Assignment 3

Deadline: 08 May 2020, 23:59h

General information

- This assignment is part of the overall assessment of this course and, therefore, your answer counts for the final grade.
- This assignment has to be solved and submitted by every student individually before the above mentioned deadline closes. Group work and group submissions are not allowed.
- The assignment has to be submitted via DTU Inside. Use the entry `Assignments` in the course menu and upload your files to the corresponding assignment. In case of technical problems with DTU Inside, please send your files to `dngk@dtu.dk` before the deadline.
- The submission must consist of a pdf-document containing the answers to the tasks below. Furthermore, program code and scripts have to be uploaded as well.
- **Name your report `<studentnumber>-<last name>-Assignment3.pdf`**

Part 1 - Solution techniques

Task 1 - L-shaped method (20 Points)

Recall the following planning problem to optimize the production of a small district heating company from the exercise on 18-02-2020 (Two-stage stochastic programming).

See Table 1 for the sets, parameters and variables. The objective function (1) minimizes the expected cost of the production taking the income from the electricity market into account. Constraints (2) and (3) limit the capacity of the CHP and heat-only productions units, respectively. Constraints (4) ensure that the heat demand is fulfilled in every period and scenarios. Finally, the non-negativity of the variables is enforced in constraints (5) and (6).

$$Min \sum_{t \in T} \left[\sum_{j \in J} \left(c_j^{CHP} q_{j,t}^{CHP} - \sum_{s \in S} \pi_s e_{t,s} \frac{1}{\phi_j} q_{j,t}^{CHP} \right) + \sum_{i \in I} \sum_{s \in S} \pi_s c_i^H q_{i,t,s}^H \right] \tag{1}$$

$$s.t. \quad q_{j,t}^{CHP} \leq Q_j^{CHP} \qquad \forall j \in J, t \in T \tag{2}$$

$$q_{i,t,s}^{H} \leq Q_i^{H} \qquad \forall i \in I, t \in T, s \in S \tag{3}$$

$$\sum_{j \in J} q_{j,t}^{CHP} + \sum_{i \in I} q_{i,t,s}^{H} = d_{t,s} \qquad \forall t \in T, s \in S \tag{4}$$

$$q_{j,t}^{CHP} \geq 0 \qquad \forall j \in J, t \in T \tag{5}$$

$$q_{i,t,s}^{H} \geq 0 \qquad \forall i \in I, t \in T, s \in S \tag{6}$$

1. Introduce an artificial full recourse to the stochastic program (1) - (6). Define the new variables and parameters that you use and write down the changed parts of the model. Explain why you should introduce a full recourse. (Hint: Look at constraints (4)).
2. Decompose the problem into master and subproblem and write down both models. The master problem should include a general cut constraint for now. Use the set of cuts $\mathcal{L}$.

Sets	
I	Heat-only production units
J	Combined heat-and-power units
T	Time periods (hours)
S	Scenarios
Parameters	
$d_{t,s}$	Heat demand in time period t and scenario s [MWh-heat]
$e_{t,s}$	Electricity price in time period t and scenario s [EUR/MWh-power]
c_j^{CHP}, c_j^H	Cost producing 1 MWh heat of unit i and j, respectively [EUR]
Q_j^{CHP}, Q_i^H	Max. heat per hour for unit i and j, respectively [MWh-heat]
ϕ_j	Heat-to-power ratio for unit j
Variables	
$q_{j,t}^{CHP}$	Heat production of unit j in period t [MWh-heat]
$q_{i,t,s}^{H}$	Heat production of unit i in period t and scenarios s [MWh-heat]

Table 1: Sets, parameters and variables

3. Define dual variables based on the constraints in the primal subproblem (i.e. define which constraint the dual variable belongs to and which indices it has). Define also the dual objective function of the subproblem based on those dual variables. You do not need to write down the entire dual subproblem (just the variables and objective function).
4. Derive the formulas to determine the aggregated cut coefficients and cut right-hand-side in the master problem based on your result in task 1.3.
5. Write down the final cut constraint for the master problem using the formulas for the coefficients and right-hand-side from task 1.4.

You do not need to solve the model.

Task 2 - (Meta)heuristics - General questions (8 Points)

Comment briefly and in your own words on the advantages and disadvantages of applying a (meta)heuristic solution approach for an optimization problem.

Part 2 - Robust optimization

Task 3 - Robust optimization - General questions (12 Points)

Answer the following questions/tasks in a few sentences using your own words.

1. What are the two goals/purposes we want to achieve when we apply a robust optimization problem?
2. Explain briefly the difference between stochastic programming and robust optimization.
3. Why can adjustable robust optimization be considered less conservative than normal robust optimization?

Task 4 - Reformulation to robust linear models (30 Points)

Consider the following basis model:

$$
\begin{array}{lrcrcrcr}
\text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\
\text{s.t.} & 5x_1 & + & 3x_2 & + & 3x_3 & \leq & 10 \\
 & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5
\end{array}
$$

$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

In the following tasks some of the parameters are defined as uncertain parameters within given uncertainty sets. In each task you need to reformulate the model to a robust linear model that can be solved using a general purpose linear programming solver.
For each task you have to document the steps of reformulation (writing down just the final robust linear model is not enough).
Furthermore, you have to report the optimal solution for each model, i.e., the objective values and the variables values of x_1, x_2 and x_3 that you get from solving the model with Julia JuMP using Gurobi. You can use the model in `RO_Initial.jl` as a starting point for your implementation.

1. Parameters $\tilde{a}$ and $\tilde{b}$ are uncertain:

$$
\begin{array}{lrcrcrcr}
\text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\
\text{s.t.} & 5x_1 & + & 3x_2 & + & \tilde{a}x_3 & \leq & 10 \\
 & 7x_1 & - & 2x_2 & - & \tilde{b}x_3 & \geq & 5
\end{array}
$$

$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

   $\tilde{a}$ and $\tilde{b}$ are uniform distributed with $\tilde{a} \sim \mathcal{U}(1,5)$ and $\tilde{b} \sim \mathcal{U}(1,3)$.

   Name your code file `<studentnumber>_RO_Task1.jl`.

2. Parameter $\tilde{c}$ is uncertain:

$$
\begin{array}{lrcrcrcr}
\text{Max} & 10x_1 & + & \tilde{c}x_2 & + & 15x_3 & & \\
\text{s.t.} & 5x_1 & + & 3x_2 & + & 3x_3 & \leq & 10 \\
 & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5
\end{array}
$$

$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

   $\tilde{c}$ is uniform distributed with $\tilde{c} \sim \mathcal{U}(5,35)$.

   Name your code file `<studentnumber>_RO_Task2.jl`.

3. Parameters $\tilde{a}_2$ and $\tilde{a}_3$ are uncertain:

$$\begin{array}{lrcrcrcr} \text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\ \text{s.t.} & 5x_1 & + & \tilde{a}_2 x_2 & + & \tilde{a}_3 x_3 & \leq & 10 \\ & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5 \end{array}$$

$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

$\tilde{a}_2$ and $\tilde{a}_3$ can take values in the uncertainty set $U = \Big\{ \tilde{a}_2 + \tilde{a}_3 \leq 8, \tilde{a}_2 + \tilde{a}_3 \geq 2, \tilde{a}_2, \tilde{a}_3 \geq 0 \Big\}$.

Name your code file `<studentnumber>_RO_Task3.jl`.

4. Parameters $\tilde{a}_1$ $\tilde{a}_2$ and $\tilde{a}_3$ are uncertain:

$$\begin{array}{lrcrcrcr} \text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\ \text{s.t.} & \tilde{a}_1 x_1 & + & \tilde{a}_2 x_2 & + & \tilde{a}_3 x_3 & \leq & 10 \\ & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5 \end{array}$$

$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

$\tilde{a}_1, \tilde{a}_2$ and $\tilde{a}_3$ are uniform distributed with $\tilde{a}_1 \sim \mathcal{U}(1,8)$, $\tilde{a}_2 \sim \mathcal{U}(2,9)$, $\tilde{a}_3 \sim \mathcal{U}(1,5)$. At most two of the three parameters will deviate from their mean value.

Name your code file `<studentnumber>_RO_Task4.jl`.

Task 5 - Adjustable Robust Optimization (30 Points)

You are working for a production company that produces an electronic device for car manufacturers. The company wants you to optimize the monthly production amounts for the next 12 months to get an overview of the production quantities on their different machines. You assume the quantities as continuous values. They are looking for a cost minimal solution while covering the demand D_t [units] in each month t. The demand is known due to the already present orders that were made. Use a $\geq$ constraint to cover the demand.
The company has different machines $I = \{1, 2\}$ to produce the device. The production cost per unit on each machine i is given by c_i [EUR/unit] ($c_1 = 4, c_2 = 2$). The capacity per month is denoted by K_i ($K_1 = 400000, K_2 = 650000$) given in units.
Machine $i = 1$ has higher cost, but is also more reliable. 100% of the devices produced on this machine are actually working. Thus, machine $i = 1$ has an efficiency of $\eta_1 = 1.0$. In contrast, machine $i = 2$ is an old machine where only between 60% and 80% are fault-free, $\eta_2 \in [0.6, 0.8]$.
In the following tasks you need to provide a general formulation of the above planning using the suggested notation in the task description. However, you can use the explicit indices of the machines in the constraint with the uncertainty. Describe all variables, sets, parameters and constraints. Besides the model formulations and solution values in your report, attach also the code files to your final submission.

1. Formulate a robust linear optimization problem that handles the uncertainty of the efficiency of machine $i = 2$ in an appropriate way.

2. Use `device.jl` as a basis for an implementation of your model from Task 5.1 and solve it. Report the objective value and production amounts per period and machine. Name your code file `<studentnumber>-RO-device.jl`.

Now the company wants to investigate what happens if they can react to the outcome of the uncertain efficiency η_2 by extending the capacity of machine $i = 2$ through extended working hours. Therefore, they consider an additional capacity $i = 3$ that represents the extended working hours. The productions costs $c_3 = 3$ [EUR/units] are higher than for production during normal hours. The additional capacity available is $K_3 = 200000$ [units]. Assume the efficiency of the extra capacity to be $\eta_3 = 0.8$.

3. Extend your model to an adjustable robust linear optimization problem that determines the extra production amount during the extended working hours, i.e., on virtual machine $i = 3$, based on the uncertain efficiency of machine $i = 2$ in an appropriate way.

4. Use `device.jl` as a basis for an implementation of your model from Task 5.3 and solve it. Report the objective value and production amounts per period and machine. Name your code file `<studentnumber>-ARO-device.jl`.