

Course 31778 – Assignment 3 – Distribution grid modelling

SOLUTION

- 1) Calculate the phase to phase voltage drop ΔV over the line and the transformer, using the complete and the approximated formula. Use both formulas twice; first for the voltage drop across the transformer and secondly for the voltage drop across the cable. Consider in both cases that the overall load is the sum of Load 1 and Load 2 and a nominal voltage equal to 400 V.

First for the voltage drop across the transformer. The Z base for the transformer on the secondary side:

$$Z_{base_2} = (0.4 \text{ kV})^2 / 630 \text{ kVA} = 0.254 \Omega$$

Multiplied with the pu values for the secondary side.

$$R_{secondary} = r_2 \cdot Z_{base} = 0.005 \cdot 0.254 \Omega = 1.270 \text{ m}\Omega, \quad R_{trafo} = R_{secondary} \cdot 2 = 2.540 \text{ m}\Omega$$

$$X_{secondary} = x_2 \cdot Z_{base} = 0.0195 \cdot 0.254 \Omega = 4.952 \text{ m}\Omega, \quad X_{trafo} = X_{secondary} \cdot 2 = 9.905 \text{ m}\Omega$$

The load is equal to the sum of load 1 and 2. Complete voltage drop formula:

$$\begin{aligned} \Delta V_{trafo} &= \frac{R_{trafo} \cdot P_{load} + X_{trafo} \cdot Q_{load}}{V_{nom}} + j \frac{X_{trafo} \cdot P_{load} - R_{trafo} \cdot Q_{load}}{V_{nom}} \\ &= \frac{0.002540 \cdot 180000 + 0.009905 \cdot 50000}{400} + j \frac{0.009905 \cdot 180000 - 0.002540 \cdot 50000}{400} \\ &= 2.38 + j4.14 \text{ V} \rightarrow |\Delta V| = 4.78 \text{ V} \end{aligned}$$

Simplified: $|\Delta V| \approx Re(\Delta V) = 2.38 \text{ V}$

Calculating the voltage drop of the cable:

$$R_{line} = 0.1556 \frac{\Omega}{\text{km}} \cdot 0.5 \text{ km} = 0.0778 \Omega$$

$$X_{line} = 0.072 \frac{\Omega}{\text{km}} \cdot 0.5 \text{ km} = 0.0360 \Omega$$

Complete voltage drop formula:

$$\begin{aligned} \Delta V &= \frac{R_{line} \cdot P_{load} + X_{line} \cdot Q_{load}}{V_{nom}} + j \frac{X_{line} \cdot P_{load} - R_{line} \cdot Q_{load}}{V_{nom}} \\ &= \frac{0.0778 \cdot 180000 + 0.036 \cdot 50000}{400} + j \frac{0.036 \cdot 180000 - 0.0778 \cdot 50000}{400} \\ &= 39.51 + j6.48 \text{ V} \rightarrow |\Delta V| = 40.04 \text{ V} \end{aligned}$$

Simplified: $|\Delta V| \approx Re(\Delta V) = 39.51 \text{ V}$

- 2) Design the system in Simulink and run the load flow calculation. Compare the voltage drop including and disregarding the voltage angles and discuss the calculated values from objective (1). Display the values of voltage, current and power of each measurement point with displays and scopes. Notes: Implement a load element for each load.

For the voltage drop across the transformer:

Disregarding angles: $|\Delta V| = (1 - 0.9929) \cdot 400 = 2.84 \text{ V}$ (taken from Simulink)

With angles: $|\Delta V| = |(1 \angle 0) - (0.9929 \angle -0.669)| \cdot 400 = 5.44 \text{ V}$ (taken from Simulink)

It is a bit more than both the extended and simplified formula since it does not take the joule losses in the cable into account. As matter of fact the power flowing through the transformer is the sum of the power consumed by the loads and the one lost in the cable.

For the voltage drop across the line:

Disregarding angles: $|\Delta V| = (0.9929 - 0.8805) * 400 = 44.96 \text{ V}$ (taken from Simulink)

With angles: $|\Delta V| = |[(0.9929\angle - 0.669) - (0.8805\angle - 1.74)]| * 400 = 45.50 \text{ V}$ (taken from Simulink)

Difference with the calculated results in 1) is because V_{nom} was used in both complete and approximated formulas.

- 3) Calculate the joule losses in Watt in both the LV cable and the transformer, based on the current from the Simulink simulation results. Also calculate and compare the losses based on the difference between the active power (calculated with the *power-block*) at the grid side, the LV transformer side, and at the load.

For the transformer:

With the current taken from Simulink: $P_{loss} = 3 * |I_{ph}|^2 * R_{trafo} = 3 * 306.2^2 * 0.00254 = 714 \text{ W}$

With the power taken from Simulink: $P_{loss} = 203.9 \text{ kW} - 201.9 \text{ kW} = 1.97 \text{ kW}$

There is a difference between the two results because the transformer also has a base consumption due to the magnetization resistance.

For the cable:

With the current taken from Simulink: $P_{loss} = 3 * |I_{ph}|^2 * R_l = 3 * 306.2^2 * 0.0778 = 21.89 \text{ kW}$

With the power taken from Simulink: $P_{loss} = 201.9 \text{ kW} - 180 \text{ kW} = 21.89 \text{ kW}$

- 4) Assume that you can change either P_2 or Q_2 in order to control the voltage at the end of the line. Calculate (using the simplified formula and considering the voltage drop across transformer and line altogether) the P_2 and Q_2 needed to achieve a new voltage drop of $\Delta V = 0.05 * V_{nom}$ and compare with the results in Simulink (consider the voltage at the main grid terminal equal to 1 pu). Change only either the active or reactive power at the time and keep the other value equal to the initial value: when using P_2 , keep $Q_2 = 0$; while when using Q_2 , keep $P_2 = 80 \text{ kW}$. Discuss why the new configuration is not able to completely fulfill the objective. Comment also on any drawback of the solution. Note: there is no need to save the results in Simulink, just use the created model to verify the results.

Firstly we need to calculate the equivalent series R and X for transformer and line R_{tl} and X_{tl} .

$$R_{tl} = R_{trafo} + R_{line} = 0.00254 \Omega + 0.0778 \Omega = 0.08034 \Omega$$

$$X_{tl} = X_{trafo} + X_{line} = 0.009905 \Omega + 0.0360 \Omega = 0.045905 \Omega$$

To make reactive power voltage control; first the Q_{load} giving the wanted voltage drop is isolated.

$$\Delta V = 0.05 * V_{nom} = \frac{R_{tl} * P_{load} + X_{tl} * Q_{load}}{V_{nom}}$$

$$0.05 * V_{nom} = \frac{R_{tl} * P_{load}}{V_{nom}} + \frac{X_{tl} * Q_{load}}{V_{nom}}$$

$$0.05 * V_{nom} - \frac{R_{tl} * P_{load}}{V_{nom}} = \frac{X_{tl} * Q_{load}}{V_{nom}}$$

$$Q_{load} = \left(0.05 * V_{nom} - \frac{R_{tl} * P_{load}}{V_{nom}} \right) * V_{nom} / X_{tl}$$

$$Q_{load} = \left(\frac{0.05 * V_{nom}^2 - R_{tl} * P_{load}}{X_{tl}} \right) = \left(\frac{0.05 * 400^2 - 0.08034 * 180e3}{0.045905} \right) = -140.8 \text{ kVar}$$

The reactive power of Q_1 is subtracted from the total reactive power to find the new Q_2 .

$$\rightarrow Q_2 = Q_{load} - Q_1 = -140.8 \text{ kVar} - 50 \text{ kVar} = -190.8 \text{ kVar}$$

Verifying the results in Simulink, for $Q_2 = -190.8 \text{ kVar}$, disregarding angles: $|\Delta V| = (1 - 0.9384) \text{ pu} = 0.0616 \text{ pu}$

The calculated voltage drop is larger compared to the expected 5% because in the simplified formula we used as reference voltage 400 V (instead of using the expected final value) and the voltage drop on the transformer should have accounted also for the cable losses. Beware of phase current (352 A) that rises beyond the thermal limit of the cable (120 mm² copper) which is 315 A.

The same can be done for active power voltage control.

$$\begin{aligned} \Delta V &= 0.05 * V_{nom} = \frac{R_{tl} * P_{load} + X_{tl} * Q_{load}}{V_{nom}} \\ 0.05 * V_{nom} &= \frac{R_{tl} * P_{load}}{V_{nom}} + \frac{X_{tl} * Q_{load}}{V_{nom}} \\ 0.05 * V_{nom} - \frac{X_{tl} * Q_{load}}{V_{nom}} &= \frac{R_{tl} * P_{load}}{V_{nom}} \\ P_{load} &= \left(0.05 * V_{nom} - \frac{X_{tl} * Q_{load}}{V_{nom}} \right) * V_{nom} / R_{tl} \\ P_{load} &= \left(\frac{0.05 * V_{nom}^2 - X_{rt} * Q_{load}}{R_{rt}} \right) = \left(\frac{0.05 * 400^2 - 0.045905 * 50e3}{0.08034} \right) = 71.0 \text{ kW} \end{aligned}$$

$$\rightarrow P_2 = P_{load} - P_1 = 71.0 \text{ kW} - 100 \text{ kW} = -29.0 \text{ kW (production)}$$

Verifying the results in Simulink, for $P_2 = -29.0 \text{ kW}$ and disregarding angles: $|\Delta V| = (1 - 0.9471) \text{ pu} = 0.0529 \text{ pu}$

Similarly as before, the calculated voltage drop is larger due to the presence of the nominal voltage value in the simplified formula and the voltage drop on the transformer should have accounted also for the cable losses. Moreover, it has to be highlighted how this type of solution would be more expensive to realize as the negative active power would require the presence of a generator (or a storage).

- 5) Set P_2 and Q_2 back to the initial values. Add a third load at the ending bus, with $P_3=0 \text{ kW}$ $Q_3(0)=0 \text{ kVar}$. Assume you can adjust linearly the reactive power between -150 and +150 kVar. Design a Q controller (Q(V) droop control) with the voltage reference equal to 0.95 pu. The voltage controller has an overall measurement delay of 0.5s and the droop gain has to be chosen to achieve the highest voltage increase without sustained oscillations. Enable the controller only $t=3 \text{ s}$ and run the simulation for 10 s. Settling time should be within 5 s (tune the controller with the usual Ziegler-Nichols method). Report in the narrative document the derived droop gain, the amount of Q that the unit will deploy and discuss on the consequences on the system. Considering the derived droop, re-analyze the system with a reference voltage for the controller equal to 1.05 pu. Discuss and compare the new results with the ones obtained with a reference voltage equal to 0.95 pu. Note: the Simulink file submitted should include all the elements and values necessary to verify this point.

The critical gain is for a droop equal to 5%. The droop selected is 10%. The droop control loop includes the base power for the load (chosen equal to 150k) and a saturation to prevent the controller to set a reactive power value larger than $\pm 150 \text{ k}$ (it is clear that the inductive contribution is unnecessary, given the under-voltage situation). At the end of the simulation $Q_3 = -67.8 \text{ kVar}$ and $V_{load} = 0.905 \text{ pu}$ (Figure 1). The current decreases from 306 to 289 A due to the improvement in the overall power factor (lower amount of reactive power) and increase of voltage (being all loads constant power, the higher the voltage, the lower the current required).

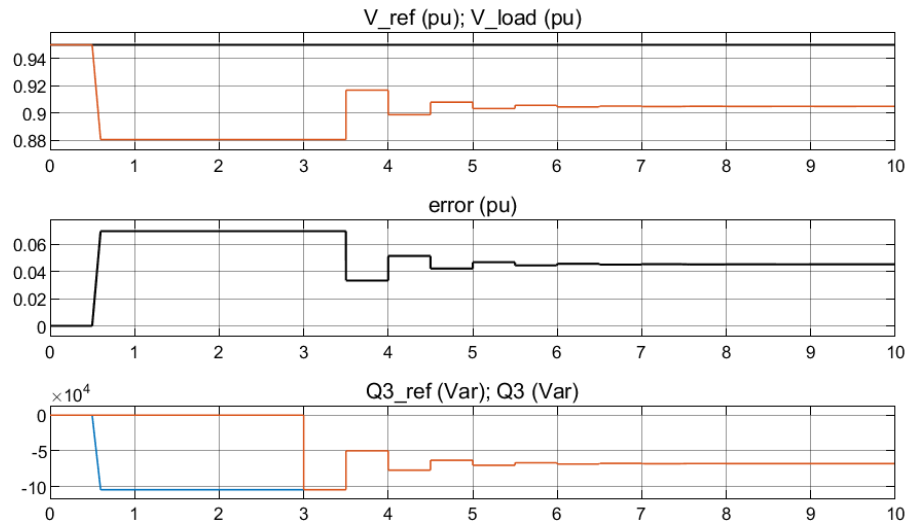


Figure 1. Voltage at the load, error and deployed and desired reactive power with reference voltage equal to 0.95 pu.

In the moment that the reference voltage is increased to 1.05 pu, it can be noted how the deployed power increases. At the end of the simulation $Q_3 = -150$ kVar and $V_{\text{load}} = 0.93$ pu (Figure 2). An eye must always be kept on the current, which increases to 319 A due to the reduction of the power factor (which from lagging becomes leading, but lower), which cannot compensate for the improvement of the voltage.

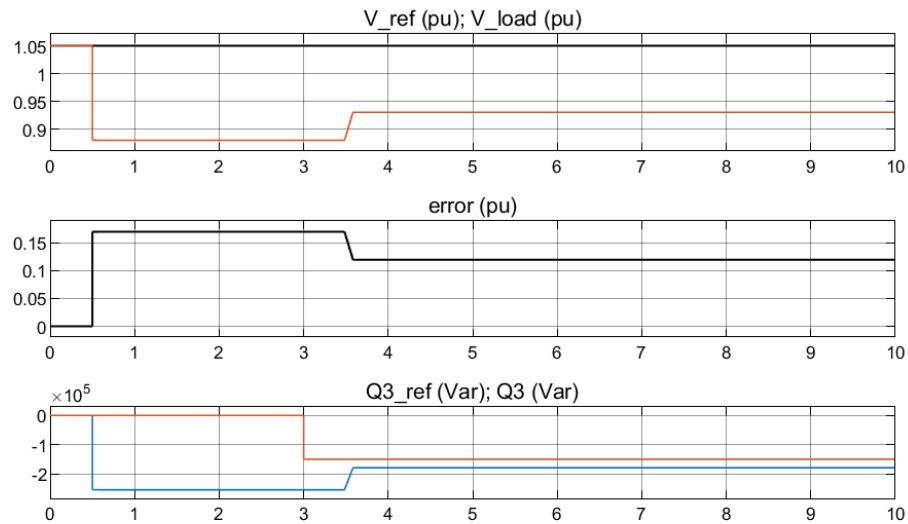


Figure 2. Voltage at the load, error and deployed and desired reactive power with reference voltage equal to 1.05 pu.