MBML 42186

Technical University of Denmark

SPRING, 2019
Campus: Lyngby

MODEL-BASED MACHINE LEARNING

Quiz 1

(Time allowed: 45 MINUTES)

Name: ______________________________
Student ID: ______

Part 1

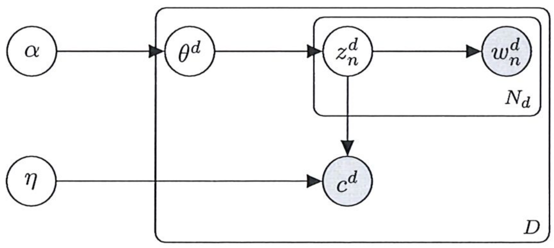


Figure 1: Model A

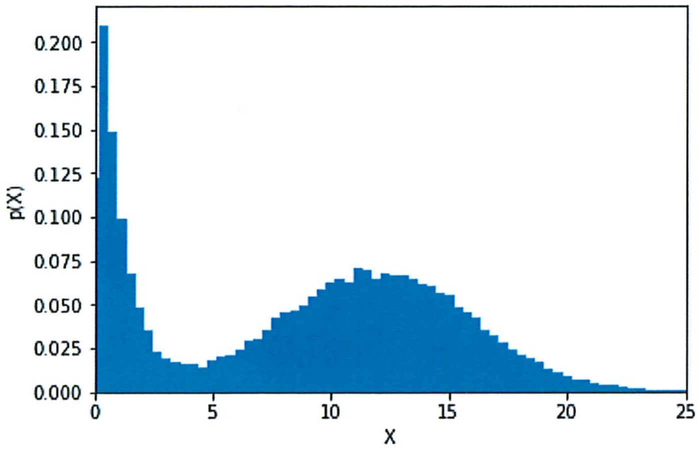
Consider the graphical model in Figure 1:

1. (20%) Write the corresponding factorization of the joint distribution.

$$p(\alpha)\, p(\eta) \prod_{d=1}^{D} p(\theta^d|\alpha)\, p(c^d|\mathbf{z}^d, \eta) \prod_{n=1}^{N^d} p(z_n^d|\theta^d)\, p(w_n^d|z_n^d)$$

this should ideally be bold $\mathbf{z}^d = \{z_1^d, \ldots z_{N^d}^d\}$, because c^d depends on all $\{z_1^d, \ldots, z_{N^d}^d\}$

Part 2

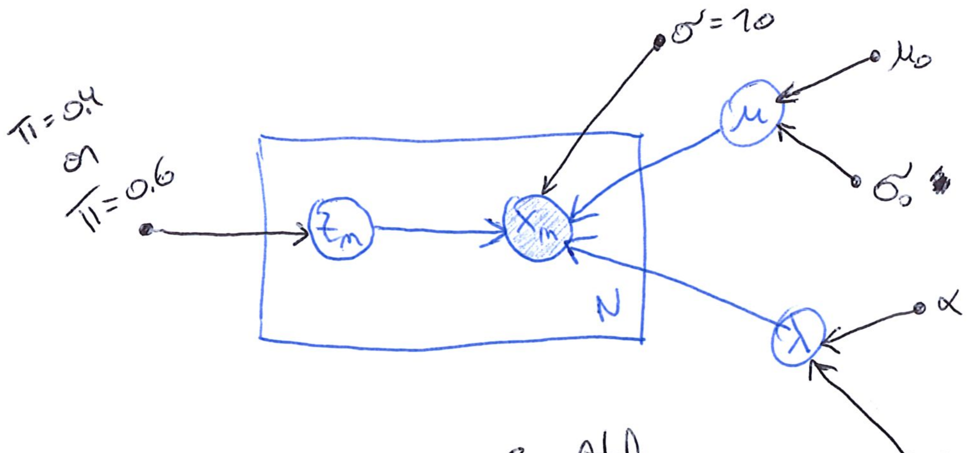


Consider the picture above

You received a univariate dataset, $\mathcal{D}$ ($\mathcal{D} = \{x_1, x_2, ..., x_N\}$) and plotted its distribution. Your domain knowledge clearly indicates that it contains a mixture of two distributions: 60% of an exponential and 40% normal, defined respectively as $\text{Exp}(\lambda)$ and $\mathcal{N}(\mu, 10)$.

You want to implement a Probabilistic Graphical Model that fits this data. You are essentially interested in inferring the distributions of μ and λ.

1. (20%) Build the corresponding graphical model



(parts in black are completely optional! they add extra information to the PGM, but you were only required to do the parts in blue!

2. (30%) Write the generative process (generative story). Don't forget to make sure to include the appropriate conjugate priors (see last page)

1) Draw $\mu \sim N(\mu \mid \mu_0 \mid \sigma_0)$

2) Draw $\lambda \sim Gamma(\lambda \mid \alpha, \beta)$

3) For each observation $n \in \{1, \dots, N\}$

   a) Draw $z_n \sim Bernoulli(z_n \mid \pi = 0.6)$

   b) If $z_n = 1$:

      Draw $x_n \sim Exp(x_n \mid \lambda)$

      If $z_n = 0$:

      Draw $x_n \sim N(x_n \mid \mu, \sigma = 10)$

Part 3

Consider the following generative process:

1. Draw $\pi \sim \text{Beta}(\pi|2, 1)$
2. Draw z, such that $z \sim \text{Bernoulli}(z|\pi)$
3. If $z = 0$
   (a) Draw x, such that $x \sim \mathcal{N}(x|-2, 2)$
   If $z = 1$
   (a) Draw x, such that $x \sim \mathcal{N}(x|3, 1)$

According to the generative process above, you know that the joint distribution, $p(\pi, z, x)$, factorizes as:

$$p(\pi, z, x) = p(\pi)\, p(z|\pi)\, p(x|z)$$

Based on the generative story and corresponding factorization of the joint distribution:

with all intermediate steps:

1. (30%) Formulate the expression for $p(x|\pi)$, using the factorization you have given above. **Simplify your expression as much as possible** (no "z" symbol in the final expression).

$$p(x|\pi) = \frac{p(x,\pi)}{p(\pi)} = \frac{\sum_z p(\pi, z, x)}{p(\pi)} = \frac{\sum_z p(\pi)p(z|\pi)p(x|z)}{p(\pi)}$$

$$= \frac{\cancel{p(\pi)} \sum_z p(z|\pi)\, p(x|z)}{\cancel{p(\pi)}} = \sum_z p(z|\pi)\, p(x|z)$$

Now, you can use the information in the generative process above to simplify:

$$\sum_{z\in\{0,1\}} p(z|\pi)\, p(x|z) = p(z=0)\, p(x|z=0) + p(z=1)\, p(x|z=1)$$
$$= (1-\pi)\cdot \mathcal{N}(x|-2,2) + \pi \cdot \mathcal{N}(x|3,1)$$