

## Exercise - Adjustable Robust optimization

14-04-2020

### Production planning continued

Assume the following robust program (1a) - (1g) for the production example from the exercise on Robust Optimization (31-03-2020).

Here the full problem statement as reminder:

*You work for a production company and support them with optimizing their capacity and production schedule for a new factory.*

*The company has  $p \in P$  different products that are produced on different machine types  $m \in M$ . Not each product can be produced on each machine, i.e., parameter  $a_{p,m} = 1$ , if product  $p$  can be produced on machine type  $m$  and  $a_{p,m} = 0$  otherwise. As you are opening a new factory, you also have to decide how many machines of type  $m$  you want to buy. The price is  $c_m^M$  for one machine of type  $m \in M$ . Each machine of type  $m \in M$  provides  $T_m$  hours of production.*

*The production costs are  $c_p^P$  for each  $p \in P$ . The targeted production quantities  $d_p$  for each product  $p \in P$  for the next year are given and should at least be covered. Because we consider the entire year, we approximate the production quantities as continuous values.*

*The production time of product  $p \in P$  is uncertain. You know that the expected production time is  $\bar{t}_p$  and the deviation (positive and negative) can be up to  $t_p^{\text{dev}}$ . From experience from other factories, we can conclude that for each machine type  $m \in M$  not more than 30% of the products that can be produced on machine type  $m$  will have a deviation from the expected production time.*

Use the following robust formulation using a box-uncertainty set.

$y_m$  Number of machines of type  $m$

$x_{p,m}$  Production amount of product  $p$  on machine type  $m$

$$\min \sum_{m \in M} \left[ c_m^M y_m + \sum_{p \in P} c_p^P x_{p,m} \right] + \sum_{p \in P} \Phi \delta_p(\zeta_p) \quad -1 \leq \zeta_p \leq 1 \quad (1a)$$

$$s.t. \sum_{m \in M} x_{p,m} + \delta_p(\zeta_p) \geq d_p \quad \forall p \in P, -1 \leq \zeta_p \leq 1 \quad (1b)$$

$$\sum_{p \in P} \bar{t}_p x_{p,m} + \sum_{p \in P} t_p^{\text{dev}} \zeta_p x_{p,m} \leq T_m y_m \quad \forall m \in M, -1 \leq \zeta_p \leq 1 \quad (1c)$$

$$x_{p,m} \leq \text{BigM}_{m,p} a_{p,m} y_m \quad \forall m \in M, p \in P \quad (1d)$$

$$y_m \geq 0 \text{ and integer} \quad \forall m \in M \quad (1e)$$

$$x_{p,m} \geq 0 \quad \forall m \in M, p \in P \quad (1f)$$

$$\delta_p(\zeta_p) \geq 0 \quad \forall p \in P, -1 \leq \zeta_p \leq 1 \quad (1g)$$

where  $\zeta_p$  describes the uncertainty in the production time of product  $p$ .  $\text{BigM}_{m,p}$  is a large constant (here:  $\frac{T_m}{\bar{t}_p - t_p^{\text{dev}}}$ ).

The objective function (1a) minimizes the cost. Constraints (1b) ensure that the targeted production amounts are fulfilled. Constraints (1c) limits the production time and constraints (1d) model the compatibility between products and machine types.

We now allow the demand constraint in (1b) to be violated, i.e., not fulfilling the demand is allowed but penalized with  $\Phi$  per unit in the objective function.  $\delta_p$  is the amount of products of type  $p$  that we are missing.

**Task**

1. Transform constraint (1c) to a linear robust formulation by transforming the box uncertainty set.
2. The missing amount  $\delta_p$  is a recourse decision and based on the uncertainty of the production time  $\zeta_p$ . Therefore, you need to formulate a Linear Decision Rule dependent on the uncertain parameter  $-1 \leq \zeta_p \leq 1$ . Reformulate the necessary parts of the model based on your Linear Decision Rule.
3. Solve the model for different values of the penalty term  $\Phi = \{100, 150, 200, 300\}$  using the model in the Julia file `production_ARO.jl` as a basis. The file has deterministic model of the problem above, i.e., a placeholder variable for  $\delta_p$  is used and constraint (1c) uses the mean value of the uncertain parameter.