Technical University of Denmark

31765: Optimization in modern power systems

Assignment 1
Economic Dispatch and DC-OPF

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Part A: Economic Dispatch & DC-OPF on a 3-bus system

3-Bus Test System. The system was tested with five different cases: Case 1 to 3 simulates the operation of a Power Exchange, Case 4 simulates the operation of an NTC-based market coupling, and Case 5 simulates the operation of a nodal market. Table 1 show the results of the five cases for both Economic Dispatch and DC-OPF solutions.

		Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
ED	Power Dispatch P_{G_1}, P_{G_2} (MW)	150, 0	100, 50	100, 50	100, 50	100, 50
	Total Generation Costs ($/h)	9000	12000	12000	12000	12000
DC-OPF	Power Dispatch P_{G_1}, P_{G_2} (MW)	150, 0	100, 50	100, 50	100, 50	50, 100
	Total Generation Costs ($/h)	9000	12000	12000	12000	15000

Table 1: Results of ED and DC-OPF optimization for cases 1 to 5

Case 1-3. Power Exchange. The results for the Power Exchange cases are the same for both Economic Dispatch and DC-OPF. Given that in the ED formulations the system acts as a copper-plate, the only constraint that changes the result is the Generator 1 maximum capacity change from 200 to 100 (comparing Case 1 vs. Case 2 and 3). Power flows and line limits are not part of the constraints for ED.

For DC-OPF, the susceptance matrix **B** is a 3x3 zero-matrix since there are no reactances for the lines. Therefore, the equality constraint $\mathbf{B} \cdot \delta = \mathbf{P_G} - \mathbf{P_D}$ becomes $\mathbf{P_G} = \mathbf{P_D}$. In that case, the equality power flow constraint is simplified to become the same as the Economic Dispatch formulation and the inequality line flow can't be computed. In our code, the result was an undefined error, since the line flow constraint divides 1 over the unavailable line reactants.

Case 4. NTC-based market coupling. The results of the NTC-based market coupling are the same for both Economic Dispatch and DC-OPF. For ED, the new reactants aren't part of the formulations (because of the copperplate assumption), therefore it gives the same results as Case 2 and 3.

For DC-OPF, all of the constraints of the formulation are computed. This results in a power flow of 50 MW in $line_1 2$, 50 MW in $line_{13}$, and 100 MW in $line_{23}$. The line limit of 70 MW in $line_{13}$ is respected. Therefore, the $line_{13}$ isn't congested and the system's generation costs remain the same as Case 2 and 3.

Case 5. Nodal market.

The results of the Nodal Market are **different** between Economic Dispatch and DC-OPF. For ED, the formulations don't change (because of the copperplate assumption), resulting in the same values as Case 2, 3, and 4. There's no congestion in the lines, since it's not part of the formulation or the input values.

For DC-OPF, the decrease in the line flow constraint of $line_{13}$ to 40 causes a rearrangement of power generation. The power flow from G_1 is divided with 10 MW in $line_{13}$ and 40 MW in $line_{12}$, thus reaching the maximum limit of the line, while G_2 is required to upraise it's production to 100 MW, compared to Case 4, resulting in a power flow of 110 MW in $line_{23}$. The congestion of $line_{13}$ demanded more generation from G_2, thus increasing the generation cost of the system.
In summary, regardless of the case or formulation method used (ED or DC-OPF), the relationship between congestion and total generation costs is positive, since we are going to have a greater generation cost if somewhere on our system there is a congestion that can't be evaded or reaches it's limit.

Section 2

Part B: DC-OPF & LMP on a 10-bus system

2.1 Lagrangian Multipliers for the Equality Constraint

Nominal Load		
Bus	**PG(MW)**	**Nodal Price($/MWh)**
1	0	36.6085
2	1200	10.9471
3	8000	27.344
4	0	27.9944
5	1182.2	29.1
6	0	31.9617
7	0	39.001
8	727.8	50
9	0	19.4899
10	1000	61.6225
Total	12110	

Table 2: Results of Nodal Prices for equality constraints at nominal load value

2.2 Nodal Prices with 70% of nominal loading

It can be observed in table 3 that the nodal prices are the same for all the nodes when the system is not fully loaded, specifically at 70 percent of the nominal load. Some generators stop working in this case and taking into account that the system is not at its full capacity, then it is a reaction that

70% Nominal Load		
Bus	**PG(MW)**	**Nodal Price($/MWh)**
1	0	24.3
2	1200	24.3
3	6277	24.3
4	0	24.3
5	0	24.3
6	0	24.3
7	0	24.3
8	0	24.3
9	0	24.3
10	1000	24.3
Total	8477	

Table 3: Results of Nodal Prices for equality constraints at 70 percent nominal load value

the prices will be the same until another generator starts working due to the increase in the load of the system.
When the systems is working at its full nominal load, the nodal prices vary depending on how far of the generators are, the cost of generation of each bus and its load.

2.3 Lagrangian Multipliers for Inequality Constraint

From table 4 it can be appreciated that almost all Lagrangian Multipliers for the Inequality Constraint tend to zero. Except from the one that goes from bus 2 to bus 10. If that line is analyzed it can be seen that it has the lowest line limit flow. So the Lambda can be an indicator of the saturation of the lines when the system is with its nominal load.

2.4 Bus 10 behaviour

Plot of LM on Bus 10 varying loads uniformity from 20% to 100% and the wind infeed from 0% to 100%:
Nodal price in bus 10 behaves as a step function. The reasons for this behaviour could be related with the demand and supply law. The generator means a change in the supply and the load in the demand. As long as the demand of the system doesn't reach the supply, the nodal price won't increase and will be the same for all nodes. Once the demand is equal or greater to the supply, the nodal price will increase or decreasing depending on the load and generation of the bus.

Section 3

From Bus	To Bus	Lower Lambda	Upper Lambda
1	3	0.00E+00	0.00E+00
1	10	-7.15E-11	-3.80E-11
2	3	-6.73E-12	-3.30E-12
2	9	-1.80E-10	0.00E+00
2	10	-8.69E+01	-1.72E-11
3	4	0.00E+00	0.00E+00
3	5	-1.93E-11	-1.35E-11
3	6	-1.57E-11	-7.49E-12
4	5	0.00E+00	0.00E+00
5	6	-2.14E-11	-2.10E-11
6	7	-4.15E-11	0.00E+00
7	8	0.00E+00	-4.96E-11
8	9	-1.69E-11	0.00E+00
8	10	-7.47E-12	-2.09E-11

Table 4: Results LPM for line inequalities constraints at nominal load value

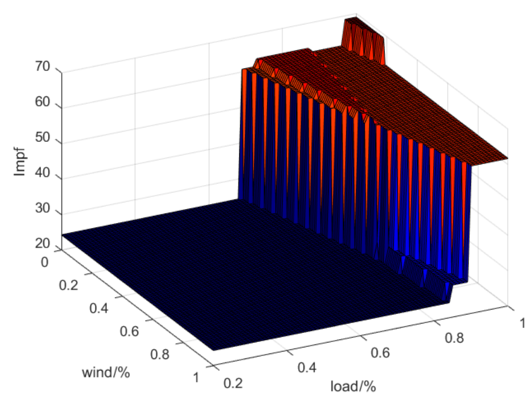

Figure 1: Nodal Prices at bus 10 varying the load and max power generation of wind farm

Lessons Learned and Sharing of Workload

Conclusions

- By computing the five cases of Part A, we learned that the system's generation costs is mainly dependant of each generator's cost for ED. For DC-OPF, a line with high reactance and low flow limit causes the system to use an expensive generator to satisfy the demand while trying to maintain generation costs low.
- On Part B, if we use 70% of nominal load (or any load lower than nominal one), we'll get the same nodal price for every node. Once we use the nominal load the marginal prices will change.
- The systems obeys the law of supply and demand. We could see that when the system is not fully

loaded to the point that it doesn't reach the maximum generation power of 1 or more generators, the nodal price will be stable and will remain the same. If the supply reaches the maximum generation power, then the nodal prices will depend on the demand of each bus and generation.Under this assumption the nodal prices can be model as a step function

Workload

Workload						
Member	**Part A**			**Part B**		
	Code		Report	Code		Report
	ED	DCOPF		DCOPF	PLOT	
s191212		X	X	X	X	X
s193450	X		X	X		X
s192184	X	X	X	X		X
s182565		X	X			

Appendix

During this analysis, we used the following linear program formulations:

$$\text{Objective function: } min \sum_i c_i P_{G_i} \tag{1}$$

Economic Dispatch

$$\text{Subject to constraints: } \begin{cases} \sum_i P_{G_i} = P_D \\ P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max} \end{cases} \tag{2}$$

Optimal Power Flow

$$\text{Subject to constraints: } \begin{cases} \mathbf{B} \cdot \delta = \mathbf{P_G} - \mathbf{P_D} \\ P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max} \\ -P_{ij,max} \leq \frac{1}{x_{ij}}(\delta_i - \delta_j) \leq P_{ij,max} \end{cases} \tag{3}$$

where:
P_{G_i} = Power Generation
$P_{G_i}^{MAX}$ = Generator Maximum Power Limit
$P_{G_i}^{MIN}$ = Generator Minimum Power Limit
$\mathbf{B}$ = Susceptance Matrix
$\mathbf{P_G}$ = Power Generation Matrix
$\mathbf{P_D}$ = Power Demand Matrix
δ = Voltage angle in radians
x_{ij} = Reactance between node i and j
This formulations can be seen in the assignment's code.