Assignment 2: AC-OPF with convex relaxations (SDP-OPF)

October 1, 2019

Deadline: Monday, November 4, 2019, at 20:00

Deliverables

- A report which will address the following questions:
  - Tasks 2, 3, 4, 5, 6, (bonus: 7) (max. 3.5 pages)
  - Lessons Learned, and Sharing of Workload (max. 0.5 pages)
- The source code (Matlab) for the implementation of all tasks and case studies

1 Introduction

The AC optimal power flow problem (AC-OPF) is central to power system operation. In this optimization problem, an objective function (e.g. the generation cost or system losses) is minimized subject to the power system constraints (on e.g. voltages, line limits or generator limits) and the AC power flow equations. Recent works in literature have achieved to relax the non-linear, non-convex AC-OPF problem to convex formulations, for example the semidefinite relaxation in [1] yields a semidefinite program (SDP). These convex relaxations of the AC-OPF have attained significant research interest as in several test cases they provably yield the global optimum to the original non-convex problem. The goal of this assignment is to implement the semidefinite relaxation of the AC optimal power flow, investigate its properties and compare it to the non-linear, non-convex AC-OPF. The goal of this assignment is to implement the semidefinite relaxation of the AC optimal power flow (AC-OPF), investigate its properties and compare it to the non-linear, non-convex AC-OPF.

The objectives to be achieved at the end of the assignment are the following:

- Understand the concept of convex relaxations and the notion of exactness of a relaxation
- Implement the semi-definite relaxation of the AC-OPF and evaluate its relaxation gap
- Compare the solution of the relaxation to the solution of the original non-convex AC-OPF problem and evaluate its feasiblity
- Decompose the solution matrix $\mathbf{W}$ and recover the optimal voltage vector
- Understand when a relaxation might fail and investigate possible methods to obtain a rank-1 solution

2 IEEE 9 bus system

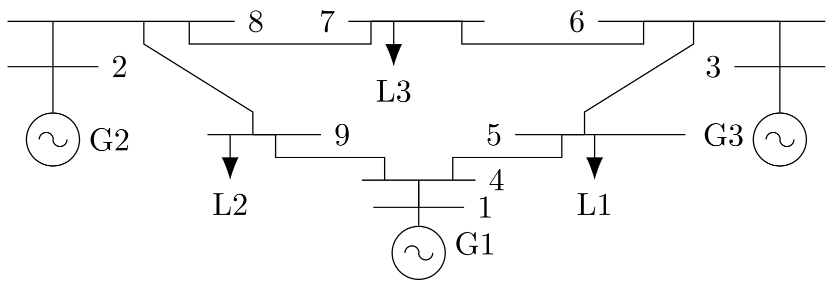

Figure 1: IEEE 9-bus system

In the first part, the IEEE 9 bus test system is used which is shown in Fig. 1. This simple meshed transmission grid has three generators and three loads. We use the specifications provided in MATPOWER [2].

3 Tasks: Implement the Semidefinite Relaxation of the AC Optimal Power Flow

- As we consider an AC power network in this assignment, you can find a file `sdpopf_pseudocode.m` with some pseudocode and hints online. The system data for the test cases is also available:
  - `case9_SDP`: IEEE 9 bus system from [2]
  - `case3_SDP`: 3 bus system from [3]

1. Implement the semidefinite relaxation of the AC-OPF based on [1] for `case9_SDP`. Include objective function and constraints in the optimization problem. Use the `makeYbus` and `makesdpmat` function of MATPOWER to calculate the bus and the line admittance matrices and the resulting auxiliary variables. To achieve a formulation linear in $\mathbf{W}$ remeber to use the reformulation based on Shur's complement.

2. Solve the semidefinite program (SDP) using MOSEK and compare objective value and resulting active and reactive bus power injections with the original non-convex AC-OPF in MATPOWER using `runopf()`.

3. Evaluate the exactness of the relaxation. The work in [4] proposes the following heuristic measure to validate the rank-1 property of $\mathbf{W}$: Compute the ratio of the 2nd and 3rd eigenvalue of $\mathbf{W}$. If this ratio is larger than 10^5, the matrix $\mathbf{W}$ is considered to have rank-1.

4. Post-process the results. Implement the decomposition of the matrix $\mathbf{W}$ into the optimal voltage vector by means of an eigendecomposition. Compare the optimal voltage vector in polar coordinates to the solution of the non-convex AC-OPF.

5. Vary network properties step-wise (e.g. further constrain line flow limits) and evaluate the exactness of the relaxation for the IEEE 9 bus system. Compare your results with the non-convex AC-OPF in terms of objective function. Comment on the feasilibity of the non-convex AC-OPF and the semidefinite relaxation of the AC-OPF. Hint: if you want to plot the eigenvalue ratio as a function of a certain limit, use a logarithmic scale; changes might then be better visible.

6. Investigate the exactness of the semidefinite relaxation for the provided 3 bus test system:

   - Assume line flow limits as provided in the `case3_SDP` file.
   - Reduce the apparent line flow limit on the line from bus 3 to bus 2 from 60 MVA to 50 MVA.
   - Compare your results to the findings from the IEEE 9 bus system. Comment on feasibility and objective value of the non-convex AC-OPF and the semidefinite relaxation of the AC-OPF.

7. **Bonus**: Include a penalty factor on the reactive power injections of generators in the objective function f based on [5]:

$$f_{\text{mod}} = f_{\text{cost}}(P_G) + \epsilon \sum_{k}^{n_G} Q_{G_k} \tag{1}$$

   The vectors P_G, Q_G denote the active and reactive generator power. The term n_G denotes the number of generators. Vary the value of penalty weight ϵ and investigate the exactness for the provided 3 bus test system. Compare your result to the non-convex AC-OPF.

4 Lesssons Learned – Reflection

During the development of your code for this assignment there were definitely several issues that came up until you got it running correctly.

In no more than half a page, please list 2-3 main points that you think you should remember for the next time you code an SDP-OPF or you have to evaluate results from an OPF.

Please list at least one issue that had to do with coding, i.e. what should you remember to do in some specific way, or avoid, next time you code an OPF? And please list at least one main takeaway from the evaluation of your results, i.e. what did you learn from evaluating the semi-definite relaxation and the non-convex AC-OPF?

5 Sharing of Workload

Please mention what were the responsibilities or the tasks that each group member carried out in order to accomplish this assignment.

References

[1] J. Lavaei and S. H. Low, "Zero duality gap in optimal power flow problem," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 92–107, 2012.

[2] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on power systems*, vol. 26, no. 1, pp. 12–19, 2011.

[3] B. C. Lesieutre, D. K. Molzahn, A. R. Borden, and C. L. DeMarco, "Examining the limits of the application of semidefinite programming to power flow problems," in *Communication, Control, and Computing (Allerton), 2011 49th Annual Allerton Conference on*. IEEE, 2011, pp. 1492–1499.

[4] D. K. Molzahn, J. T. Holzer, B. C. Lesieutre, and C. L. DeMarco, "Implementation of a large-scale optimal power flow solver based on semidefinite

programming," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3987–3998, 2013.

[5] R. Madani, S. Sojoudi, and J. Lavaei, "Convex relaxation for optimal power flow problem: Mesh networks," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 199–211, 2015.