Playtime 1:

$$p(\beta|y,X,\lambda,\sigma) \propto \overbrace{\mathcal{N}\big(\underset{\substack{\downarrow\\ x}}{\beta}\,\big|\,\underset{\substack{\downarrow\\ \mu}}{0}, \underset{\substack{\downarrow\\ \Lambda^{-1}}}{\lambda I}\big)}^{\text{prior}} \; \overbrace{\mathcal{N}\big(\underset{\substack{\downarrow\\ y}}{y}\,\big|\,\underset{\substack{\downarrow\\ A}}{X}\underset{\substack{\downarrow\\ x}}{\beta}, \underset{\substack{\downarrow\\ L^{-1}}}{\sigma^2 I}\big)}^{\text{likelihood}}$$

$$= \mathcal{N}(\beta \,|\, \mu, \Sigma)$$

where

$$\Sigma = \left(\lambda^{-1} I + X^T \sigma^{-2} I X\right)^{-1}$$

$$= \left(\lambda^{-1} I + \sigma^{-2} X^T X\right)^{-1}$$

$$\mu = \Sigma\left(X^T \sigma^{-2} I (y - 0)\right)$$

$$= \Sigma\left(\sigma^{-2} X^T y\right)$$

Playtime 2:

Joint distribution:

$$p(a,b,c,d) = p(b)\,p(c)\,p(a|c,b)\,p(d|a)$$

We want: $p(d|b=1)$

$$p(d|b=1) = \frac{\sum_{a,c} p(a,b=1,c,d)}{p(b=1)}$$

$$= \frac{\sum_{a,c} p(d|a)\,p(a|c,b=1)\,p(c)\,\cancel{p(b=1)}}{\cancel{p(b=1)}} \quad \text{(re-order)}$$

$$= \sum_a p(d|a) \underbrace{\sum_c p(a|c,b=1)\,p(c)}_{f_c(a)} \quad \text{(eliminate } c\text{) (and } b\ldots\text{)}$$

$$= \underbrace{\sum_a p(d|a)\,f_c(a)}_{f_a(d)} \quad \text{(eliminate } a\text{)}$$

, where $f_c(a)$:

$a=0$	$a=1$
$0.5\times0.7+0.1\times0.3 = 0.38$	$0.5\times0.3+0.9\times0.3 = 0.62$

where $f_a(d)$:

$d=0$	$d=1$
$\sum_a p(d=0\|a)\,f_c(a)$ $= p(d=0\|a=0)\,f_c(a=0) + p(d=0\|a=1)\,f_c(a=1)$ $= 0.6\times0.38 + 0.2\times0.62$ $= 0.352$	$p(d=1\|a=0)\,f_c(a=0) + p(d=1\|a=1)\,f_c(a=1)$ $= 0.4\times0.38 + 0.8\times0.62$ $= 0.648$

Playtime 3:

Initialization step:

$\pi(c) = p(c)$

$c=0$	$c=1$
0,7	0.3

$\pi(d)$

$d=0$	$d=1$
0	1

$\lambda(b) = 1$

$b=0$	$b=1$
1	1

$\lambda(e)$

$e=0$	$e=1$
0	1

Calculating messages:

$$\pi_{c,a}(c) = \pi(c) = p(c)$$

$$\pi_{d,a}(d) = \pi(d)$$

$$\pi(a) = \sum_{c,d} p(a|c,d)\ \pi_{c,a}(c)\ \pi_{d,a}(d)$$

$$= \sum_{c,d} p(a|c,d)\ \pi(c)\ \pi(d)$$

because $\pi(d) = 1$ only when $d=1$ (see $\pi(d)$ table ...)

$$= \sum_{c} p(a|c, d=1)\ \pi(c)$$

$$= p(a|c=0, d=1)\,\pi(c=0) + p(a|c=1, d=1)\,\pi(c=1)$$

For $a=1$:

$$\pi(a=1) = p(a=1|c=0, d=1)\,\pi(c=0) + p(a=1|c=1, d=1)\,\pi(c=1)$$

$$= 0.1 \times 0,7 + 1 \times 0.3 = \underline{\underline{0.37}}$$, $\pi(a=0) = \underline{\underline{0.63}}$

$\pi(a)$

$a=0$	$a=1$
0.63	0.37

$$\lambda_{e,a}(a) = \sum_{e} \lambda(e)\ p(e|a)$$

$$= \lambda(e=0)\,p(e=0|a) + \lambda(e=1)\,p(e=1|a)$$

$$= \underline{\underline{p(e=1|a)}}$$

$a=0$	0,8
$a=1$	0.3

$$\pi_{a,b}(a) = \pi(a)\ \lambda_{e,a}(a)$$

$$= \pi(a)\ p(e=1|a)$$

$$\pi_{a,b}(a=0) = 0.63 \times 0.8 = 0.504$$

$$\pi_{a,b}(a=1) = 0.37 \times 0.3 = 0.111$$

$$\pi(b) = \sum_a p(b|a)\, \pi_{a,b}(a)$$

$$= \sum_a p(b|a)\, \pi(a)\, p(e=1|a)$$

For $b=1$:

$$\pi(b=1) = p(b=1|a=0)\, \pi_{a,b}(a=0) + p(b=1|a=1)\, \pi_{a,b}(a=1)$$

$$= 0.7 \times 0.504 + 1 \times 0.111 = 0.4638$$

$$\pi(b=0) = p(b=0|a=0)\, \pi_{a,b}(a=0) + p(b=0|a=1)\, \pi_{a,b}(a=1)$$

$$= 0.3 \times 0.504 + 0 \times 0.111 = 0.1512$$

$$\boxed{Bel(b) \propto \pi(b) \underbrace{\lambda(b)}_{=1} = \pi(b)}$$

$Bel(b) \propto \pi(b)$

$b=0$	$b=1$
0.1512	0.4638

we must re-normalize! $\longrightarrow$

$Bel(b)$

$b=0$	$b=1$
0.246	0.754

$$\lambda_{b,a}(a) = \sum_b \underbrace{\lambda(b)}_{=1} p(b|a) = \sum_b p(b|a) = 1$$

$$\lambda(a) = \underbrace{\lambda_{b,a}(a)}_{=1} \lambda_{e,a}(a) = \lambda_{e,a}(a) \longrightarrow$$

$a=0$	$a=1$
0.8	0.3

$$\boxed{Bel(a) \propto \pi(a)\, \lambda(a)} \longrightarrow$$

$a=0$	$a=1$
$0.63 \times 0.8 = 0.504$	$0.37 \times 0.3 = 0.111$

normalize

$a=0$	$a=1$
0.82	0.18

$$\lambda_{a,c}(c) = \sum_a \lambda(a) \sum_d p(a|c,d) \underbrace{\pi_{d,a}(d)}_{= \pi(d)}$$

$$= \sum_a \lambda(a) \sum_d p(a|c,d)\, \pi(d)$$

$$= \sum_a \lambda(a)\, p(a|c, d=1) \longrightarrow$$

$c=0$	$c=1$
$0.8 \times 0.9 + 0.3 \times 0.1 = 0.75$	$0 + 0.3 \times 1 = 0.3$

$$\lambda(c) = \lambda_{a,c}(c)$$

$$\boxed{Bel(c) \propto \pi(c)\, \lambda(c)}$$

$c=0$	$c=1$
$0.7 \times 0.75 = 0.525$	$0.3 \times 0.3 = 0.09$

$\xrightarrow{\text{normalize}} \approx$

$c=0$	$c=1$
0.85	0.15