Say yes to the guess:
Ensembles for prediction with scantily clad data[*]

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Abstract

Bla bla blub

1   Introduction

Although accurate prediction of future events is not the primary goal for most social sciences, recent years have witnessed spreading of systematic forecasting from more traditional topics (e.g., GDP growth and unemployment) to many new domains (e.g., elections and mass killings) . Several factors have motivated this increase. To begin with, testing systematic predictions about future events against observed outcomes is generally seen as the most stringent validity check of statistical and theoretical models. In addition, forecasting of important political, economic, and social events is of great interest to policymakers and the general public who are generally less interested testing theories of the world than correctly anticipating and altering the future.

With the proliferation of forecasting efforts, however, comes a need for sensible methods to aggregate and utilize the various scholarly efforts. One attractive solutions to this problem is to combine various prediction models and create an ensemble forecast. Combining forecasts reduces reliance on any single data source or methodology, but also allows for the incorporation of more information than any one model is likely to include in isolation. Across subject domains, ensemble predictions are usually more accurate than any individual component model. Second, they are signicantly less likely to make dramatically incorrect predictions (Bates and Granger 1969; Armstrong 2001; Raftery et al. 2005).

The idea of ensemble learning itself has a long history in the machine learning and nonparametric statistics community. The most thorough treatment is found in Hastie, Tibshirani and Friedman (2009). A wide range of statistical approaches including neural nets, bagging, random forests, additive regression trees, and boosting and more may be properly considered ensemble approaches.

One ensemble method advocated recently for forecasting is ensemble Bayesian model averaging (EBMA). This methods was first proposed by Raftery et al. (2005) and recently forwarded as a useful method for the social sciences by **?**. In essence, EBMA creates a finite mixture model that creates a kind of weighted average of forecasts. EBMA mixture models seek to collate the good parts of existing forecasting models while avoiding over-fitting to past observations or over-estimating our certainty about the future. The hope is for greater accuracy as both the knowledge and implied uncertainty of a variety of approaches are integrated into a combined predictive probability distribution.

However, there are several challenges for creating ensemble predictions for many social science applications. To begin with amount and quality of data for calibrating ensembles is far from ideal. EBMA was first developed for use in weather forecasting where measurement of outcomes is fairly precise and data is relatively abundant. Predicting, for instance, water surface temperatures in 200 locations across five days provides 1,000 observations by which model weights can be calibrated. Forecasting quarterly GDP growth in the United States for five *years* only provides 20.

A second and related issue is that there tends to be a lot more forecasts than observations. For example, the economic forecasting survey has like a billion experts. blah, balh

A final issue is the inconsistency with which forecasts are issued. Given the lengthy time periods involved, of any given time window there are many missing forecasts. Moreover, we cannot assume

that forecasts for any time period from a specific model or team are missing at random. Particularly unsuccessful forecasts may be suppressed. Moreover, forecasts have tended to accumulate with more observations being available for more proximate time periods.

One example of forecasting that combines all of these issues in the prediction of U.S. Presidential elections. Table 1 represents nearly entirety of scholarly forecasts which produced more than one forecast for elections in the 20th century [1]. In this instance we have only five observations by which to calibrate a model while we have nine forecasts. Moreover, several of the individual forecasts are missing for a significant portion of the data. The forecast of Cuzan, for instance, is missing for 60% of the elections in the dataset.

Table 1: Pre-election forecasts of the percent of the two-party vote going to the incumbent party in U.S. Presidential elections

	F	A	C	H	LBRT	L	Hol	EW	Cuz
1992	55.7	46.3	49.7	48.9	47.3				
1996	49.5	57.0	55.5	53.5	53.3		57.2	55.6	
2000	50.8	53.2	52.8	54.8	55.4	60.3	60.3	55.2	
2004	57.5	53.7	52.8	53.2	49.9	57.6	55.8	52.9	51.1
2008	48.1	45.7	52.7	48.5	43.4	41.8	44.3	47.8	48.1

Forecasts were published prior to each election by **F**air, **A**bramowitz, **C**ampbell, **H**ibbs, **L**ewis-**B**eck and **R**ice (1992), Lewis-Beck and **T**ien (1996-2008), **L**ockerbie, **Hol**brook, **E**rikson and **W**lezien and **Cuz**an. Data taken from CITE SILVER POST HERE.

While particularly egregious for presidential forecasting, these data issues are endemic across the social sciences. Blah, blah

In this paper, we explore several adjustments to the basic EBMA model as specified in Montgomery, Hollenbach and Ward (2012) that can help applied researchers create ensemble forecasts even in the presence of these kinds of data-quality issues. Specifically, we show EBMA can be adjusted to easily accomodate missing forecasts. In addition, we propose an alteration to the basic model. Below, we briefly introduce the basic EBMA model in Section 2. We outline modifications to the model for missingness and small samples in Sections 3 and 4. In Section 5, we apply the adjusted EBMA model to unemployment data as well as presidential forecasting models shown in Table 1.

2   Notation and basic EBMA model

Assume a quantity of interest to forecast, $\mathbf{y}^{t^*}$, in some future period $t^* \in T^*$. Further assume that we have extant forecasts for events $\mathbf{y}^t$ for some past period $t \in T$ that were generated from K forecasting models or teams, $M_1, M_2, \ldots, M_K$, for which have a prior probability distribution $M_k \sim \pi(M_k)$. The PDF for $\mathbf{y}^t$ is denoted $p(\mathbf{y}^t|M_k)$. Under this model, the predictive PDF for the quantity of interest is $p(\mathbf{y}^{t^*}|M_k)$, the conditional probability for each model is $p(M_k|\mathbf{y}^t) =$

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[1]Citations. Also something here about how we previously calibrated based on in-sample, but here we are focused on pure out-of-sample forecasts

$p(\mathbf{y}^t|M_k)\pi(M_k)/\sum_{k=1}^{K} p(\mathbf{y}^t|M_k)\pi(M_k)$ and the and the marginal predictive PDF is $p(\mathbf{y}^{t^*}) = \sum_{k=1}^{K} p(\mathbf{y}^{t^*}|M_k)p(M_k|\mathbf{y}^t)$. This can be viewed as the weighted average of the component PDFs where the weights are determined by each model's performance within the already-observed period T.

2.1 Dynamic ensemble forecasting

The EBMA procedure assumes K forecasting throughout the training (T') calibration (T) and test (T^*) periods. The goal is to estimate the parameters for the ensemble prediction model using $\mathbf{f}_k^t$ for some period T. It is then possible to generate true ensemble forecasts $(\mathbf{f}_k^{t^*})$ for observations in the test period $t^* \in T^*$.

Let $g_k(\mathbf{y}|\mathbf{f}_k^{s|t,t^*})$ represent the predictive PDF of component k, which may be the original prediction from the forecast model or the bias-corrected forecast. The EBMA PDF is then a finite mixture of the K component PDFs, denoted $p(\mathbf{y}|\mathbf{f}_1^{s|t}, \ldots, \mathbf{f}_K^{s|t}) = \sum_{k=1}^{K} w_k g_k(\mathbf{y}|\mathbf{f}_k^{s|t})$, where $w_k \in [0,1]$ are model probabilities, $p(M_k|\mathbf{y}^t)$, and $\sum_{k=1}^{K} w_k = 1$. The ensemble predictive PDF with this notation is is then $p(y|f_1^{t^*}, \ldots, f_K^{t^*}) = \sum_{k=1}^{K} w_k g_k(y|f_k^{t^*})$.

Past applications have statistically post-process the predictions for out-of-sample bias reduction and treat these adjusted predictions as a component model. Raftery et al. (2005) propose approximating the conditional PDF as a normal distribution centered at a linear transformation of the individual forecast, $g_k(\mathbf{y}|\mathbf{f}_k^{s|t}) = N(a_{k0} + a_{k1}\mathbf{f}_k^t, \sigma^2)$. However, in the presence of sparse data, including the additional $\mathbf{a}$ parameters risks over-fitting and reduced predictive performance. We therefore use a simpler formulation where $g_k(\mathbf{y}|\mathbf{f}_k^t) = N(\mathbf{f}_k^t, \sigma^2)$. Thus, the ultimate predictive distribution for some observation y^{t^*} is

$$p(y|f_1^{s|t^*}, \ldots, f_K^{s|t^*}) = \sum_{k=1}^{K} w_k N(f_k^{t^*}, \sigma^2). \tag{1}$$

This, is a mixture of K normal distributions each of whose mean is determined by $f_k^{t^*}$ and which is scaled by the model weights w_k.

2.2 Parameter estimation

Since the component model forecasts, $f_1^t, \ldots, f_k^t$, are pre-determined, EBMA model is fully specified by estimating model weights, $w_1, \ldots, w_k$ and the common variance parameter σ^2. We estimate these by maximum likelihood methods (Raftery et al. 2005), although Vrugt, Diks and Clark (2008) have proposed estimation via Markov chain Monte Carlo metods. The log likelihood function is

$$\mathcal{L}(w_1, \ldots, w_k, \sigma^2) = \sum_t log\left(\sum_{k=1}^{K} w_k N(f_k^t, \sigma^2)\right). \tag{2}$$

This function cannot be maximized analytically, so Raftery et al. (2005) propose an EM algorithm which explicitly expresses EBMA as a finite mixture model (CITE: McLachlan and Peel 200 and

the new Imai AJPS). We introduce the unobserved quantities z_k^t, which represents the probabilty that observation y^t is "best" predicted by model k. The E step involves calculating estimates for these unobserved quantities using the formula

$$\hat{z}_k^{(j+1)t} = \frac{\hat{w}_k^{(j)} p^{(j)}(y|f_k^t)}{\sum\limits_{k=1}^{K} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t)}, \tag{3}$$

where the superscript j refers to the jth iteration of the EM algorithm.

$w_k^{(j)}$ is the estimate of w_k in the jth iteration and $p^{(j)}(.)$ is shown in (1). Assuming these estimates of $z_k^{s|t}$ are correct, it is then straightforward to derive the maximizing value for the model weights. Thus, the M step estimates these as

$$\hat{w}_k^{(j+1)} = \frac{1}{n} \sum_t \hat{z}_k^{(j+1)t}, \tag{4}$$

where n represents the number of observations in the validation dataset. Finally,

$$\hat{\sigma}^{2(j+1)} = \frac{1}{n} \sum_t \sum_{k=1}^{K} \hat{z}_k^{(j+1)t} (y - f_k^t)^2. \tag{5}$$

The E and M steps are iterated until the improvement in the log-likelihood is no larger than some pre-defined tolerance. We initiate the algorithm with the assumption that all models are equally likely, $w_k = \frac{1}{K} \ \forall \ k \in [1, \ldots, K]$ and $\sigma^2 = 1$.

3   Missing forecasts

To accomodate missing ensemble values, (Fraley, Raftery and Gneiting 2010) modify the EM algorithm as follows. [2] Define

$$\mathcal{A}^t = \{i | \text{ensemble member i available at time t}\}.$$

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which is simply the indicators of the list of components that provide forecasts for observation y_t. For convenience, define $\tilde{z}_k^{(j+1)t} \equiv \sum\limits_{k \in A^t} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t) / \sum\limits_{k \in A^t} w_k^{(j)}$. Equation 3 is then replaced with

$$\hat{z}_k^{(j+1)t} = \begin{cases} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t) / \tilde{z}_k^{(j+1)t} \text{ if } k \in \mathcal{A}^t \\ 0 \text{ if } k \notin \mathcal{A}^t \end{cases} \tag{6}$$

The M steps in Equations 4 and 5 are likewise replaced with

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[2]In future versions of this paper, we hope to compare alternative methods for handling missing data including imputation gaussian copulas (CITATIONS).

$$\hat{w}_k^{(j+1)} = \frac{\sum\limits_t \hat{z}_k^{(j+1)t}}{\sum\limits_t \sum\limits_{k=1}^{K} \hat{z}_k^{(j+1)t}} \tag{7}$$

and

$$\hat{\sigma}^{2(j+1)} = \frac{\sum\limits_t \sum\limits_{k=1}^{K} \hat{z}_k^{(j+1)t}(y - f_k^t)^2}{\sum\limits_t \sum\limits_{k=1}^{K} \hat{z}_k^{(j+1)t}}. \tag{8}$$

4   Small sample adjustment

When ensembles are calibrated on very few observations, there is an increased chance that EBMA may over-weight high performing models in a way that reduces out of sample performance. Thus, we introduce a "wisdom of crowds" parameter, $c \in [0, 1]$, that reflects our prior belief that all models should receive some weight. In essence, we rescale z_k^t to have a minimum value $\frac{c}{K}$. This essentially states that there is, at a minimum, a $\frac{c}{K}$ probability that the observation is correctly represented by each model k. Since $\sum\limits_{k=1}^{K} z_k^t = 1$, this implies that $z_k^t \in [\frac{c}{K}, (1-c)]$. To achieve this, we replace Equation 4 above with

$$\hat{z}_k^{(j+1)t} = \frac{c}{K} + (1-c)\frac{\hat{w}_k^{(j)}p^{(j)}(y|f_k^t)}{\sum\limits_{k=1}^{K} \hat{w}_k^{(j)}p^{(j)}(y|f_k^t)}. \tag{9}$$

Note that when $c = 1$, that all models are considered equally informative about the outcome and $w_k = \frac{1}{K} \forall K$. Thus, we see that the arithmetic mean or median of component forecasts for time period t represents a special case of EBMA where $c = 1$.[3] Likewise, the general EBMA discussed in Montgomery, Hollenbach and Ward (2012) represents special case of this more general model where $c = 0$.

5   Applications

Blah, blah transition paragraph here

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[3]The mean or median would be equivalent depending on if the posterior mean or median is used to make a point prediction.

5.1   Predicting unemployment

Blah, blah. Short lit review of this topic here

The eight metrics selected are mean absolute error (MAE), root mean squared error (RMSE), median absolute deviation (MAD), root mean squared logarithmic error (RMSLE), mean absolute percentage error (MAPE), median absolute percentage error (MEAPE), median relative absolute error (MRAE) and percent worse (PW). The latter two metrics are measured relative to a naive model simply predicting the future rate of unemployment as being the same as the current rate of unemployment. Although these metrics are all correlated, there are significant differences (Brandt, Freeman and Schrodt (2011) for a more thorough discussion of these prediction metrics). The correlations range from a low of 0.2 (MRAE and MAE) to a high of .98 (MAE and RMSE).

The EBMA model uses only 163 models out of a 293 forecasts available over this period.

Table 2: clever caption here

	MAE	RMSE	MAD	RMSLE	MAPE	MEAPE	MRAE	PW
EBMA (c=0)	0.54	0.74	0.37	0.009	8.37	6.49	**0.73**	**27.36**
EBMA (c=0.05)	**0.54**	0.74	**0.37**	**0.009**	**8.33**	**6.30**	0.75	**27.36**
EBMA (c=0.1)	0.54	0.74	0.35	0.009	8.40	6.44	0.76	28.30
EBMA (c=1)	0.61	0.80	0.46	0.010	9.72	8.92	0.95	46.23
Green Book	0.57	**0.73**	0.43	0.009	9.37	8.81	1.00	45.28
Forecast Median	0.62	0.81	0.47	0.011	9.83	8.87	0.98	47.17
Forecast Mean	0.61	0.80	0.46	0.010	9.71	9.06	0.93	46.23

The model with the lowest score for each metric are shown in bold.

5.2   Predicting presidential elections

Blah, Blah

6   Discussion

Super blah, blah.

In future drafts of this paper we hope to (1) compare alternative methods of handling missing data (2) discuss how to select window of time for calibration and (c) conduct some simulation studies to explore settings for c parameter.

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Figure 1: Clever caption here

Comparing EBMA with components

% of models EBMA equals or beats by metric
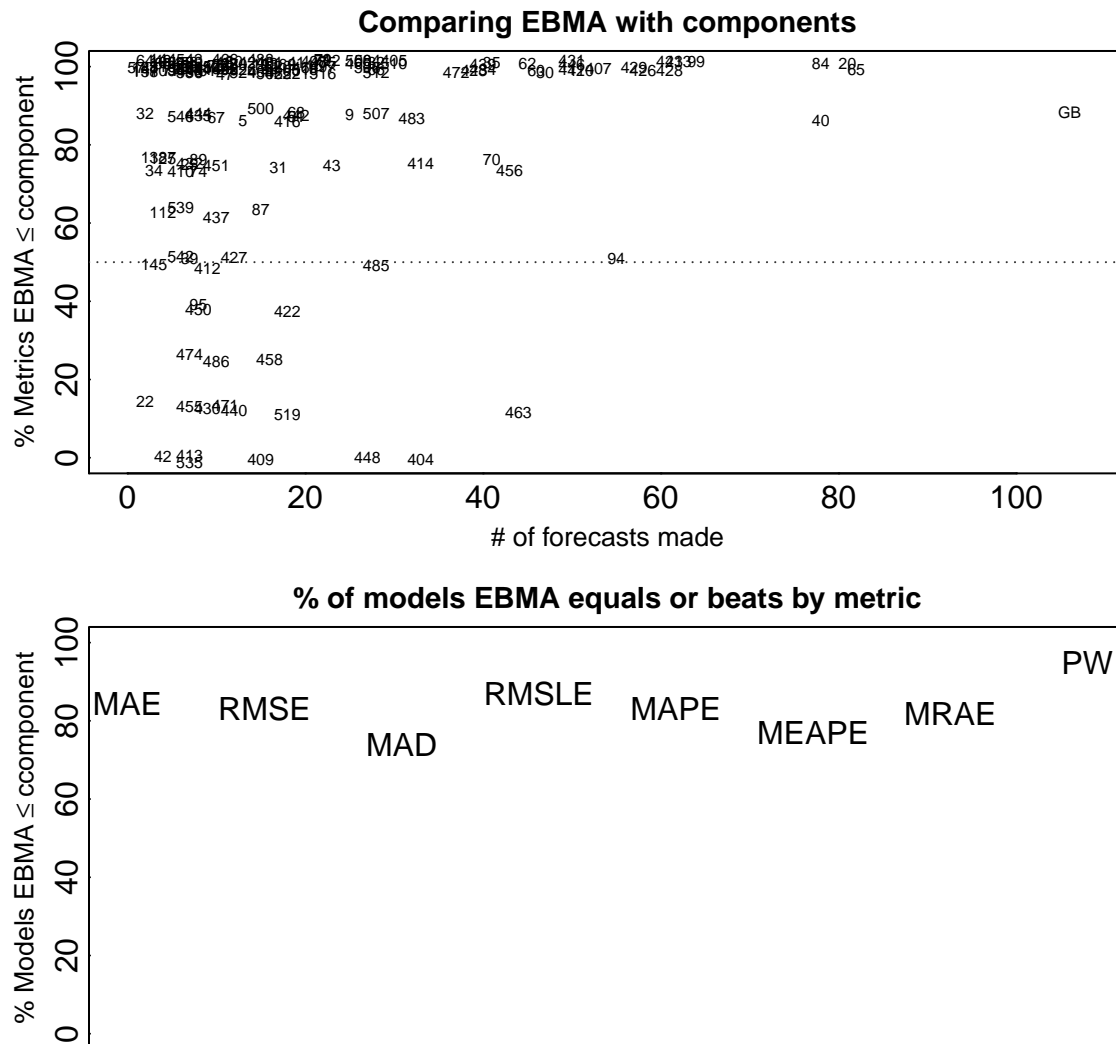
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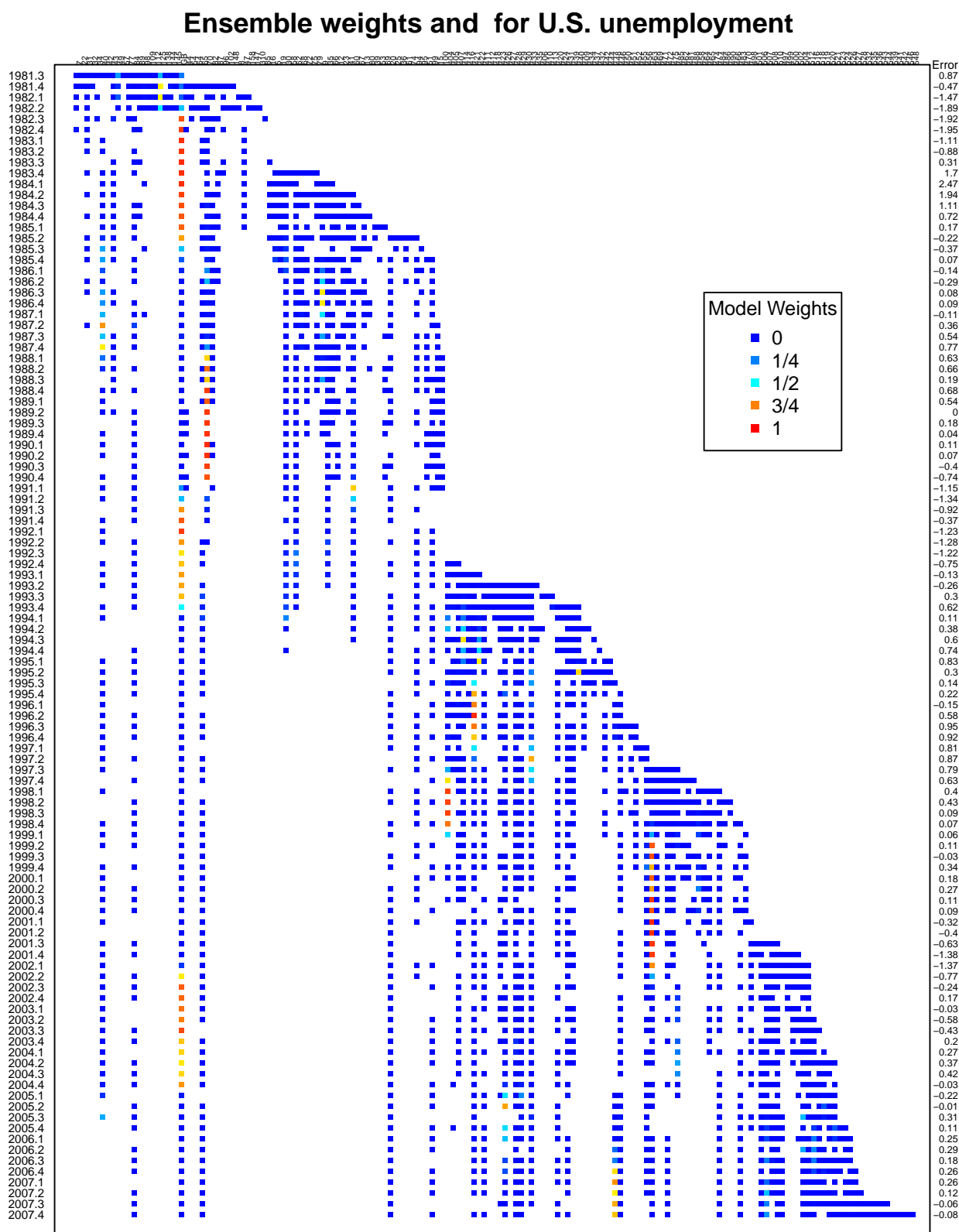
Figure 2: Clever caption here

Ensemble weights and for U.S. unemployment

Figure 3: Clever caption here

Observed and forecasted U.S. unemployment (1981–2007)