Scientific Computing for Differential Equations 1

Lecture 02 - The Explicit Euler Method

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Ordinary Differential Equations

The Explicit Euler Method

Ordinary Differential Equation (ODE)

$$\frac{d}{dt}x(t) = f(t, x(t)) \quad \text{or} \quad \dot{x}(t) = f(t, x(t)) \tag{1}$$

Solution method by integration

$$x(t_{k+1}) - x(t_k) = \int_{x(t_k)}^{x(t_{k+1})} dx(t) = \int_{t_k}^{t_{k+1}} f(t, x(t))dt \tag{2}$$

Let $x_k = x(t_k)$. Then (2) can be approximated as

$$x_{k+1} = x_k + \Delta t_k f(t_k, x_k) \tag{3}$$

The Explicit Euler Method

Ordinary Differential Equation (ODE)

$$\frac{d}{dt}x(t) = f(t, x(t)) \quad \text{or} \quad \dot{x}(t) = f(t, x(t)) \tag{4}$$

Solution method by differentiation

$$\frac{x(t_{k+1}) - x(t_k)}{\Delta t_k} \approx \frac{d}{dt}x(t_k) = f(t_k, x(t_k)) \tag{5}$$

Let $x_k = x(t_k)$. Then (2) can be approximated as

$$x_{k+1} = x_k + \Delta t_k f(t_k, x_k) \tag{6}$$

Initial Value Problem and the Explicit Euler Method

Initial value problem

$$x(t_0) = \bar{x}_0 \tag{7a}$$

$$\frac{d}{dt}x(t) = f(t, x(t)) \qquad t_0 \le t \le t_N \tag{7b}$$

Fixed time step

$$\Delta t = \frac{t_N - t_0}{N} = \frac{t_b - t_a}{N} \tag{8}$$

Euler's explicit method

$$x_0 = \bar{x}_0 \tag{9a}$$

$$x_{k+1} = x_k + \Delta t f(t_k, x_k) \qquad k = 0, 1, \ldots, N-1 \tag{9b}$$

Matlab Implementation of Euler's Explicit Method

Euler's Explicit Method

$$x_0 = \bar{x}_0 \tag{10a}$$

$$x_{k+1} = x_k + \Delta t f(t_k, x_k) \qquad k = 0, 1, \ldots, N-1 \tag{10b}$$

```
function [T,X] = ExplicitEulerFixedStepSize(fun,ta,tb,N,x0,varargin)
% Compute step size and allocate memory
dt = (tb-ta)/N;
nx = size(x0,1);
X = zeros(nx,N+1);
T = zeros(1,N+1);
% Eulers Explicit Method
T(:,1) = t0;
X(:,1) = x0;
for k=1:N
f = feval(fun,T(k),X(:,k),varargin{:});
T(:,k+1) = T(:,k) + dt;
X(:,k+1) = X(:,k) + dt*f;
end
% Form a nice table for the result
T = T';
X = X';
```

Exercises

- Implement Euler's method with fixed step-size
- Test it for various step size using the following system
  - $\dot{x}(t) = \lambda x(t)$, $x(t_0) = 1$
  - $\dot{x}(t) = cos(t)x(t)$, $x(t_0) = 1$
  - The van der Pol problem
  - The prey-predator problem
- Compute the error (the global error) at a given time say the end time as function of the step size. Plot the results
- Compute the error (the local error) at the first step as function of the step size. Comment on the plots
- Does the local and global error behave as you expect