Solution - Stochastic programming

18-02-2020

Heat production planning

A small district heating company approaches you to get your help on optimizing their production. The company has three heat producing units to fulfill the heat demand in the connected district heating network, namely one combined heat and power plant (CHP), a gas boiler (GB) and a wood chip boiler (WCB). The CHP plant is a back-pressure unit and produces heat and power simultaneously. For each produced MWh power ϕ MWh heat are produced. For all units the maximum production of heat per hour is limited to the capacity Q^{Max}. The cost to produce one MWh heat is given by c.
Your task is to obtain the optimal production of all units for every hour $t \in T$ of the next day ($T = \{1, \ldots, 24\}$). As the CHP unit has the longest start up time and the electricity production should be sold on the day-ahead market, the production of the CHP unit has to be determined one day in advance. The GB and WCP are more flexible and can be started spontaneously. The goal is to obtain a cost minimal production schedule, while taking the income from the day-ahead electricity market into account.
The heat demand $d_{t,s}$ and electricity price $e_{t,s}$ are uncertain on the day before, but the company has a forecasting tool to obtain scenarios $s \in S$ for the next day. A scenario s has the probability π_s.

Tasks

1. Formulate a general two-stage stochastic program to obtain the optimal production schedule. Hint: Split up the production units in two sets (one for combined heat and power units, one for heat-only units). You can change the notation of the parameters, if you need to (e.g. other indices).
2. Download `heat.zip` from DTUInside. Solve the model for the given input data in `data.xlsx` using GAMS. Use the GAMS-file `heat.gms` as it already contains the data input from `data.xlsx`, so that you just have to input the variables, constraints and solve statement.
3. Assume you can introduce a thermal storage with a capacity of K MWh heat to the system, which allows to store heat between hours (with losses). Adapt your stochastic program from Task 1 and solve it again with GAMS for a storage size of 7 MWh. How does the solution change and why?

Solution

1)

Sets	
I	Heat-only production units
J	Combined heat-and-power units
T	Time periods (hours)
S	Scenarios
Parameters	
$d_{t,s}$	Heat demand in time period t and scenario s [MWh-heat]
$e_{t,s}$	Electricity price in time period t and scenario s [EUR/MWh-power]
c_j^{CHP}, c_j^H	Cost producing 1 MWh heat of unit i and j, respectively [EUR]
Q_j^{CHP}, Q_i^H	Max. heat per hour for unit i and j, respectively [MWh-heat]
ϕ_j	Heat-to-power ratio for unit j
Variables	
$q_{j,t}^{CHP}$	Heat production of unit j in period t [MWh-heat]
$q_{i,t,s}^H$	Heat production of unit i in period t and scenarios s [MWh-heat]

$$Min \sum_{t\in T}\left[\sum_{j\in J}\left(c_j^{CHP}q_{j,t}^{CHP} - \sum_{s\in S}\pi_s e_{t,s}\frac{1}{\phi_j}q_{j,t}^{CHP}\right) + \sum_{i\in I}\sum_{s\in S}\pi_s c_i^H q_{i,t,s}^H\right] \quad (1)$$

$$\begin{aligned}
s.t.\quad & q_{j,t}^{CHP} \le Q_j^{CHP} && \forall j\in J, t\in T && (2)\\
& q_{i,t,s}^{H} \le Q_i^{H} && \forall i\in I, t\in T, s\in S && (3)\\
& \sum_{j\in J} q_{j,t}^{CHP} + \sum_{i\in I} q_{i,t,s}^{H} = d_{t,s} && \forall t\in T, s\in S && (4)\\
& q_{j,t}^{CHP} \ge 0 && \forall j\in J, t\in T && (5)\\
& q_{i,t,s}^{H} \ge 0 && \forall i\in I, t\in T, s\in S && (6)
\end{aligned}$$

The objective function (1) minimizes the expected cost of the production taking the income from the electricity market into account. Constraints (2) and (3) limit the capacity of the CHP and heat-only productions units, respectively. Constraints (4) ensure that the heat demand is fulfilled in every period and scenarios. Finally, the non-negativity of the variables is enforced in constraints (5) and (6).

2)

Implementation see GAMS file `heat.gms`.

Objective value: 55422.88

Heat production values:

T	GB			WCB			CHP
	s1	s2	s3	s1	s2	s3	
t1	0.02	0	0	4	4	0.85	4.88
t2	0	0.38	0	4	4	2.54	4.7
t3	0	1.96	0.52	4	4	4	3.02
t4	0	2.26	0.52	4	4	4	3.02
t5	0	2.06	0.52	4	4	4	3.02
t6	0	2.04	0	2.89	4	4	3.14
t7	0	0	0	1.05	4	2.06	5.08
t8	0	1.84	0	2.39	4	4	3.44
t9	0	0.08	0	2.39	4	4	3.34
t10	0	0	0.02	2.91	4	4	3.12
t11	0	0.08	0	2.49	4	4	3.24
t12	0	1.02	0	3.6	4	4	3.54
t13	0	0	0	2.98	4	3.18	4.36
t14	0	0	0	2.98	4	2.78	4.36
t15	0	0.82	0	3.8	4	4	3.54
t16	0	0	0	3.08	4	2.88	4.36
t17	0	0.82	0	4	4	2.39	3.54
t18	0	3.44	0	4	4	2.79	3.14
t19	0	3.04	0	4	4	2.69	3.24
t20	0	3.04	0	4	4	2.85	3.44
t21	0	2.84	0	4	4	2.75	3.44
t22	1.15	3.79	0	4	4	4	2.39
t23	0	3.04	0	4	4	3.15	3.24
t24	0	2.84	0	4	4	3.25	3.34

3)

Sets	
I	Heat-only production units
J	Combined heat-and-power units
T	Time periods (hours)
S	Scenarios
Parameters	
$d_{t,s}$	Heat demand in time period t and scenario s [MWh-heat]
$e_{t,s}$	Electricity price in time period t and scenario s [EUR/MWh-power]
c_j^{CHP}, c_j^H	Cost producing 1 MWh heat of unit i and j, respectively [EUR]
Q_j^{CHP}, Q_i^H	Max. heat per hour for unit i and j, respectively [MWh-heat]
ϕ_j	Heat-to-power ratio for unit j
K	Storage capacity [MWh-heat]
Variables	
$q_{j,t}^{CHP}$	Heat production of unit j in period t [MWh-heat]
$q_{i,t,s}^H$	Heat production of unit i in period t and scenarios s [MWh-heat]
$s_{t,s}^{in}$	Inflow to storage in period t in scenario s [MWh-heat]
$s_{t,s}^{out}$	Outflow of storage to district heating in period t and scenario s [MWh-heat]
$s_{t,s}$	Storage level in period t and scenario s [MWh-heat]

$$Min \sum_{t\in T}\left[\sum_{j\in J}\left(c_j^{CHP}q_{j,t}^{CHP} - \sum_{s\in S}\pi_s e_{t,s}\frac{1}{\phi_j}q_{j,t}^{CHP}\right) + \sum_{i\in I}\sum_{s\in S}\pi_s c_i^H q_i^H\right] \tag{7}$$

$$
\begin{aligned}
s.t. \quad & q_{j,t}^{CHP} \leq Q_j^{CHP} && \forall j \in J, t \in T && (8)\\
& q_{i,t,s}^H \leq Q_i^H && \forall i \in I, t \in T, s \in S && (9)\\
& \sum_{j\in J}(q_{j,t}^{CHP}) + \sum_{i\in I}(q_{i,t,s}^H) - s_{t,s}^{in} + s_{t,s}^{out} = d_{t,s} && \forall t \in T, s \in S && (10)\\
& s_{t,s} = s_{t-1,s} + s_{t,s}^{in} - s_{t,s}^{out} && \forall t \in T\{\backslash 1\}, s \in S && (11)\\
& s_{1,s} = s_{1,s}^{in} - s_{1,s}^{out} && \}, s \in S && (12)\\
& s_{t,s} \leq K && \forall t \in T, s \in S && (13)\\
& q_{j,t}^{CHP} \geq 0 && \forall j \in J, t \in T && (14)\\
& q_{i,t,s}^H \geq 0 && \forall i \in I, t \in T, s \in S && (15)\\
& s_{t,s} \geq 0 && \forall t \in T, s \in S && (16)\\
& s_{t,s}^{out} \geq 0 && \forall t \in T, s \in S && (17)\\
& s_{i,t,s}^H \geq 0 && \forall i \in I, t \in T, s \in S && (18)\\
& s_{j,t}^{CHP} \geq 0 && \forall j \in J, t \in T && (19)
\end{aligned}
$$

The objective function (7) minimizes the expected cost of the production taking the income from the electricity market into account. Constraints (8) and (9) limit the capacity of the CHP and heat-only productions units, respectively. Constraints (10) ensure that the heat demand is fulfilled in every period and scenario while at the same time managing the inflow and outflow to the heat storage. This means the constraint balances flow, i.e., the production is going to fulfill the demand or going to the storage and the demand is coming from the storage or the the production units. The storage level per period is updated with in- and outflow in constraints (11) and for the special case of the first period in constraints (12). The capacity of the heat storage is limited by constraints (13). Finally, the non-negativity of the variables is enforced in constraints (14) to (19).

Solution:
Objective value: 54239.23
Implementation see `heat-with-storage.gms`.
The district heating company saved 1183.65 by using a storage. As we can see from the solution, we use periods with expected high electricity prices to fill the storage and can therefore reduce the production of the GB and CHP in some other periods. In general, the production can be shifted to more favorable time periods.

	Heat production							**Storage level**		
	GB			**WCB**			**CHP**			
	s1	s2	s3	s1	s2	s3		s1	s2	s3
t1	0	0	0	4	4	3.13	9.6	4.7	4.72	7
t2	0	4.61	0	4	4	3.05	0	0	4.25	2.81
t3	0	0	0	4	4	4	7.73	4.71	7	7
t4	0	3.95	0	4	4	4	1.33	3.02	7	4.79
t5	0	1.23	0	4	4	4	0	0	3.15	1.25
t6	0	0	0	4	4	4	2.03	0	0	0.14
t7	0	0	0	0	4	4	10	3.87	4.92	7
t8	0	0	0	0.89	4	4	3.44	2.37	3.08	7
t9	0	4	0	4	4	4	3.34	3.98	7	7
t10	0	1.08	0	4	4	4	0	1.95	4.96	3.86
t11	0	0	0	4	4	4	2.92	3.14	4.56	3.54
t12	0	0	0	4	4	4	0	0	0	0
t13	0	0	0	4	4	4	10	6.66	5.64	6.46
t14	0	0	0	3.24	4	3.24	4.44	7	5.72	7
t15	0	0	0	0.78	4	0.54	0	0.44	1.36	0
t16	0	0	0	4	4	2.63	10	7	7	5.39
t17	0	0.82	0	4	4	4	3.54	7	7	7
t18	0	6.58	0	4	4	0.86	0	3.86	7	1.93
t19	0	0	0	4	4	4	0	0.62	0.72	0
t20	0	2.94	0	4	4	2.22	9.82	7	7	5.75
t21	0	0	0	4	4	4	3.44	7	4.16	7
t22	0	2.02	0	4	4	1.78	0	3.46	0	2.39
t23	0	6.28	0	4	4	4	0	0.22	0	0
t24	0	3.06	0	4	4	3.47	3.12	0	0	0