Exercise - Robust optimization

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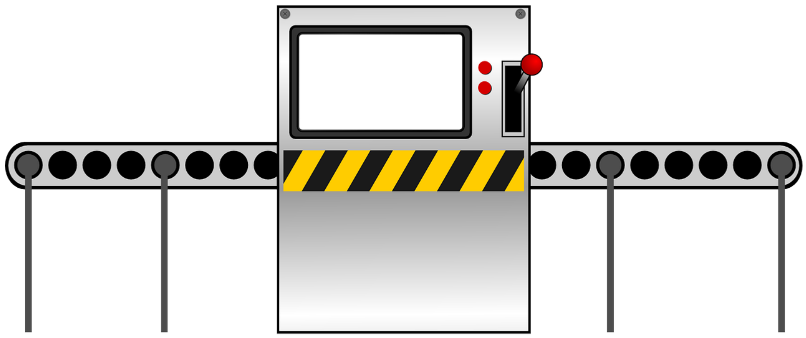
Production planning



You work for a production company and support them with optimizing their capacity and production schedule for a new factory.

The company has $p \in P$ different products that are produced on different machine types $m \in M$. Not each product can be produced on each machine, i.e., parameter $a_{p,m} = 1$, if product p can be produced on machine type m and $a_{p,m} = 0$ otherwise. As you are opening a new factory, you also have to decide how many machines of type m you want to buy. The price is c^M_m for one machine of type $m \in M$. Each machine of type $m \in M$ provides T_m hours of production.

The production costs are c^P_p for each $p \in P$. The targeted production quantities d_p for each product $p \in \mathcal{P}$ for the next year are given and should at least be covered. Because we consider the entire year, we approximate the production quantities as continuous values.

The production time of product $p \in P$ is uncertain. You know that the expected production time is $\bar{t}_p$ and the deviation (positive and negative) can be up to t^{dev}_p. From experience from other factories, we can conclude that for each machine type $m \in M$ not more than 30% of the products that can be produced on machine type m will have a deviation from the expected production time.

Have a look at the following model formulation

y_m	Number of machines of type m
$x_{p,m}$	Production amount of product p on machine type m

$$\min \sum_{m \in M} \left[c^M_m y_m + \sum_{p \in P} c^P_p x_{p,m} \right] \tag{1a}$$

$$s.t. \sum_{m \in M} x_{p,m} \geq d_p \qquad \forall p \in P \tag{1b}$$

$$\sum_{p \in P} \tilde{t}_p x_{p,m} \leq T_m y_m \qquad \forall m \in M, \tilde{t}_p \in [\bar{t}_p - t^{\text{dev}}_p, \bar{t}_p + t^{\text{dev}}_p] \tag{1c}$$

$$x_{p,m} \leq BigM_{m,p} a_{p,m} y_m \qquad \forall m \in M, p \in P \tag{1d}$$

$$y_m \geq 0 \text{ and integer} \qquad \forall m \in M \tag{1e}$$

$$x_{p,m} \geq 0 \qquad \forall m \in M, p \in P \tag{1f}$$

where $\tilde{t}_p$ represents the uncertain production time and $BigM_{m,p}$ a large enough constant (here: $\frac{T_m}{\bar{t}_p - t^{\text{dev}}_p}$).

The objective function (1a) minimizes the cost. Constraints (1b) ensure that the targeted production amounts are fulfilled. Constraints (1c) limits the production time and constraints (1d) model the compatibility between products and machine types.

Task

The model above is the robust counterpart that does not consider the uncertainty set in a linear formulation. Furthermore, it does not include a budget of uncertainty.
Your tasks is to transform the model to a robust linear formulation using the budget of uncertainty Γ_m per machine type $m \in M$. (Hint: We are mainly talking about constraint (1c).)

Solve your robust model with Julia JuMP and Gurobi using the file `production.jl` which already has some data input and part of the model formulation. The part about the production time is missing. You have to add the missing constraints and variable needed for the budget of uncertainty formulation. Compare the solution to the following cases:

- no deviation from the expected production time is included in the model ($\Gamma_m = 0$)
- full deviation from the expected production time is included in the model (all products on that machine type can deviate at the same time)