Technical University of Denmark

31765 Optimization in modern power systems

Assignment 2
AC-OPF with convex relaxations (SDP-OPF)

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1 SDP-OPF vs. Non-convex AC-OPF

The results obtained from the SDP-OPF vs. Non-convex AC-OPF are **exactly the same** in terms of objective function value, active power injections and reactive power injections. As discussed in the following sections, the only difference is on the handling of the SDP problem and the resulting eigenvalues and, therefore, optimal voltage vector. We solved the SDP in two different ways, one with the normal formulation from class, and another by constraining the slack bus voltage angle to 0, i.e. setting $\mathbf{W}$(N+1,:)= 0 and $\mathbf{W}$(:,N+1)= 0, with $N = 9$ and Node 1 as the slack bus. This change gives exactly the same results as doing the SDP without changing the $\mathbf{W}$ matrix.
The objective function value is **5,297.41 \$/hr** for all three cases (i.e. normal SDP, SDP with slack bus voltage angle = 0, and non-convex AC-OPF). The power injections can be seen on **Table 1**.[1]

	SDP		SDP with slack angle = 0		Non-convex AC-OPF	
	P	Q	P	Q	P	Q
Node 1	89.8	12.9	89.8	12.9	89.8	12.9
Node 2	134.3	0.1	134.3	0.1	134.3	0.1
Node 3	94.2	-22.6	94.2	-22.6	94.2	-22.6
Node 4	0.0	0.0	0.0	0.0	0.0	0.0
Node 5	-90.0	-30.0	-90.0	-30.0	-90.0	-30.0
Node 6	0.0	0.0	0.0	0.0	0.0	0.0
Node 7	-100.0	-35	-100	-35	-100	-35
Node 8	0.0	0.0	0.0	0.0	0.0	0.0
Node 9	-125	-50	-125	-50	-125	-50

Table 1: Active and Reactive Power [units in MW and MVAR]

2 Exactness of the Relaxation

Figure 1 and **Figure 2** show the eigenvalues of the $\mathbf{W}$ matrix for both cases of the SDP-OPF. **Figure 1** shows two eigenvalues that are non-zero and 16 zero-eigenvalues. In this case, the rank of the $\mathbf{W}$ matrix is 2 if the relaxation is exact. While in **Figure 2** only one eigenvalue is non-zero with 17 zero-eigenvalues. In this case, the rank of the $\mathbf{W}$ matrix is 1 if the relaxation is exact. The relevant eigenvalues to calculate the ratio are ($\rho_2 = 5.3614$ and $\rho_3 = 5.59E^{-}9$) for the normal SDP and ($\rho_1 = 10.7241$ and $\rho_2 = 1.74E^{-}8$) for the SDP with slack bus voltage angle = 0.

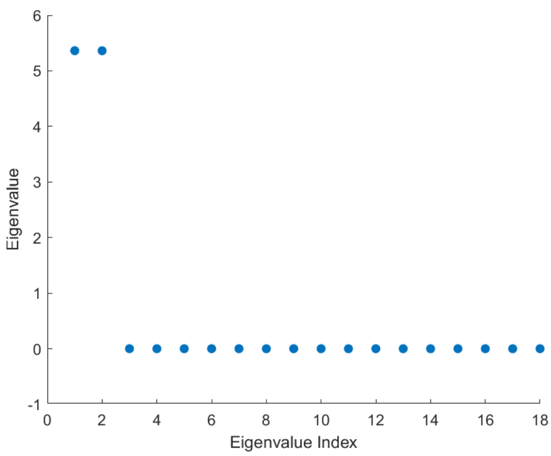

Figure 1: Eigenvalues - W-rank 2

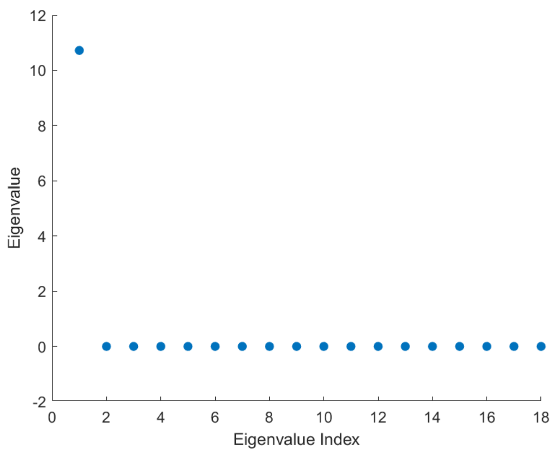

Figure 2: Eigenvalues - W-rank 1

[1]Note that Node 1 to 3 refers to the active and reactive power injections from the generators, while node 5, 7, and 9 refers to the active and reactive power consumption from the loads.

To evaluate the satisfaction of the rank condition, the work by Molzahn[2] proposes a heuristic measure to validate the rank condition of **W**. In the case of the normal SDP, the ratio of the second and third largest eigenvalue is $9.77E^8$ and thus indicates a solution with zero duality gap. In the case of the SDP with slack bus voltage angle = 0, the ratio of the largest non-zero eigenvector and the second-largest eigenvector is $6.18E^8$ and thus also indicates a solution with zero duality gap. Ergo, both are exact solutions.

3 Eigendecomposition and Optimal Voltage Vector

After obtaining the **W** matrix, the optimal voltage vector was obtained using an eigendecomposition.[3] For the normal SDP, we used the formula from the lectures on SDP, i.e. $X_{opt} = \sqrt{\rho_1^{opt}} * E_1^{opt} + \sqrt{\rho_2^{opt}} * E_2^{opt}$, to obtain the complex bus voltage matrix $X_{opt} = [\Re(V)\Im(V)]^T$. While for the angle-constrained SDP, we used almost the same formula but only using the largest non-zero eigenvalue $X_{opt} = \sqrt{\rho_1^{opt}} * E_1^{opt}$. From **Table 2**, the magnitude of the bus voltage are all the same for the SDP, angle-constrained SDP and non-convex AC-OPF. However, the angles are different only for the normal SDP. This is because the slack bus voltage angle isn't constrained to be zero, since the system has one additional degree of freedom; but both largest eigenvalues are the same to numerical precision.[4]

	SDP		SDP with slack angle = 0		Non-convex AC-OPF	
	Magnitude	Angle	Magnitude	Angle	Magnitude	Angle
Node 1	1.1000	-63.34	1.1000	0.00	1.1000	0.00
Node 2	1.0975	-58.45	1.0975	4.89	1.0976	4.89
Node 3	1.0868	-60.09	1.0868	3.25	1.0868	3.25
Node 4	1.0942	-65.80	1.0942	-2.46	1.0942	-2.46
Node 5	1.0845	-67.32	1.0845	-3.98	1.0844	-3.983
Node 6	1.1000	-62.74	1.1000	0.60	1.1000	0.60
Node 7	1.0895	-64.54	1.0895	-1.19	1.0895	-1.19
Node 8	1.1000	-62.43	1.1000	0.90	1.1000	0.90
Node 9	1.0718	-67.95	1.0718	-4.61	1.0717	-4.61

Table 2: Voltage Magnitudes and Angles at every node [magnitude units in p.u. and angles in degrees]

4 Sensitivity Analysis: Network Constraints

	Objective Function Value						
	100%	90%	80%	70%	60%	50%	40%
SDP-OPF ($/hr)	5297	5297	5297	5297	5297	5310	0
AC-OPF ($/hr)	5297	5297	5297	5297	5297	5310	inf.

Table 3: Objective Function Value when changing network constraints

It can be observed that when the line limit decreases to around 50% of the nominal capacity, the total cost starts to raise due to congestion and around 40% of decrease the eigenvalue ratio drops to less than $10E^5$ making the relaxation not exact. The non-convex solution for the system start to change at around 50% and become non feasible at 40%. It can also be observed that exactness of relaxation of the Eigenvalue ratio is more stable when the rank of the matrix W is 1. Although, the matrix W with rank 2, presents higher eigenvalues ratios. It can be said that both options can get to optimal solutions in the range of 100% to 45% because they have a zero relaxation gap.

[2]D. K. Molzahn, J. T. Holzer, B. C. Lesieutre, and C. L. DeMarco, Implementation of a large-scale optimal power flow solver based on semidefinite programming," IEEE Transactions on Power Systems, vol. 28, no. 4, pp. 3987-3998, 2013.

[3]Ídem

[4]From SDP tutorials at class.

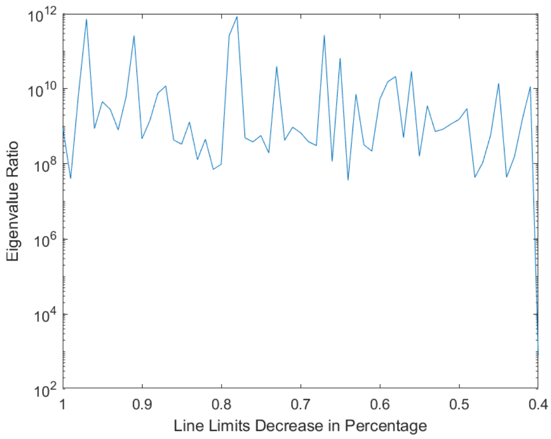

Figure 3: Exactness of Relaxation - W-rank 2

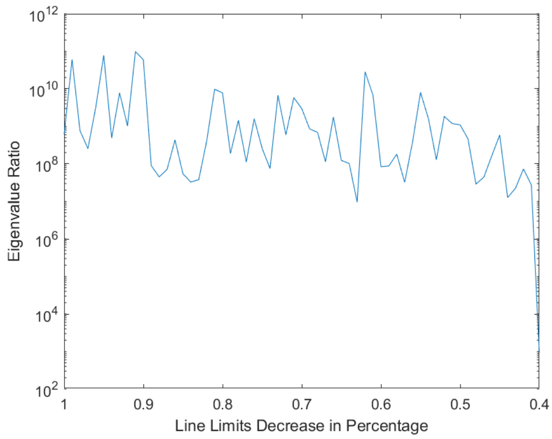

Figure 4: Exactness of Relaxation - W-rank 1

5 3-bus System

The results obtained from the SDP-OPF vs. Non-convex AC-OPF differ by 0.14 % in terms of objective function value when using the nominal value of the line from bus 3 to bus 2 and 1.06 % when reducing the line limit from 60 MVA to 50 MVA. To solve this case we solved the SDP in the with the same method as the example from IEEE 9. When constraining the slack bus voltage angle to 0, then the matrix W is rank 1.

The objective function value for nominal line limit is **6,782.80 $/hr** for SDP with slack bus voltage angle = 0 and **6,792.11 $/hr** for non-convex AC-OPF. For the case were the line limit is 50 MVA the objective function value is **6,824.60 $/hr** for SDP with slack bus voltage angle = 0 and **6,897.64 $/hr** for non-convex AC-OPF. In both cases it can be observed that the magnitude of the objective function value for the SDP is smaller than the one from the non-convex AC-OPF. This is due to the fact that the relaxation helps reaching the global minimum, and as the ratio of eigenvalues is greater than $10E^8$, then the relaxation is exact. Therefore the values from the SDP are closer to the global minimum and the values of the AC-OPF are located in a local minimum. A optimal global minimum can be obtained if a penalty for system losses is applied to the Shur constraint. The solution obtained will be more physically meaningful[5].

6 Bonus

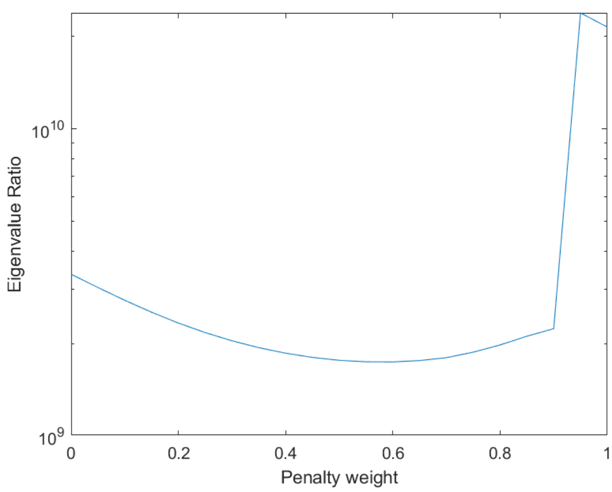

Figure 5: Exactness of the relaxation vs. Penalty Factor

[5] Venzke, A., Halilbasic, L., Markovic, U., Hug, G., Chatzivasileiadis, S. (2017). Convex Relaxations of Chance Constrained AC Optimal Power Flow. I E E E Transactions on Power Systems, 33(3), 2829 - 2841. https://doi.org/10.1109/TPWRS.2017.2760699

By including a penalty term in the objective function, the SDP is enforced to remove the non-zero eigenvalues and obtain a rank-1 W matrix. This is because the lowest reactive loss enforces the optimal point (P_G, Q_G) to take into account this penalty. Therefore, the optimal solution vector is guided by also increasing the weighted sum of the real parts of the off-diagonal matrix of W, i.e. the real and imaginary parts of the voltage vector. As can be seen in **Figure 5**, the relaxation is always exact while we increase the penalty factor. In terms of objective function value, the non-convex AC-OPF results in a value of 6792.1 $/hr, while the SDP-OPF goes from 6782.8 $/hr to 6781.4 $/hr, i.e. the objective value decreases with the penalty factor increase. This can be argued because the constraint of reactive power isn't satisfied anymore, thus decreases the objective function value but isn't a feasible solution. In theory it should increase the value of the objective function because of the penalty variable.

7 Lessons Learned and Sharing of Workload

Regarding coding issues, we struggled some time with the objective function and it's corresponding positive semi-definite constraint and also with the several iterations we had to do to test the exactness of the relaxation. For future implementation is important to use the Shur Complement because that is what enables the relaxation of non-convex problems. We learned how to efficiently use the matrices and get the correct vector lengths for the objective function (alphas); therefore, next time we'll be more cautious when using big matrices and vectors.
Regarding the results, we learned that testing the exactness of the SDP relaxation is crucial to obtain the optimal objective function value. By testing the exactness we can be sure that the function reaches a global minimum and it doesn't get stuck in a local minimum as the AC-OPF could do. There are also data preparation that can help to reduce achieve a zero relaxation gap.This can be done by adding a small resistance to the transformer and also set a slack bus angle to 0 that enables a rank 1 for the optimization matrix. With this the constraints fit better in order to achieve a global minimum. It is also important to notice that the SDP can be used to identify the global solution in a convex problem. For example, in the case with 9 buses all objective function values were the same and having high eigenvalue ratio, meaning that all the solutions were global minimum. In the 3 bus case, the SDP and non-convex values were different. Meaning that there was a relaxation between those two, in this case in order to reduce the gap a penalty factor can be introduced in the objective function in order to compensate power losses that the non convex solver takes into account.

8 Sharing of Workload

Jorge - I was responsible of implementing the SDP relaxation in Matlab, comparing the results with the AC-OPF, evaluating the exactness of the relaxation and post-processing the results. I assisted my colleagues on their codes and wrote several parts of the report.
Andrés - I was responsible of implementing the SDP in the 3 bus case and evaluating the results. I assisted in the implentation of the SDP relaxation in Matlab and worked in the report.
Andrei - I was responsible of doing bonus exercise, initial try of SDP in the 9 bus case and worked in the report.
Miriam - SDP relaxation in Matlab, task 5.