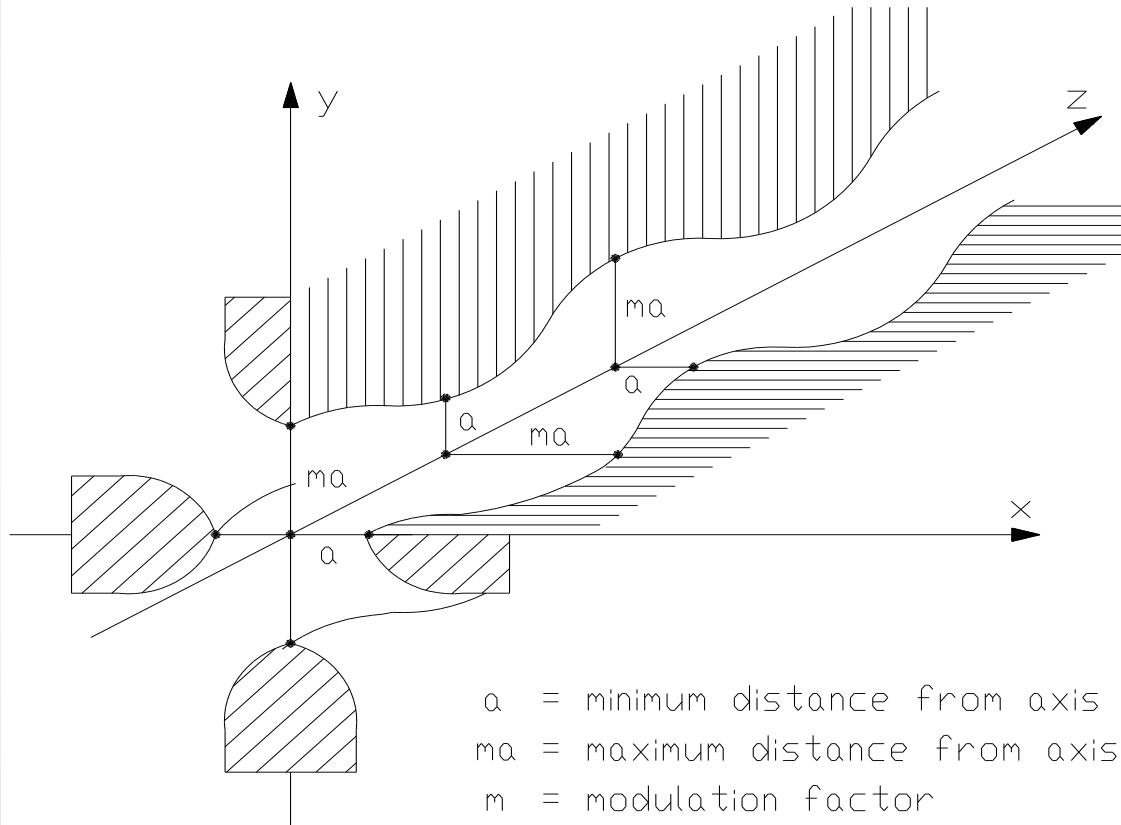
alternating gradient
focussing structure with
period length $\beta\lambda$
(in half RF period the
particles have travelled a
length $\beta\lambda/2$)



transverse field in an RFQ



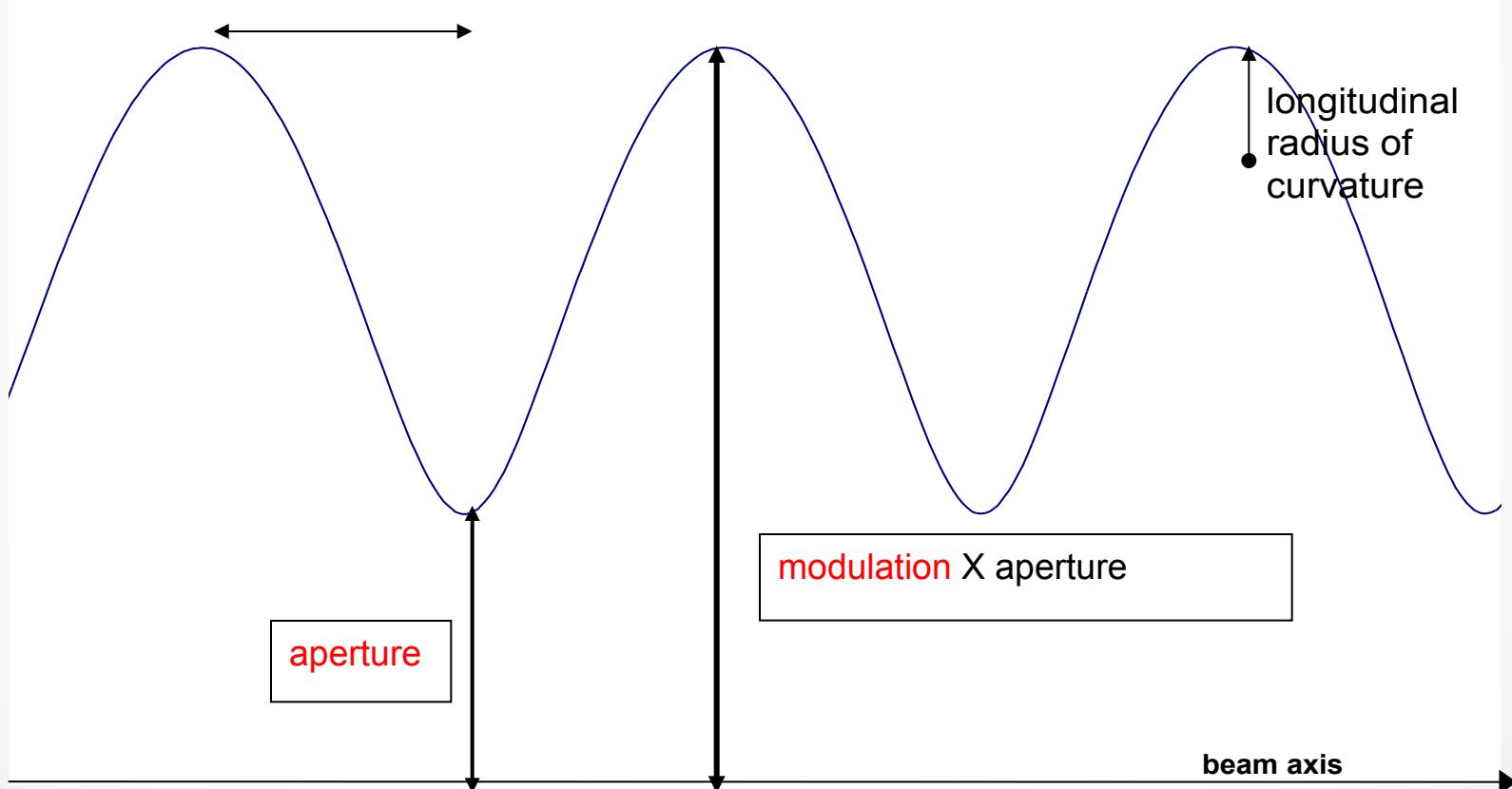
acceleration in RFQ



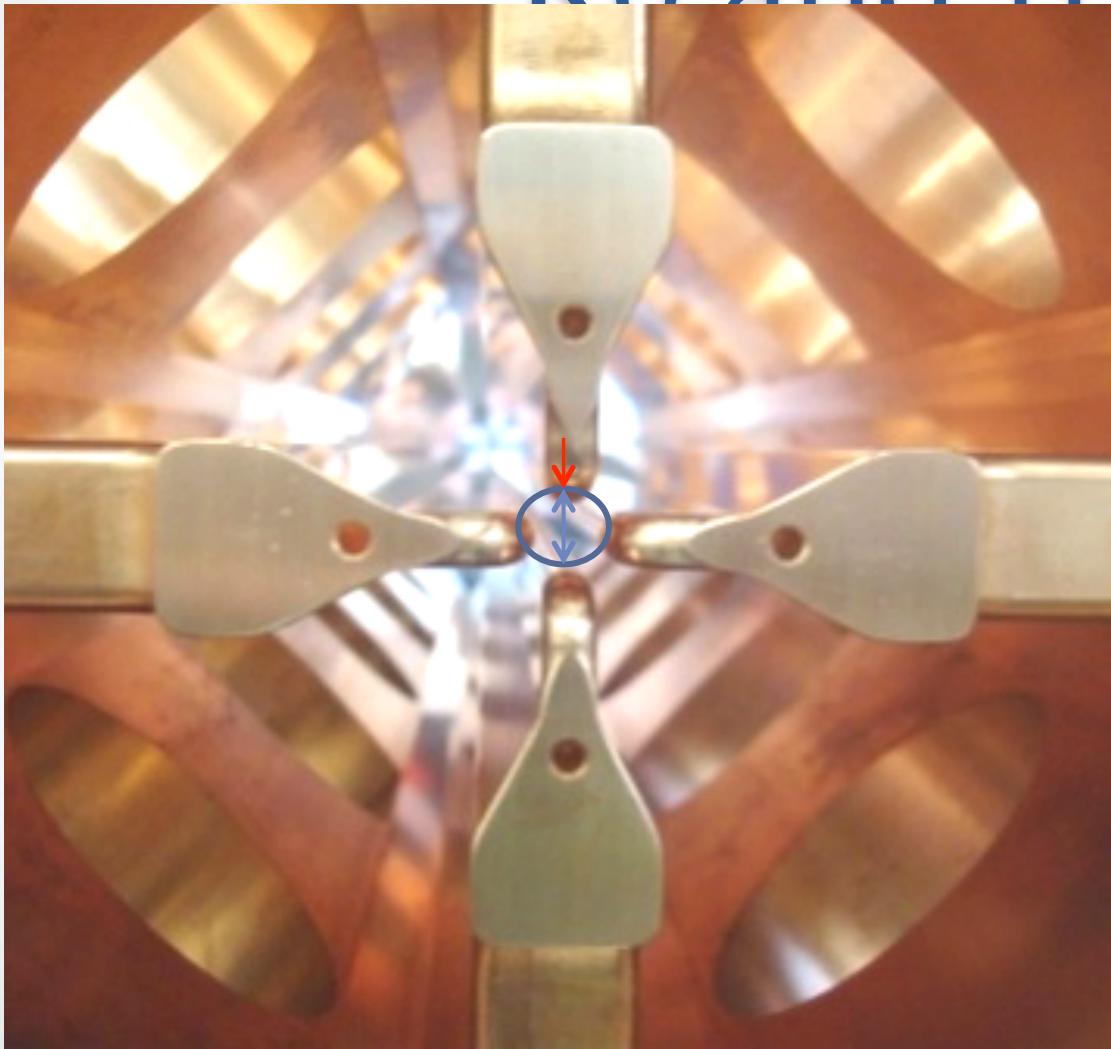
longitudinal modulation on the electrodes creates a longitudinal component in the TE mode

acceleration in an RFQ

$$\frac{\beta\lambda}{2} \left(1 - \frac{\Delta\varphi}{2\pi}\right)$$



R_0 and ρ



$2R_0$ = average
distance between
opposite vanes

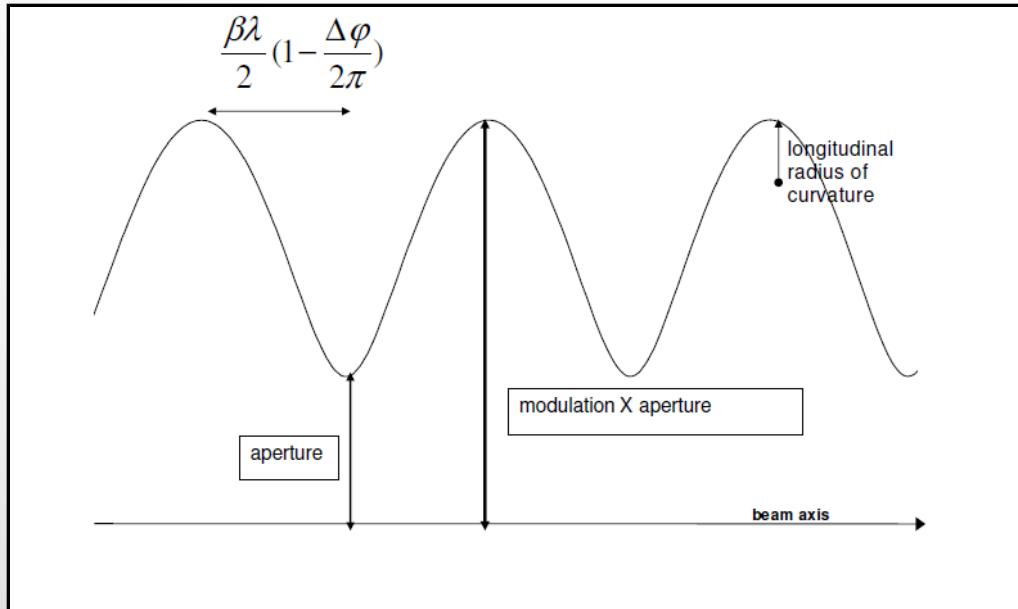
↓
 ρ : Transverse
radius of curvature
of the vane-tip.

Beam goes into the paper
in between the 4
vane tips

Modulation and Rhøl



Looking in from the RF port : these are adjacent



important parameters of the RFQ

$$B = \left(\frac{q}{m_0} \right) \left(\frac{V}{a} \right) \left(\frac{1}{f^2} \right) \frac{1}{a} \left(\frac{I_o(ka) + I_o(mka)}{m^2 I_o(ka) + I_o(mka)} \right)$$

type of particle

limited by
sparking

Transverse field distortion due to
modulation (=1 for un-modulated
electrodes)

$$E_0 T = \frac{m^2 - 1}{m^2 I_o(ka) + I_o(mka)} \cdot V \frac{2}{\beta \cdot \lambda} \frac{\pi}{4}$$

Accelerating efficiency : fraction of the field
deviated in the longitudinal direction
(=0 for un-modulated electrodes)

cell
length

transit
time factor

....and their relation

$$\left(\frac{I_o(ka) + I_o(mka)}{m^2 I_o(ka) + I_o(mka)} \right) + \frac{m^2 - 1}{m^2 I_o(ka) + I_o(mka)} \cdot I_0(ka) = 1$$

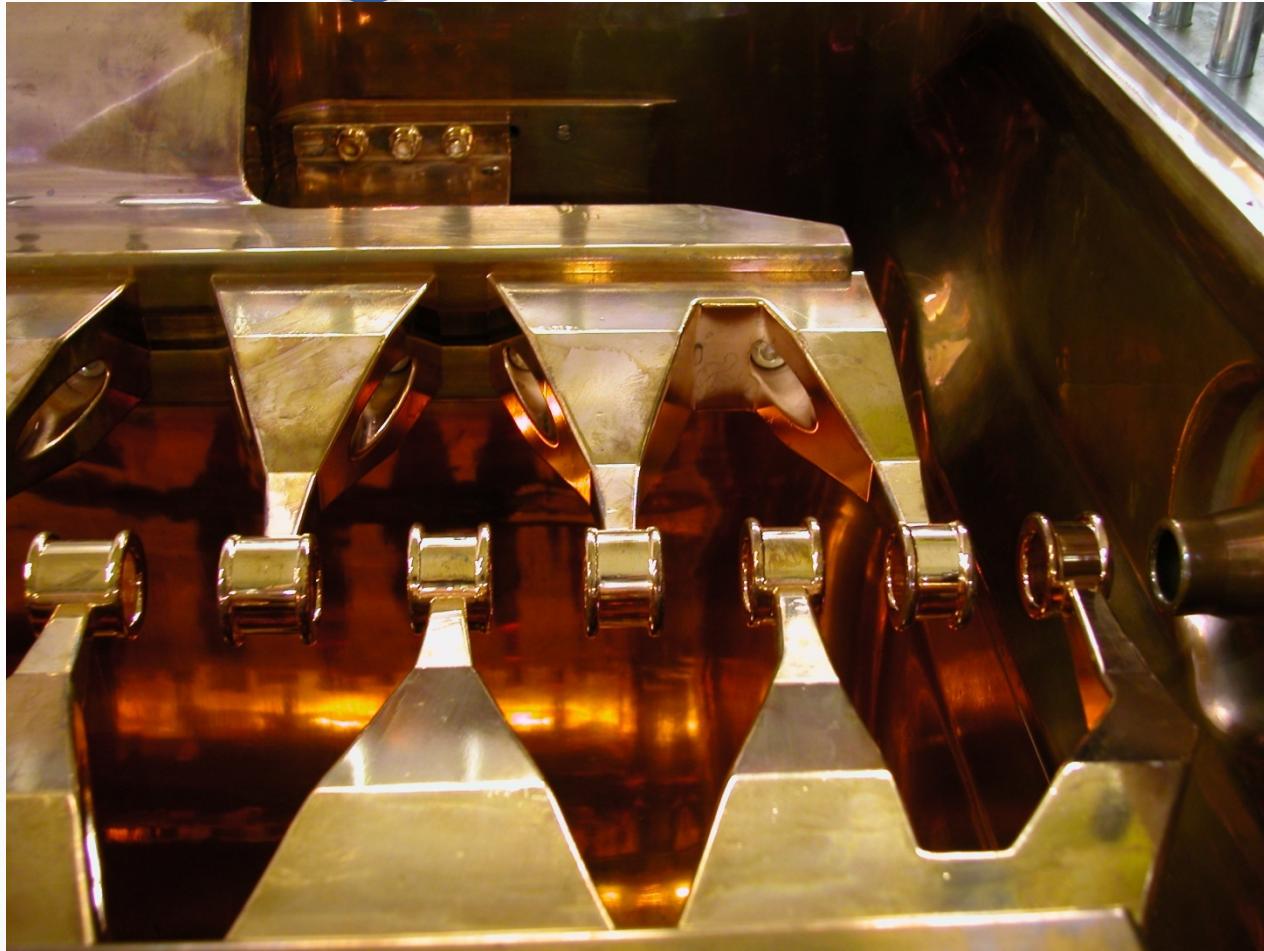
|
focusing efficiency |
accelerating efficiency

a =bore radius, β, γ =relativistic parameters, c =speed of light, f = rf frequency,
 I_0, I_1 =zero,first order Bessel function, k =wave number, λ =wavelength,
 m =electrode modulation, m_0 =rest q =charge, r = average transverse beam
dimension, r_0 =average bore, V =vane voltage

RFQ

- The resonating mode of the cavity is a focusing mode
- Alternating the voltage on the electrodes produces an alternating focusing channel
- A longitudinal modulation of the electrodes produces a field in the direction of propagation of the beam which bunches and accelerates the beam
- Both the focusing as well as the bunching and acceleration are performed by the RF field
- Not very efficient accelerator
- The RFQ is the only linear accelerator that can accept a low energy CONTINUOUS beam of particles
- 1970 Kapchinskij and Teplyakov propose the idea of the radiofrequency quadrupole (I. M. Kapchinskii and V. A. Teplvakov, Prib.Tekh. Eksp. No. 2, 19 (1970))

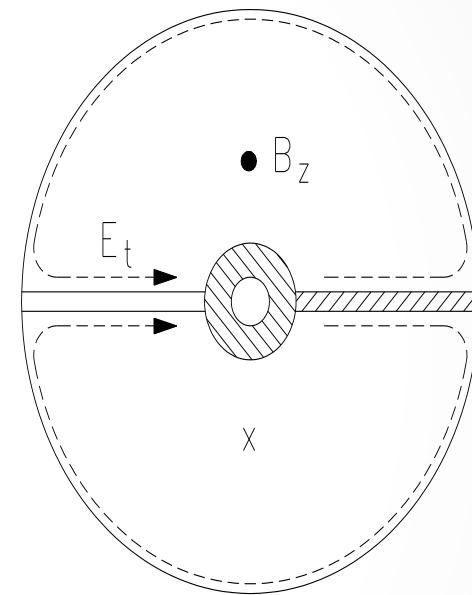
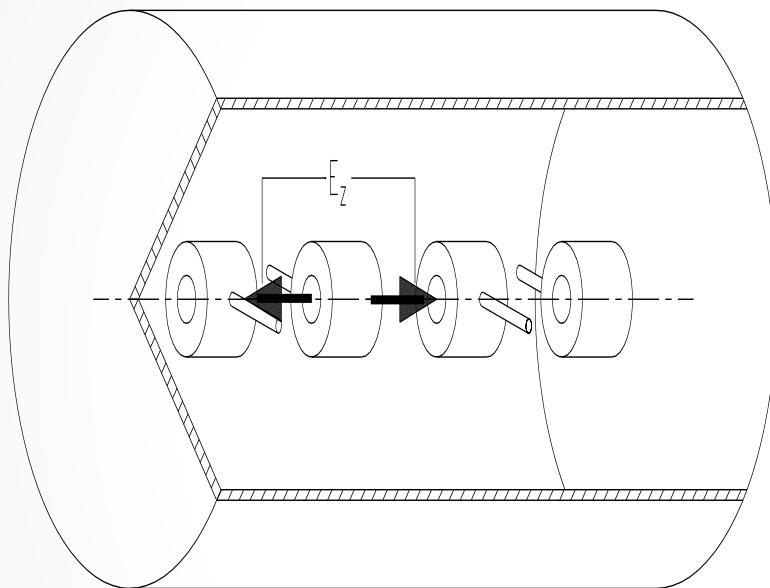
Interdigital H structure



CNAO IH

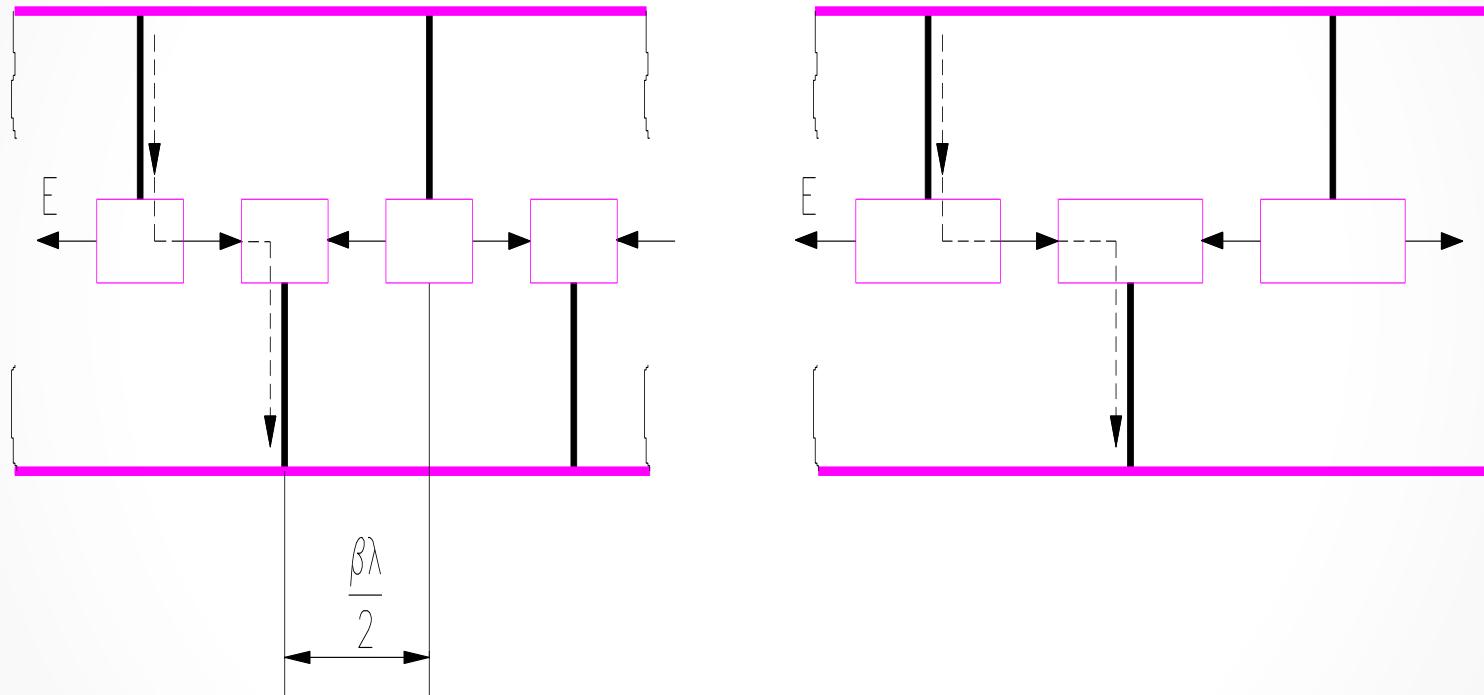


Interdigital H structure



the mode is the TE110

Interdigital H structure



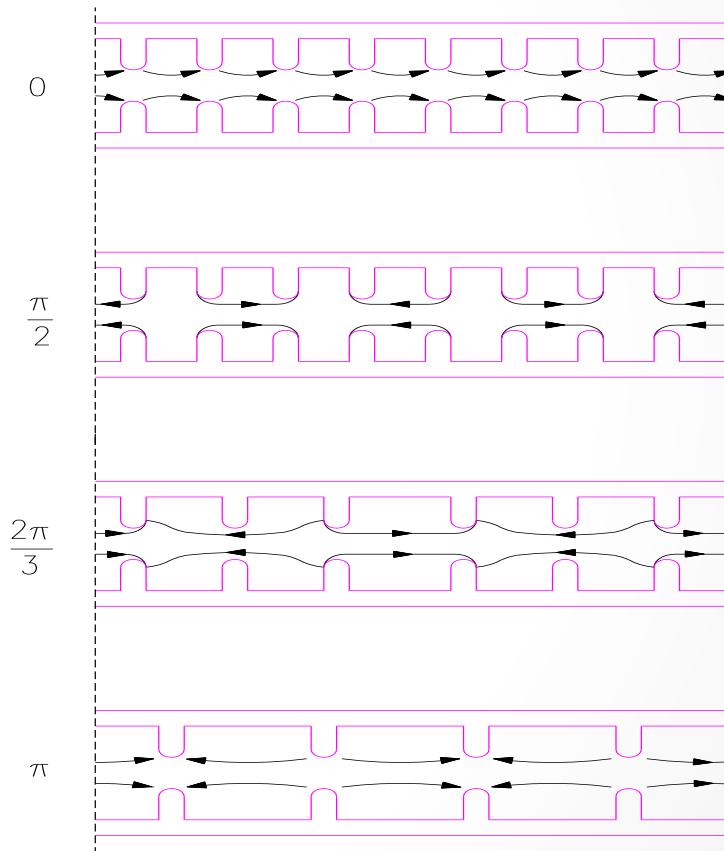
- stem on alternating side of the drift tube force a longitudinal field between the drift tubes
- focalisation is provided by quadrupole triplets places OUTSIDE the drift tubes or OUTSIDE the tank

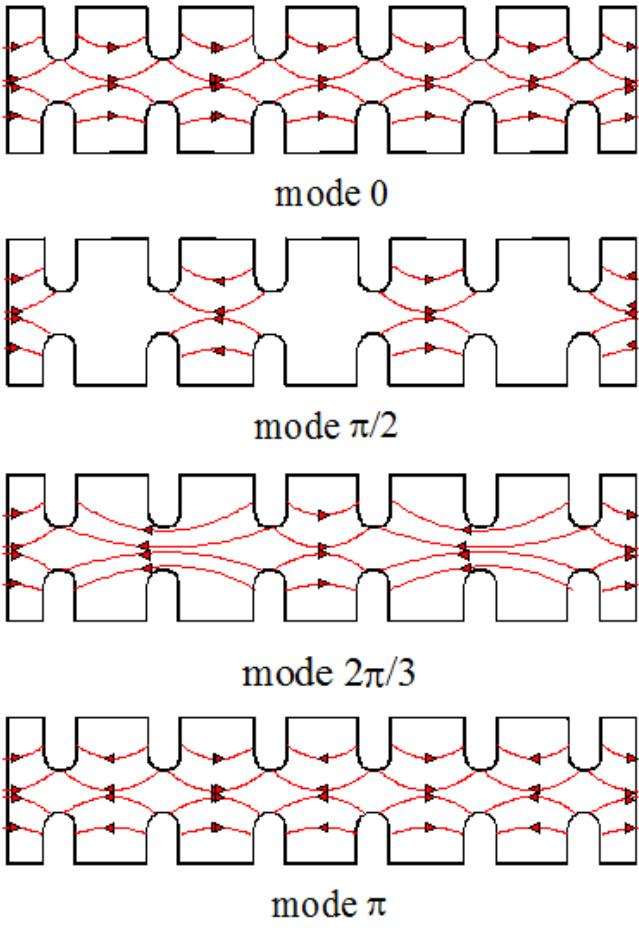
IH

- The resonating mode of the cavity is a dipole mode mode
- The cavity is equipped with thin drift tubes.
- Alternating the stems on each side of the drift tubes produces a field in the direction of propagation of the beam which accelerates the beam
- Focusing is provided by quadrupole triplets located inside the tank in a dedicated section
- Very efficient in the low beta region ($(\beta \approx 0.02 \text{ to } 0.08)$) and low frequency (up to 200MHz)
- not for high intensity beam due to long focusing period
- ideal for low beta ion acceleration

cavity modes

- **0-mode** Zero-degree phase shift from cell to cell, so fields adjacent cells are in phase. Best example is DTL.
- **π -mode** 180-degree phase shift from cell to cell, so fields in adjacent cells are out of phase. Best example is multicell superconducting cavities.
- **$\pi/2$ mode** 90-degree phase shift from cell to cell. In practice these are biperiodic structures with two kinds of cells, accelerating cavities and coupling cavities. The CCL operates in a $\pi/2$ structure mode. This is the preferred mode for very long multicell cavities, because of very good field stability.





Named from the phase difference between adjacent cells.

Mode 0 also called mode 2π .

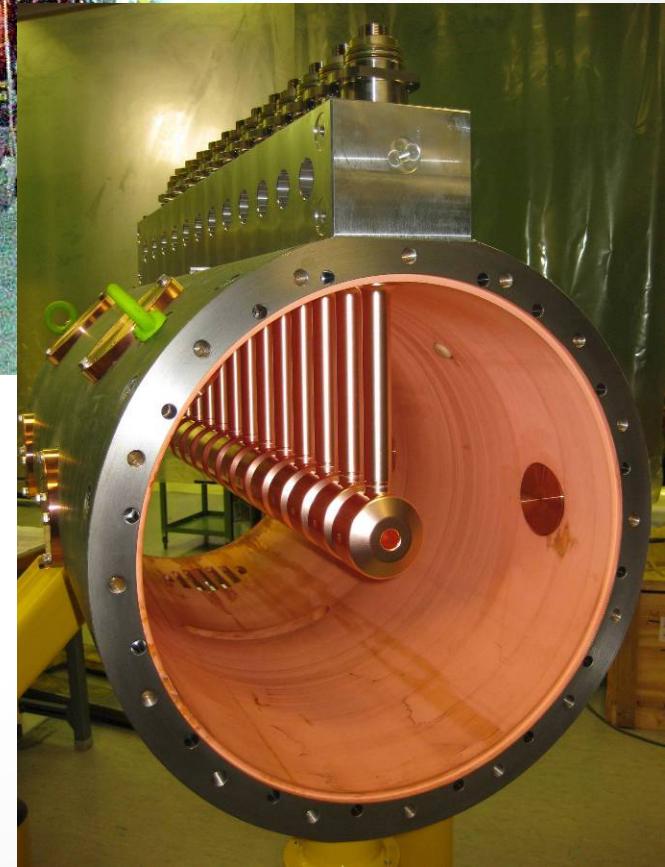
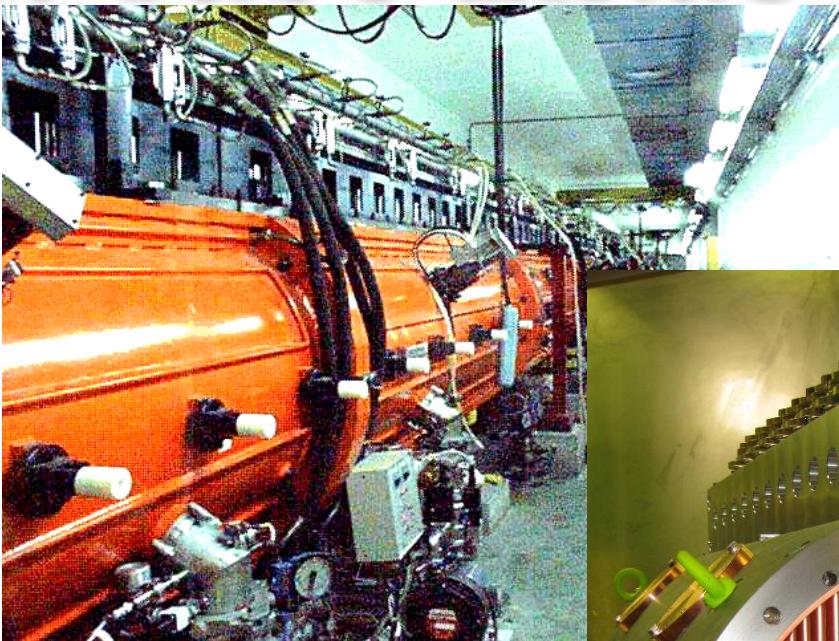
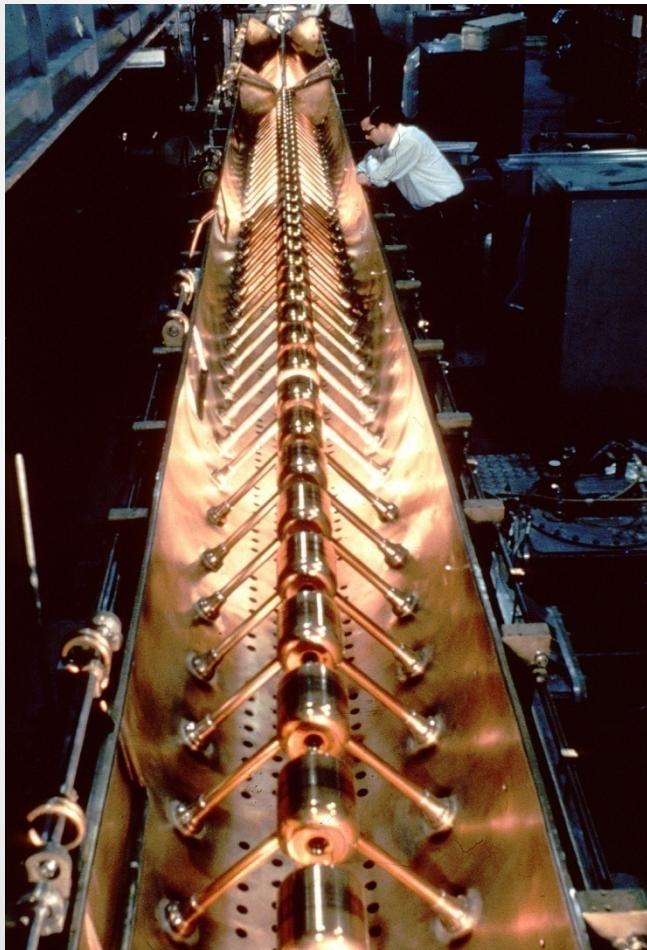
For synchronicity and acceleration, particles must be in phase with the E field on axis (will be discussed more in details in part.3).

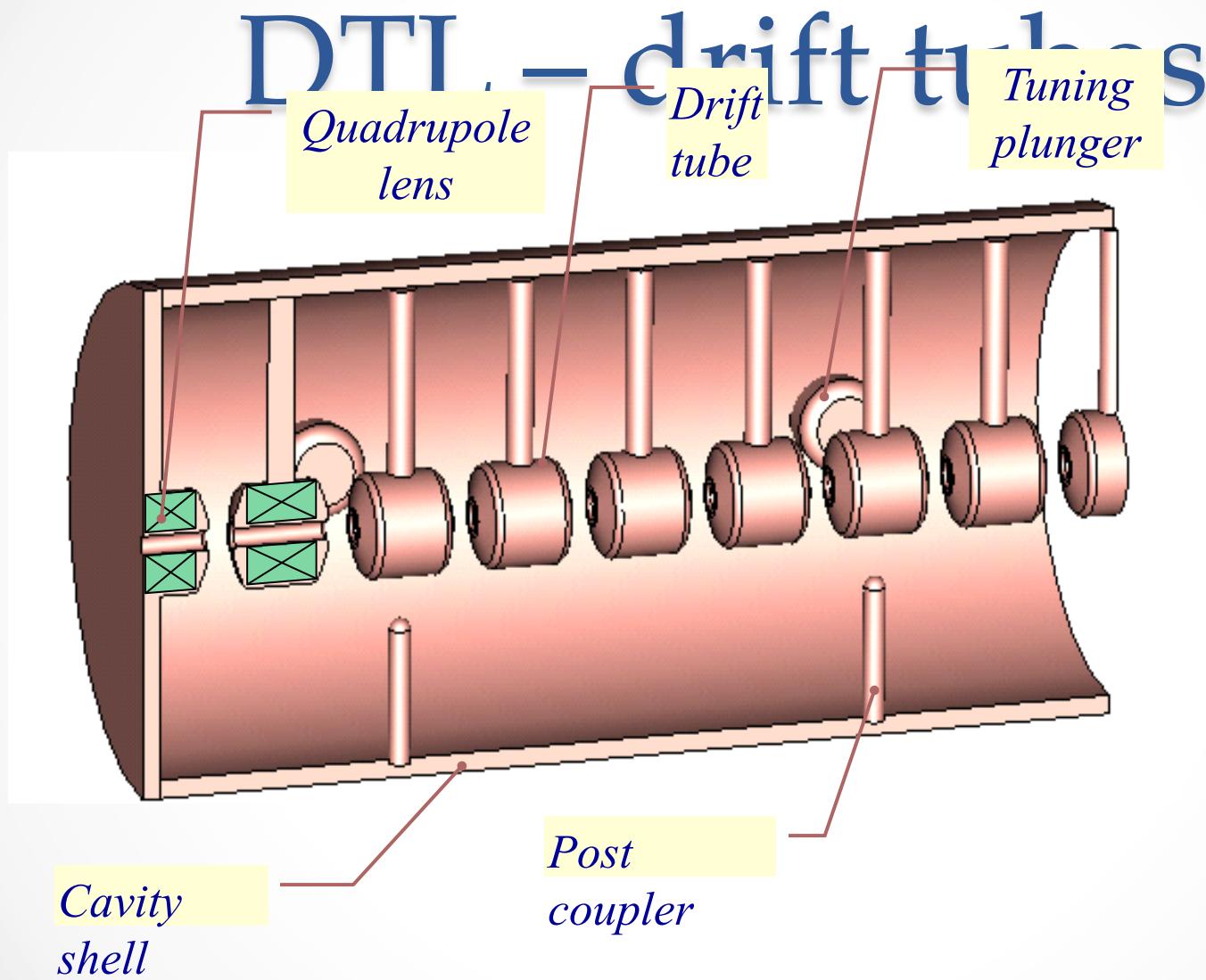
During 1 RF period, the particles travel over a distance of $\beta\lambda$.

The cell L length should be:

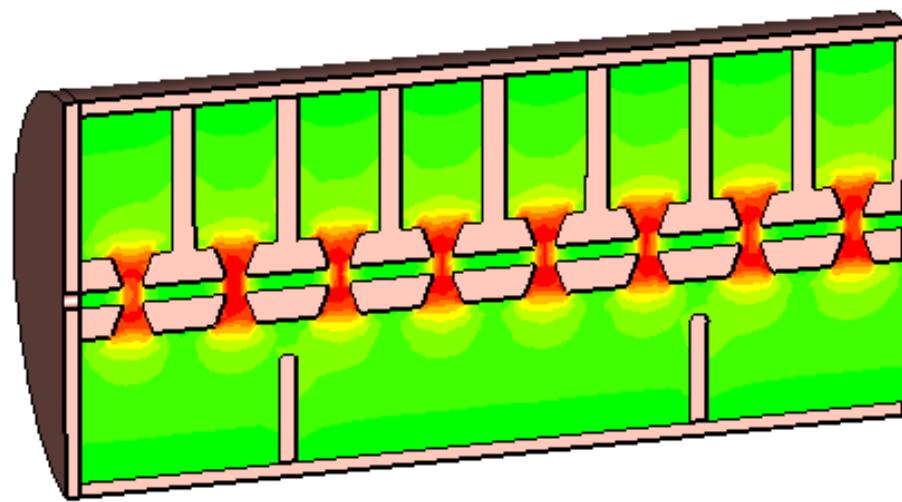
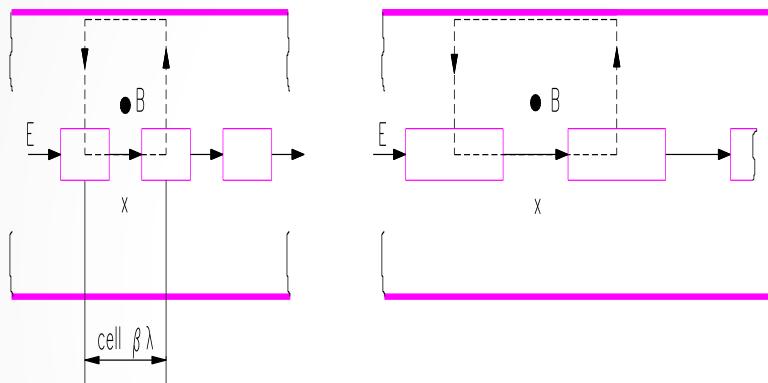
Mode	L
2π	$\beta\lambda$
$\pi/2$	$\beta\lambda/4$
$2\pi/3$	$\beta\lambda/3$
π	$\beta\lambda/2$

Drift Tube Linac



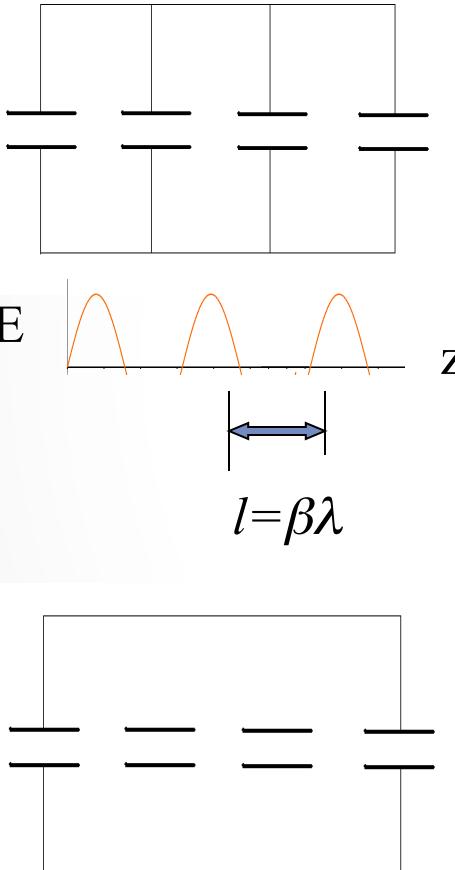


DTL : electric field



Mode is TM010

DTL



The DTL operates in **0 mode** for protons and heavy ions in the range $\beta=0.04-0.5$ (750 keV - 150 MeV)

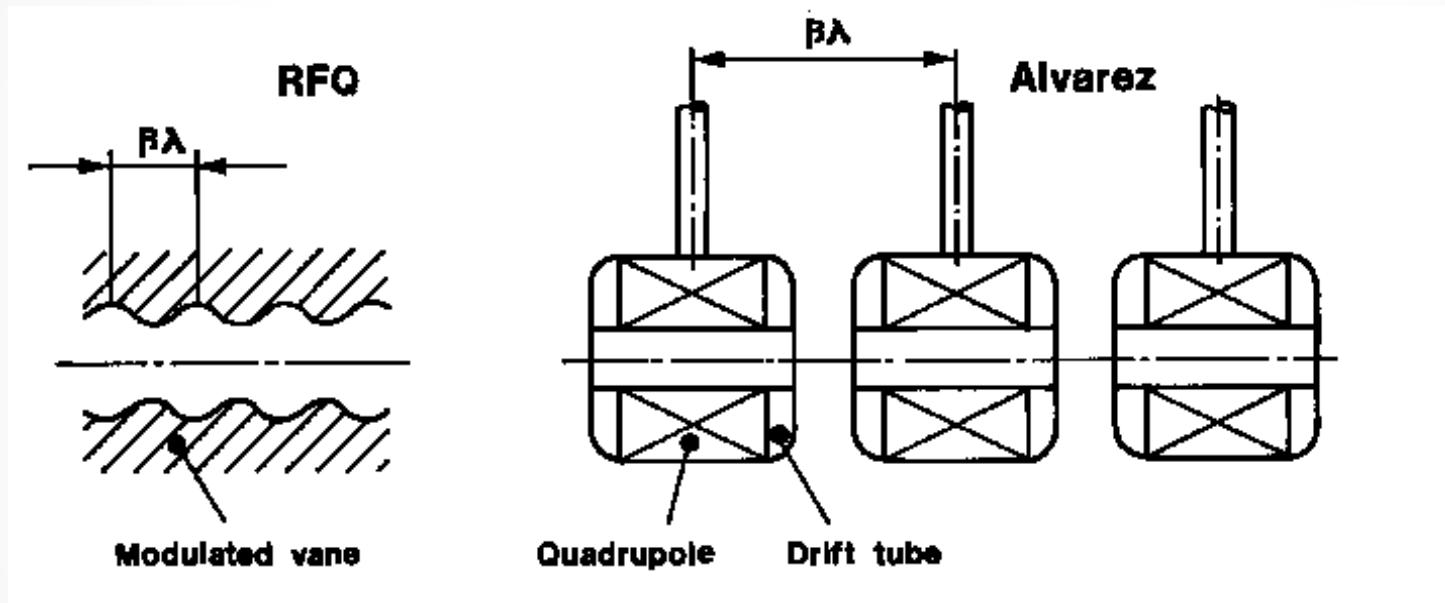
Synchronism condition (0 mode):

$$l = \frac{\beta c}{f} = \beta\lambda$$

The beam is inside the “drift tubes” when the electric field is decelerating

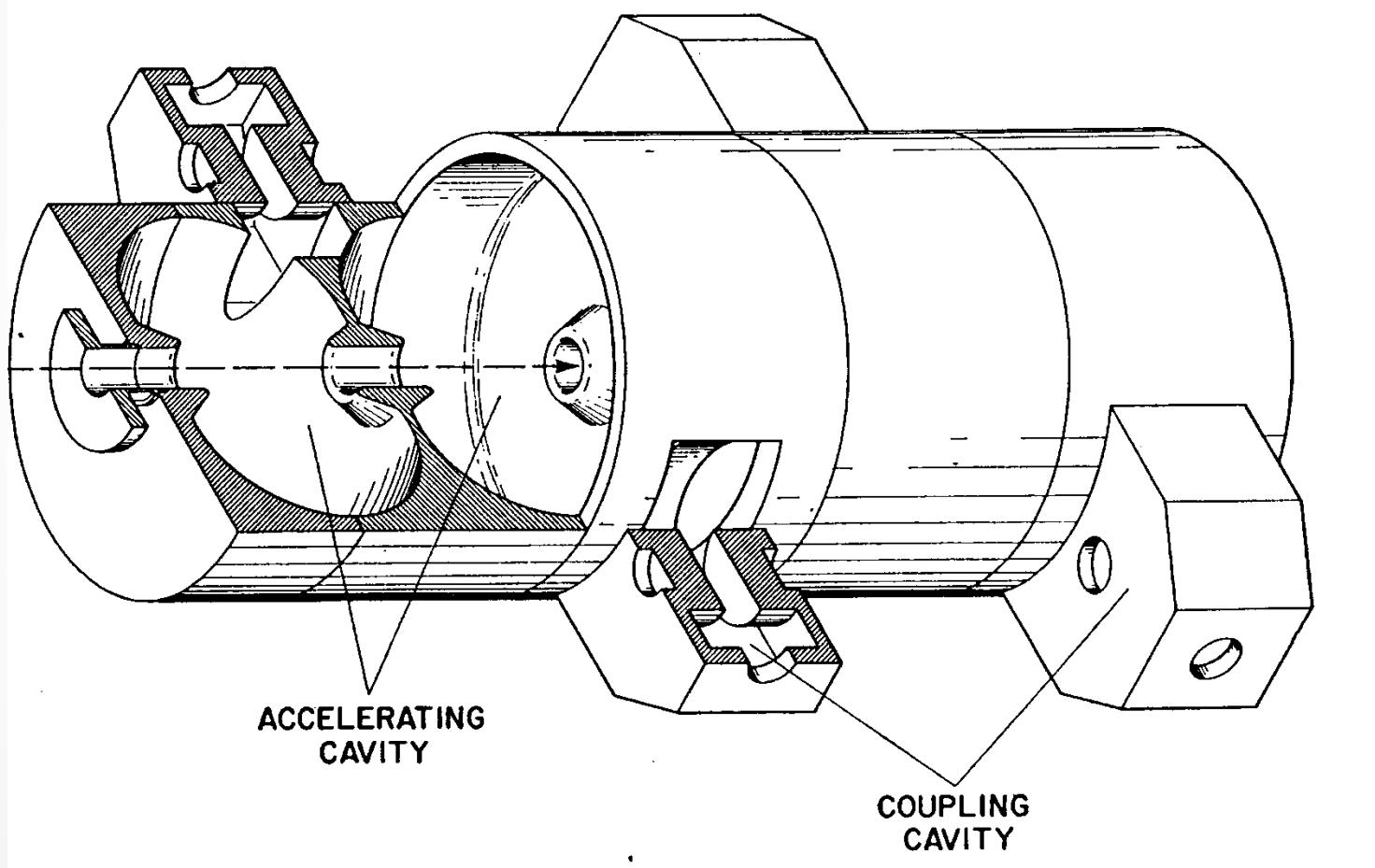
The fields of the 0-mode are such that if we eliminate the walls between cells the fields are not affected, but we have less RF currents and higher shunt impedance

RFQ vs. DTL

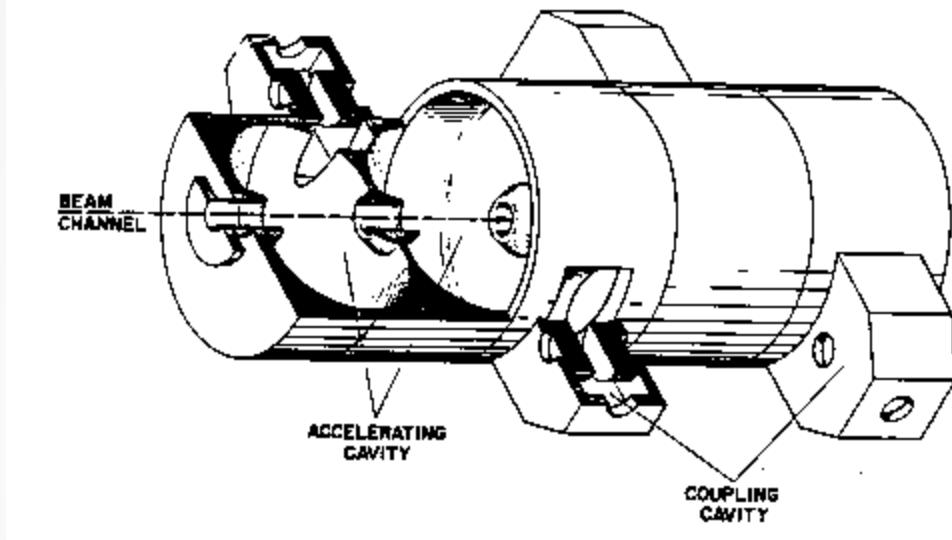


- DTL can't accept low velocity particles, there is a minimum injection energy in a DTL due to mechanical constraints

Side Coupled Linac



The Side Coupled Linac



multi-cell Standing Wave structure in **$\pi/2$ mode**
frequency 800 - 3000 MHz
for protons ($\beta=0.3 - 1$)

Rationale: high beta \Rightarrow cells are longer \Rightarrow advantage for high frequencies

- at high f , high power (> 1 MW) klystrons available \Rightarrow long chains (many cells)
- long chains \Rightarrow high sensitivity to perturbations \Rightarrow operation in $\pi/2$ mode

Side Coupled Structure:

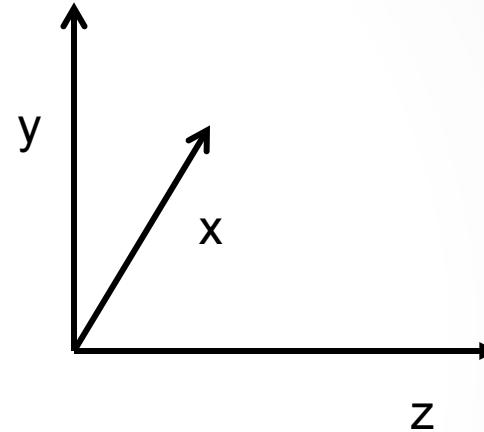
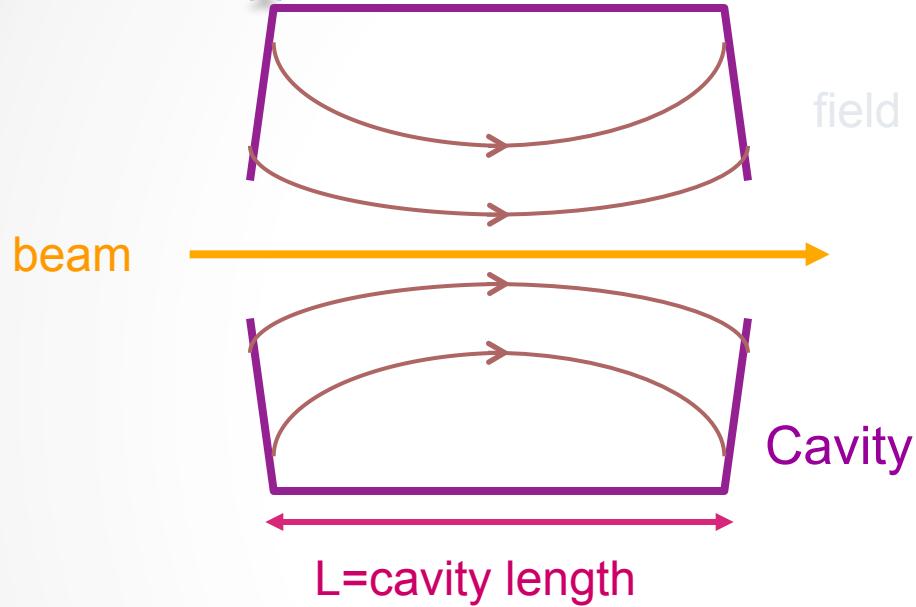
- from the wave point of view, $\pi/2$ mode
- from the beam point of view, π mode

SDTL -mix DTL and SCL



Choices choices

cavity geometry and related parameters definition



- 1-Maximum field/average field
- 2-Shunt impedance
- 3-Quality factor
- 4-Filling time
- 5-Transit time factor
- 6-Effective shunt impedance

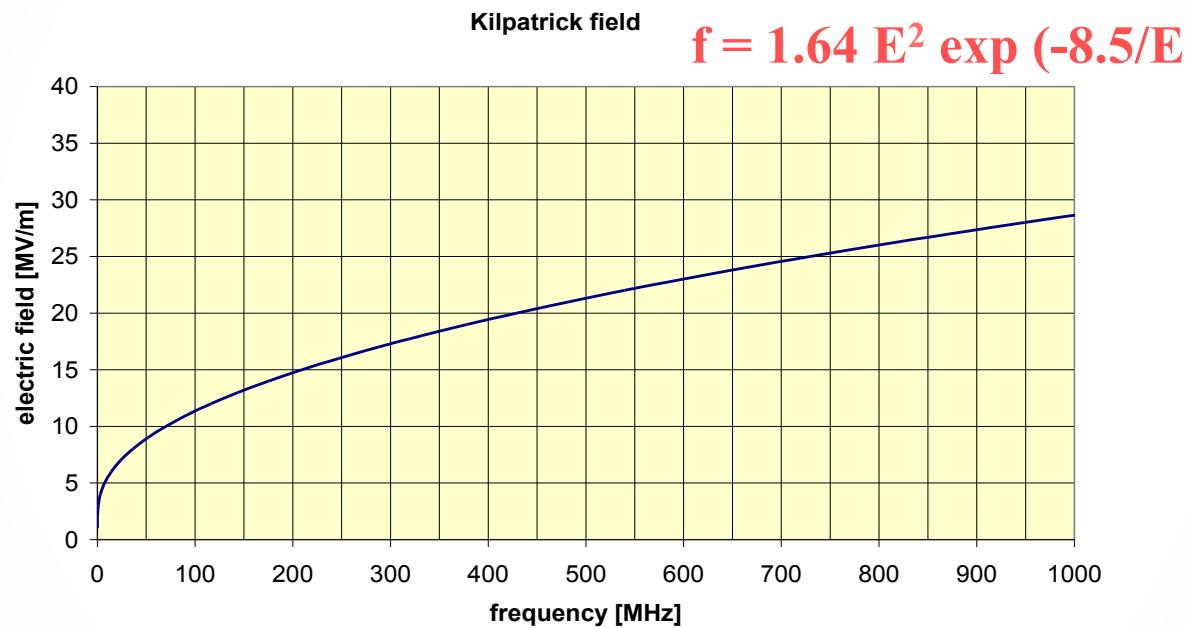
Average electric field

- Average electric field (E_0 measured in V/m) is the space average of the electric field along the direction of propagation of the beam in a given moment in time when $F(t)$ is maximum.

$$E_0 = \frac{1}{L} \int_0^L E_z(x=0, y=0, z) dz \quad E(x, y, z, t) = E(x, y, z) \cdot e^{-j\omega t}$$

- physically it gives a measure how much field is available for acceleration
- it depends on the cavity shape, on the resonating mode and on the frequency

Kilpatrick sparking criterion



GUIDELINE

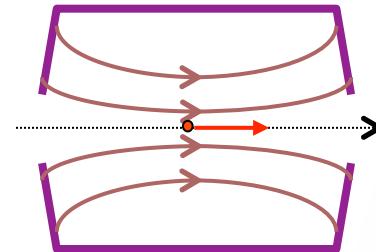
nowadays : peak
surface field up to
2*kilpatrick field

Quality factor for normal
conducting cavity is $E_{\text{peak}} / E_0 T$

Transit time factor

- transit time factor (T, dimensionless) is defined as the maximum energy gain per charge of a particles traversing a cavity over the average voltage of the cavity.
- Write the field as

$$E_z(x, y, z, t) = E_z(x, y, z) e^{-i(\omega t)}$$



- The energy gain of a particle entering the cavity on axis at phase ϕ is

$$\Delta W = \int_0^L q E_z(o, o, z) e^{-i(\omega t + \phi)} dz$$

Transit time factor

- assume constant velocity through the cavity (APPROXIMATION!!) we can relate position and time via

$$z = v \cdot t = \beta c t$$

- we can write the energy gain as

$$\Delta W = qE_0 LT \cos(\phi)$$

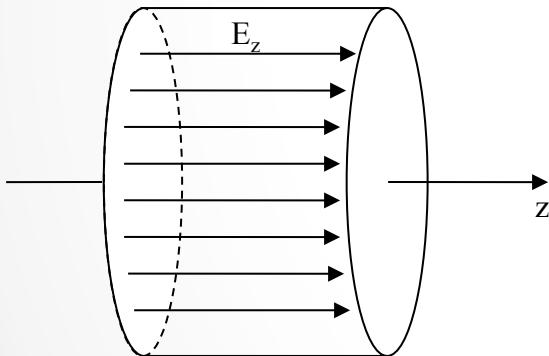
- and define transit time factor as

$$T = \frac{\left| \int_0^L E_z(z) e^{-j\left(\frac{\alpha z}{\beta c}\right)} dz \right|}{\int_0^L E_z(z) dz}$$

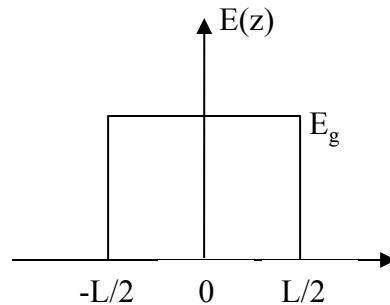
T depends on the particle velocity and on the gap length. IT DOESN'T depend on the absolute value field

Transit time factor

- NB : Transit time factor depends on x,y (the distance from the axis in cylindrical symmetry). By default it is meant the transit time factor on axis
- Exercise!!! If $E_z = E_0$ then



TM_{010} mode in a pillbox cavity



Square-wave electric field distribution

$$T = \frac{\sin\left(\frac{\pi L}{\beta\lambda}\right)}{\left(\frac{\pi L}{\beta\lambda}\right)}$$

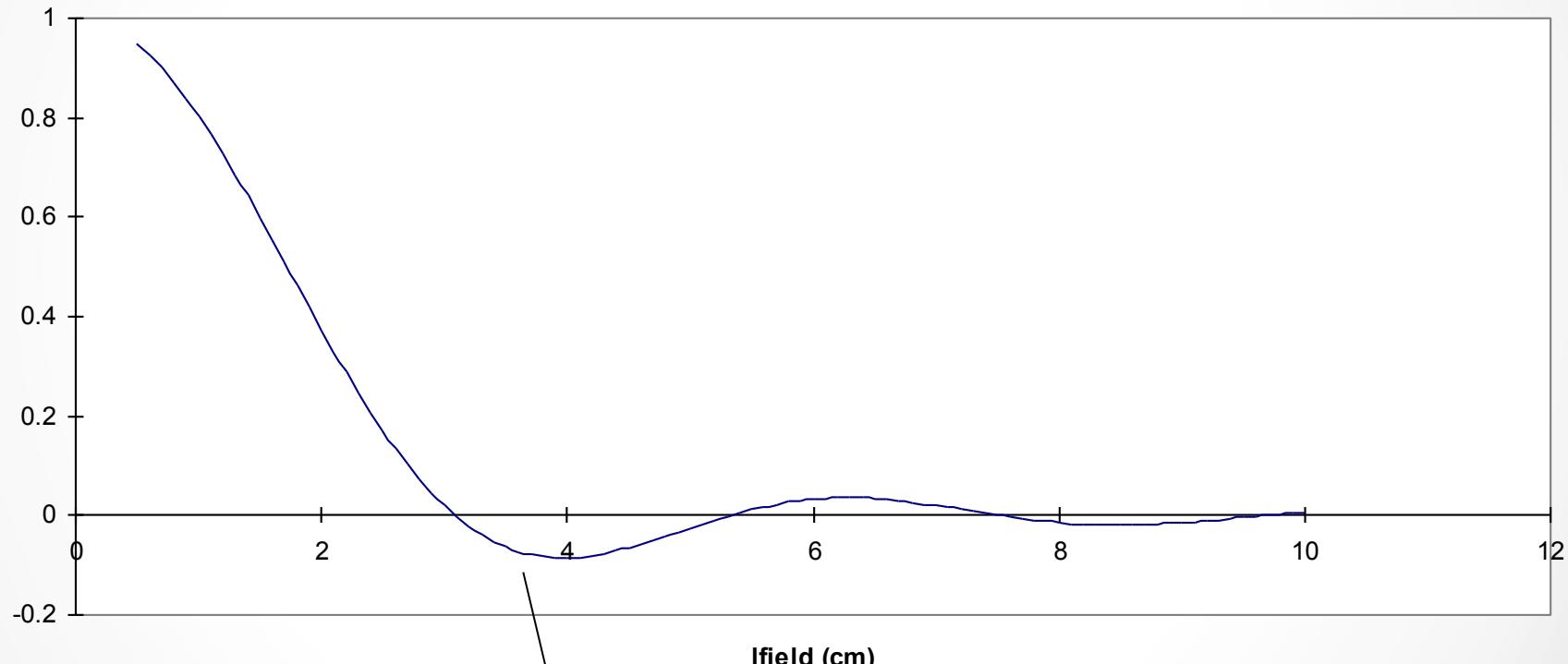
L =gap length

β =relativistic parametre

λ =RF wavelenght

Transit time factor

ttf for 100 keV protons, 200 MHz., parabolic distribution



if we don't get the length right we can end up decelerating!!!

effective shunt impedance

- Effective shunt impedance (Z measured in Ω /m) is defined as the ratio of the average effective electric field squared ($E_0 T$) to the power (P) per unit length (L) dissipated on the wall surface.
- it is independent of the field level and cavity length, it depends on the cavity mode and geometry and on the velocity of the particle to be accelerated

$$ZTT = \frac{(E_0 T)^2}{P} \cdot \frac{L}{P}$$

measure if the structure is optimized and adapted to the velocity of the particle to be accelerated

Measure of how much energy a charged particle can gain for 1 w of power when travelling over 1 m of structure.

overview

take with
CAUTION!

	Ideal range of beta	frequency	Effective gradient	
RFQ	Low!!! - 0.05	40-400 MHz	1 MV/m (350MHz)	tons
IH	0.02 to 0.08	40-200 MHz	4.5 MV/m (200MHz)	Ions and also protons
DTL	0.04-0.5	100-400 MHz	3.5 MV/m (350MHz)	Ions / protons
SCL	Ideal Beta=1 But as low as beta 0.3	800 - 3000 MHz	20 MV/m (3000MHz)	protons

What is a linac-cont'ed

$$\frac{d}{dt} \left(\gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left(\vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right)$$

type of particle :
charge couples with the field, mass slows the acceleration

Relativistic or not

type of focusing

type of RF structure

Magnetic quadrupoles

ElectroMQ



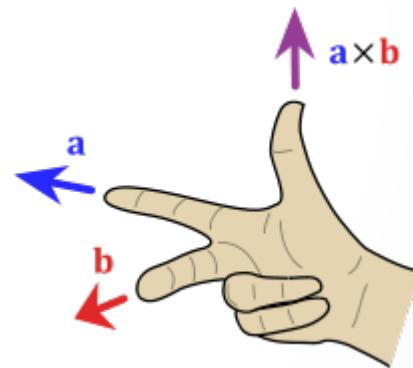
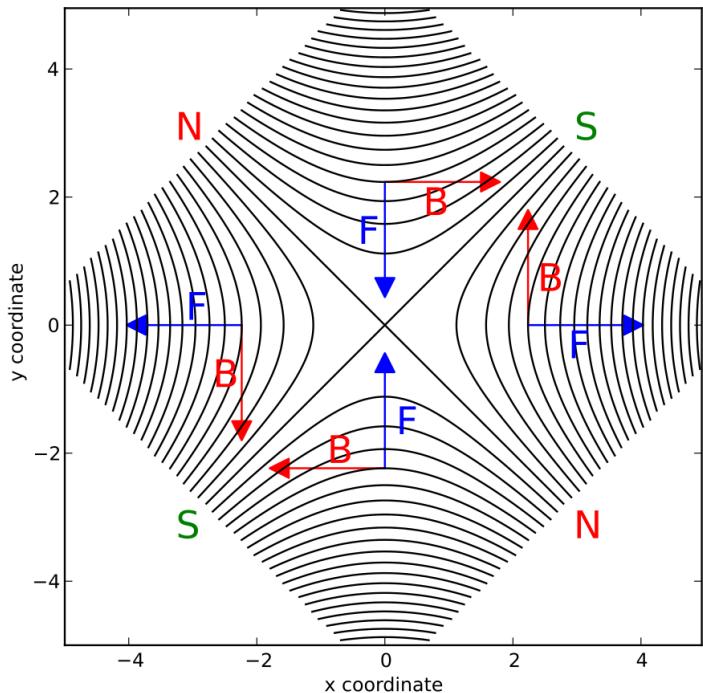
Permanent MQ



Focusing force

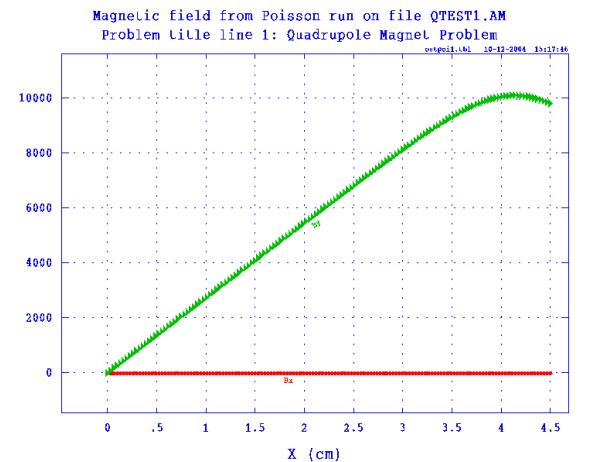
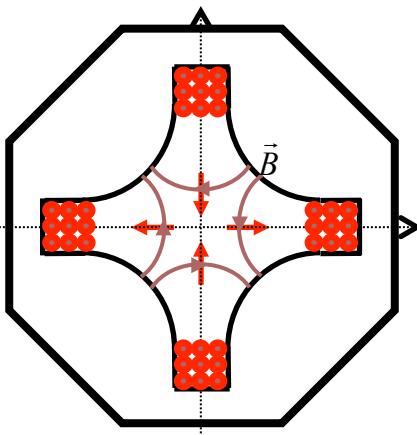
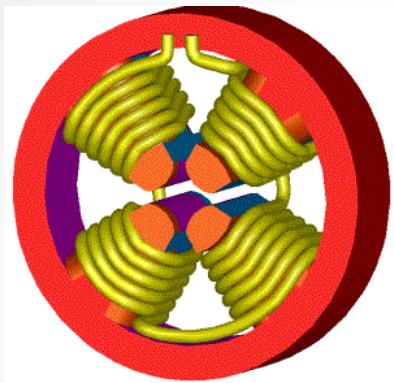
B =magnetic field/ F =force

Positively charged particles going into the screen

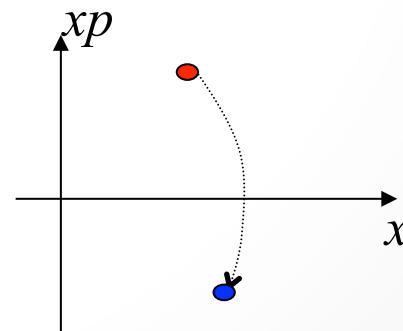
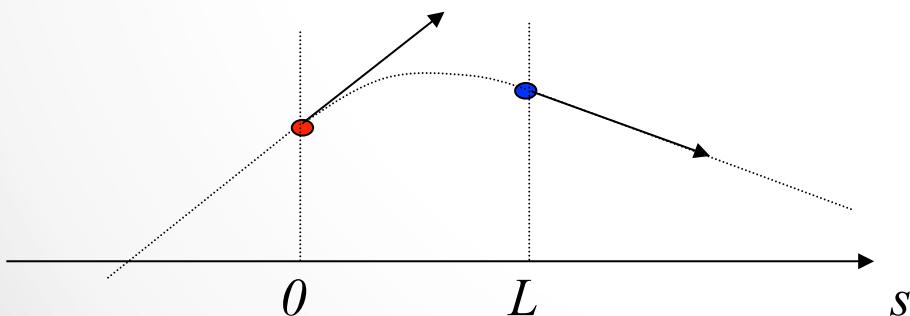


POLARITY
CHANGED WRT
PREVIOUS SLIDE

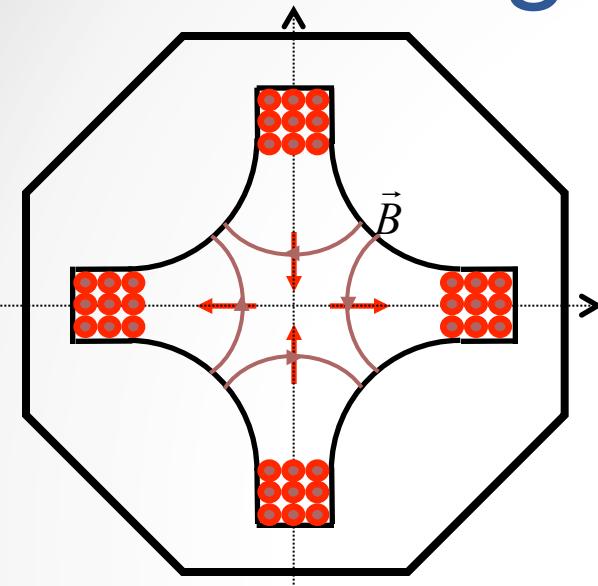
Quadrupole



Length L ; gradient G=B/a ; a = aperture



Magnetic quadrupole



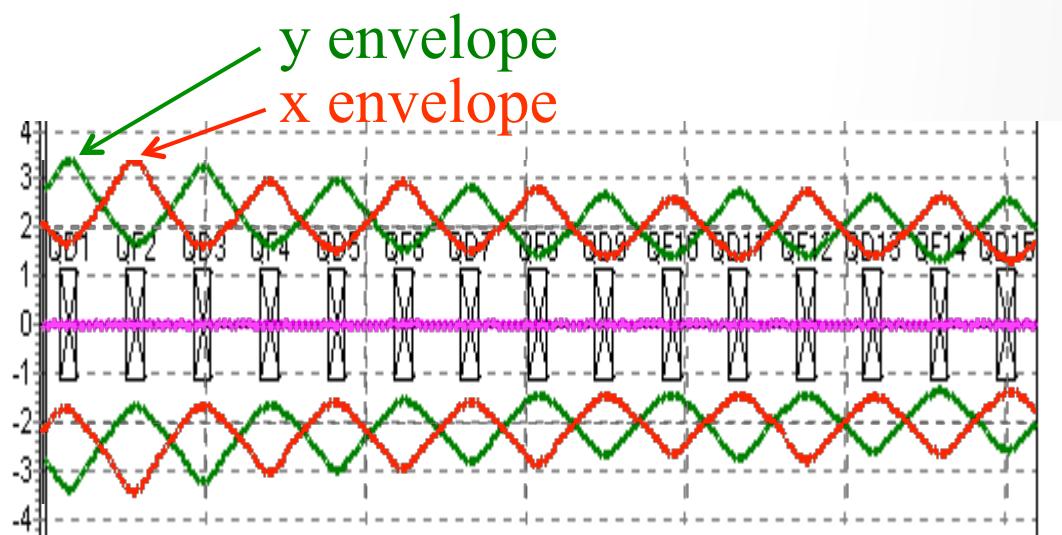
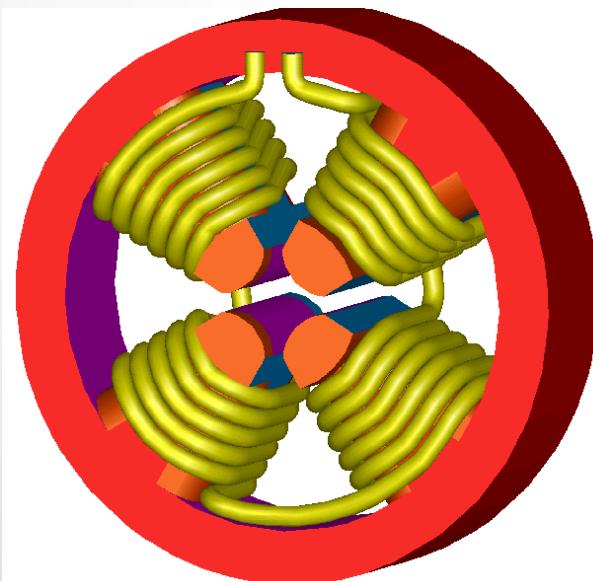
Magnetic field

$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases}$$

Magnetic force

$$\begin{cases} F_x = -q \cdot v \cdot G \cdot x \\ F_y = q \cdot v \cdot G \cdot y \end{cases}$$

Focusing in one plan, defocusing in the other



sequence of focusing and defocusing quadrupoles