Assignment 1

Deadline: 6th March 2020, 18:00h

General information

- This assignment is part of the overall assessment of this course and, therefore, your answer counts for the final grade.
- This assignment has to be solved and submitted by every student individually before the above mentioned deadline closes. Group work and group submissions are not allowed.
- The assignment has to be submitted via DTU Inside. Use the entry `Assignments` in the course menu and upload your files to the corresponding assignment. In case of technical problems with DTU Inside, please send your files to `dngk@dtu.dk` before the deadline.
- The submission must consist of a pdf-document containing the answers to the tasks below. Furthermore, program code and scripts have to be uploaded as well.

Part I - Modelling and duality

Task 1 - Modelling a mixed-integer linear program (14 Points)

You help a flower nursery to distribute the space between different types of flowers for the next season. In total, the nursery has 5000 m^2 space to distribute. There are three flower types: roses, garden pinks and dahlia. The different types require different amounts of working hours, water and fertilizer per year (see table). The selling prices per m^2 at the end of the season per type are also given in the table. The nursery can use a total of 20000 working hours, 700000 l water and 2500 kg fertilizer per year.

Type	**Selling price [EUR/m^2]**	**Working hours [h/m^2]**	**Water [l/m^2]**	**Fertilizer [kg/m^2]**
Roses	25	2.5	90	0.3
Dahlia	30	5	100	0.5
Garden pinks	20	2	120	0.2

For Tasks 1.1-1.4: This is a modelling exercise, you do not need to solve the model. In all tasks, declare and describe all variables and constraints. In case you need additional parameters, such as big-M, please specify a reasonable value based on the data above (including justification). For tasks 1.2.-1.4.: You can state the changes to the previous model.

1. Formulate an **explicit** mathematical program for the above mentioned data to maximize the income from selling the flowers at the end of season. (For an example of an explicit model formulation see Lecture 01 - Recap Mathematical Optimization, slide 14)
2. Extend your model to capture the following requirement. If the nursery plants roses, they need to build a greenhouse at cost of 20000 EUR. The greenhouse does not give additional space, but takes up 700m^2 of the initial 5000m^2. Roses can only be planted in the greenhouse. Remaining space in the greenhouse can be filled with any flower type.
3. Extend your model to capture the following requirement. If the nursery builds the greenhouse, they have use it for at least 200m^2 of roses.
4. Extend your model to capture the following requirement. The remaining space of the greenhouse can only be used for roses and garden pinks but not dahlia.

Task 2 - Primal-dual-transformation (8 Points)

Transform the following primal LP to its dual LP. You do not need to solve the model.

$$
\begin{array}{lrcrcrcr}
\text{Min} & -5x_1 & + & 4x_2 & - & 3x_3 & & \\
\text{s.t.} & 2x_1 & - & 3x_2 & - & x_3 & \leq & 5 \\
 & 4x_1 & - & x_2 & + & 2x_3 & \geq & 11 \\
 & -3x_1 & + & 4x_2 & + & 2x_3 & \leq & 8 \\
 & 6x_1 & - & 5x_2 & + & x_3 & = & 1 \\
 & & & & & & & 0 \leq x_1, x_2 \\
 & & & & & & & x_3 \in \mathbb{R}
\end{array}
$$

Part II - Stochastic Programming

Task 3 - General questions (12 Points)

Answer the following questions/tasks in a few sentences using your own words.

1. Explain the general decision variable structure of a two-stage program.
2. What is the advantage of stochastic programming compared to a deterministic optimization?
3. Which are the more important decisions in a two-stage stochastic program: here-and-now decisions or wait-and-see decision? Justify your answer. (In other words: Which decisions are we more interested in when we solve a two-stage stochastic program)

Task 4 - Modelling a two-stage stochastic program (42 Points)

The furniture producer UKEA is expanding to a new region by opening new production facilities. Your task is to support them to decide the locations and capacities of the new facilities. The company is producing a set of different furniture pieces $\mathcal{F}$. Based on their knowledge from already covered regions, they provide you with set of demand scenarios $\mathcal{S}$ for the markets $\mathcal{M}$ in the region.
UKEA has to make the following decisions before opening the new production facilities. First, they have to select the locations from a set of possible locations $\mathcal{I}$. Second, the capacity of each facility can be decided based on a set of possible capacity levels $\mathcal{L}$. Third, it needs to be decided which market is covered by which facility. Each market is assigned to only one facility. Furthermore, you should take the following information into account:

- The planning horizon $\mathcal{T}$ has a length of 10 years divided in 10 time periods t with a length of one year.
- The parameter $b_{m,f,t,s}$ gives you the forecasted demand per market m, product f, time period t and scenario s.
- Transporting the furniture from the facility to the market costs c^T per km. The distance between location and market is given by $d^M_{i,m}$.
- The locations have different yearly operational costs of c_i.
- The distance $d^L_{i,j}$ between two opened locations has to be at least D km.
- The capacity levels are defined by different building costs (has to be paid once when the facility is opened) c^B_l as well as production capacities k^P_l and storage capacities k^S_l per period (both given in units of furniture).
- Furniture can be stored in the storage from one period to the next while respecting the installed storage capacities.
- It is possible to transport products between two opened locations at a cost of c^T per km and product unit.
- As we are planning for a long planning horizon, you can assume the production and transportation amounts as continuous amounts.
- Each scenario s has a given probability π_s.
- You are looking for the cost minimal solution.

```
Explored 1 nodes (476 simplex iterations) in 0.46 seconds
Thread count was 4 (of 4 available processors)

Solution count 2: 107253 107836

Optimal solution found (tolerance 1.00e-04)
Best objective 1.072529221755e+05, best bound 1.072529221755e+05, gap 0.0000%
<Further output can be here
and here .... >
Location:1,open,3
Location:2,not_open,0
Location:3,not_open,0
Market:1,5
Market:2,5
Market:3,5
Market:4,1
Objective:107252.922
julia>
```

Figure 1: Example output at the end of the program

1. Formulate a **general** two-stage stochastic program for the furniture producer. Define and describe all necessary sets, variables, parameters and constraints in your report. (For an example of an general model formulation see Lecture 01 - Recap Mathematical Optimization, slide 14)

2. Implement and solve your model using the `furniture.jl` file and the data file `demand.csv`. Solve it with standard settings of the Gurobi solver. The data is given in the `furniture.jl` file. Make sure the file `demand.csv` is in the same directory as `furniture.jl`. Extend the `furniture.jl` file by implementing your model. Comment the program code. Furthermore, you have to add the following output, so that your program prints out the following last rows:

   `Location:<id of location i>,<open/not_open>,<level>`

   ... for all locations

   `Market:<id of market m>,<assigned location id>`

   ... for all markets

   `Objective:<objective value>`

   Use 0 for the level, if the location is not open. You can have further output before the lines stated above. An example for 3 locations and 4 markets is given in Fig 1.

   Report this information also in your report.

   Rename the file to `furniture-<studentnumber>.jl` and submit it along with the report. Please also add your name and student number as a comment at the top of the file.

Task 5 - Evaluation of stochastic programs (24 Points)

Evaluate the two-stage stochastic program in `heat.jl` with respect to the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS). Use the data given in the file.

Calculate the EVPI and VSS and interpret the two values in your own words also with respect to the problem setting (what do the numbers mean).

State also the expected values of the wait-and-see solution and expected value solution that you used for calculation.

Upload your scripts (the last part of the file name must be your student number.
Example: `heat-vss-<studentnumber>.jl`).

The file contains a model for the following planning problem:
A small district heating company uses the following model to optimize their production. The company has two heat producing units $\mathcal{I}$ to fulfill the heat demand in the connected district heating network, namely a gas boiler (GB) and a wood chip boiler (WCB). Each production unit has maximum production of heat Q_i. The cost to produce one MWh heat for each unit is given by c_i. The district heating provider has thermal storage of size K MWh that can store heat from one period to the next.

The model aims for the optimal production of all units for every hour $t \in T$ of the next day ($T = \{1, \ldots, 24\}$) which has to be determined one day in advance.The goal is to obtain a cost minimal production schedule.
The heat demand $d_{t,s}$ is uncertain on the day before, but the company has a forecasting tool to obtain scenarios $s \in S$ for the next day. If the district heating network can not fulfill the heat demand the missing heat is penalized with 10000 EUR in the objective function.

Sets	
I	Heat production units
T	Time periods (hours)
S	Scenarios
Parameters	
$d_{t,s}$	Heat demand in time period t and scenario s [MWh-heat]
c_i	Cost producing 1 MWh heat of unit i and j, respectively [EUR]
Q_i	Max. heat per hour for unit i and j, respectively [MWh-heat]
Variables	
$q_{i,t}$	Heat production of unit i in period t [MWh-heat]
$s^{in}_{t,s}$	Inflow to storage in period t in scenario s [MWh-heat]
$s^{out}_{t,s}$	Outflow of storage to district heating in period t and scenario s [MWh-heat]
$s_{t,s}$	Storage level in period t and scenario s [MWh-heat]
$m_{t,s}$	Missing heat in period t and scenario s [MWh-heat]

$$Min \sum_{i \in I} \sum_{t \in T} c_i q_{i,t} + \sum_{i \in I} \sum_{t \in T} \sum_{s \in S} \pi_s 10000 m_{t,s} \tag{1}$$

$$
\begin{aligned}
s.t. \quad & q_{i,t} \leq Q_i && \forall i \in I, t \in T && (2)\\
& \sum_{i \in I} (q_{i,t}) - s^{in}_{t,s} + s^{out}_{t,s} = d_{t,s} - m_{t,s} && \forall t \in T, s \in S && (3)\\
& s_{t,s} = s_{t-1,s} + s^{in}_{t,s} - s^{out}_{t,s} && \forall t \in T\{\backslash 1\}, s \in S && (4)\\
& s_{1,s} = s^{in}_{1,s} - s^{out}_{1,s} && \forall s \in S && (5)\\
& s_{t,s} \leq K && \forall t \in T, s \in S && (6)\\
& q_{i,t} \geq 0 && \forall i \in I, t \in T && (7)\\
& s_{t,s} \geq 0 && \forall t \in T, s \in S && (8)\\
& s^{out}_{t,s} \geq 0 && \forall t \in T, s \in S && (9)\\
& s^{in}_{t,s} \geq 0 && \forall t \in T, s \in S && (10)\\
& m_{t,s} \geq 0 && \forall t \in T, s \in S && (11)
\end{aligned}
$$

The objective function (1) minimizes the expected cost of the production taking the penalty costs into account. Constraints (2) limit the capacity of the productions units, respectively. Constraints (3) ensure that the heat demand is fulfilled in every period and scenario while at the same time managing the inflow and outflow to the heat storage. This means the constraint balances flow, i.e., the production is going to fulfill the demand or going to the storage and the demand is coming from the storage or the the production units. The storage level per period is updated with in- and outflow in constraints (4) and for the special case of the first period in constraints (5). The capacity of the heat storage is limited by constraints (6). Finally, the non-negativity of the variables is enforced in constraints (8) to (11).