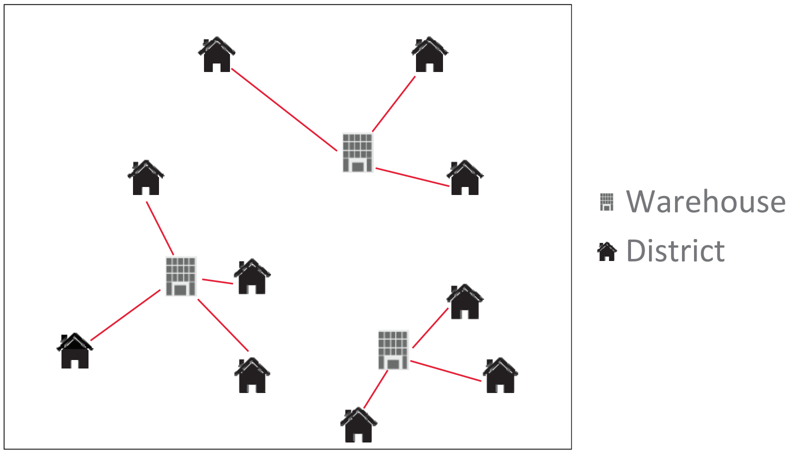
Solution - Multi-stage stochastic programming

25-02-2020

Parcel delivery service



You are helping a parcel delivery company that recently acquired a new area where it will deliver parcels. The company already rented several warehouses $\mathcal{W}$ from which the deliveries tours will start in the morning. Now, the company has to hire drivers to deliver parcels to the different districts $\mathcal{D}$. Furthermore, it has to be decided which warehouse takes care of which district.
Your task is to formulate a optimization model to assign each district to a warehouse and decide how many drivers to hire at each warehouse. Both decisions are long-term decisions and we plan for the next two years. Thus, our periods are $\mathcal{T} = \{0, 1, 2\}$ were 0 represents here-and-now. Take the following information into account:

- Each driver works H hours per year.
- The number of drivers per warehouse w per year should be not more than K_w.
- The demand $A_{d,t,s}$ of each district d per year is measured in hours and uncertain. However, the company provides you with demand scenarios $\mathcal{S}$ for each district. After one year, the scenarios get updated to adjust to development in the previous year. (This means you have one demand value for year $t = 1$ and one for year $t = 2$).
- The assignment of warehouses to districts and initial hiring of drivers has to be done once at $t = 0$.
- After one year (at $t = 1$), the company can reallocate the drivers between the warehouses. Furthermore, additional drivers can be hired (but no one gets fired).
- The goal is to find a cost-minimal solution. The personal costs are given by C^P EUR per driver per year. Furthermore, we have transportation costs $C^T_{w,d}$ for the two years that depend on the distance between warehouse w and district d.
- Each driver can cover several districts that are assigned to their warehouse.
- Not fulfilling the demand in $t = 1$ and $t = 2$ in the districts has to be modeled so that we can always obtain a feasible solutions, even if the demand is too high in some scenarios. However, each hour of missing demand should be avoided and is therefore penalized with a factor of ϕ.

Some further information regarding the data:
We consider two scenarios (high demand and low demand). After one year, it becomes clear which of those scenarios realized and therefore we result in two updated scenarios that either correspond to high demand (very high, moderately high) or low demand (very low, moderately low). The probabilities are distributed as follows:

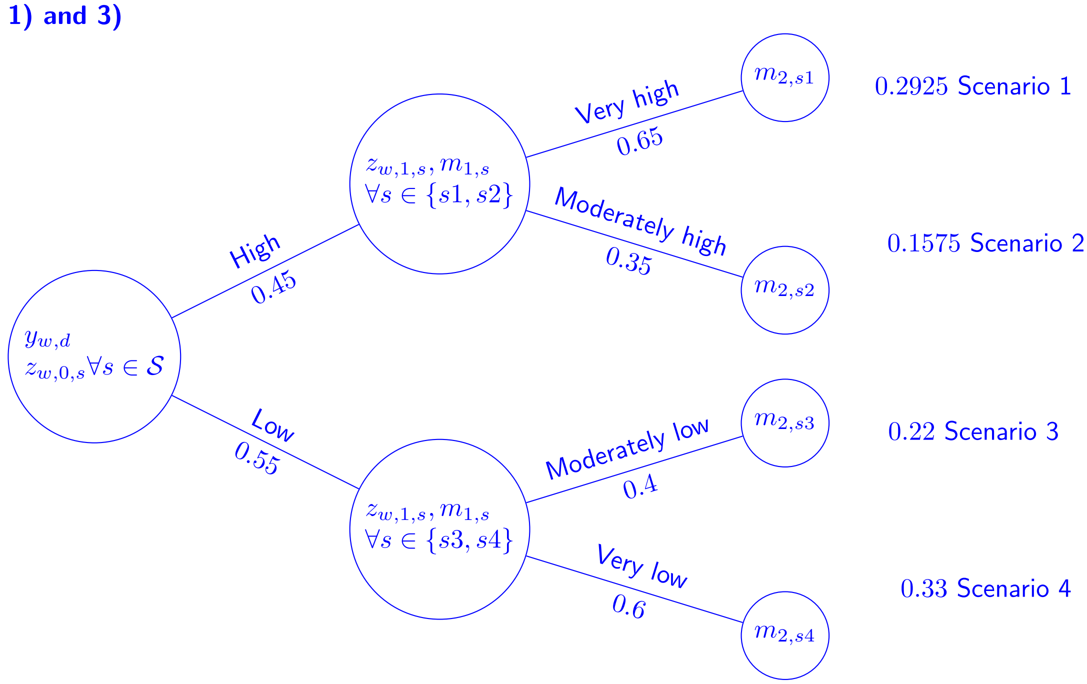
Scenario	Year 1 Probability	Scenario	Year 2 Probability
High demand	0.45	Very high demand	0.65
		Moderately high demand	0.35
Low demand	0.55	Moderately low demand	0.4
		Very low demand	0.6

Tasks:

1. Draw the scenario tree. Tag the branches on each stage with the case they represent and calculate the overall scenario probability.
2. Model the above mentioned planning problem as multi-stage stochastic program.
3. Extend the scenario tree in Task 1 with decision variables by assigning the decision variables to the nodes in the scenario tree. (Give only the indices for time periods and scenarios explicitly. All warehouses and districts can be simply stated by the index w and d, respectively.)
4. Solve the model using the files `parcel.jl` and `demand.csv` with already given data-input (both files have to be in the same directory). You can extend the sets, parameters and variables if necessary. You are allowed to model the non-anticipativity constraints with explicit values. Report the objective, warehouse-district assignments and number of drivers per period and scenario.

Solution

1) and 3)



2)

Sets	
$\mathcal{W}$	Warehouses $w \in \mathcal{W}$
$\mathcal{D}$	Districts $d \in \mathcal{D}$
$\mathcal{T}$	Time periods $t \in \mathcal{T}$ (here $\{0,1,2\}$)
$\mathcal{T}^H$	Hiring periods $t \in \mathcal{T}^H$ (here $\{0,1\}$)
$\mathcal{T}^D$	Delivery periods $t \in \mathcal{T}^D$ (here $\{1,2\}$)
$\mathcal{S}$	Scenarios $s \in \mathcal{S}$
$\Omega_{s,t}$	Scenarios $\omega \in \Omega_{s,t}$ that have to be considered for non-anticipativity with $s \in \mathcal{S}$ at period/stage $t \in \mathcal{T}$
Parameters	
π_s	Probability of scenarios s
$A_{d,t,s}$	Demand in district d in period t and scenario s [hours]
H	Working hours per driver per year [hours]
K_w	Maximum number of drivers at warehouse w
ϕ	Penalty factor for one hour unfulfilled demand [EUR/hour]
$C^T_{w,d}$	Transportation cost for assigning district d to warehouse w for two years [EUR]
C^P	Yearly personnel cost for assigning district d to warehouse d [EUR]
Variables	
$y_{w,d}$	Binary variable: 1, if warehouse $w \in \mathcal{W}$ is assigned to district $d \in \mathcal{D}$, 0, otherwise
$z_{w,t,s}$	Number of drivers at warehouse $w \in \mathcal{W}$ in period $t \in \mathcal{T}^H$ and scenarios $s \in \mathcal{S}$
$m_{t,s}$	Missing demand in period $t \in \mathcal{T}^D$ and scenarios $s \in \mathcal{S}$

Objective function to minimize cost and penalties:

$$\text{Min} \sum_{w \in \mathcal{W}} \sum_{d \in \mathcal{D}} C^T_{w,d} y_{w,d} + \sum_{s \in \mathcal{S}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}^H} \pi_s C^P z_{w,t,s} + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}^D} \pi_s \phi m_{t,s} \tag{1}$$

Each district has to be assigned to exactly one warehouse:

$$\sum_{w \in \mathcal{W}} y_{w,d} = 1 \qquad \forall d \in \mathcal{D} \tag{2}$$

Demand fulfillment for each district depending on the assigned warehouse and number of drivers (Note that the number of drivers has to be done in $t-1$ to fulfill the demand in t):

$$H z_{w,t-1,s} \geq \sum_{d \in \mathcal{D}} A_{d,t,s} y_{w,d} - m_{t,s} \qquad \forall w \in \mathcal{W}, t \in \mathcal{T}^D, s \in \mathcal{S} \tag{3}$$

Maximum number of drivers per warehouse is limited:

$$z_{w,t,s} \leq K_w \qquad \forall w \in \mathcal{W}, t \in \mathcal{T}^H, s \in \mathcal{S} \tag{4}$$

Reallocation of drivers and hiring of new drivers in the next period, but no firing.

$$\sum_{w \in \mathcal{W}} z_{w,t-1,s} \leq \sum_{w \in \mathcal{W}} z_{w,t,s} \qquad \forall t \in \mathcal{T}^H \backslash \{1\}, s \in \mathcal{S} \tag{5}$$

Non-anticipativity of z-variables:

$$z_{w,t,s} = z_{w,t,\omega} \qquad \forall w \in \mathcal{W}, t \in \mathcal{T}^H, s \in \mathcal{S}, \omega \in \Omega_{s,t} \tag{6}$$

Non-anticipativity of m-variables (note $\Omega_{s,t}$ contains only the scenario s at stage $t=3$):

$$m_{t,s} = m_{t,\omega} \qquad \forall t \in \mathcal{T}^D, s \in \mathcal{S}, \omega \in \Omega_{s,t} \tag{7}$$

Non-negativity and variable domains:

$$y_{w,d} \in \{0,1\} \qquad \forall w \in \mathcal{W}, d \in \mathcal{D} \tag{8}$$

$$z_{w,t,s} \in \mathbb{Z}^+ \qquad \forall w \in \mathcal{W}, t \in \mathcal{T}^H, s \in \mathcal{S} \tag{9}$$

$$m_{t,s} \in \mathbb{R}^+ \qquad \forall t \in \mathcal{T}^D, s \in \mathcal{S} \tag{10}$$

4)

See `parcel.gms`.
Objective value: 11606057.617

Assignments

Warehouse	Districts
w1	d1, d2, d5
w2	d3, d4, d9
w3	d6, d7, d8, d10

Drivers

Period	t1				t2			
Scenario	**s1**	**s2**	**s3**	**s4**	**s1**	**s2**	**s3**	**s4**
w1	9				9		9	
w2	10				10		12	
w3	15				15		13	

Missing demand

Period	Scenario	Hours
t1	s1	0
t1	s2	0
t1	s3	0
t1	s4	0
t2	s1	0
t2	s2	0
t2	s3	0
t2	s4	0
t3	s1	305.93
t3	s2	0
t3	s3	0
t3	s4	0